Volumes and Participating Media

Applications

1. Clouds, smoke, water, ...
2. Subsurface scattering: paint, skin, ...
3. Scientific and medical visualization: CT, MRI, ...

Topics

- Volume representations
- Absorption
- Scattering and phase functions
- Volume rendering equation
- Ray tracing volumes

Volume Representations

3D arrays (uniform rectangular)

- CT data

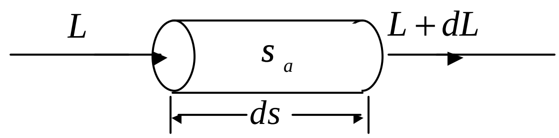
3D meshes

- CFD, mechanical simulation

Simple shapes with solid texture

- Ellipsoidal clouds with sum-of-sines densities
- Hypertexture

Absorption



$$dL = -s_a L\, ds$$

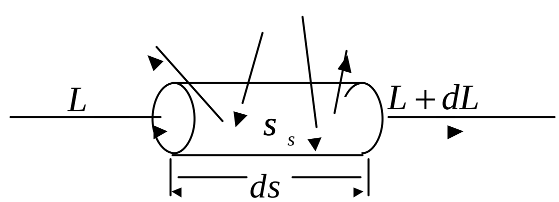
Beer's Law

$$L(s) = L(0)\, e^{-s_a s}$$

Absorption probability

$$L(s) = L(0)\, e^{-\int_0^s s_a(s')\, ds'}$$

Scatter



$$dL = s_s \left[\int_{S^2} p(w' \to w) L(w')\, dw' - \int_{S^2} p(w \to w') L(w)\, dw \right]$$

$$= s_s \left[\int_{S^2} p(w' \to w) L(w')\, dw' - L(w) \right]$$

Cross-sections

Total cross-section $s_t = s_a + s_s$

Albedo $W = \frac{s_s}{s_t} = \frac{s_s}{s_a + s_s}$

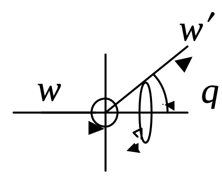
Micro vs. macro $\Sigma = r\,s$

$$\left[\frac{1}{m}\right] = \left[\frac{1}{m^3}\right]\left[m^2\right]$$

Phase Functions

Phase angle $\cos q = w \bullet w'$

Phase functions
(from the phase of the moon) $2p\int_0^p p(\cos q)\,dq = 1$



1. Isotropic
   -simple
2. Rayleigh
   -molecules
3. Mie scattering
   - small spheres

... Huge literature ...

$p(\cos q) = \frac{1}{4p}$

$p(\cos q) = \frac{3}{4}\frac{1+\cos^2 q}{l^4}$

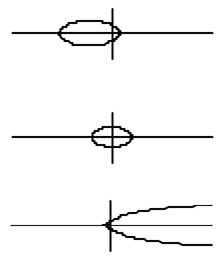
Henyey-Greenstein Phase Function

Empirical phase function

$$p(\cos\theta) = \frac{1}{4\pi} \frac{1 - g^2}{\left(1 + g^2 - 2g\cos\theta\right)^{3/2}}$$

$$2\pi \int_0^{\pi} p(\cos\theta)\cos\theta \, d\theta = g$$

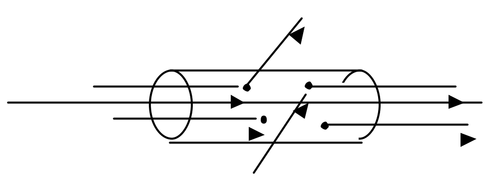
g: average phase angle



Volume Balance Equation

[change in radiance along a direction] =

[emission] - [absorption] + [scattered in] - [scattered out]



The Volume Rendering Equation

Integro-differential equation

$$\frac{\partial L(x,w)}{\partial s} = -s_t(x)L(x,w) + s_s(x)\int_{S^2} p(w' \to w)L(x,w')\,dw'$$

Integro-integral equation

$$L(x,w) = \int_0^{\infty} e^{-\int_0^{s'} s_t(x+s''w)\,ds''} \left[s_s(x+s'w)\int_{S^2} p(w' \to w)L(x+s'w,w')\,dw' \right] ds'$$

Attenuation: Absorption and scattering

Source: Scatter (+ emission)

RGBA Formulation

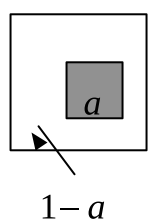
Assume color and alpha are defined in the volume

- Unassociated

$$\left(r(x), g(x), b(x), a(x)\right) = \left(C(x), a(x)\right)$$

- Associated (premultiplied)

$$\left(c(x), a(x)\right) = \left(a(x)C(x), a(x)\right)$$



Use compositing operator

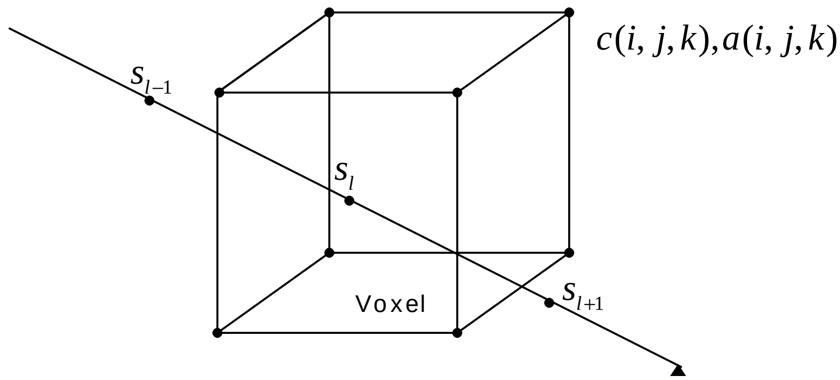
$$T(0) = 1 - a(0) \qquad A(0) = a(0)$$

$$L(0) = c(0) \qquad L(0) = c(0)$$

$$L(x+1) = L(x) + T(x)c(x) \qquad L(x+1) = L(x) + (1 - A(x))c(x)$$

$$T(x+1) = T(x)(1 - a(x+1)) \qquad A(x+1) = A(x) + (1 - A(x))a(x+1)$$

Ray Marching



$$c(s_{l+1}) = c(s_l) + (1 - a(s_l))c_{l+1}$$
$$a(s_{l+1}) = a(s_l) + (1 - a(s_l))a_{l+1}$$

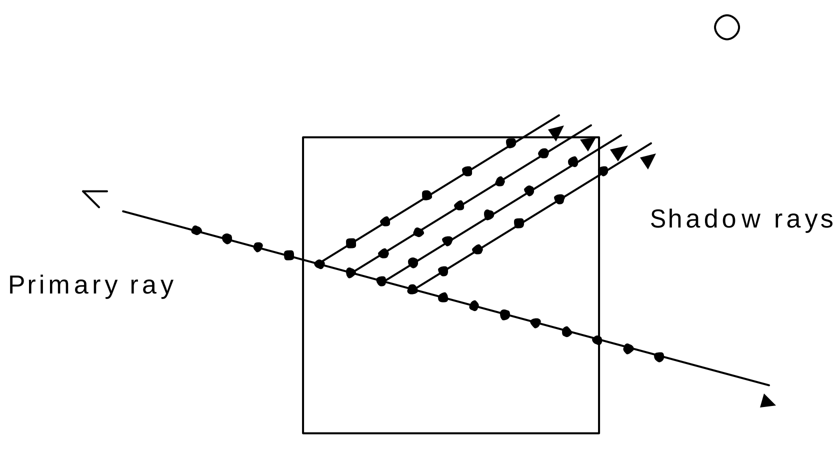
$$c(s_l) = \text{trilinear}(c, i, j, k, x(s_l))$$

Should use premultiplied colors

M. Levoy, Ray tracing volume densities

CS348B Lecture 18 Pat Hanrahan, Spring 2000

Ray Marching with Shadows



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Simple Fog Model

$$\frac{\partial L(s)}{\partial s} = -s_a L(s) + c_f$$

$$L(s) = \left(1 - e^{-s_a s}\right) c_f + e^{-s_a s} c_s$$

Examples

Participating media

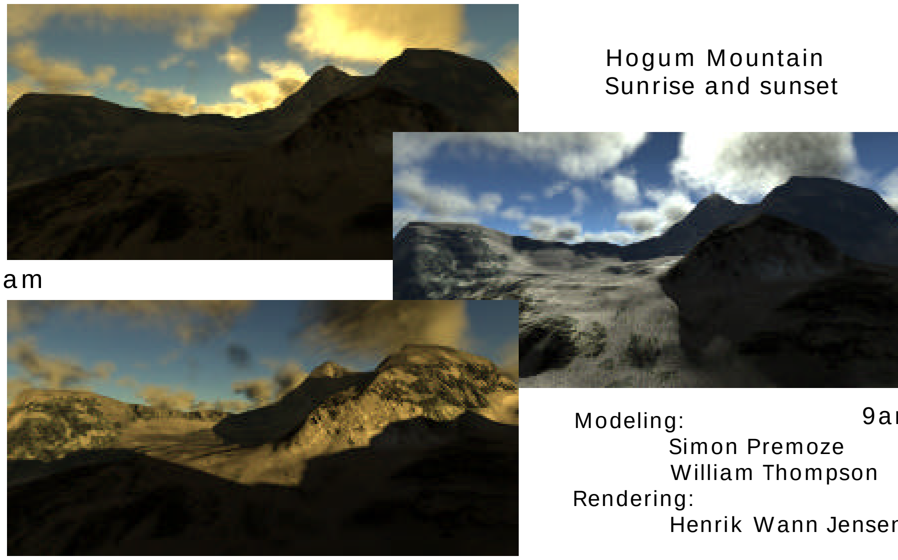
- Sunset with beams of light
- Bohren example in the shower
- Henrik clouds
- Texels
- Hypertextures

Visualization

- Visible human
- Finite element

Clouds and Atmospheric Phenomena

Hogum Mountain
Sunrise and sunset



7am

9am

6:30pm

Modeling:
Simon Premoze
William Thompson

Rendering:
Henrik Wann Jensen