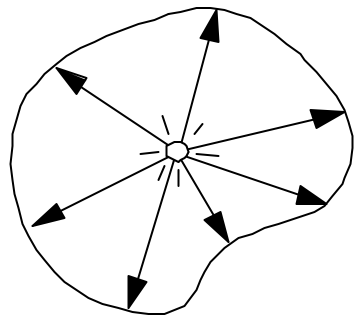
Radiant and Luminous Intensity

Definition: The *radiant* (*luminous*) *intensity* is the power per unit solid angle from a point.

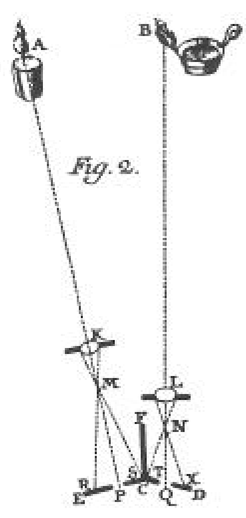


$$\frac{d\Phi}{dw} = I(w)$$

$$\Phi = \int_{S^2} I(w)\,dw$$

$$\left[\frac{W}{sr}\right]\left[candela = cd = \frac{lm}{sr}\right]$$

The Invention of Photometry



Bouguer's Classic experiment

Compare two light sources

One is a candle

Definition of a standard candle

- Originally "standard" candle
- Currently

  550 nm laser with 1/683 W/sr

Luminance of Common Sources

Sky

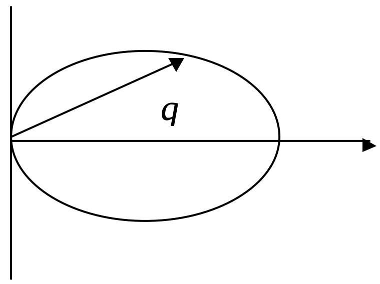
Surface of the sun	2,000,000,000. cd/m^2
Sunlight clouds	30,000.
Clear day	3,000.
Overcast day	300.
Moonlight	0.03
Moonless	0.00003

Light Sources

Properties

- Spectral
  - Blackbody (incandescent)
  - Flourescent
- Point or area
- Directional distribution – goniometric diagram

Warn’s Spotlight

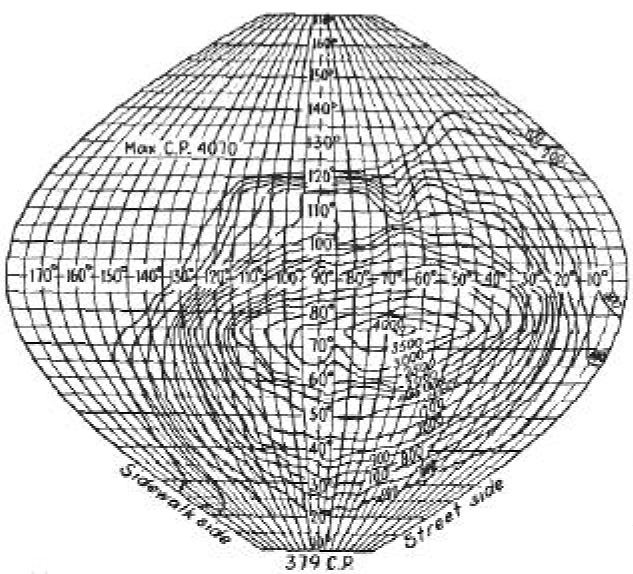


$$I(w) = \cos^s q$$

$$\Phi = \int_0^{2p}\int_0^1 I(w)\, d\cos q\; dj = 2p\int_0^1 \cos^s q\; d\cos q = \frac{2p}{s+1}$$

$$I(w) = \Phi\,\frac{s+1}{2p}\cos^s q$$

Goniometric Diagrams



Isocandle diagram for Novalux sodium luminaire

From Parry Moon
The Scientific Basic of Illuminating Engineering
p. 236

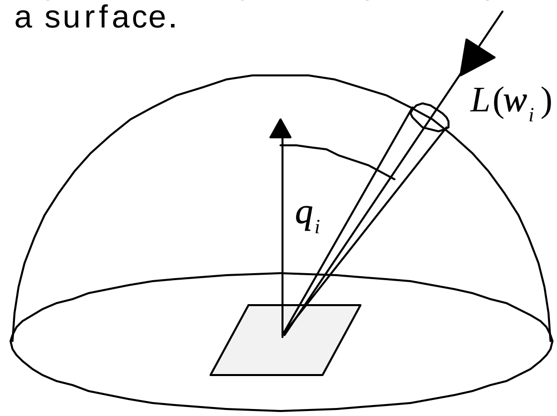
Irradiance and Illuminance

Definition: The *irradiance* (*illuminance*) is the power per unit area incident on a surface.

$$dE(x) = L(w_i)\cos q_i \, dw_i$$

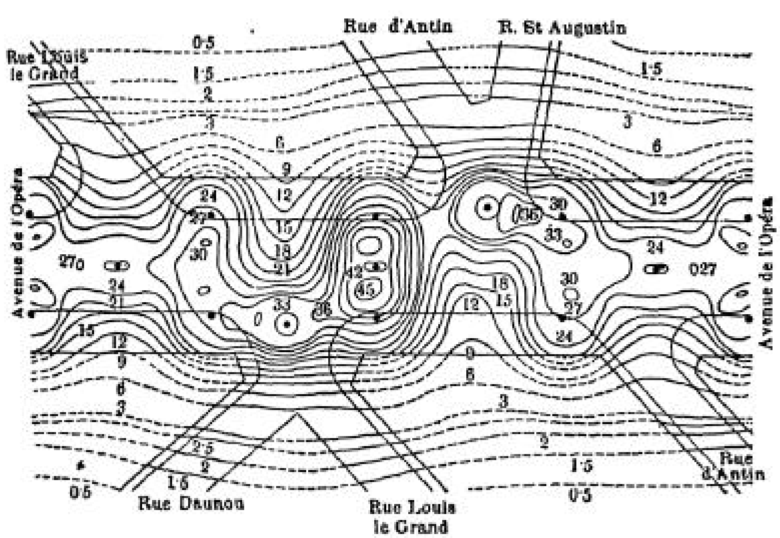
$$E(x) = \int_{H^2} L(w_i)\cos q_i \, dw$$

$$\left[\frac{W}{m^2}\right]\left[Lux = \frac{lm}{m^2}\right]$$



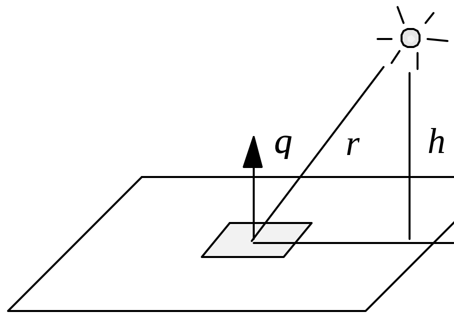
This is sometimes referred to as the radiant *and* luminous incidence*.*

Irradiance Distribution



Isolux contours

Isotropic Point Sources

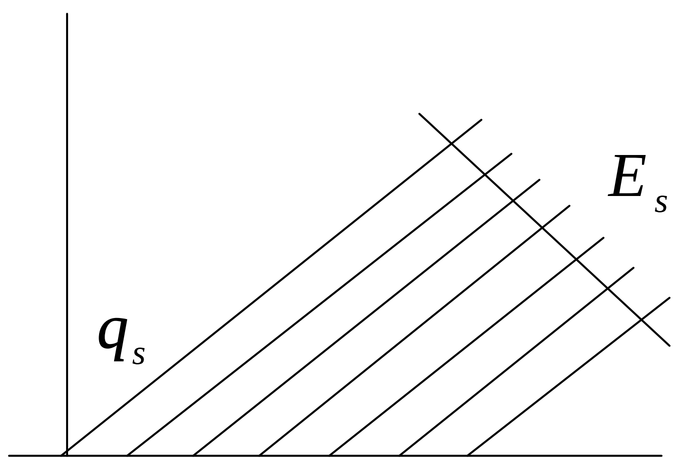


$$I(w) = \frac{\Phi}{4p}$$

$$d\Phi = E\,dA = I\,dw = \frac{\Phi}{4p}\frac{\cos q}{r^2}dA = \frac{\Phi}{4p}\frac{\cos^3 q}{h^2}dA$$

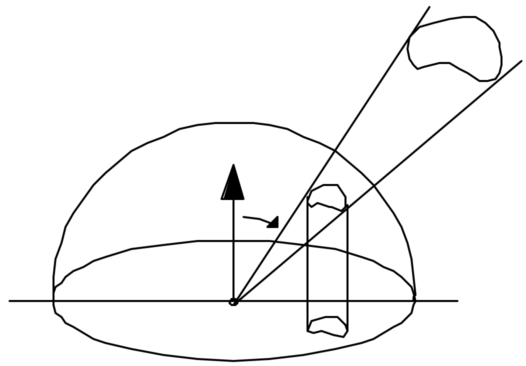
- Note inverse square law fall off.
- Note cosine dependency

Distant Source



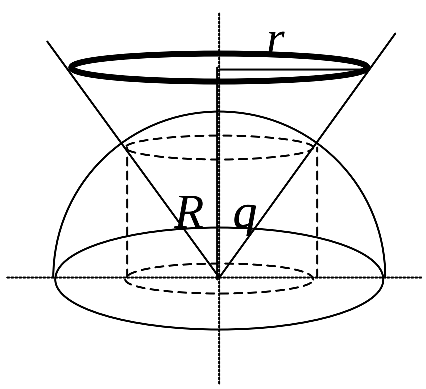
$$E = E_s \cos q_s$$

Hemisphere: Projected Solid Angle



$$\int_{?^{2}} \cos q \; dw = p$$

Disk



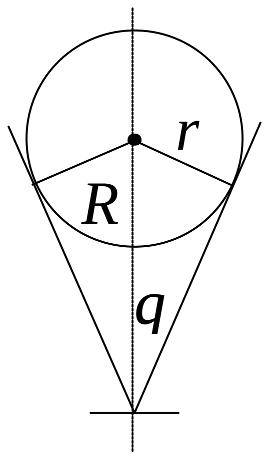
$$E = \int_{1}^{\cos q_d} \int_{0}^{2p} L \cos q \; df \; d\cos q$$

$$= 2pL \left. \frac{\cos^2 q}{2} \right|_{1}^{\cos q_d}$$

$$= Lp \sin^2 q_d$$

$$= Lp \frac{r^2}{r^2 + R^2}$$

Sphere



$$E = \int L \cos q \, dw$$
$$= Lp \sin^2 q$$
$$= Lp \frac{r^2}{R^2}$$

CS348B Lecture 5 Pat Hanrahan, Spring 2000

The Sun

Solar constant (normal incidence at zenith)

- Irradiance 1353 W/m^2
- Illuminance 127,500 Lumen/m^2 = 127.5 Kilo-Lux

Solar angle

- a = .25 degrees = .004 radians (half angle)
- w = p sin^2 a = 6 x 10^{-5} steradians

Radiance

$$L = \frac{E}{w} = \frac{1.353\times10^3 W/m^2}{6\times10^{-5} sr} = 2.25\times10^7 \frac{\mathrm{W}}{m^2\cdot sr}$$

- Pluto (6 tera-meters) 50 Lux - read a newspaper
- Deep space -> 20 micro-lux (see, but not read!)

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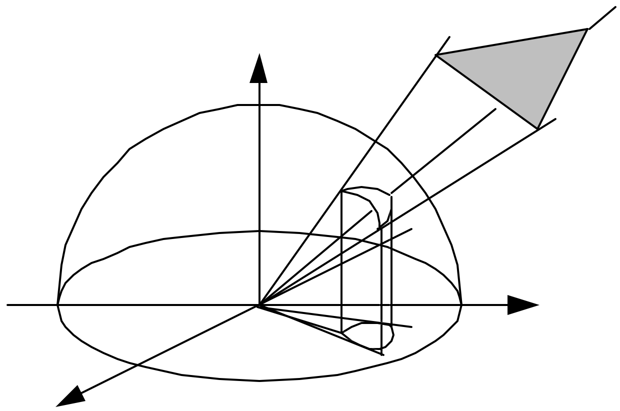
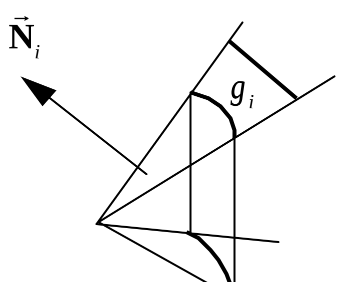
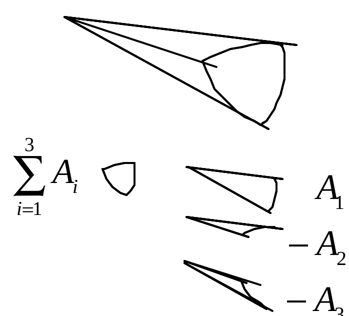
Polygonal Source



Lambert's Formula

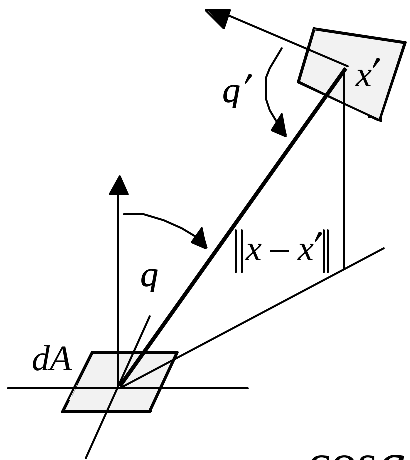


$$A_i = g_i \, \vec{\mathbf{N}} \bullet \vec{\mathbf{N}}_i$$



$$\sum_{i=1}^{n} A_i = \sum_{i=1}^{n} g_i \, \vec{\mathbf{N}} \bullet \vec{\mathbf{N}}_i$$

Form Factor



$$dw = \frac{\cos q'}{\left\| x - x' \right\|^2} dA'$$

$$\cos q \; dw = \frac{\cos q \; \cos q'}{\left\| x - x' \right\|^2} dA'$$

$$T = \int_A \int_{A'} \frac{\cos q' \cos q}{p \left\| x - x' \right\|^2} dA' dA$$

Form Factors

Differential-differential $$F_{dA_i, dA_j} = \frac{\cos q_o' \cos q_i}{p \left\| x - x' \right\|^2} dA_j$$

Differential-finite $$F_{dA_i, A_j} = \int_{A_j} \frac{\cos q_o' \cos q_i}{p \left\| x - x' \right\|^2} dA'$$

Finite-finite $$F_{A_i, A_j} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos q_o' \cos q_i}{p \left\| x - x' \right\|^2} dA' dA$$

Form Factor Properties

Form factor is the percentage of light transferred between surfaces

1. Reciprocity

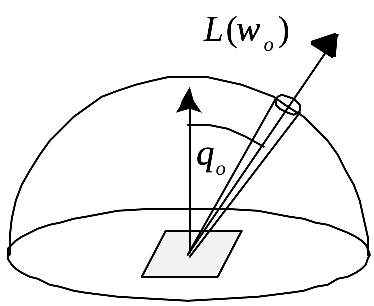
$$T_{ij} = A_i F_{ij} = \int_{A_i}\int_{A_j} \frac{\cos\theta_o' \cos\theta_i}{\pi \left\| x - x' \right\|^2}\, dA'\, dA = T_{ji} = A_j F_{ji}$$

2. Summation

$$\sum_j F_{ij} = \sum_i F_{ji} = 1$$

Radiosity and Luminosity

<u>Definition</u>: The *radiosity* (*luminosity*) is the energy per unit area leaving a surface.



$$B(x) = \int_{H^2} L(\omega_o) \cos\theta_o \, d\omega_o$$

$$\left[\frac{W}{m^2}\right]\left[Lux = \frac{lm}{m^2}\right]$$

This is officially referred to as the radiant *and* luminous exitance.

Uniform Diffuse Source

$$B = \int L \cos q \, dw$$
$$= L \int \cos q \, dw$$
$$= pL$$

$$L = \frac{B}{p}$$

$blondel = apostilb = \frac{1}{p} nit = \frac{1}{p} cd / m^2 \quad (skot = 10^{-3} apostilb)$

$lamberts = \frac{1}{p} cd / cm^2$

$foot - lamberts = \frac{1}{p} cd / ft^2 \quad (glim = 10^{-3} foot - lambert)$

Radiometric and Photometric Terms

Physics	Radiometry	Photometry
Energy	Radiant Energy	Luminous Energy
Flux (Power)	Radiant Power	Luminous Power
Flux Density	Irradiance Radiosity	Illuminance Luminosity
Angular Flux Density	Radiance	Luminance
Intensity	Radiant Intensity	Luminous Intensity

Photometric Units

Photometry	Units		
	MKS	CGS	British
Luminous Energy	Talbot		
Luminous Power	Lumen		
Illuminance Luminosity	Lux	Phot	Footcandle
Luminance	Nit Apostilb, Blondel	Stilb Lambert	Footlambert
Luminous Intensity	Candela (Candle, Candlepower, Carcel, Hefner)		

"Thus one nit is one lux per steradian is one candela per square meter is one lumen per square meter per steradian. Got it?" *Kajiya*