

## Reflection Models

Tuesday

- Reflection models
- The reflection equation and the BRDF
- Ideal reflection, refraction and diffuse

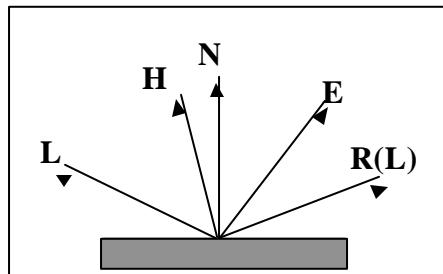
Today

- Glossy reflection models
- Rough surfaces
- Microfacets
- Self-shadowing
- Anisotropic reflection models

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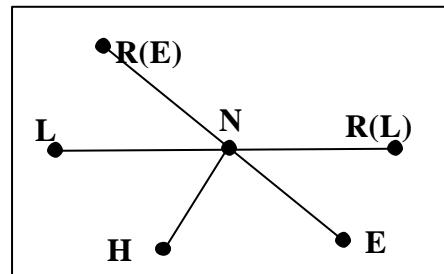
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## Reflection Geometry



$$\cos q_i = \vec{\mathbf{L}} \cdot \vec{\mathbf{N}}$$

$$\cos q_r = \vec{\mathbf{R}} \cdot \vec{\mathbf{N}}$$



$$\cos q_g = \vec{\mathbf{E}} \cdot \vec{\mathbf{L}}$$

$$\cos q_s = \vec{\mathbf{E}} \cdot \mathbf{R}(\vec{\mathbf{L}}) = \mathbf{R}(\vec{\mathbf{E}}) \cdot \vec{\mathbf{L}}$$

$$\cos q_s' = \vec{\mathbf{H}} \cdot \vec{\mathbf{N}}$$

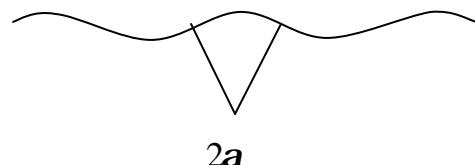
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## Reflection of the Sun from the Sea



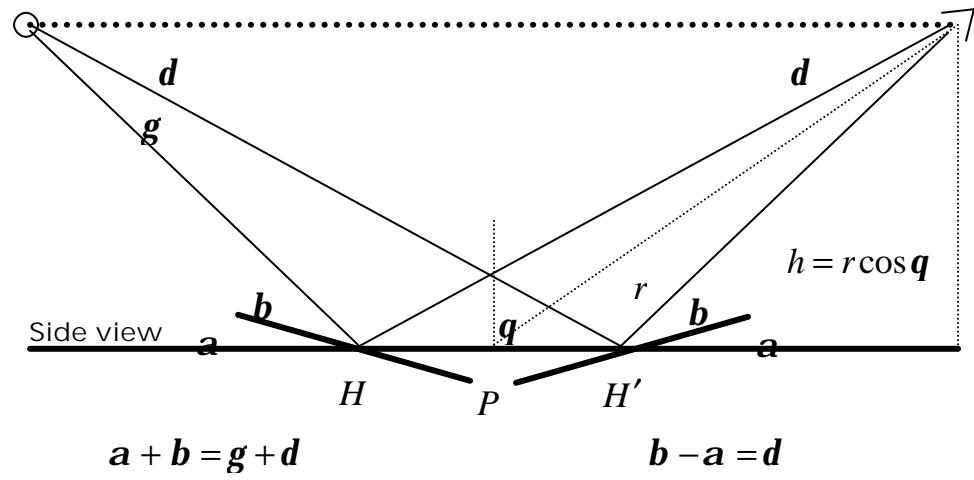
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## Reflection Angles

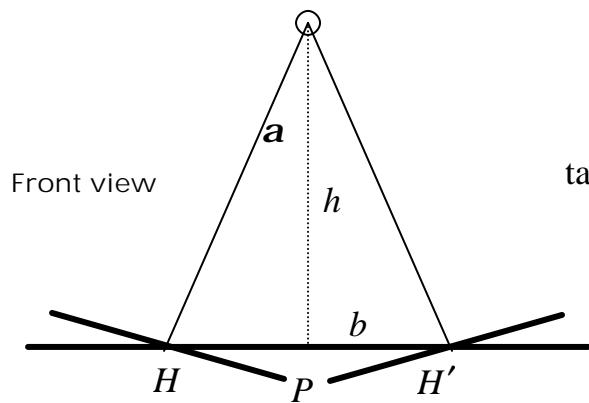


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## Reflection Angles

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$$\tan y = \frac{b}{r} = \tan a \cos q$$

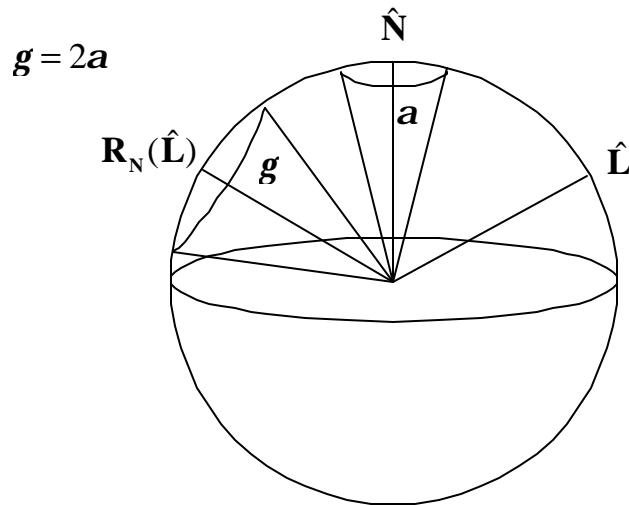
$$\tan a = \frac{b}{h}$$

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## Analysis on the Sphere

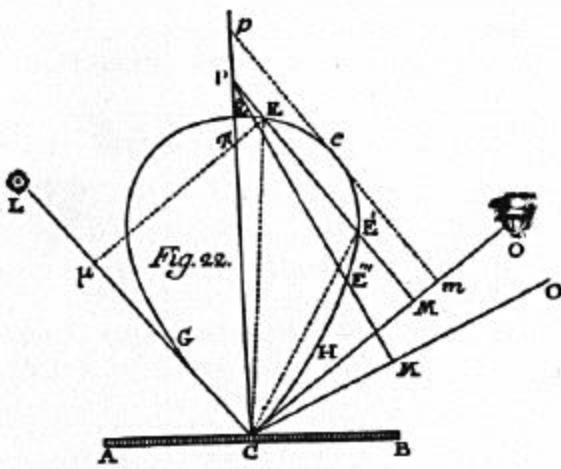
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## Bouguer's "little faces"

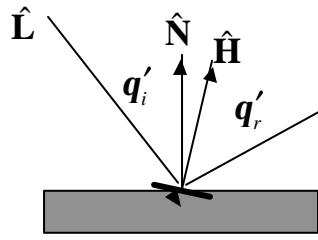


P. Bouguer, *Treatise on Optics*, 1760

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## Torrance-Sparrow Model



$$\begin{aligned} d^2\Phi_h &= L_i(\mathbf{w}_i) \cos q'_i d\mathbf{w}_i dA' \\ &= L_i(\mathbf{w}_i) \cos q'_i d\mathbf{w}_i D(\mathbf{w}_h) d\mathbf{w}_h dA \\ d^2\Phi_r &= d^2\Phi_m \end{aligned}$$

$$dA' = D(\mathbf{w}_h) d\mathbf{w}_h dA$$

$$d^2\Phi_r = dL_r(\mathbf{w}_i \rightarrow \mathbf{w}_r) \cos q_r d\mathbf{w}_r dA$$

$$\cos q_i = \hat{\mathbf{L}} \cdot \hat{\mathbf{N}}$$

$$dL_r(\mathbf{w}_i \rightarrow \mathbf{w}_r) \cos q_r d\mathbf{w}_r dA$$

$$\cos q'_i = \hat{\mathbf{L}} \cdot \hat{\mathbf{H}}$$

$$= L_i(\mathbf{w}_i) \cos q'_i d\mathbf{w}_i D(\mathbf{w}_h) d\mathbf{w}_h dA$$

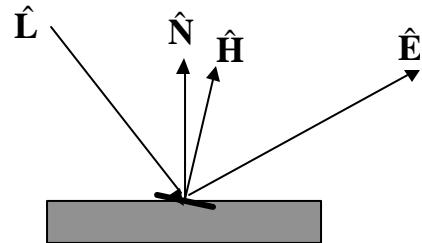
$$d\mathbf{w}'_i = d\mathbf{w}_i$$

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## Torrance-Sparrow Model

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$$\begin{aligned}
 f_r(\mathbf{w}_i \rightarrow \mathbf{w}_r) &= \frac{dL_r(\mathbf{w}_i \rightarrow \mathbf{w}_r)}{dE_i(\mathbf{w}_i)} \\
 &= \frac{L_i(\mathbf{w}_i) \cos \mathbf{q}' d\mathbf{w}_i D(\mathbf{w}_h) d\mathbf{w}_h dA}{(\cos \mathbf{q}_r d\mathbf{w}_r dA)(L_i(\mathbf{w}_i) \cos \mathbf{q}_i d\mathbf{w}_i)} \\
 &= \frac{D(\mathbf{w}_h)}{\cos \mathbf{q}_i \cos \mathbf{q}_r} \cos \mathbf{q}' \frac{d\mathbf{w}_h}{d\mathbf{w}_r} \\
 &= \frac{D(\mathbf{w}_h)}{4 \cos \mathbf{q}_i \cos \mathbf{q}_r}
 \end{aligned}$$


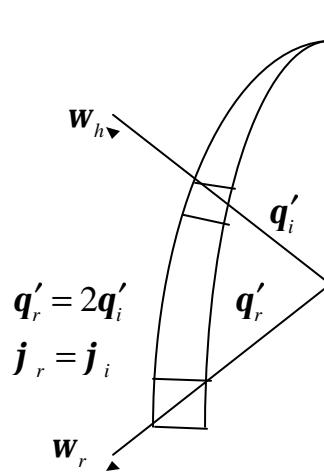
The diagram illustrates the geometry of the Torrance-Sparrow model. A light source  $\hat{L}$  is positioned above a horizontal gray rectangular plane representing a surface. At a specific point on the surface, a vertical arrow labeled  $\hat{N}$  indicates the normal vector. A curved surface labeled  $\hat{H}$  represents the hemisphere above the point, bounded by the normal  $\hat{N}$ . A second vector labeled  $\hat{E}$  extends from the surface point, representing the direction of the reflected ray.

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## Solid Angle Distributions

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The diagram shows a coordinate system with axes  $\mathbf{w}_h$ ,  $\mathbf{w}_r$ , and  $\mathbf{j}_r$ . A vector  $\mathbf{q}'_i$  is shown originating from the origin. A curved surface represents a hemisphere. A vector  $\mathbf{q}'_r$  is shown within this hemisphere. The angle between  $\mathbf{q}'_i$  and  $\mathbf{q}'_r$  is labeled  $2\mathbf{q}'_i$ . The angle between  $\mathbf{q}'_r$  and the vertical axis  $\mathbf{w}_h$  is also labeled  $2\mathbf{q}'_i$ .

$$\begin{aligned}
 d\mathbf{w}_r &= \sin \mathbf{q}'_r d\mathbf{q}'_r d\mathbf{j}_r \\
 &= (\sin 2\mathbf{q}'_i) 2d\mathbf{q}'_i d\mathbf{j}_i \\
 &= (2 \sin \mathbf{q}'_i \cos \mathbf{q}'_i) 2d\mathbf{q}'_i d\mathbf{j}_i \\
 &= 4 \cos \mathbf{q}'_i d\mathbf{w}_i
 \end{aligned}$$

$$\frac{d\mathbf{w}_h}{d\mathbf{w}_r} = \frac{1}{4 \cos \mathbf{q}'_i}$$

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## Normalizing Microfacet Distributions

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$$dA' = D(\mathbf{w}_h) d\mathbf{w}_h dA$$

$$\int_{H^2} \cos \mathbf{q}_h dA' = \int_{H^2} D(\mathbf{w}_h) d\mathbf{w}_h dA = dA$$

$$\int_{H^2} D(\mathbf{w}_h) \cos \mathbf{q}_h d\mathbf{w}_h = 1$$

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## Microfacet Distribution Functions

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Isotropic distributions  $D(\mathbf{w}) = D(\mathbf{a})$

Characterize by half-angle  $D(\mathbf{b}) = \frac{1}{2}$

Examples:

■ Blinn  $D_1(\mathbf{a}) = \cos^{c_1} \mathbf{a}$   $c_1 = \frac{\ln 2}{\ln \cos \mathbf{b}}$

■ Torrance-Sparrow  $D_2(\mathbf{a}) = e^{-(c_2 \mathbf{a})^2}$   $c_2 = \frac{\sqrt{2}}{\mathbf{b}}$

■ Trowbridge-Reitz  $D_3(\mathbf{a}) = \frac{c_3^2}{(1 - c_3^2) \cos^2 \mathbf{a} - 1}$   $c_3 = \left( \frac{\cos^2 \mathbf{b} - 1}{\cos^2 \mathbf{b} - \sqrt{2}} \right)^{\frac{1}{2}}$

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## Self-Shadowing: V-Groove Model

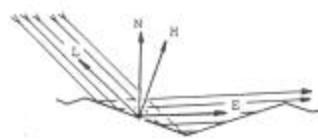
Assumptions (Torrance-Sparrow)

1. Symmetric, longitudinal, isotropically-distributed

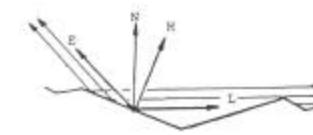
2. Upper edges lie in plane  $G = \min(G_a, G_b, G_c)$



$$G_a = 1$$



$$G_b = \frac{2(\mathbf{N} \cdot \mathbf{H})(\mathbf{N} \cdot \mathbf{E})}{(\mathbf{H} \cdot \mathbf{E})}$$

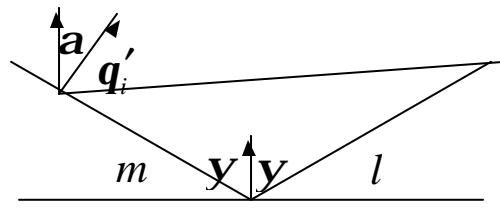


$$G_c = \frac{2(\mathbf{N} \cdot \mathbf{H})(\mathbf{N} \cdot \mathbf{L})}{(\mathbf{H} \cdot \mathbf{L})}$$

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## Self-Shadowing: V-Groove Model



$$\frac{m}{l} = \frac{\sin m}{\sin l}$$

$$\begin{aligned}\sin l &= \cos q'_i \\ \cos l &= \sin q'_i \\ \sin y &= \cos a \\ \cos y &= \sin a\end{aligned}$$

$$\begin{aligned}\sin m &= \sin l + 2y \\ &= \sin l \cos 2y + \cos l \sin 2y \\ &= \cos q'_i \cos 2y + \sin q'_i \sin 2y \\ &= \cos q'_i (1 - 2 \sin^2 y) + \sin q'_i 2 \cos y \sin y \\ &= \cos q'_i (1 - 2 \cos^2 a) + \sin q'_i 2 \cos a \sin a \\ &= \cos q'_i - 2 \cos a (\cos a \cos q'_i - \sin a \sin q'_i) \\ &= \cos q'_i - 2 \cos a \cos(a + q'_i) \\ &= \cos q'_i - 2 \cos a \cos q_r \\ &= \mathbf{H} \cdot \mathbf{E} - 2(\mathbf{N} \cdot \mathbf{H})(\mathbf{N} \cdot \mathbf{E})\end{aligned}$$

$$\begin{aligned}G &= 1 - \frac{m}{l} \\ &= 1 - \frac{\sin m}{\sin l} \\ &= \frac{\mathbf{H} \cdot \mathbf{E} - \mathbf{H} \cdot \mathbf{E} + 2(\mathbf{N} \cdot \mathbf{H})(\mathbf{N} \cdot \mathbf{E})}{\mathbf{H} \cdot \mathbf{E}} \\ &= \frac{2(\mathbf{N} \cdot \mathbf{H})(\mathbf{N} \cdot \mathbf{E})}{\mathbf{H} \cdot \mathbf{E}}\end{aligned}$$

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## Gaussian Rough Surface

Beckmann

$$p(z) = \frac{1}{\sqrt{2\pi s^2}} e^{-\frac{z^2}{2s^2}}$$

$$D(\alpha) = \frac{1}{\sqrt{\pi m^2 \cos^2 \alpha}} e^{-\frac{\tan^2 \alpha}{m^2}}$$

Smith

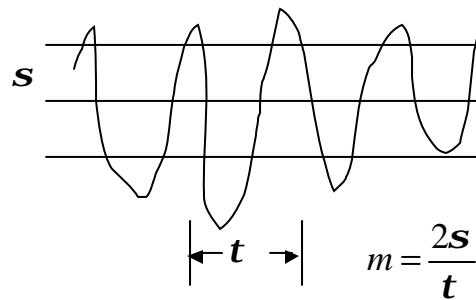
Derives shadowing function probabilistically

Self-consistency condition

The sum of the areas of the illuminated surface projected onto the plane normal to the direction of incidence is independent of the roughness of the surface, and equal to the projected area of the underlying mean plane.

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## Shadows on Rough Surfaces



Without self-shadowing



With self-shadowing

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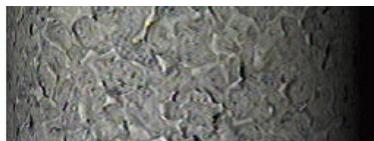
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## S. Nayar's BTF Experiments

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Complex interplay between texture and brdf

Self-shadowing a major effect

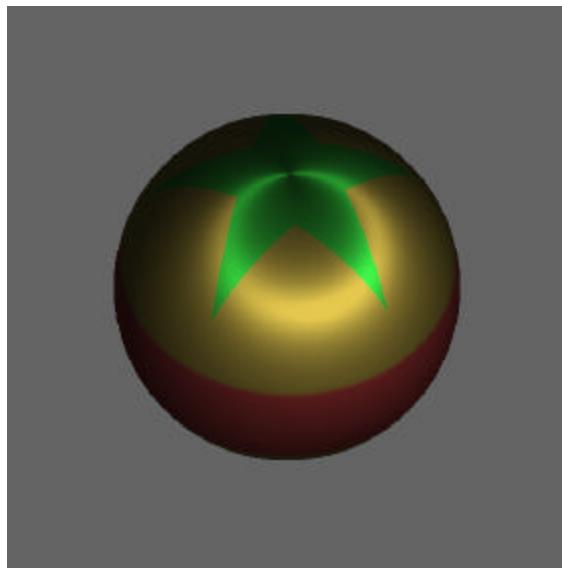


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## Anisotropic Reflection

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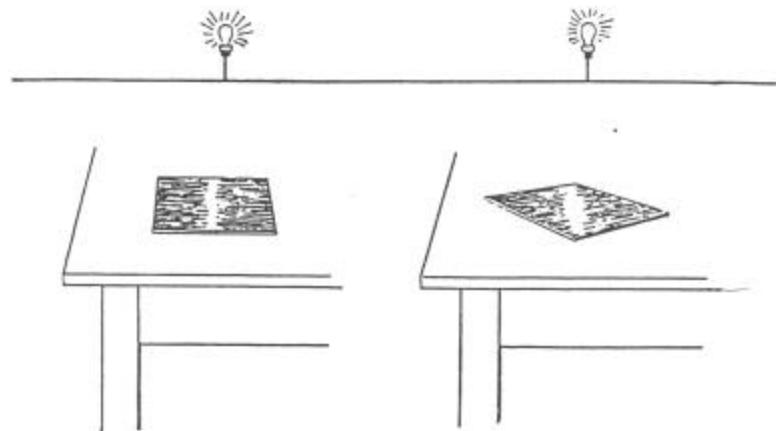


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## Anisotropic Reflection

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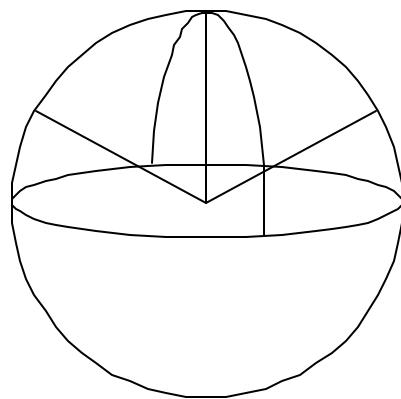


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## Anisotropic Reflection

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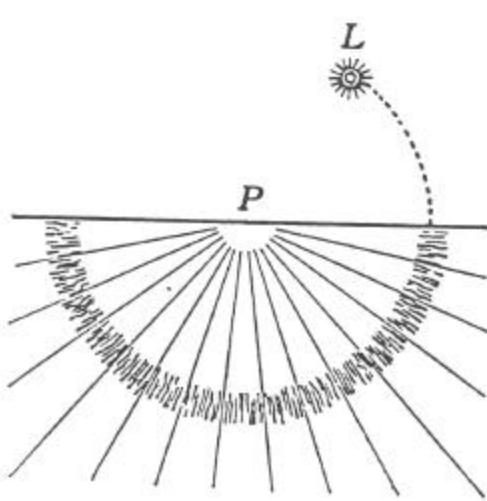


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## Anisotropic Reflection

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