Wigner Distributions and How They Relate to the Light Field

Zhengyun Zhang, Marc Levoy
Stanford University

IEEE International Conference on Computational Photography 2009

Light Fields and Wave Optics

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Why Study Light Fields Using Wave Optics?

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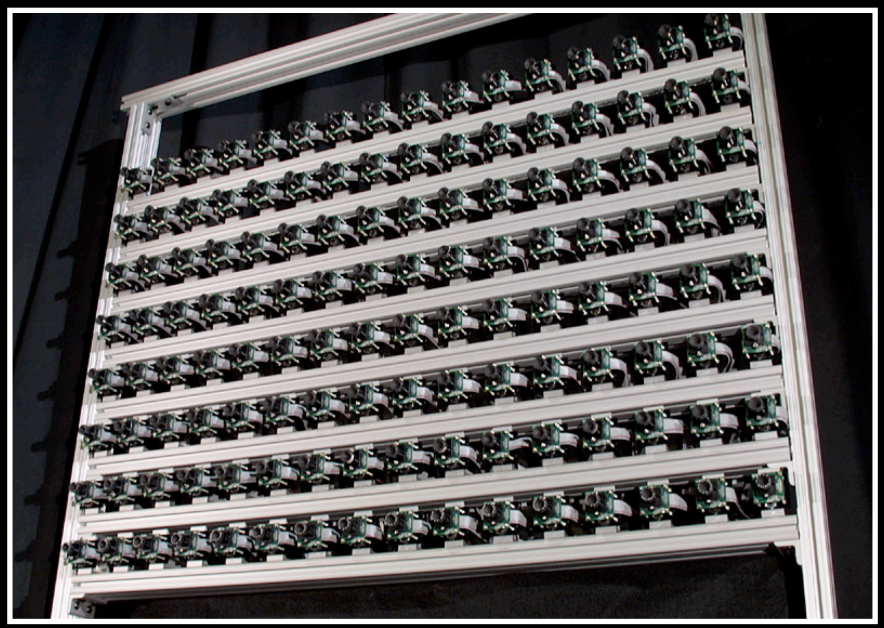
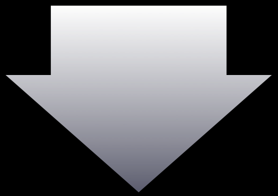
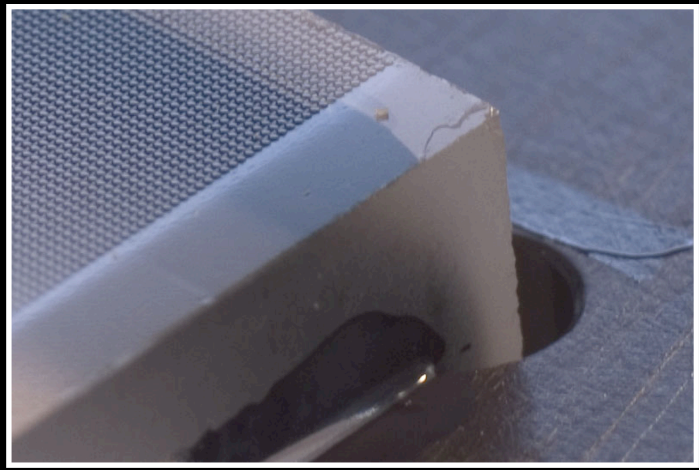


macro



micro



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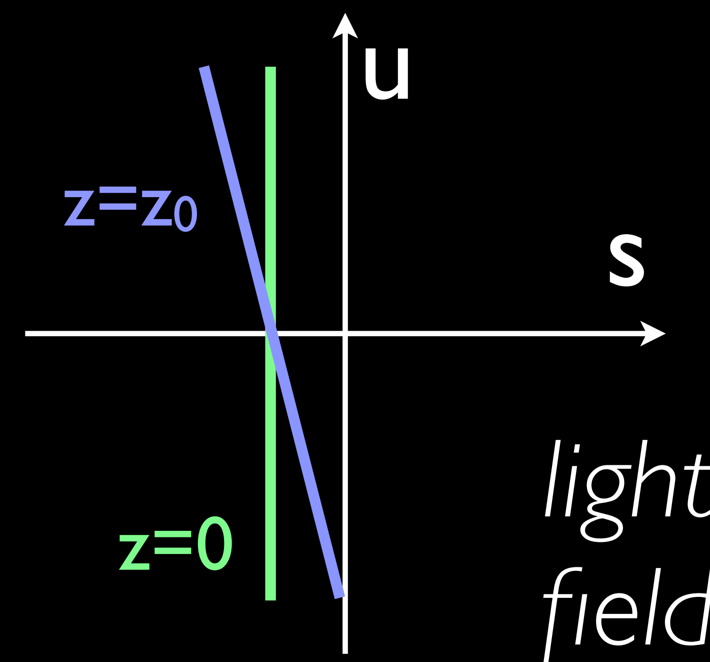

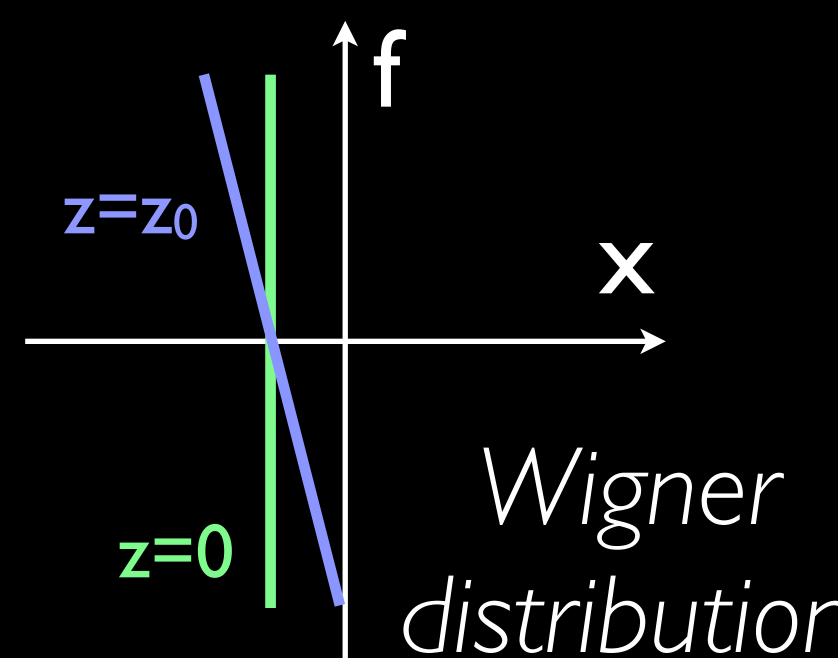

Outline

- review light fields and wave optics
- observable light field and
  the Wigner distribution
- applications

Light Fields

- radiance per ray
- ray parametrization:
  - position (s)
  - direction (u)

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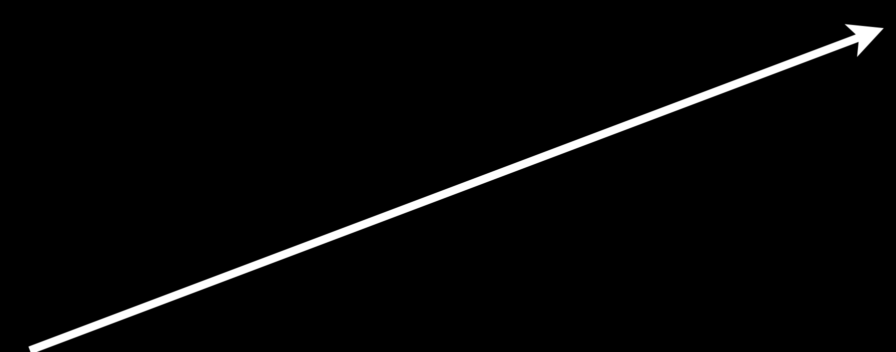

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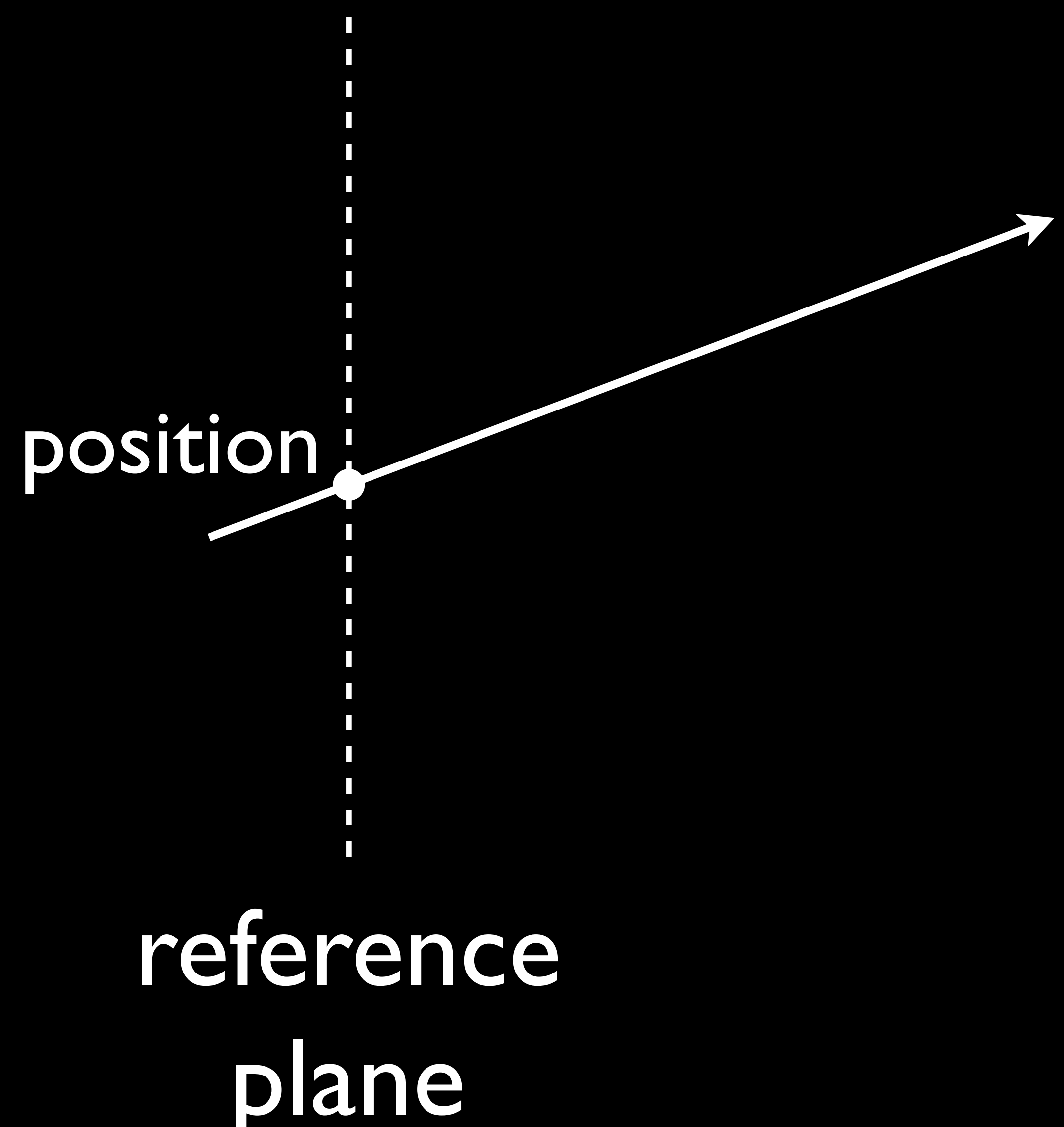

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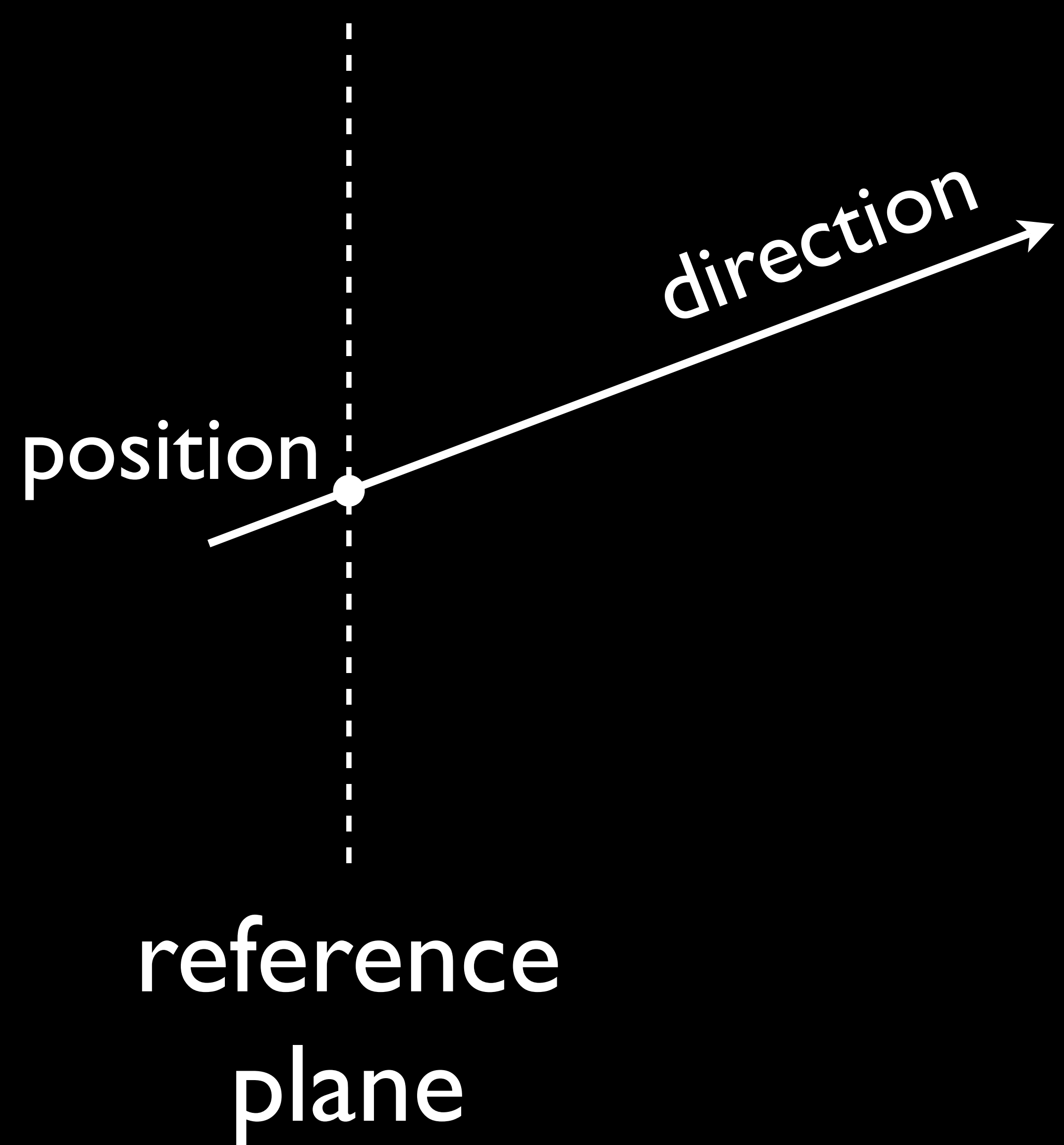
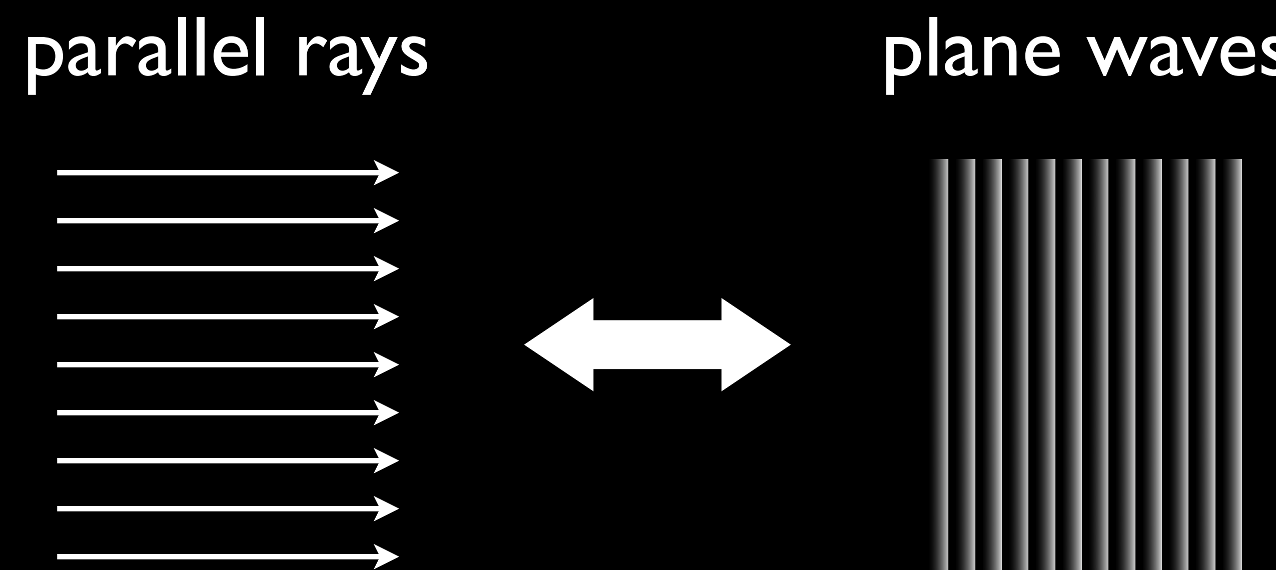


Wave Optics

- waves instead of rays
- interference, diffraction
- plane of point emitters (Huygen's principle)
- each emitter has amplitude and phase



(coherent and flatland)

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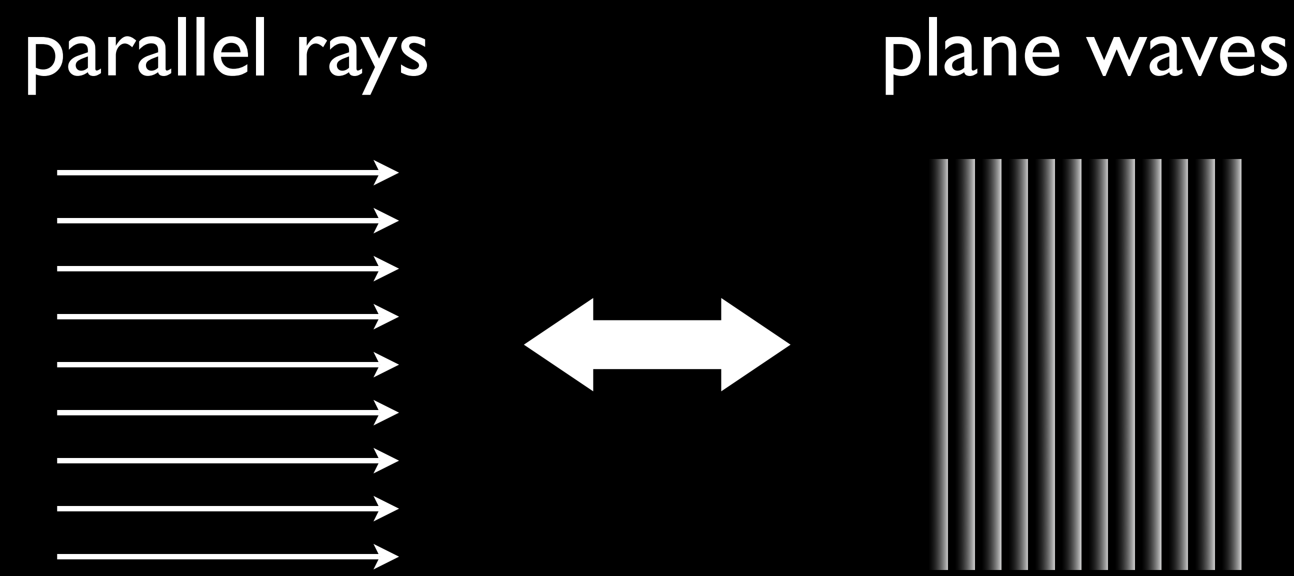

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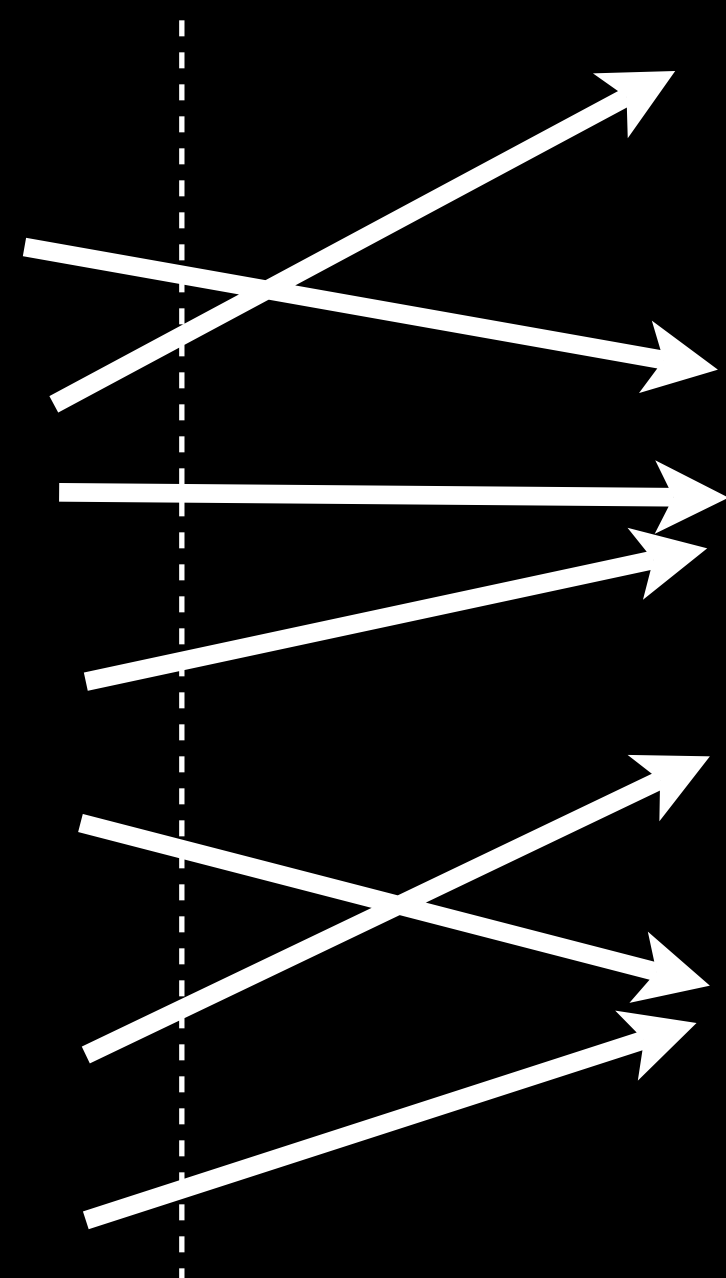

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$$U(x) = A(x)e^{j\phi(x)}$$

Position and Direction in Wave Optics

- recall: light field describes how power is spread over position and direction
- point emitters on plane have amplitude and phase
- positional spread is amplitude squared

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Position and Direction in Wave Optics

- direction
  - **axial**
  - oblique
  - more oblique

Position and Direction in Wave Optics

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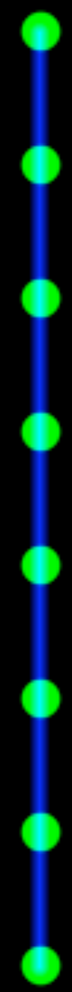

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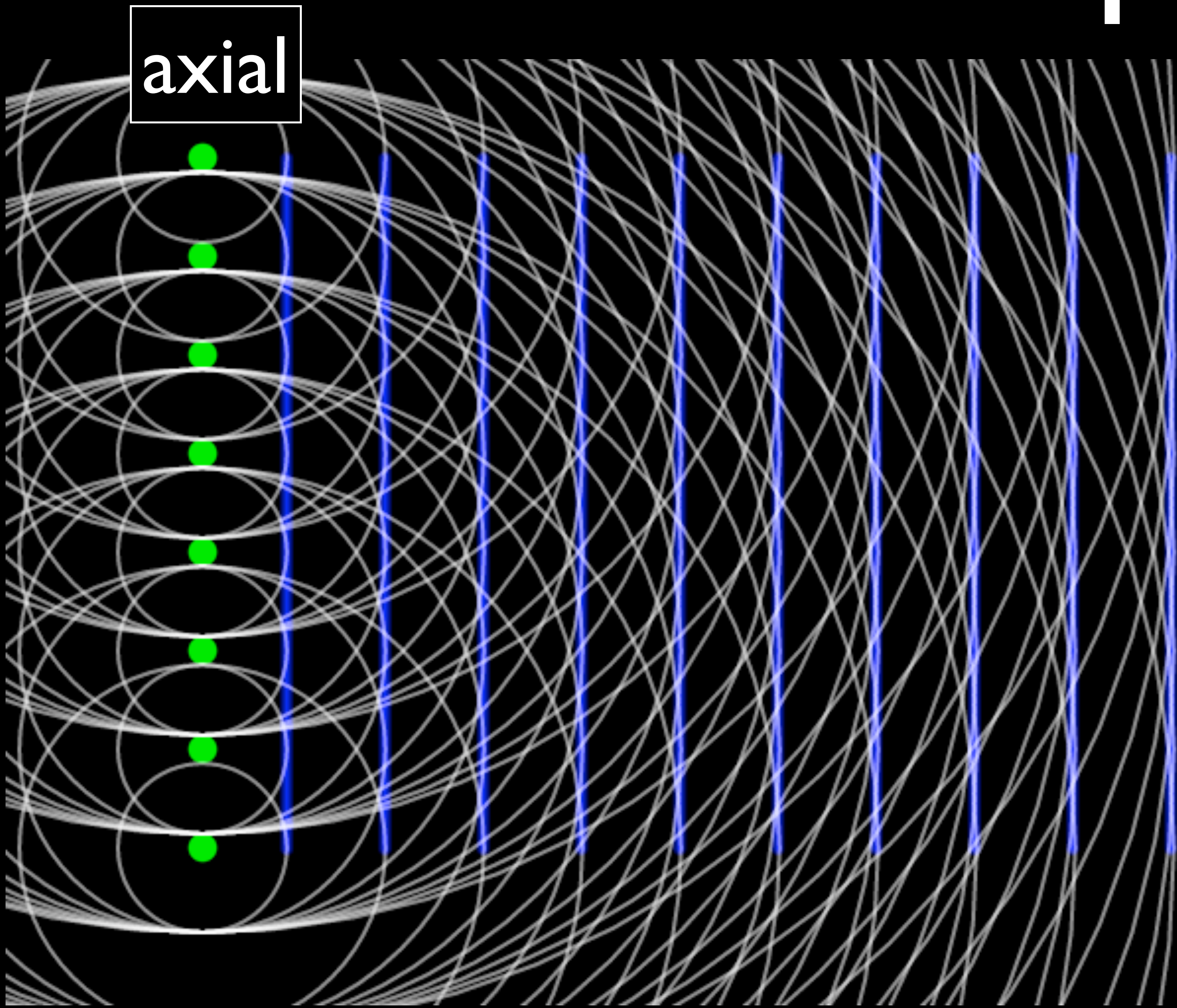

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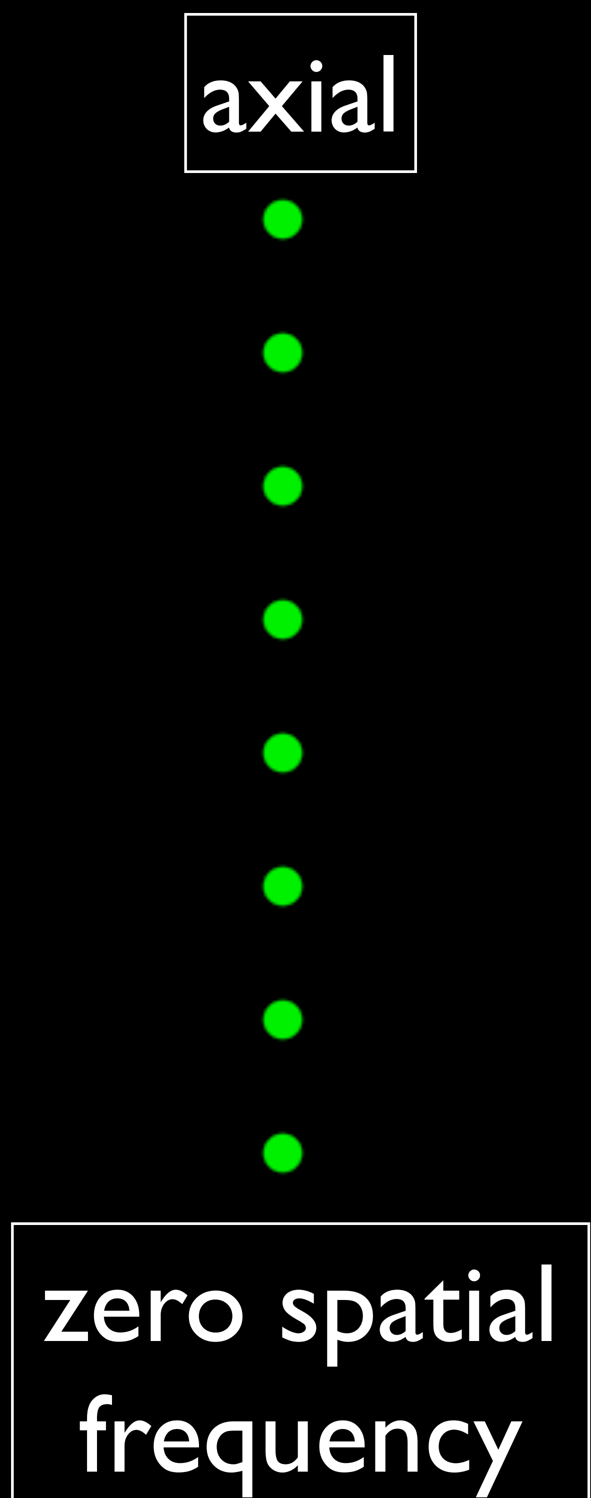

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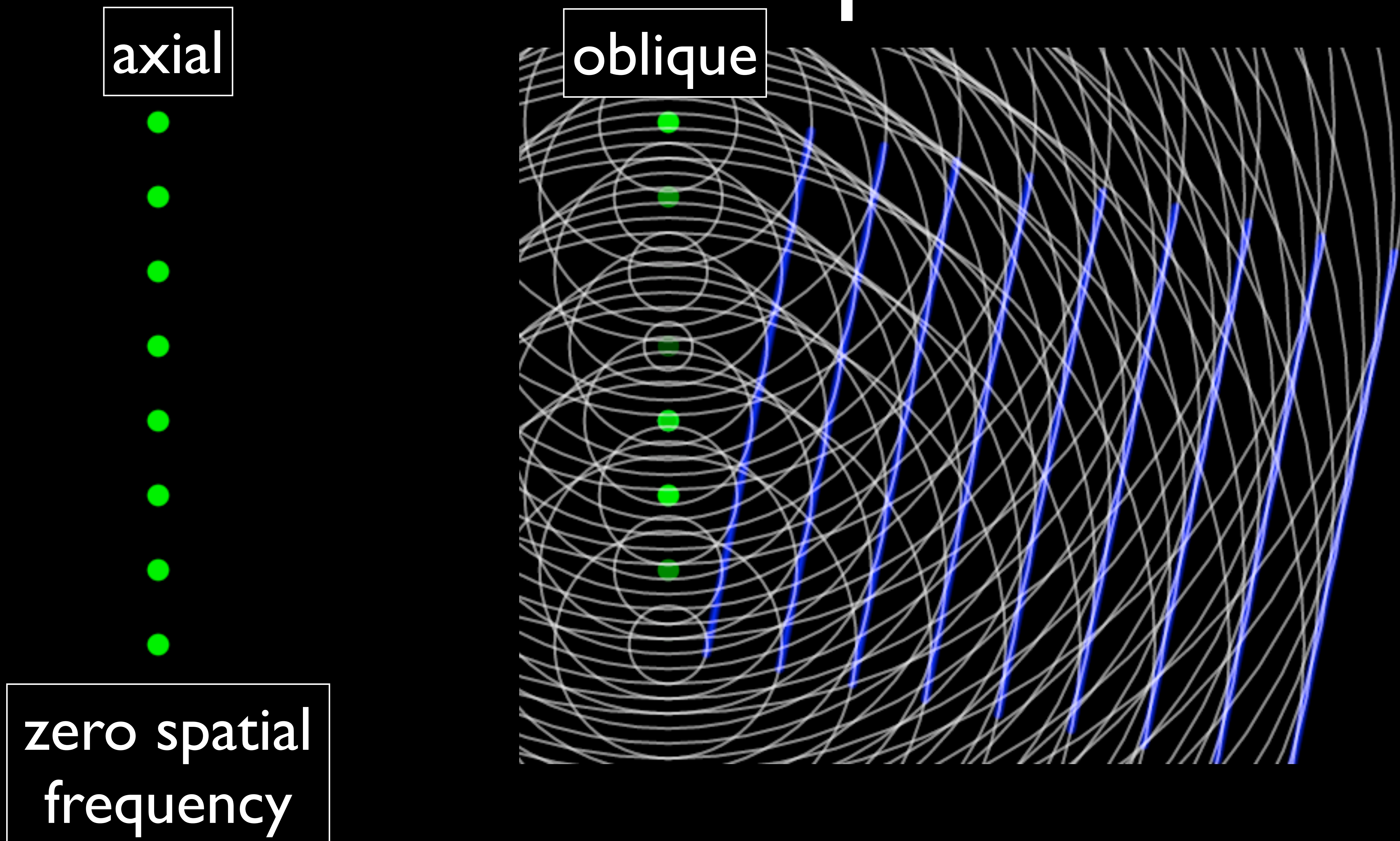

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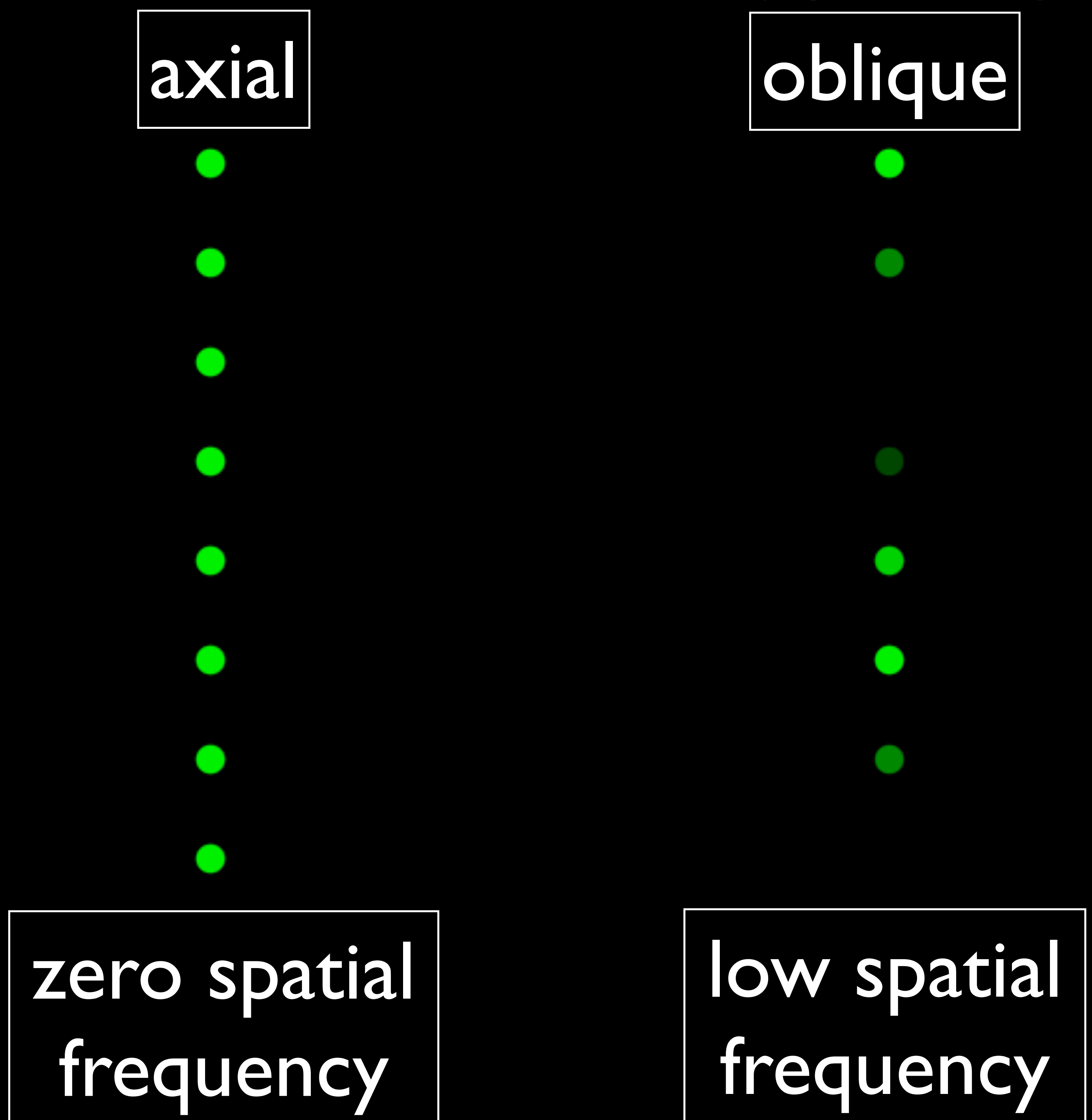

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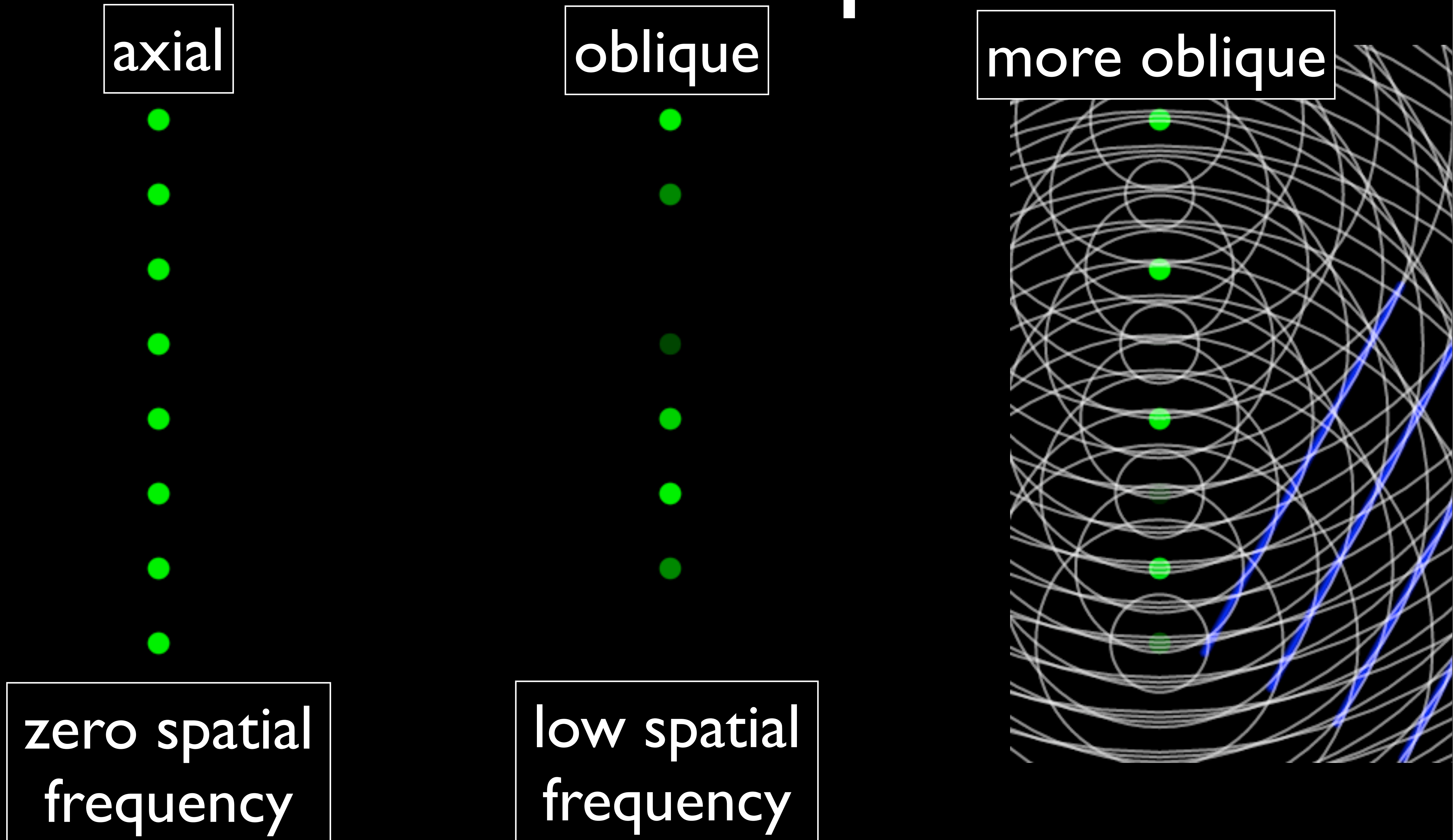

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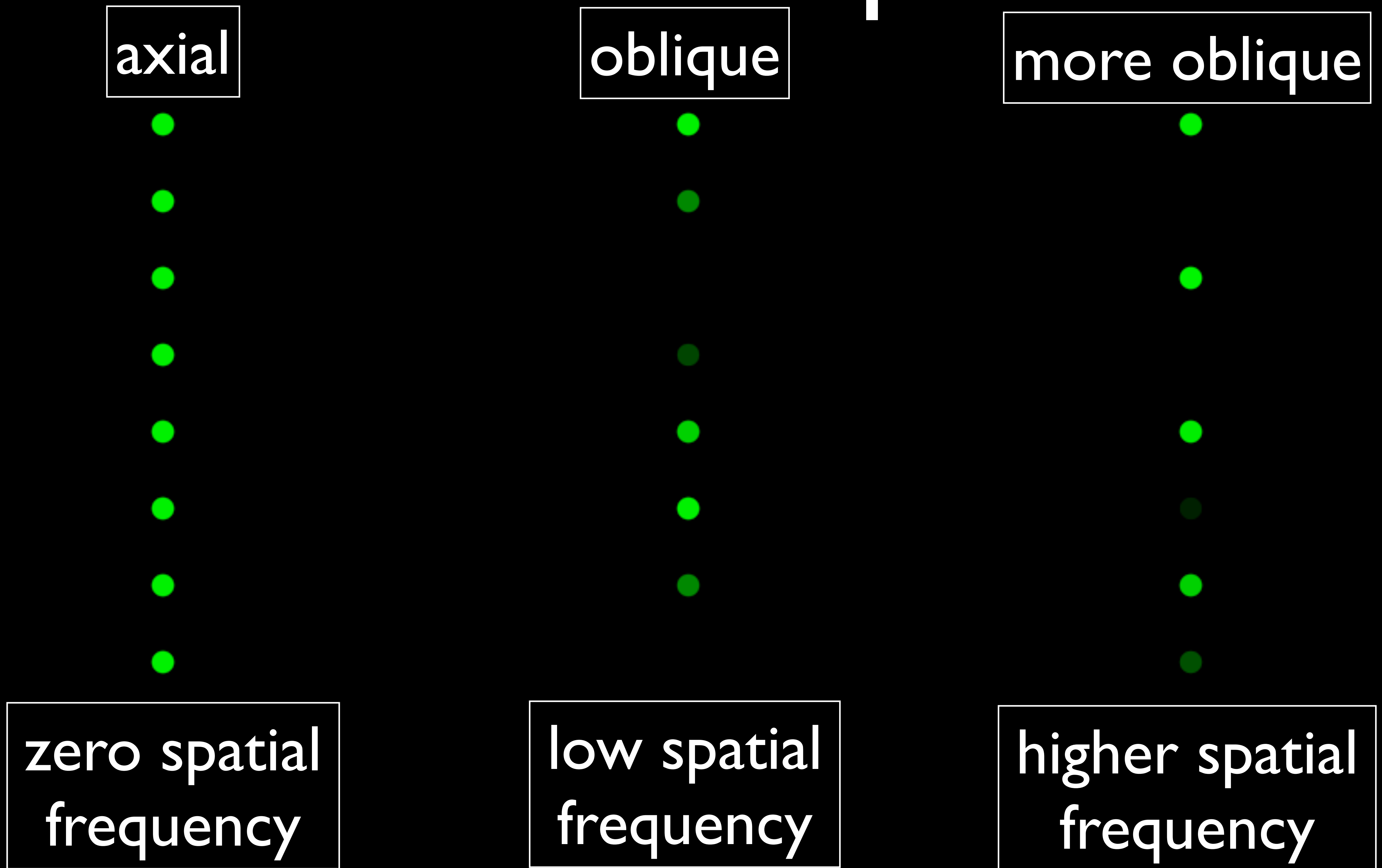

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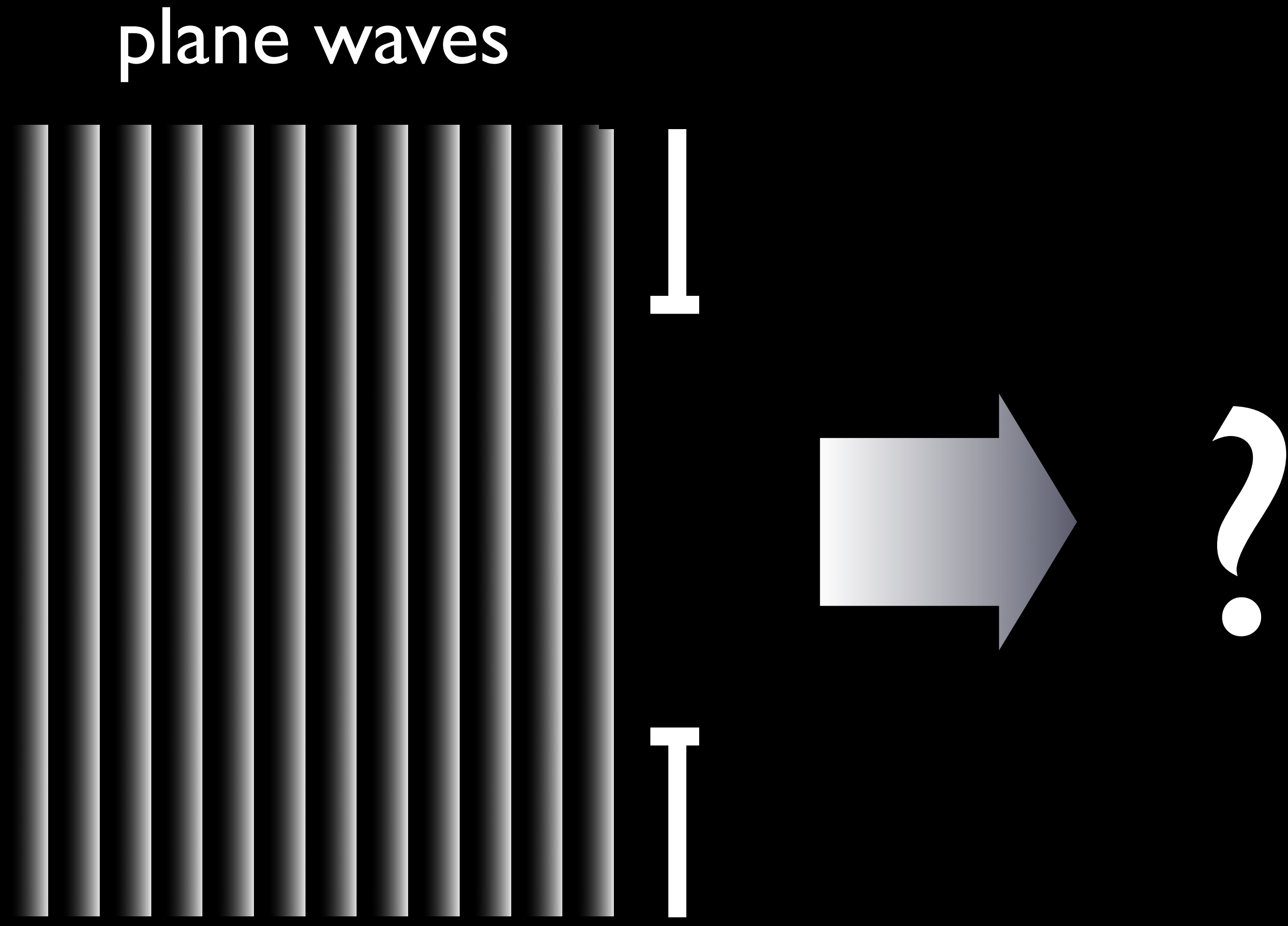

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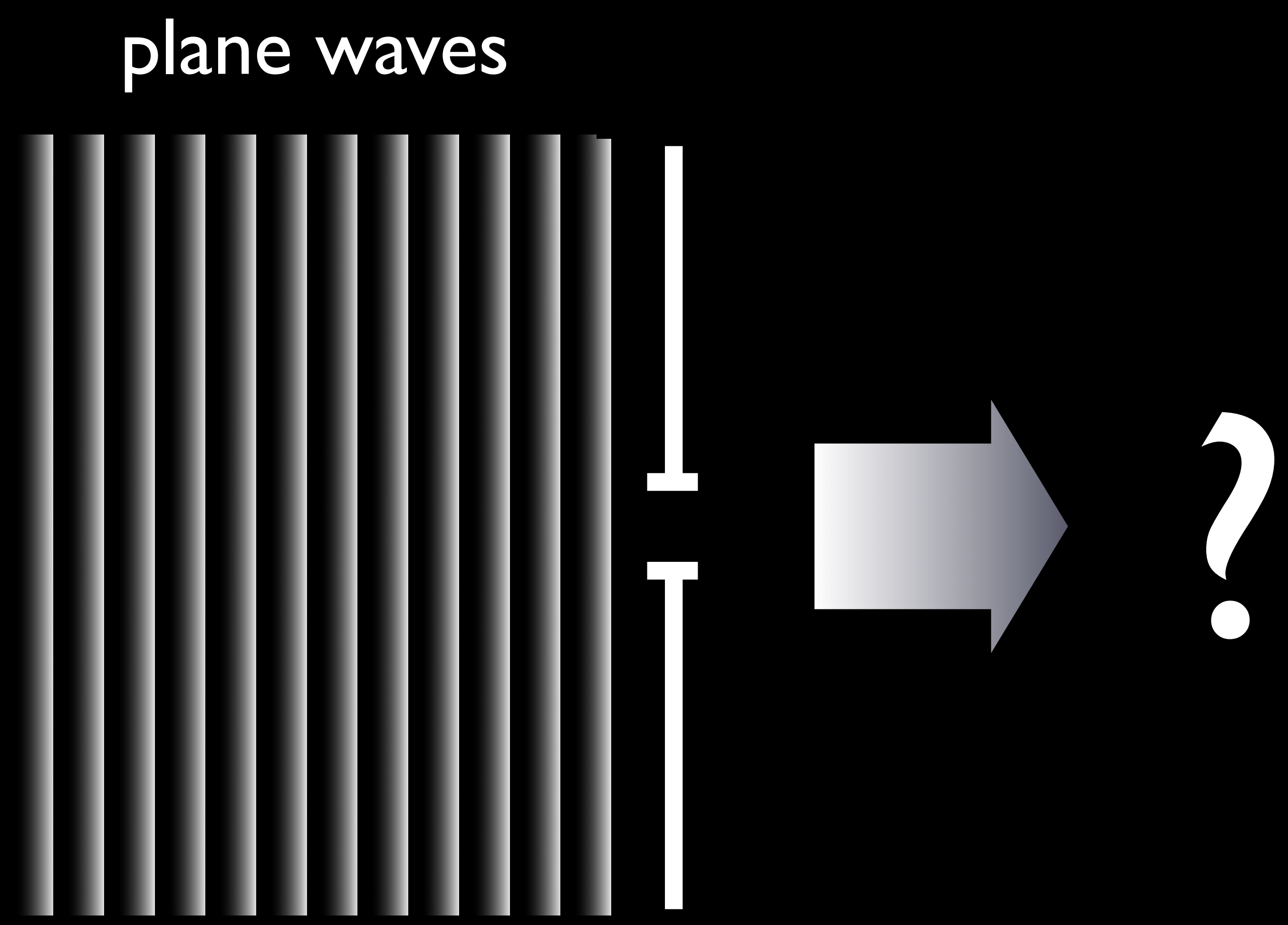

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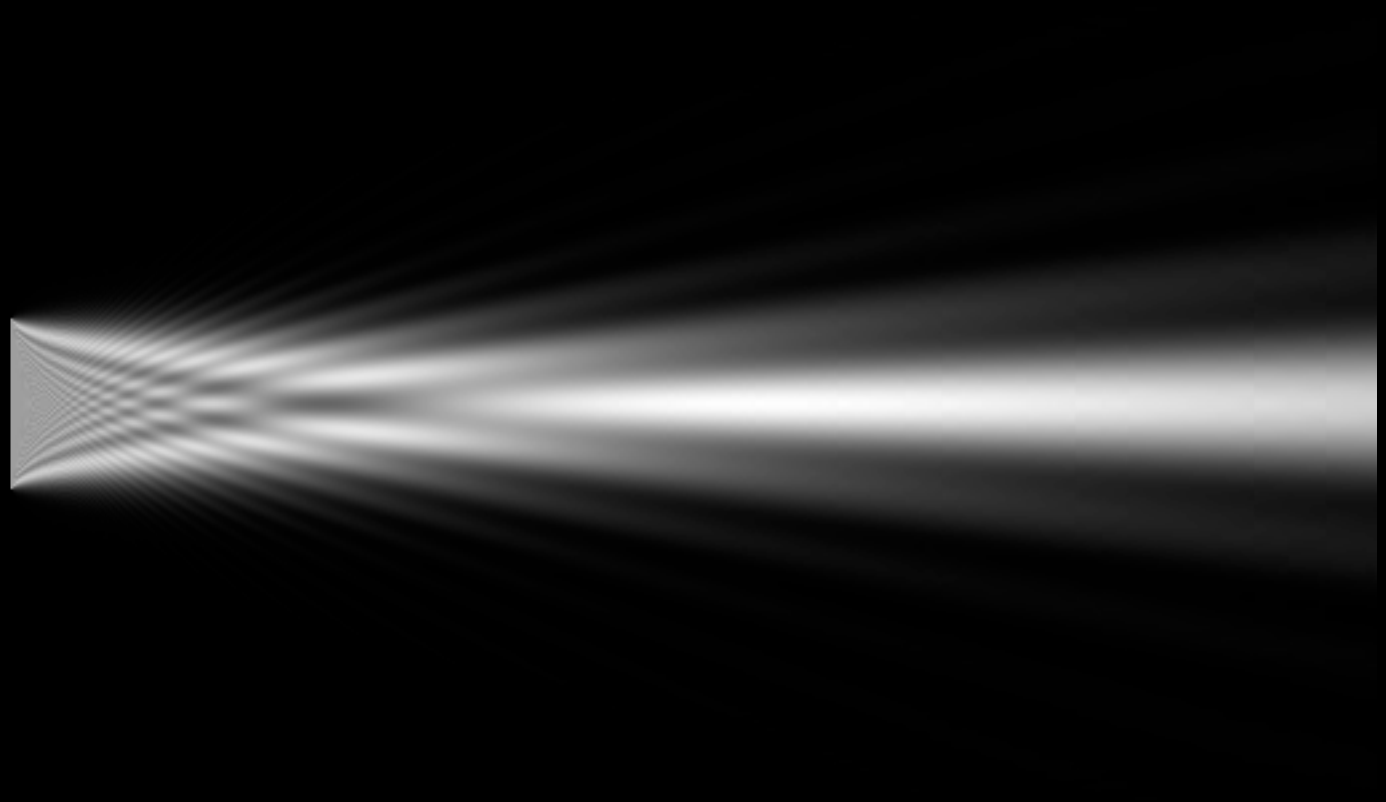


aperture = 128 wavelengths

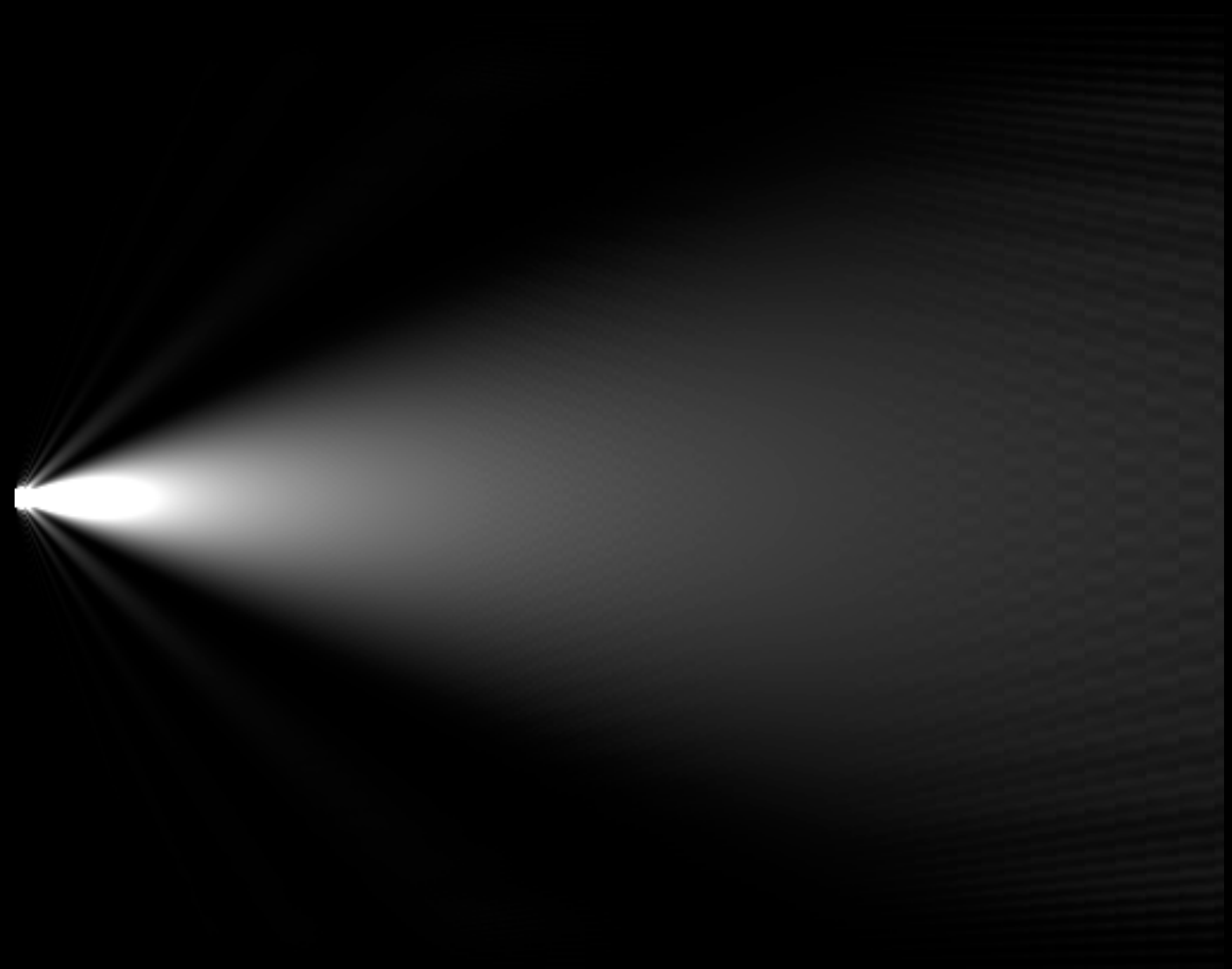
Position and Direction in Wave Optics

aperture = 64 wavelengths

Position and Direction in Wave Optics

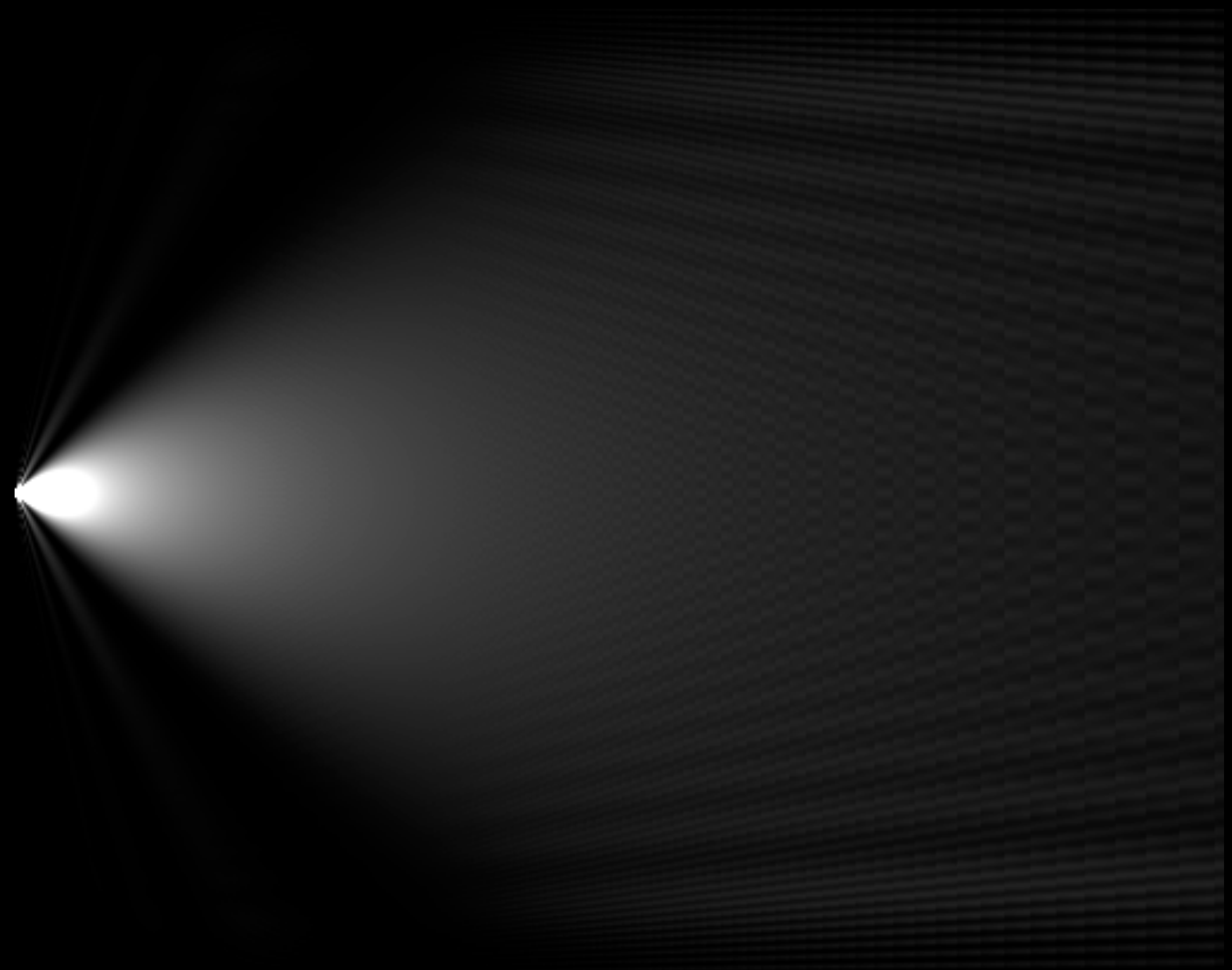
aperture = 32 wavelengths

Position and Direction in Wave Optics



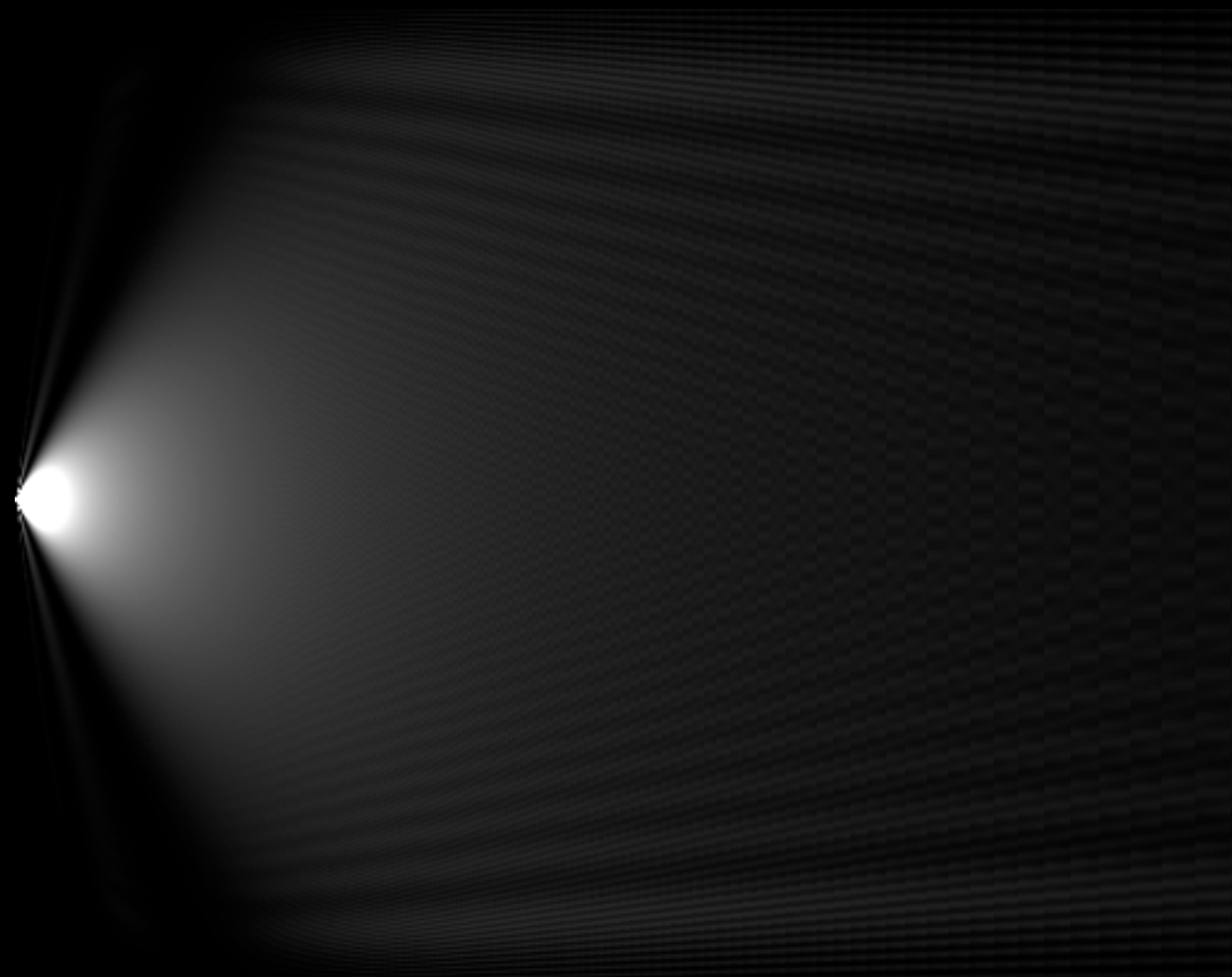
aperture = 16 wavelengths

Position and Direction in Wave Optics



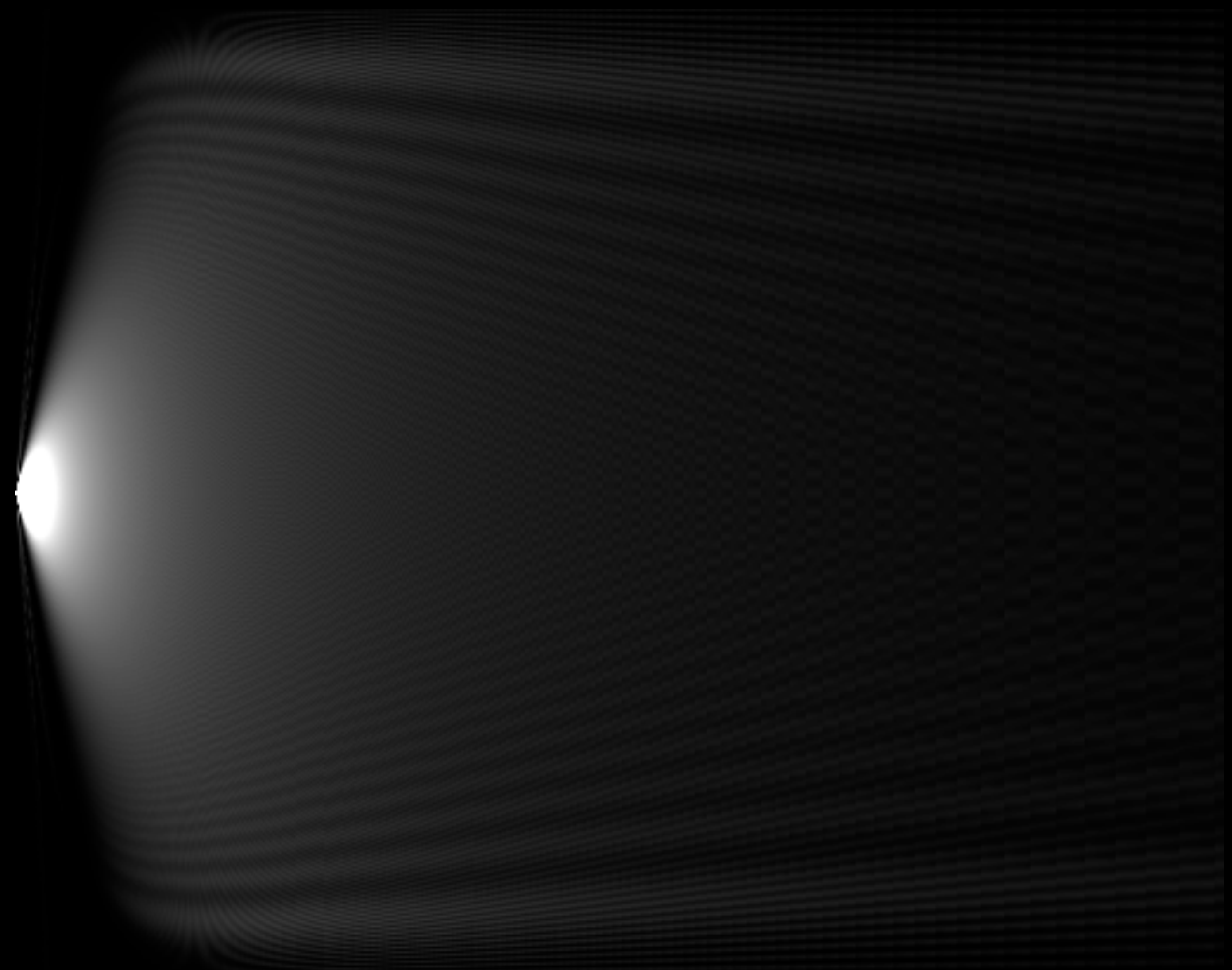
aperture = 8 wavelengths

Position and Direction in Wave Optics



aperture = 4 wavelengths

Position and Direction in Wave Optics



aperture = 2 wavelengths

Position and Direction in Wave Optics

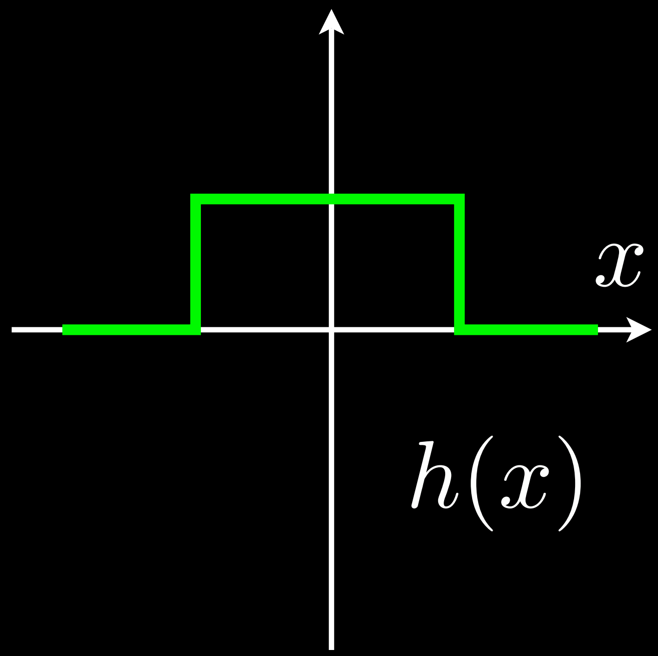
Recap

ray optics	position	direction
wave optics	position	spatial frequency

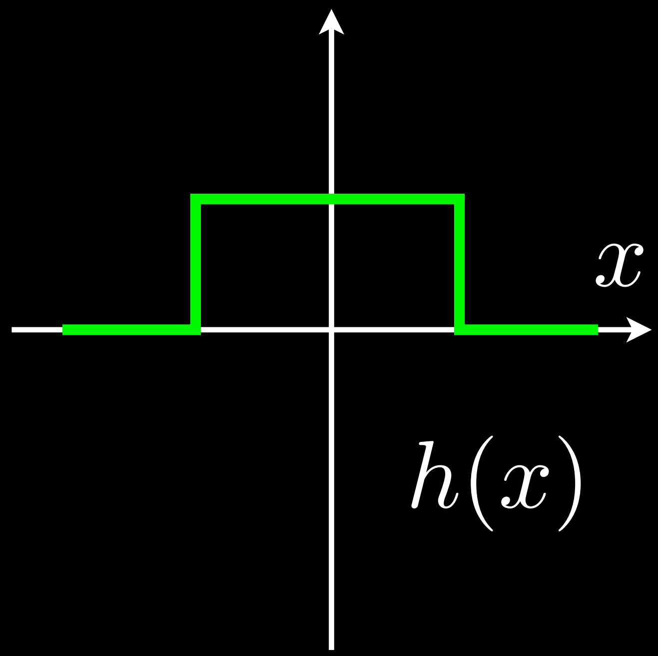
- to determine both position and spatial frequency, need to look at a window of finite (nonzero) width

2D Wigner Distribution

2D Wigner Distribution

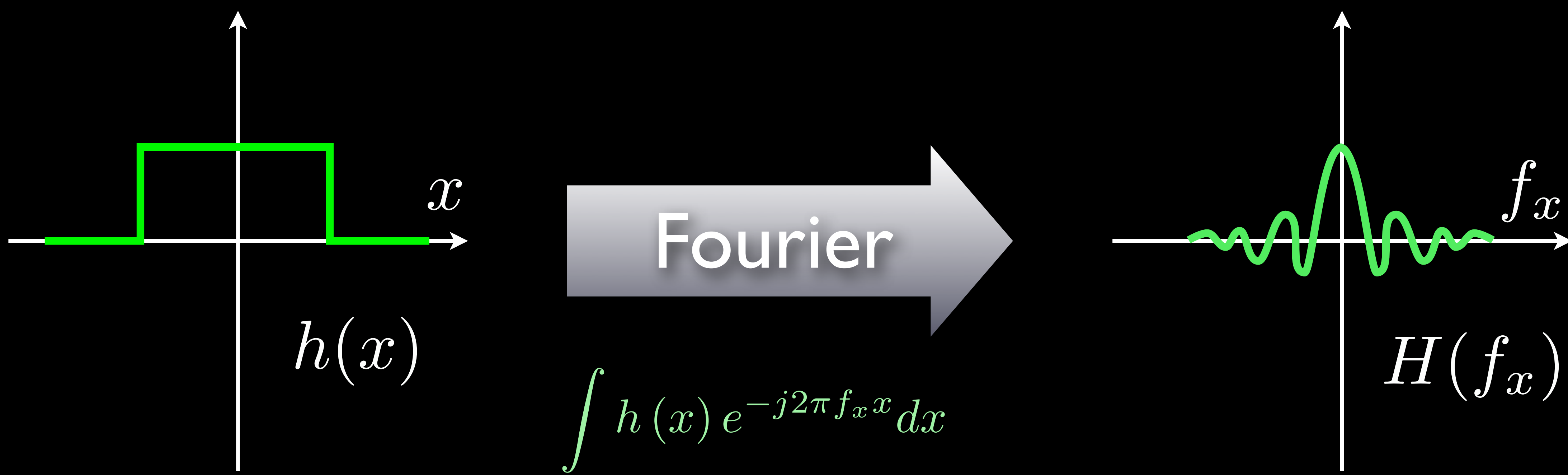


2D Wigner Distribution



$$\int h(x)\, e^{-j2\pi f_x x} dx$$

2D Wigner Distribution



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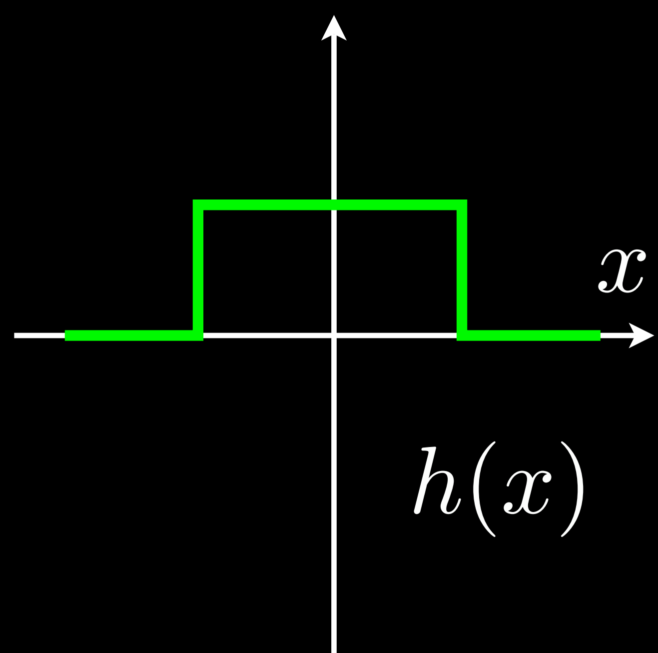

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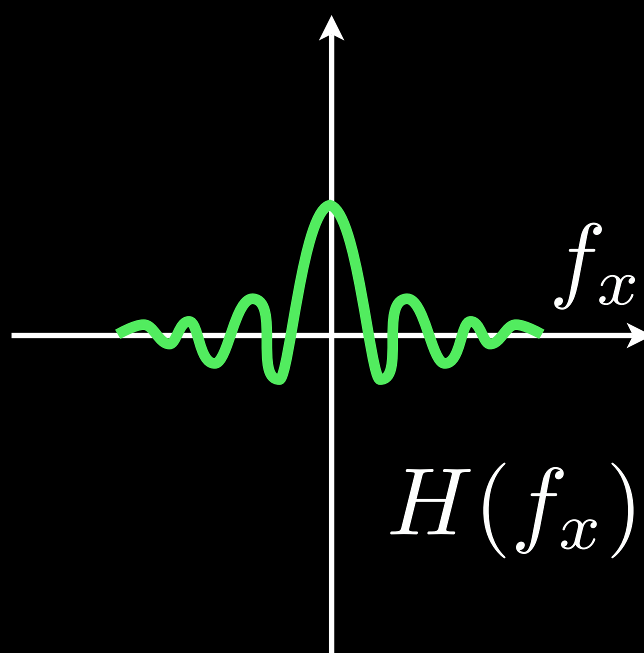

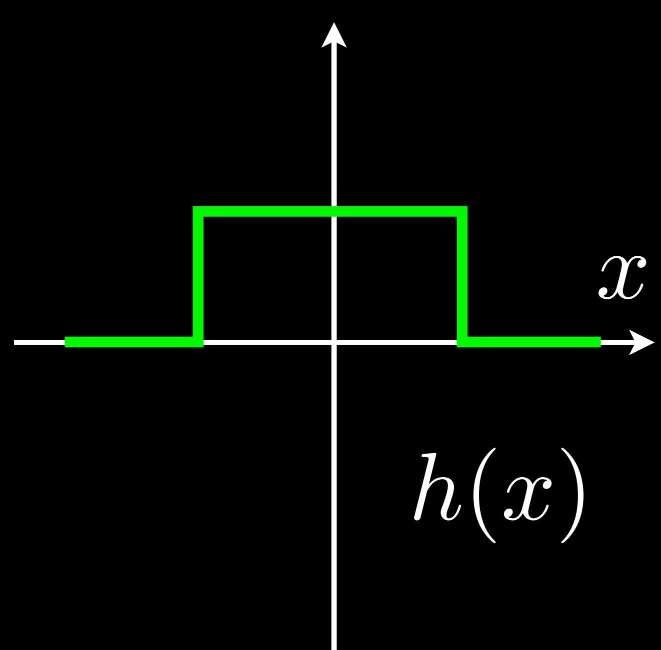

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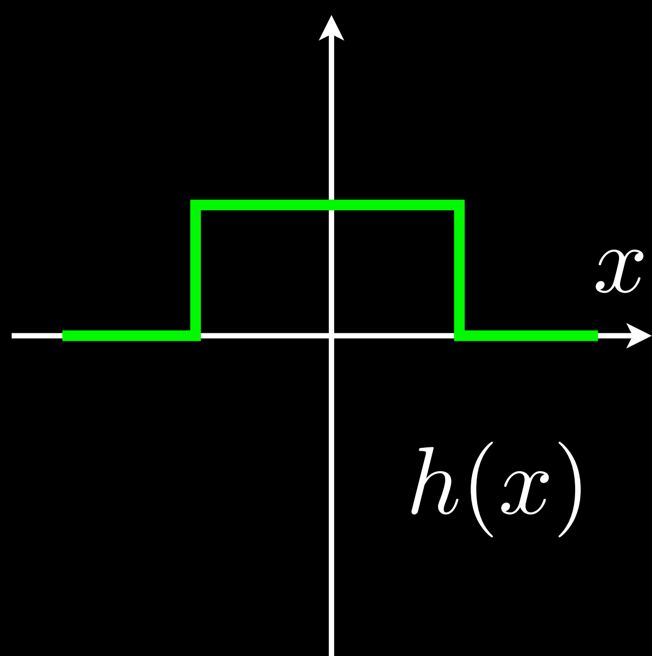

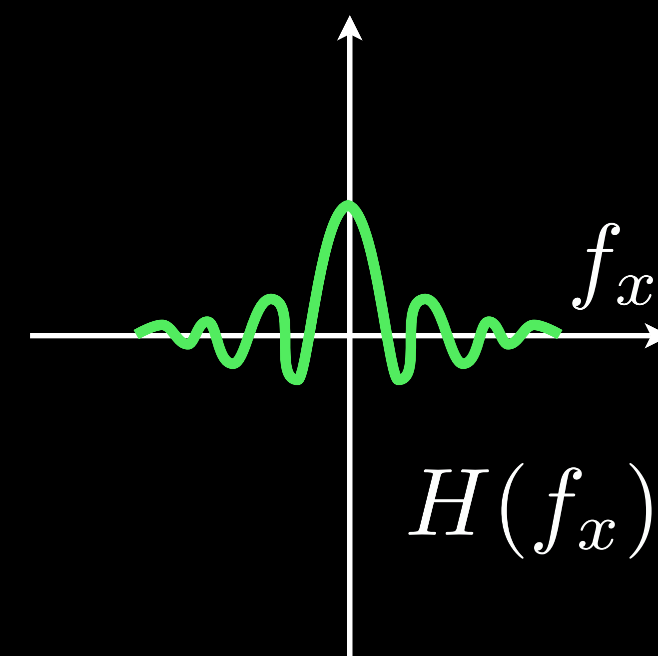

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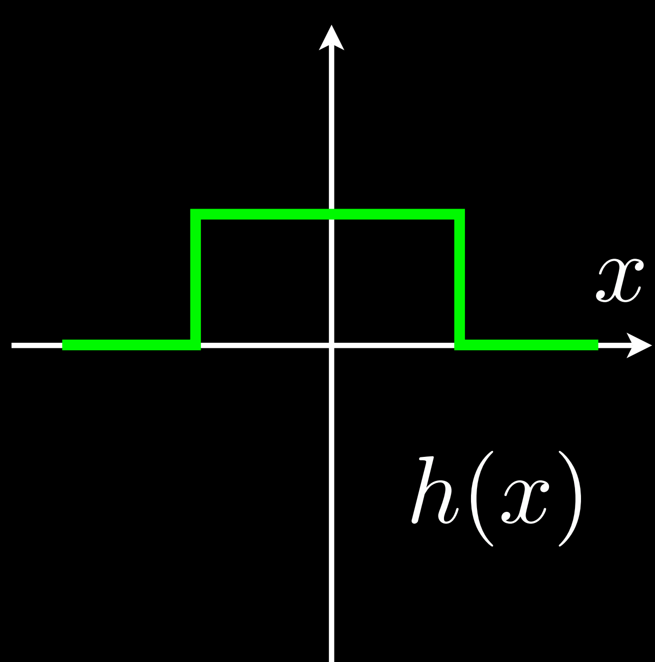

$$\int h\left(x+\frac{\xi}{2}\right) h^*\left(x-\frac{\xi}{2}\right) e^{-j2\pi f_\xi \xi} d\xi$$

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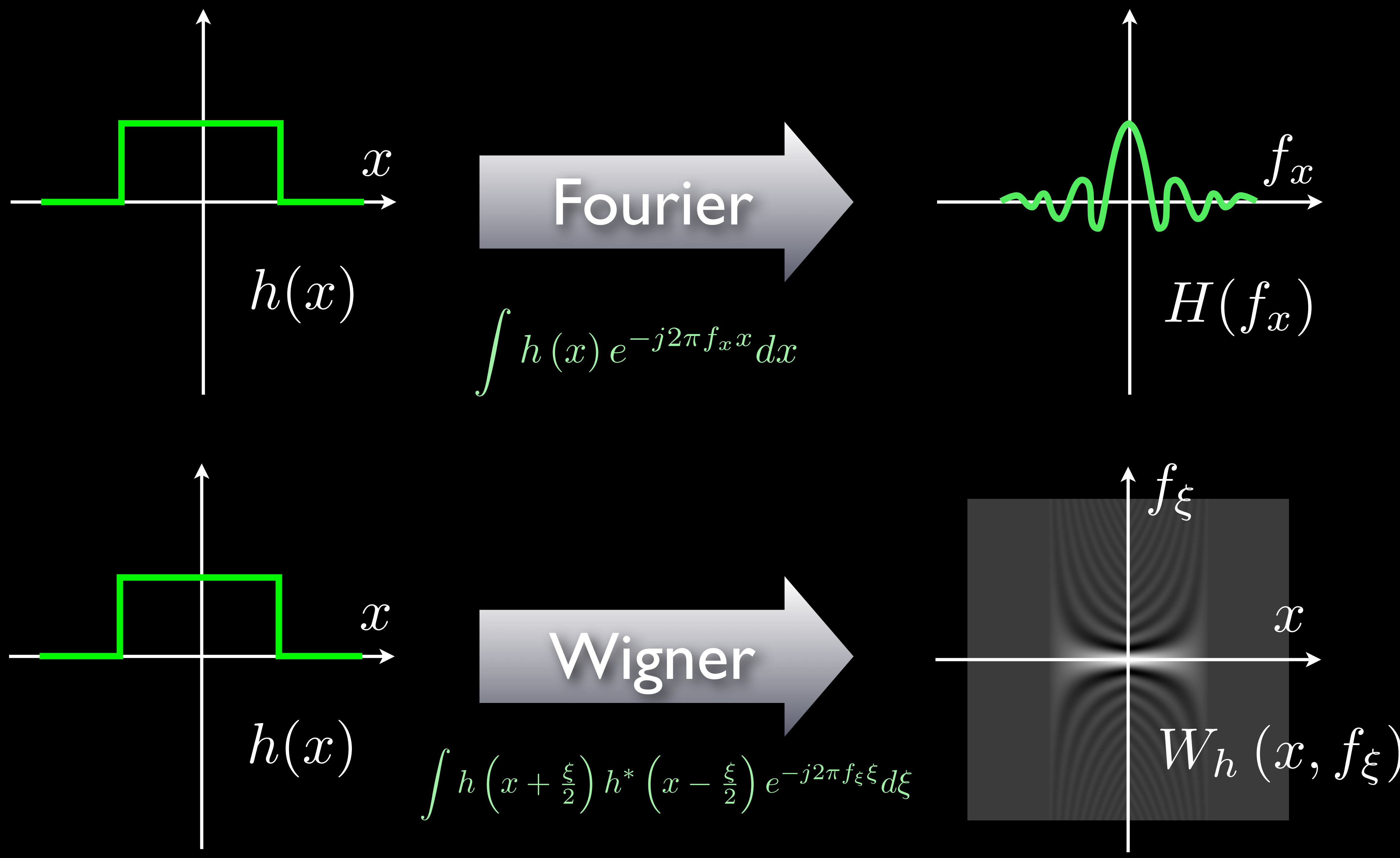


2D Wigner Distribution

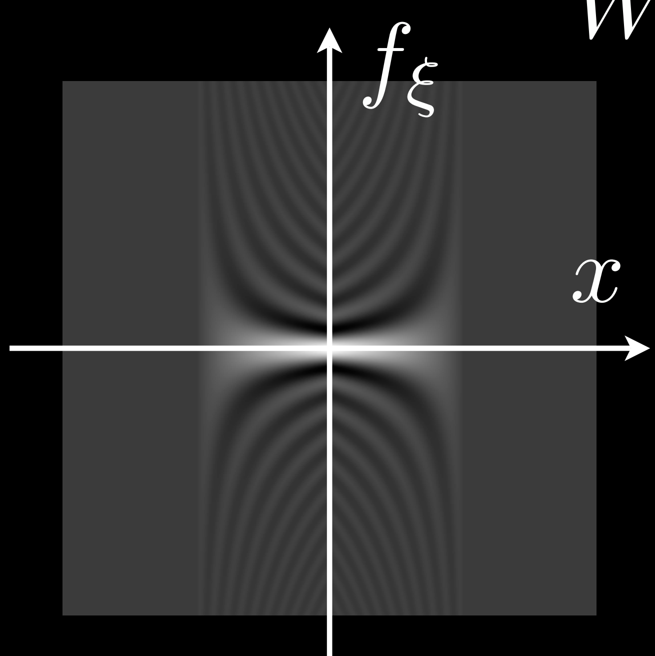
$$W_h(x, f_\xi) = \int h\left(x + \frac{\xi}{2}\right) h^*\left(x - \frac{\xi}{2}\right) e^{-j2\pi f_\xi \xi} d\xi$$

- input: one-dimensional function of position
- output: two-dimensional function of position and frequency
- (some) information about spectrum at each position

2D Wigner Distribution

- projection along frequency yields power
- projection along position yields spectral power

$W_h\left(x, f_\xi\right)$



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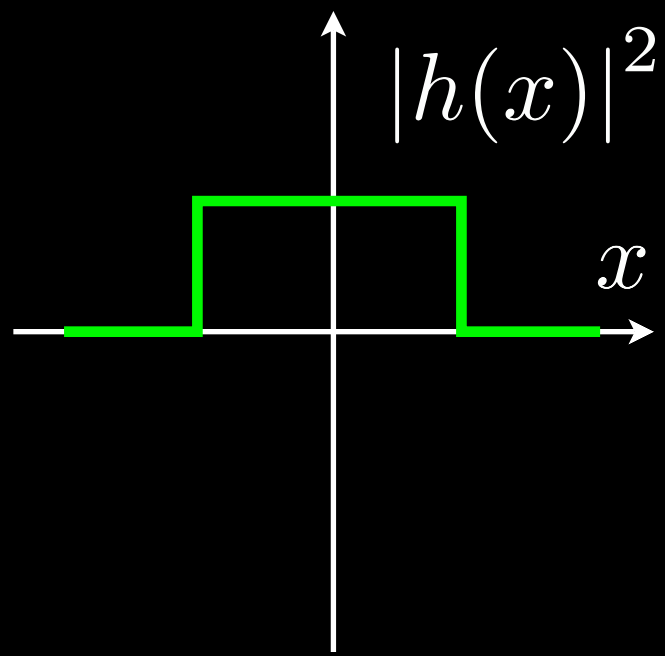

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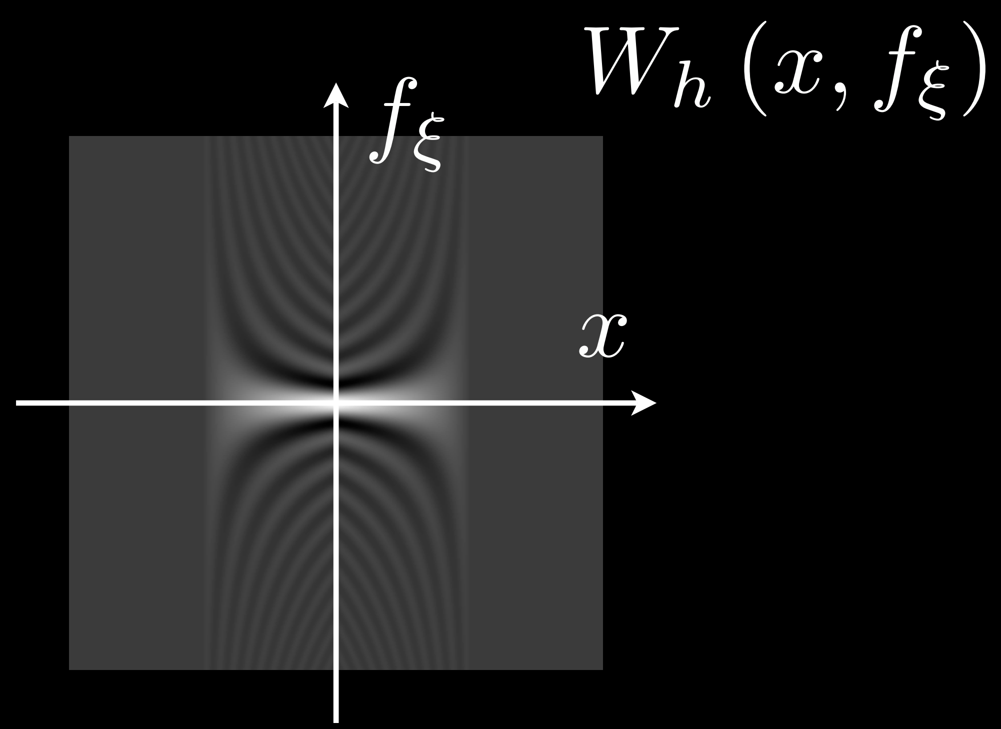

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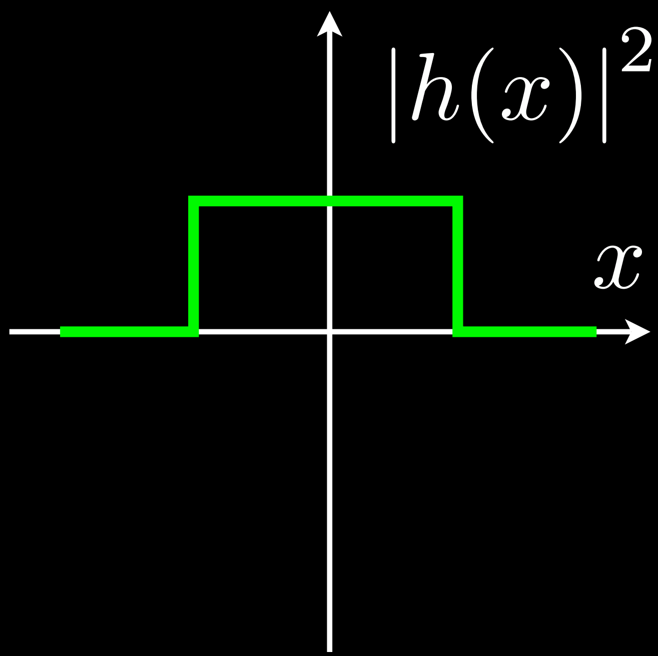

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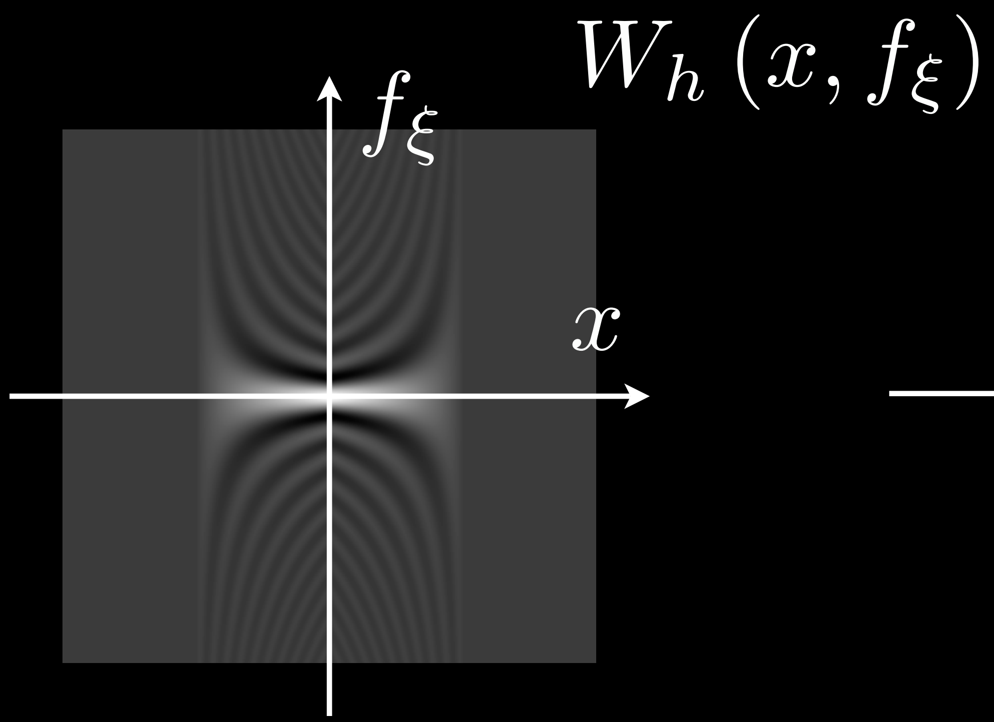

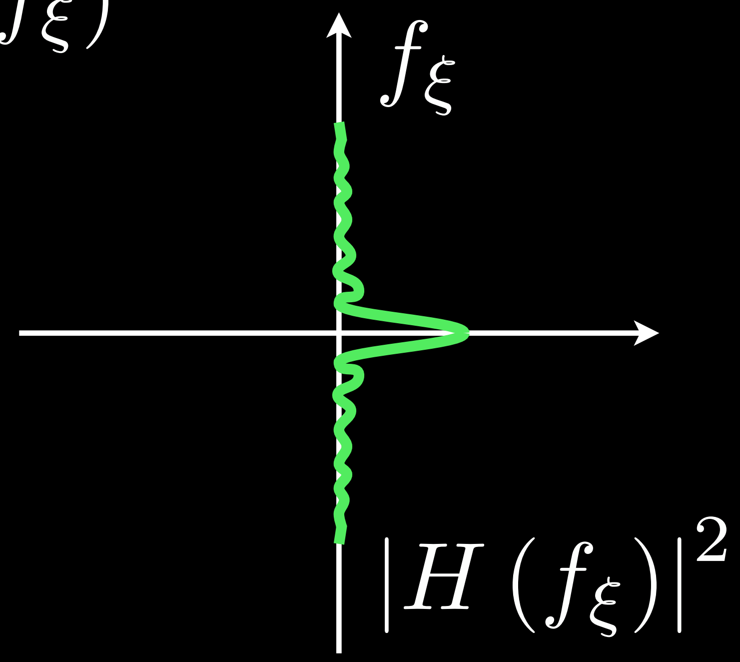

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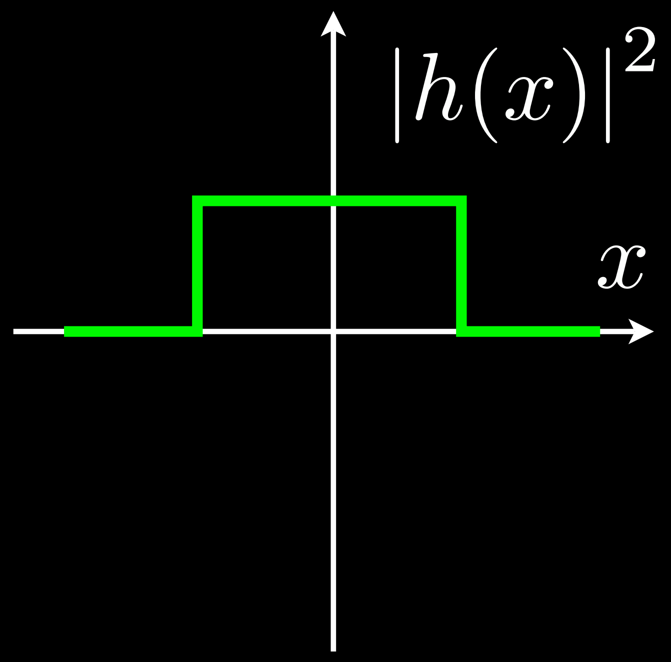
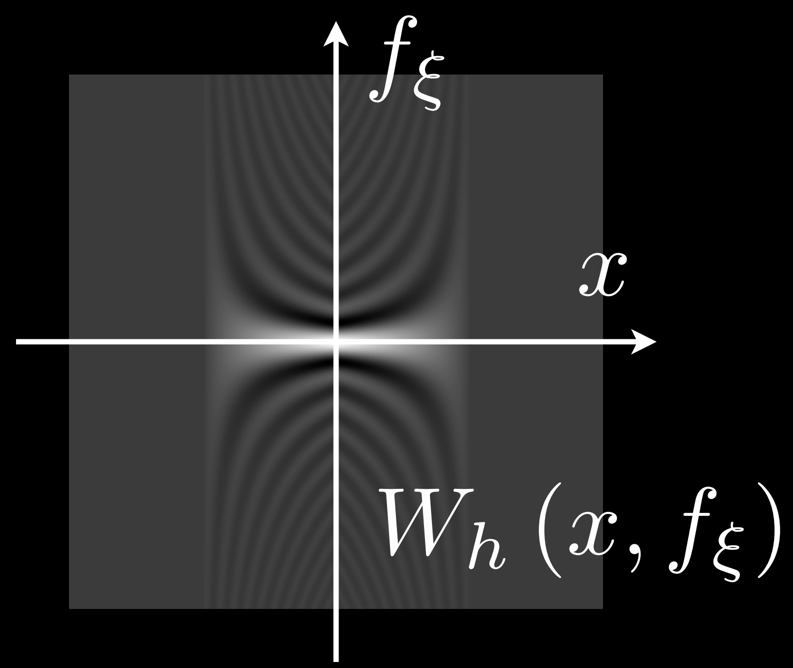
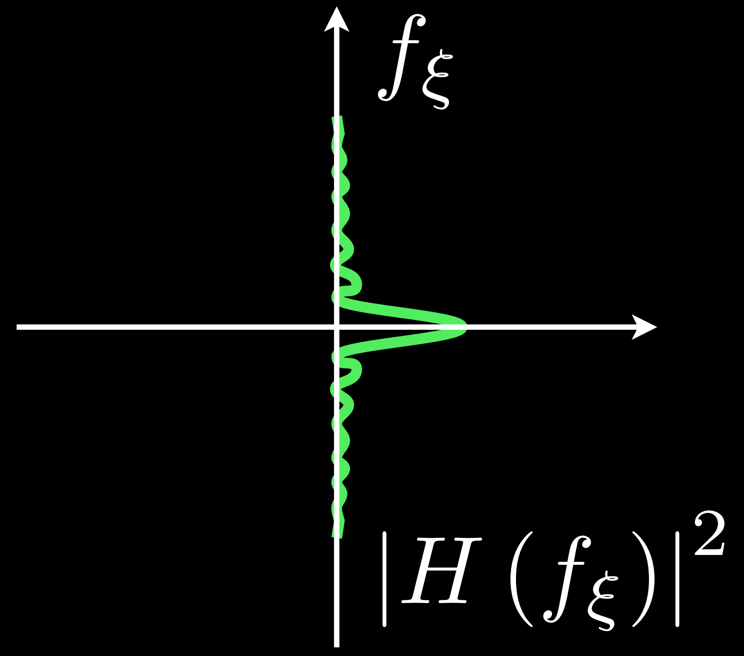


- tradeoff between width and height (fixed “area” or space-bandwidth product)
- uncertainty principle





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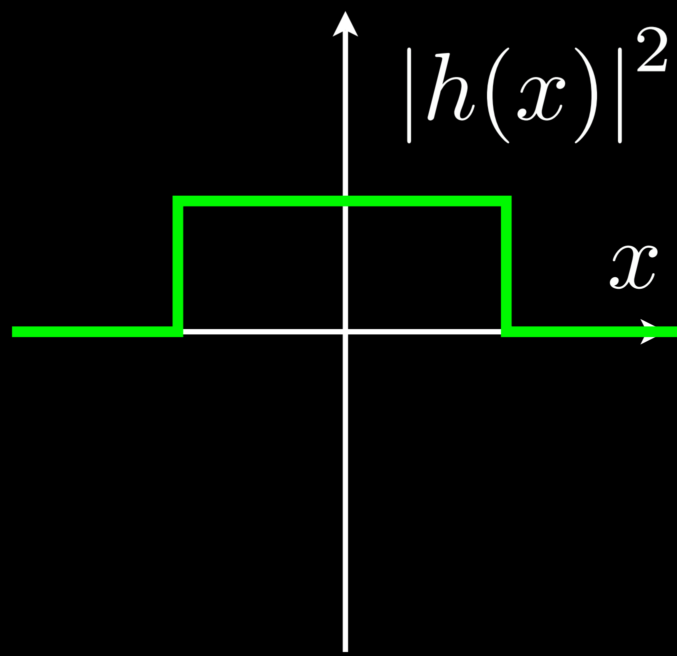

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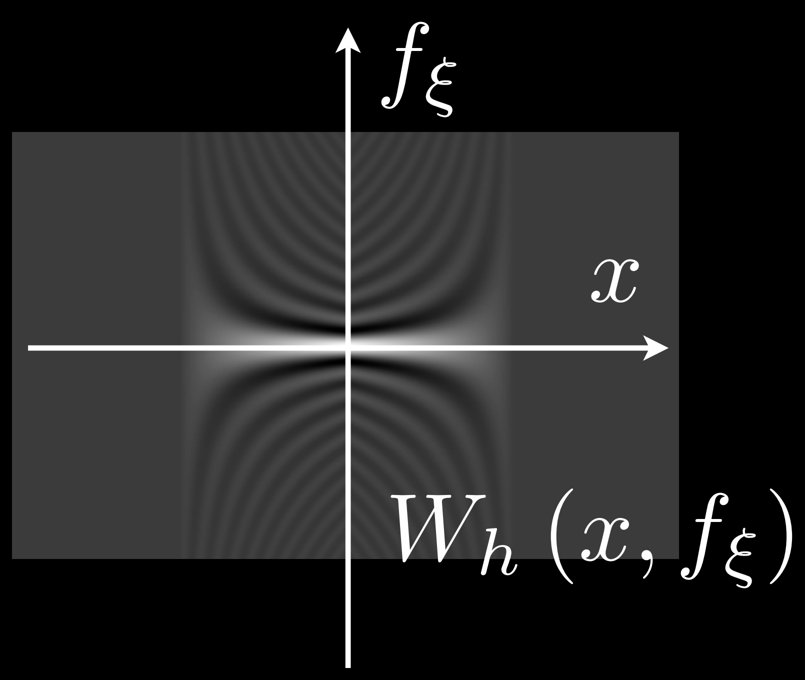

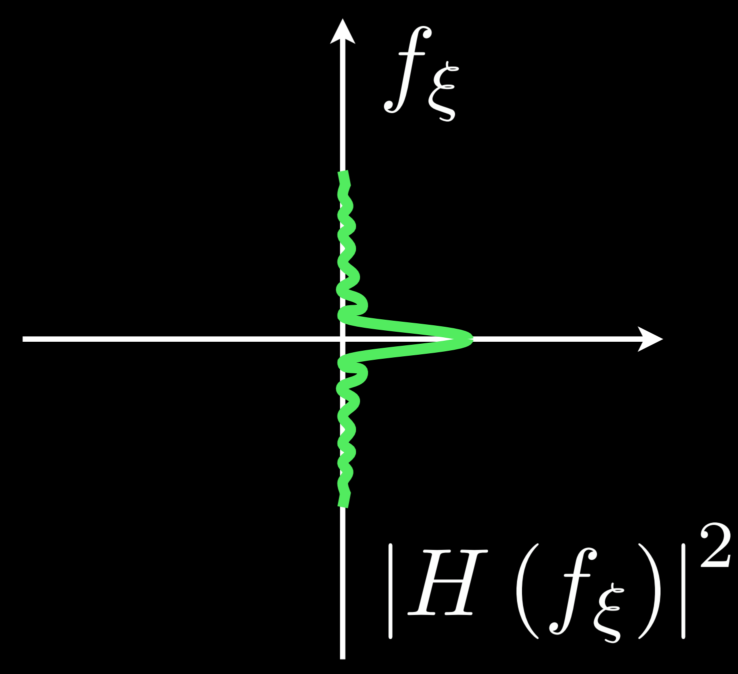

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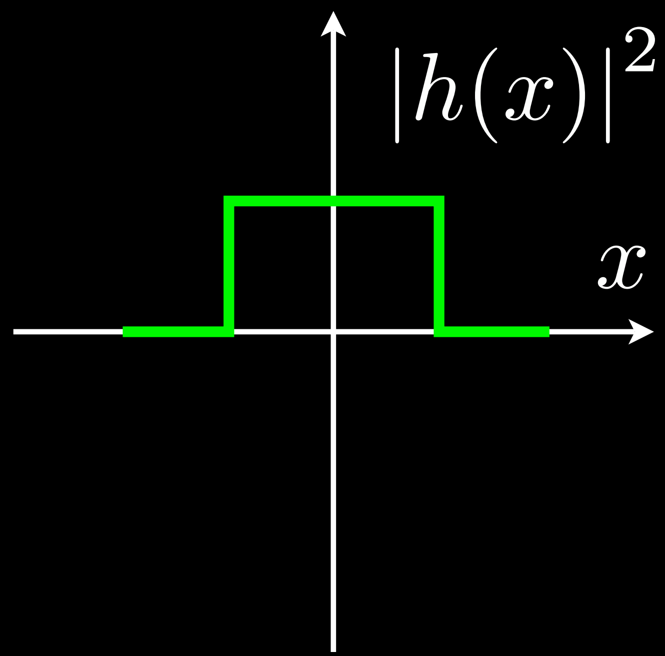

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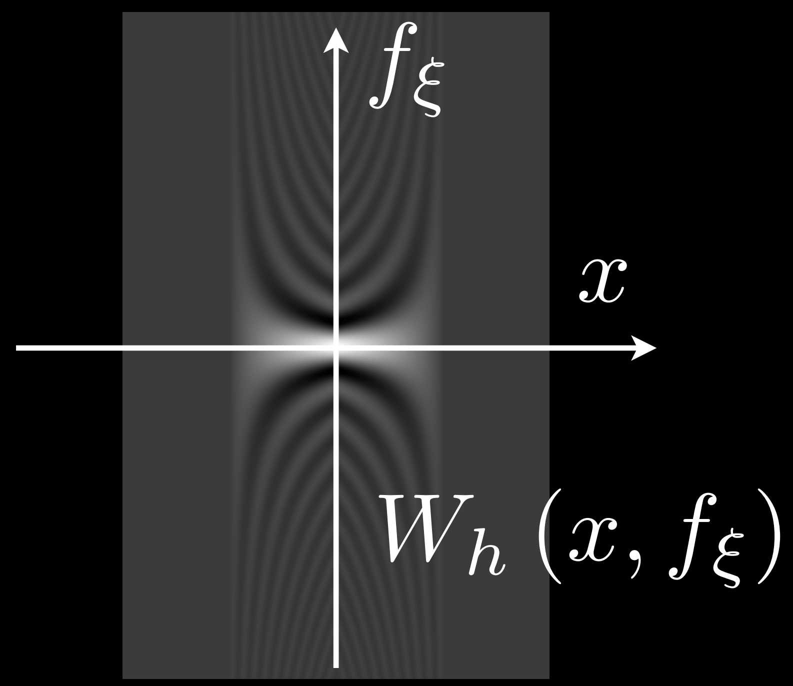

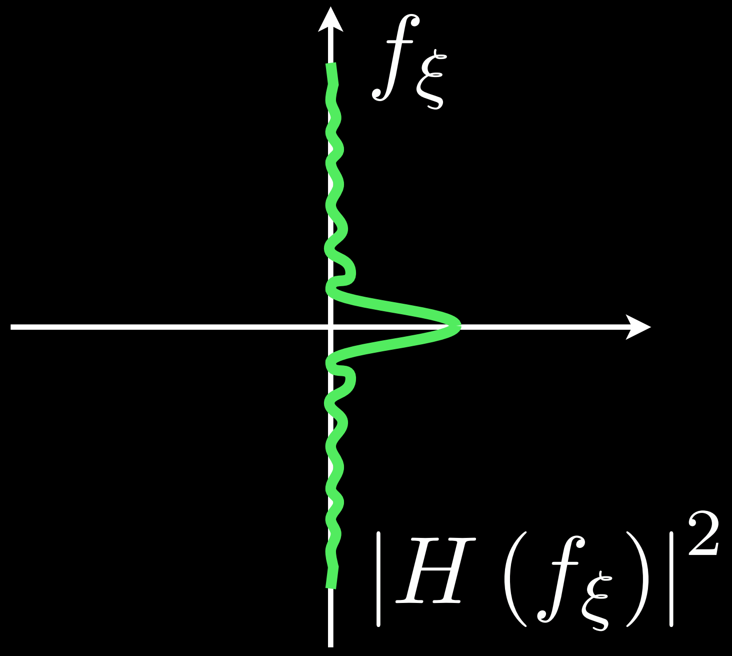

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- information about both position and frequency
- fixed space-bandwidth product

Observable Light Field

- move aperture across plane
- look at directional spread
- continuous form of plenoptic camera

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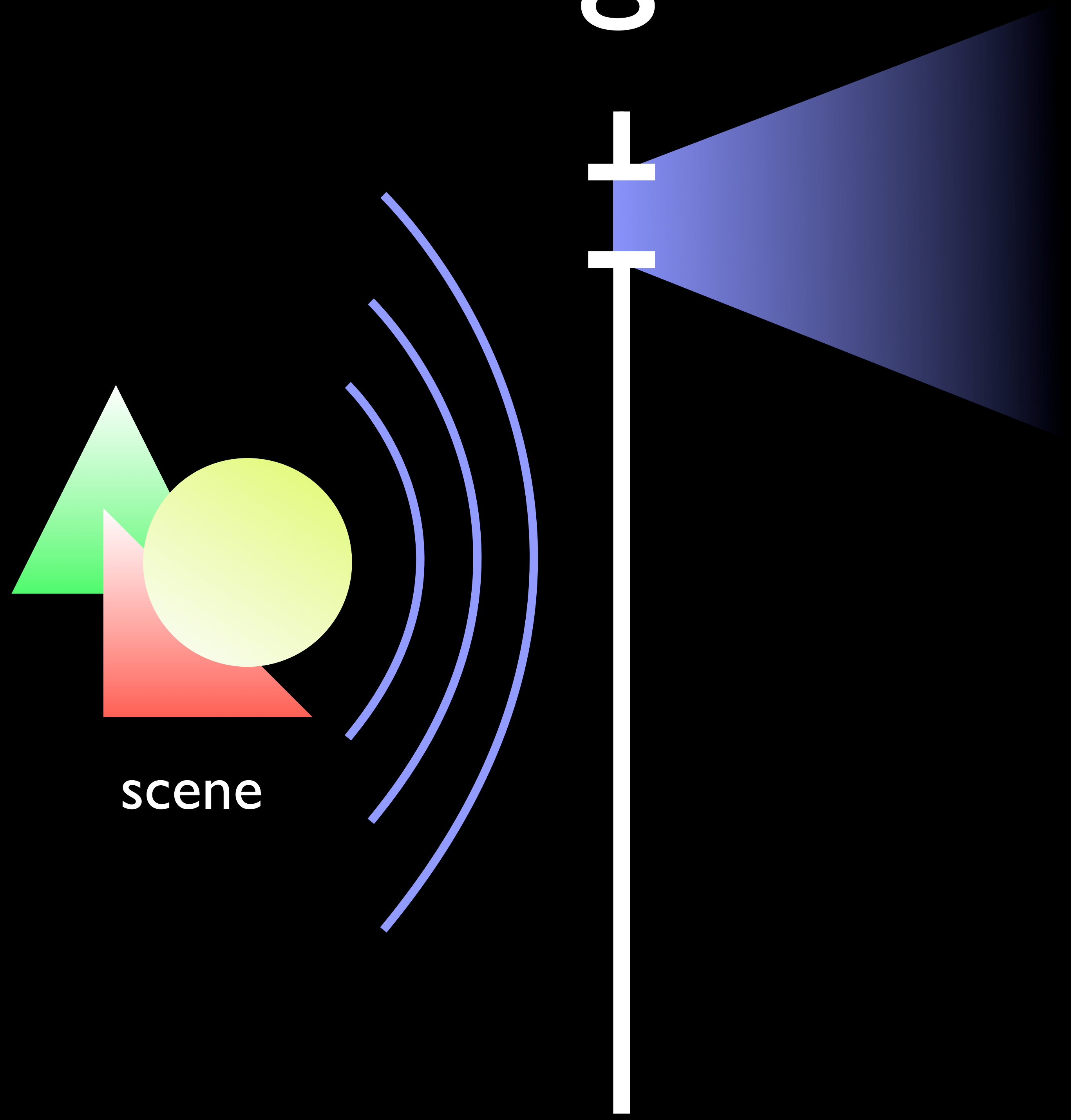

Observable Light Field

- move aperture across plane
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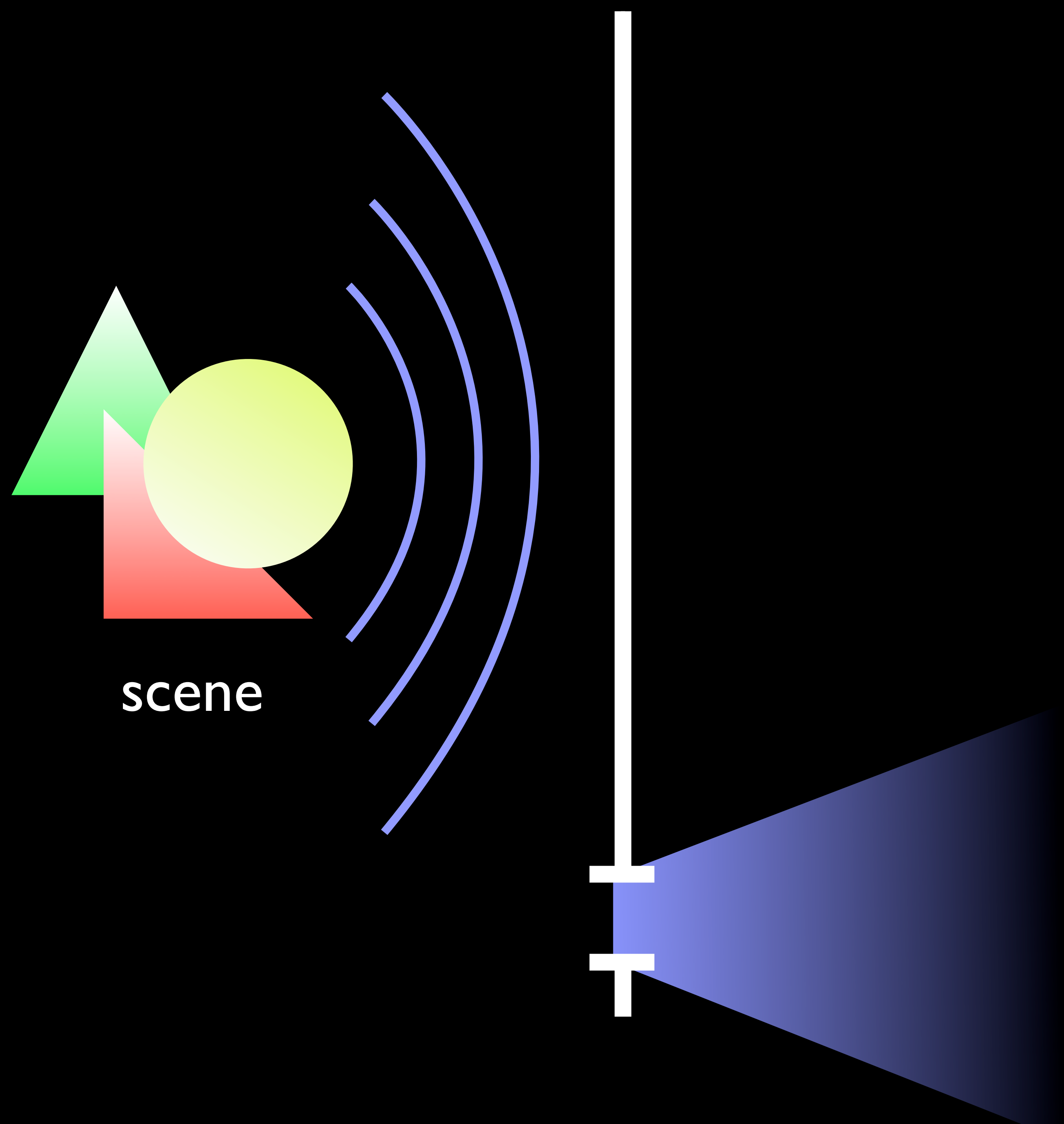

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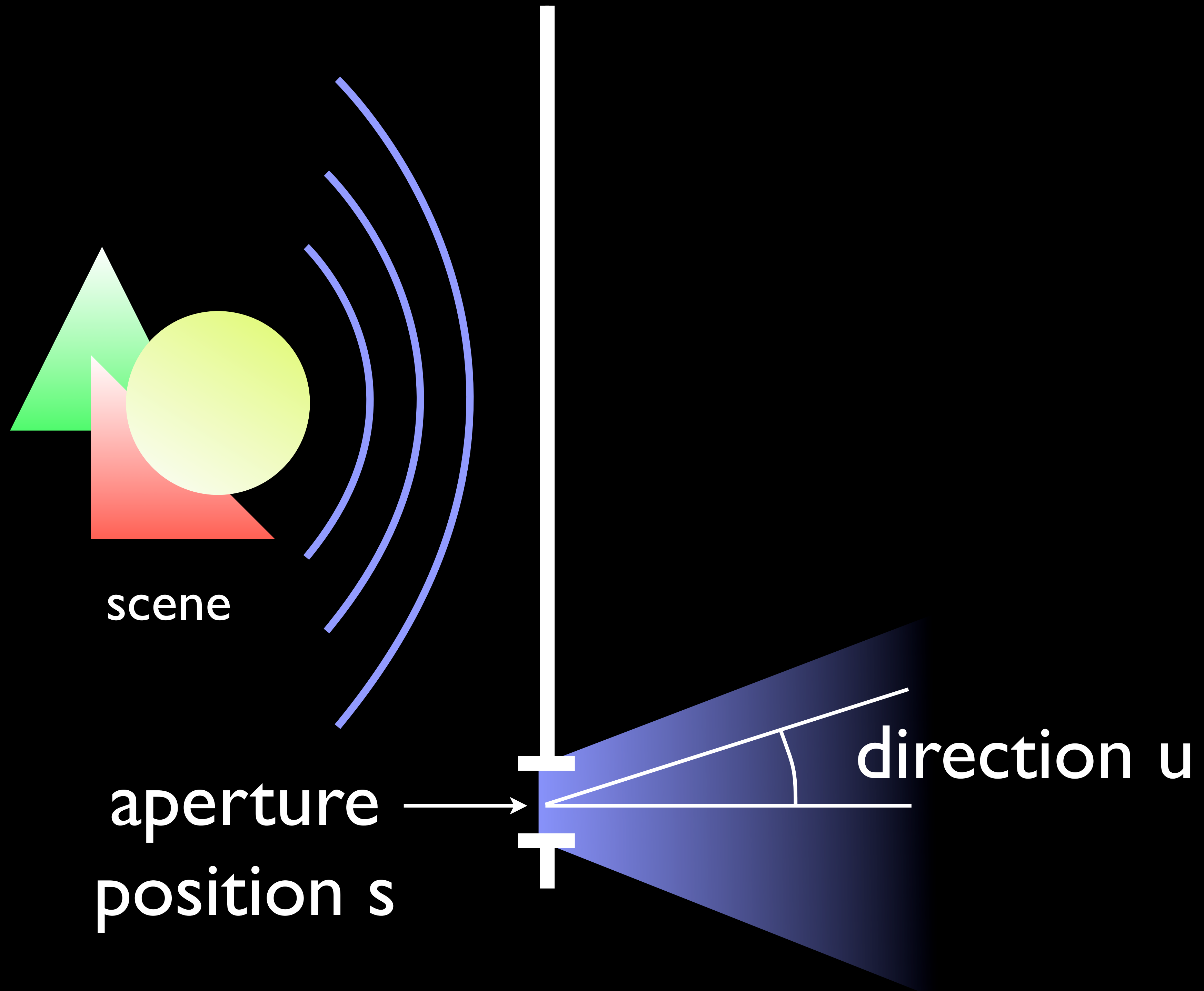

Observable Light Field

$$l_{obs}^{(T)}(s,u) = \left| \int U(x)T(x-s)e^{-j2\pi \frac{u}{\lambda}x}dx \right|^2$$

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Fourier transform

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wave

Fourier transform

Observable Light Field

aperture window

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power

wave

Fourier transform

Observable Light Field

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$$l_{obs}^{(T)}(s,u) = W_U\left(s,\frac{u}{\lambda}\right) \otimes W_T\left(-s,\frac{u}{\lambda}\right)$$

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Wigner distribution of wave function

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Wigner distribution of wave function

Wigner distribution of aperture window

Observable Light Field

$$l_{obs}^{(T)}(s,u) = \left| \int U(x)T(x-s)e^{-j2\pi\frac{u}{\lambda}x}dx \right|^2$$

blur trades off resolution in position with direction

$$l_{obs}^{(T)}(s,u) = W_U\left(s, \frac{u}{\lambda}\right) \otimes W_T\left(-s, \frac{u}{\lambda}\right)$$

Wigner distribution of wave function

Wigner distribution of aperture window

Observable Light Field

at zero wavelength limit
(regime of ray optics)

$$l_{obs}^{(T)}(s,u) = W_U\left(s, \frac{u}{\lambda}\right) \otimes W_T\left(-s, \frac{u}{\lambda}\right)$$

Wigner distribution of wave function

Wigner distribution of aperture window

Observable Light Field

at zero wavelength limit
(regime of ray optics)

$$l_{obs}^{(T)}(s,u) = W_U\left(s, \frac{u}{\lambda}\right) \otimes \delta(-s,u)$$

Wigner distribution
of wave function

Observable Light Field

at zero wavelength limit
(regime of ray optics)

$$l_{obs}^{(T)}(s,u) = W_U\left(s, \frac{u}{\lambda}\right)$$

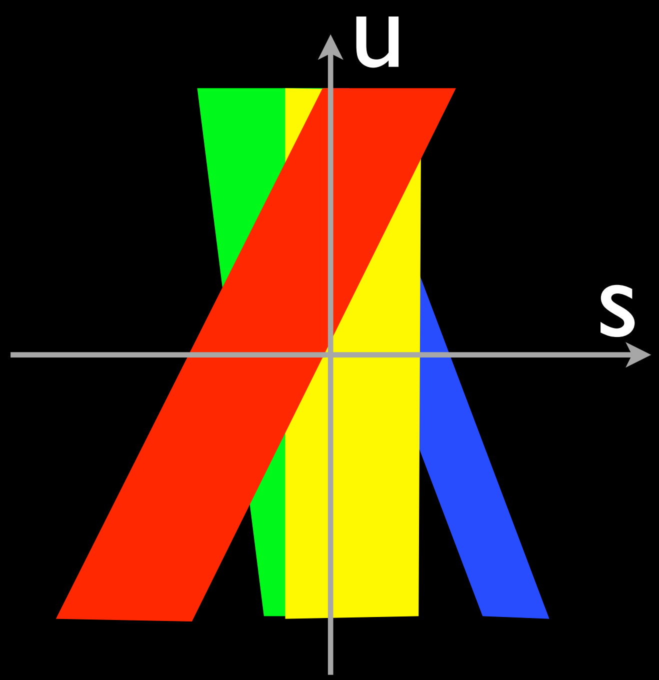
observable light field and Wigner equivalent!

Observable Light Field

- observable light field is a blurred Wigner distribution with a modified coordinate system
- blur trades off resolution in position with direction
- Wigner distribution and observable light field equivalent at zero wavelength limit

Application - Refocusing

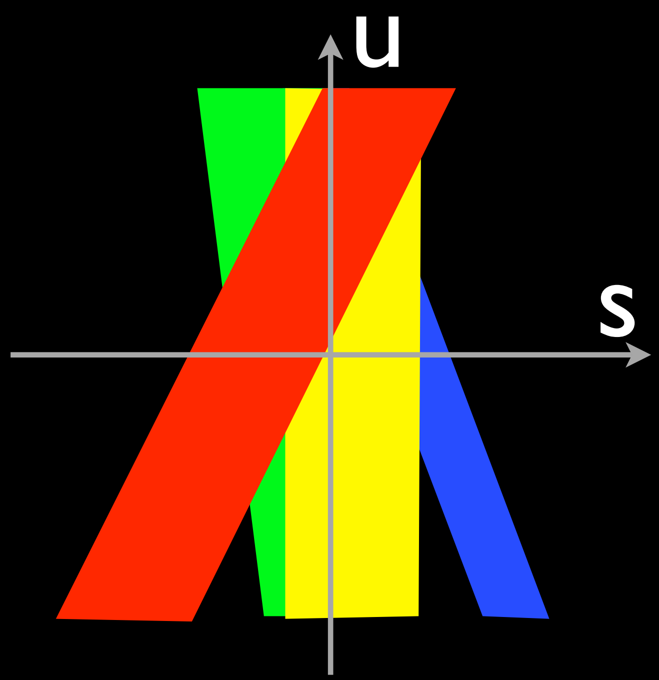
light
field



Application - Refocusing

Isaksen et. al 2000

light field



Application - Refocusing

Isaksen et. al 2000

light field

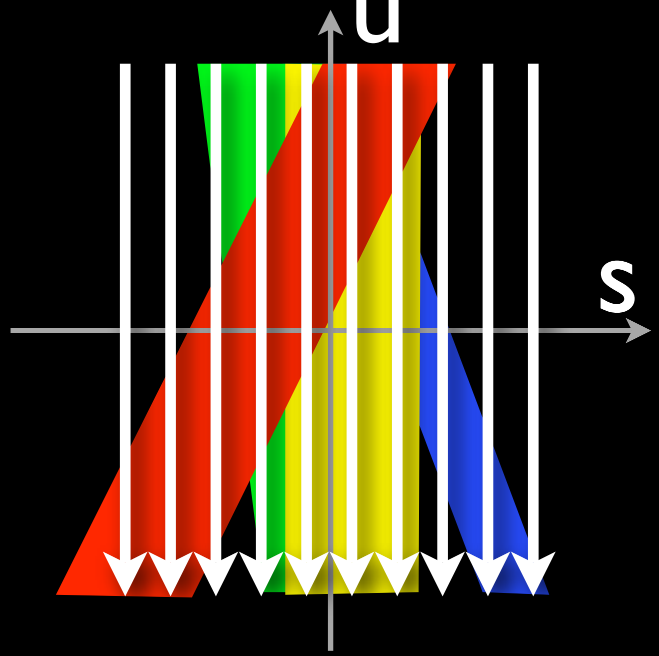


image at z=0

Application - Refocusing

Isaksen et. al 2000

light field

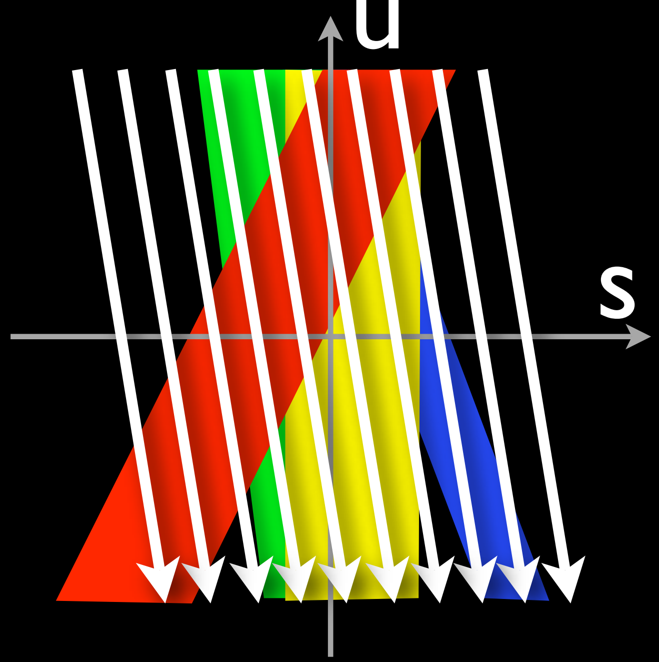
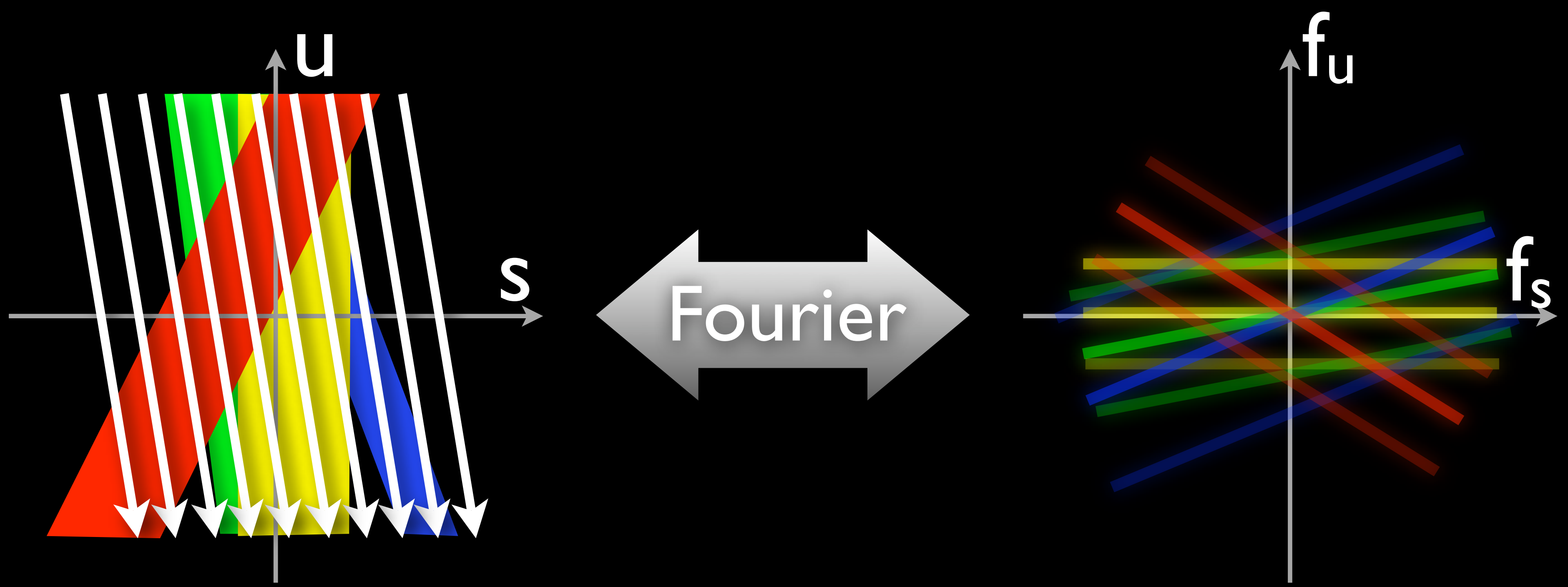


image at $z=z_0$

Application - Refocusing

Isaksen et. al 2000

light field

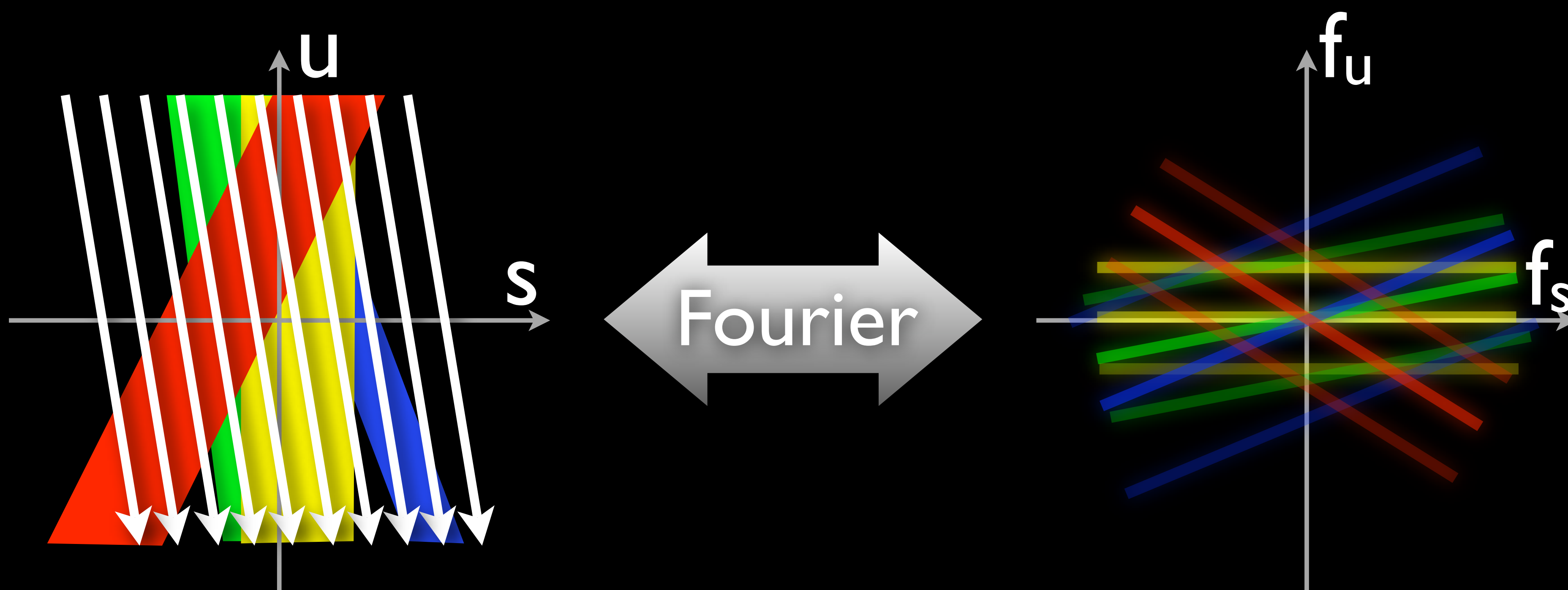


light field spectrum

Application - Refocusing

Isaksen
et. al
2000

light
field



Ng
2005

light
field
spectrum

Application - Refocusing

Isaksen
et. al
2000

light
field

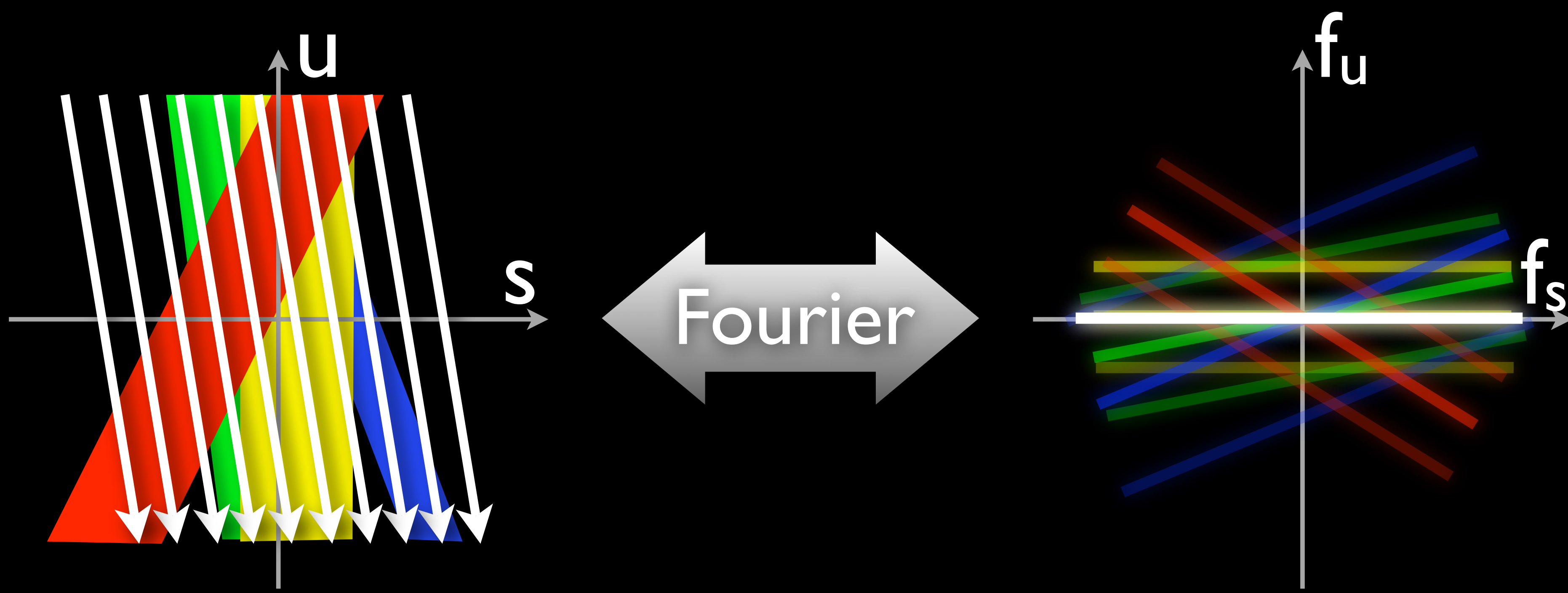


image at z=0

Ng
2005

light
field
spectrum

Application - Refocusing

Isaksen et. al 2000

light field

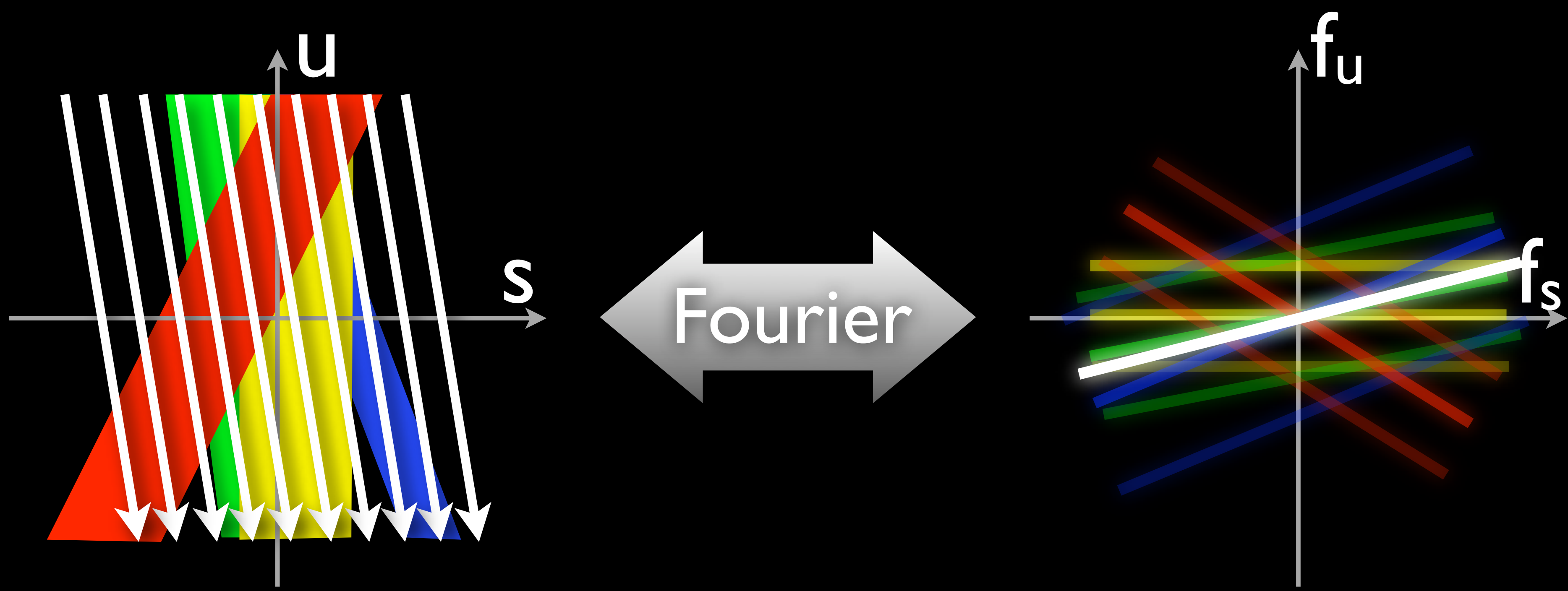


image at $z=z_0$

Ng 2005

light field spectrum

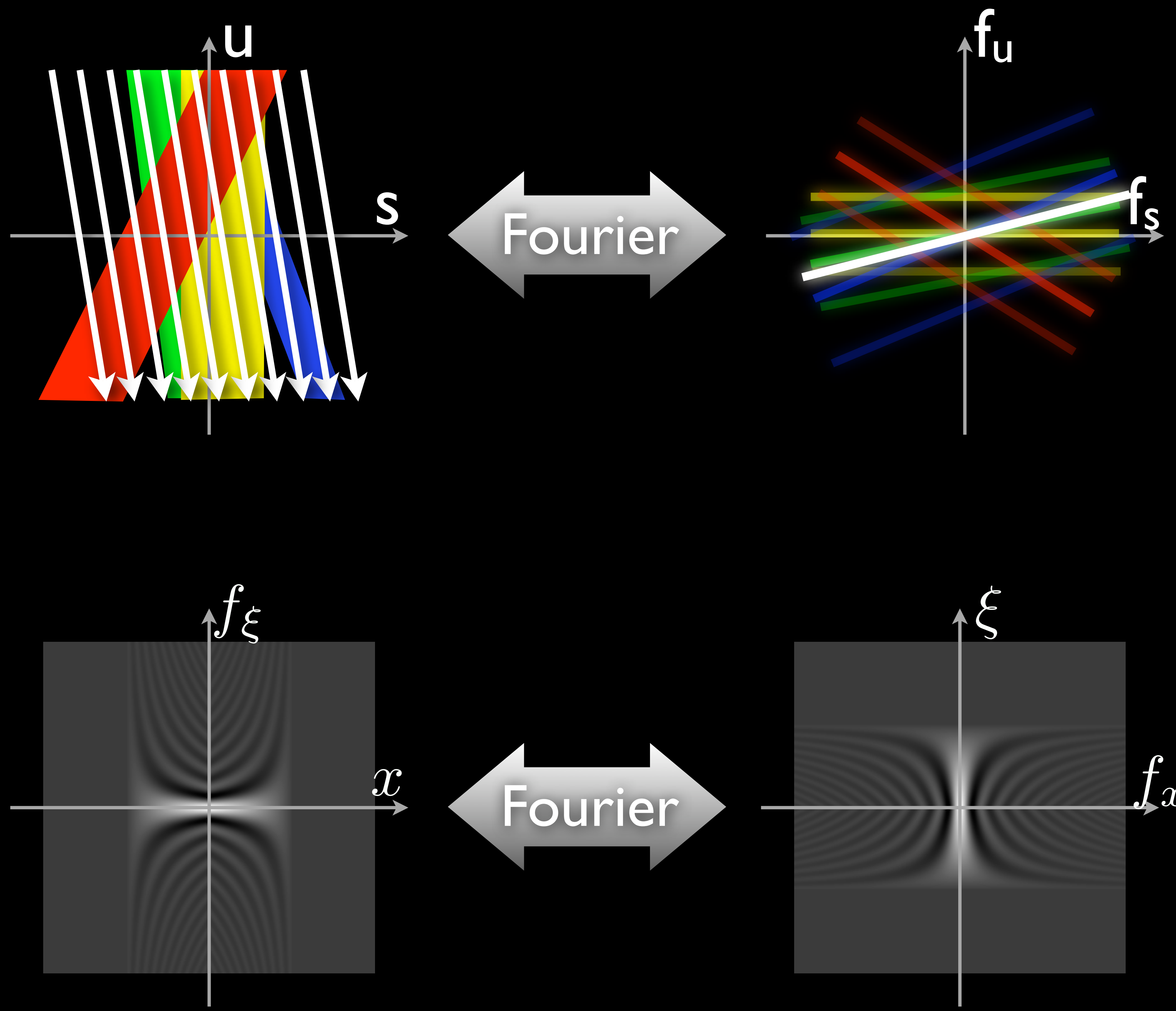
Application - Refocusing

Isaksen
et. al
2000

light
field

Ng
2005

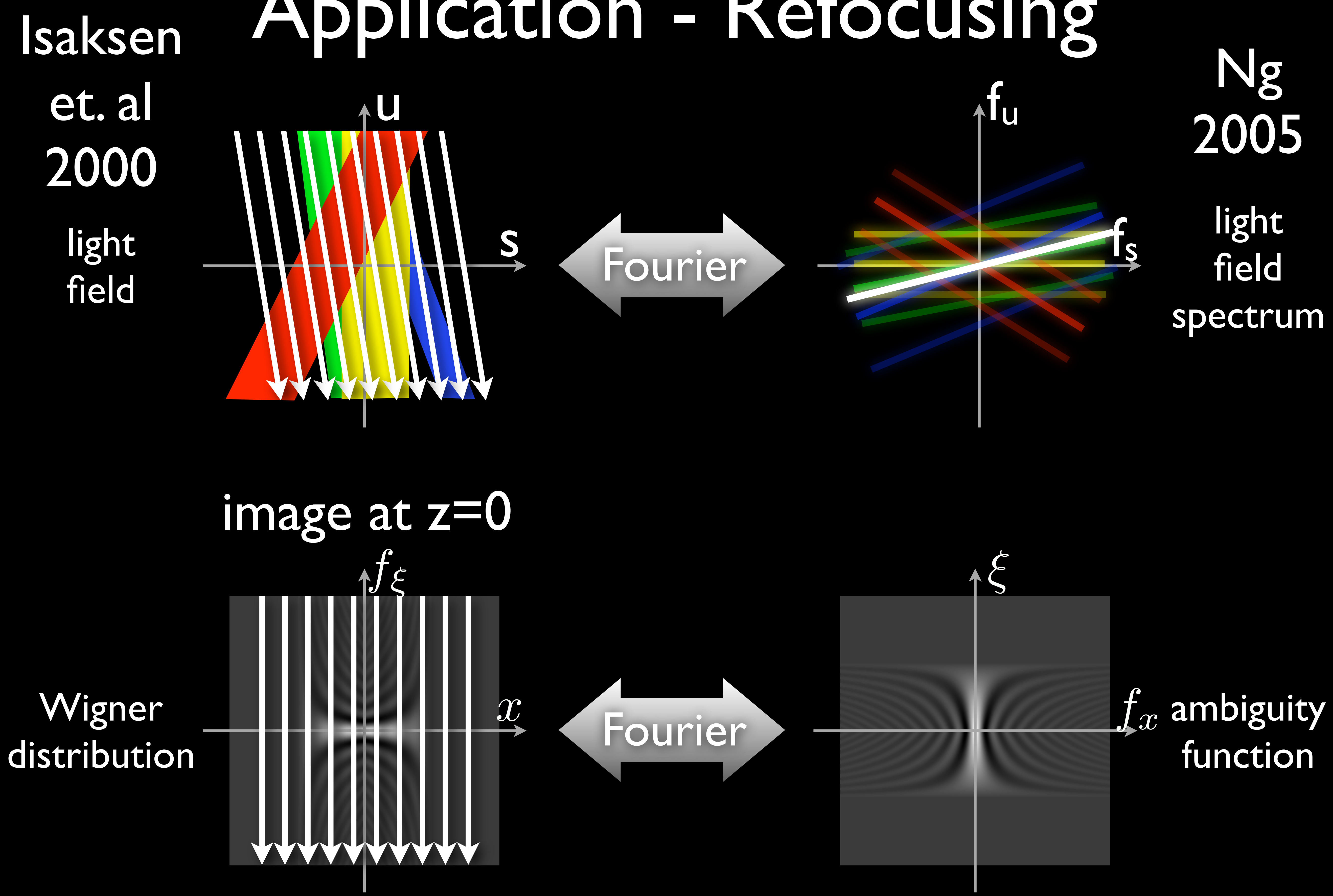
light
field
spectrum



Wigner
distribution

ambiguity
function

Application - Refocusing



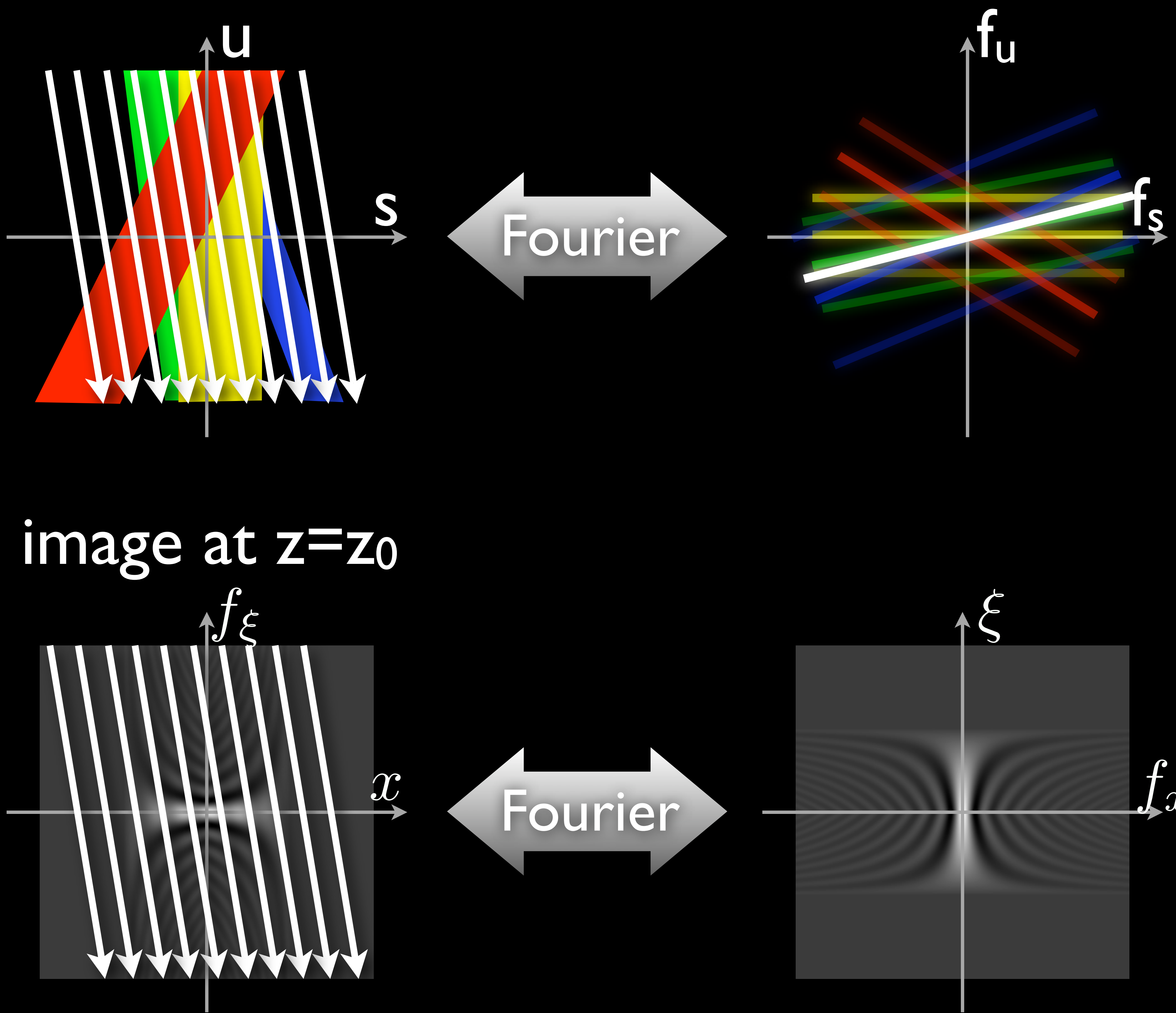
Application - Refocusing

Isaksen
et. al
2000

light
field

Ng
2005

light
field
spectrum



Wigner
distribution

ambiguity
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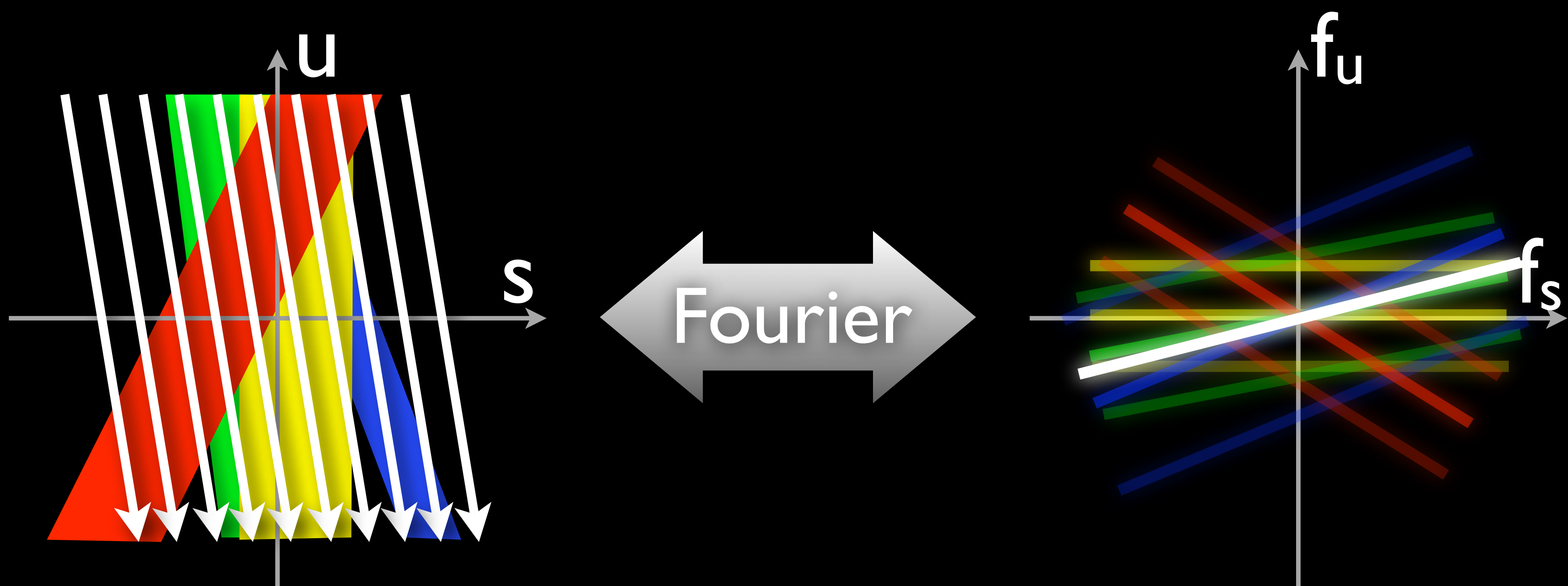
Application - Refocusing

Isaksen et. al 2000

light field

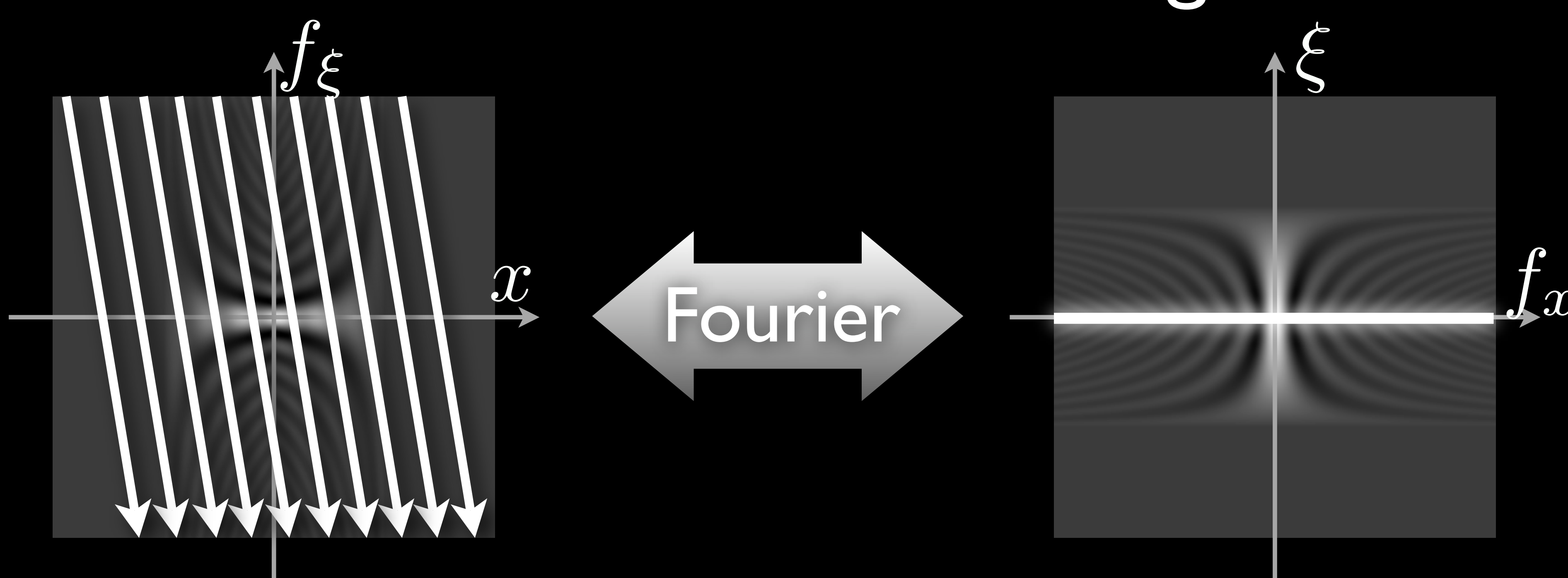


Ng 2005

light field spectrum

image at z=0

Wigner distribution



ambiguity function

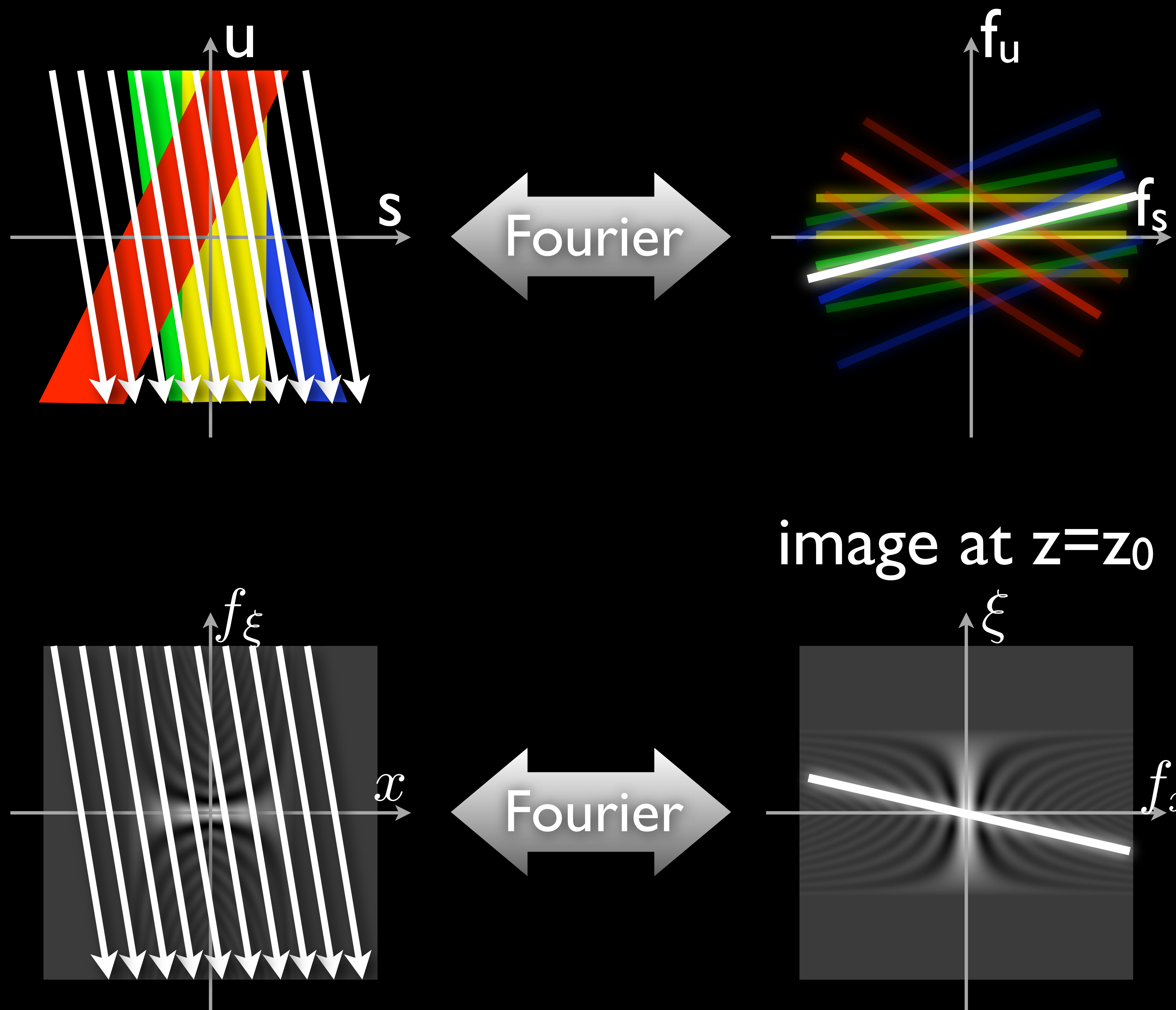
Application - Refocusing

Isaksen et. al 2000

light field

Ng 2005

light field spectrum



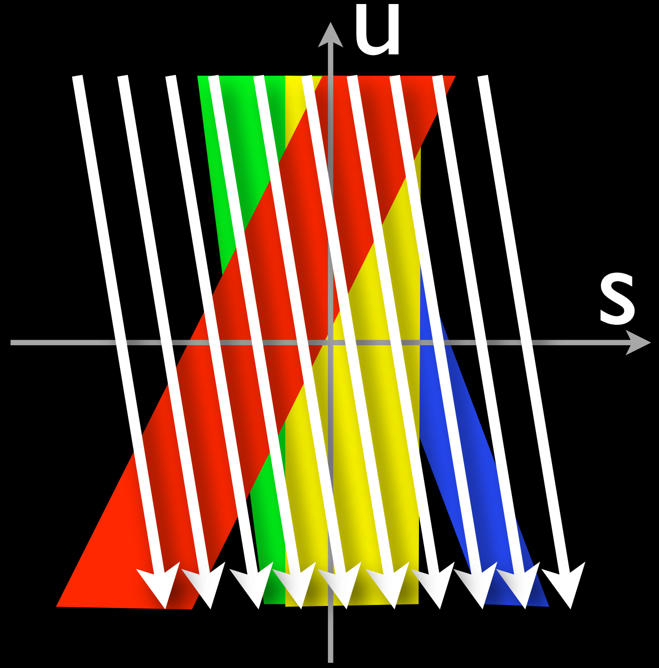
Wigner distribution

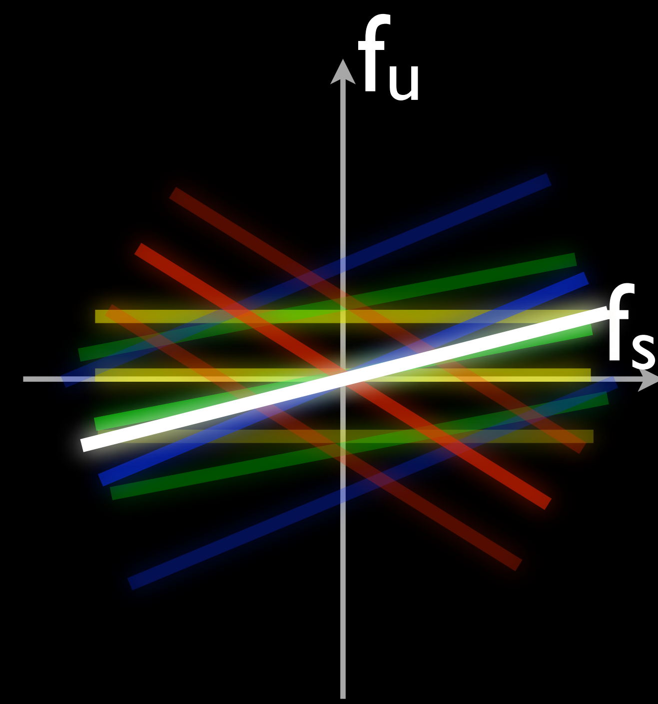
ambiguity function

Application - Refocusing

Isaksen
et. al
2000

light
field

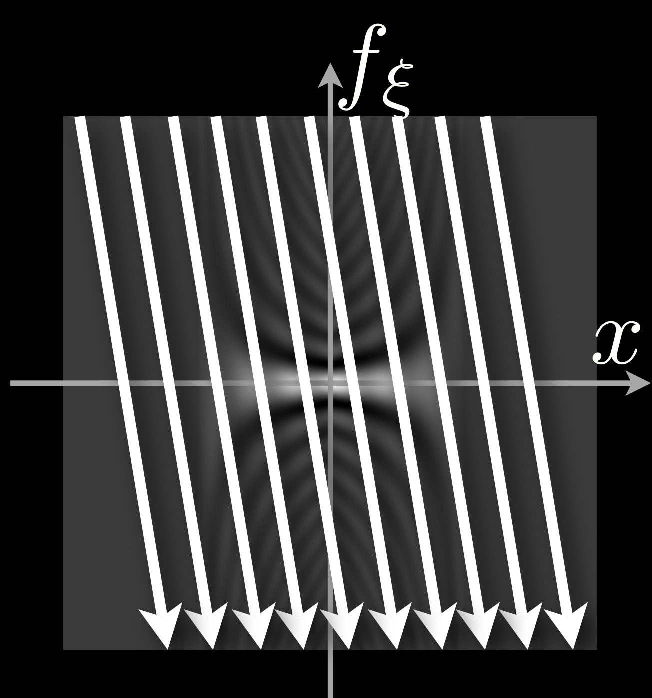


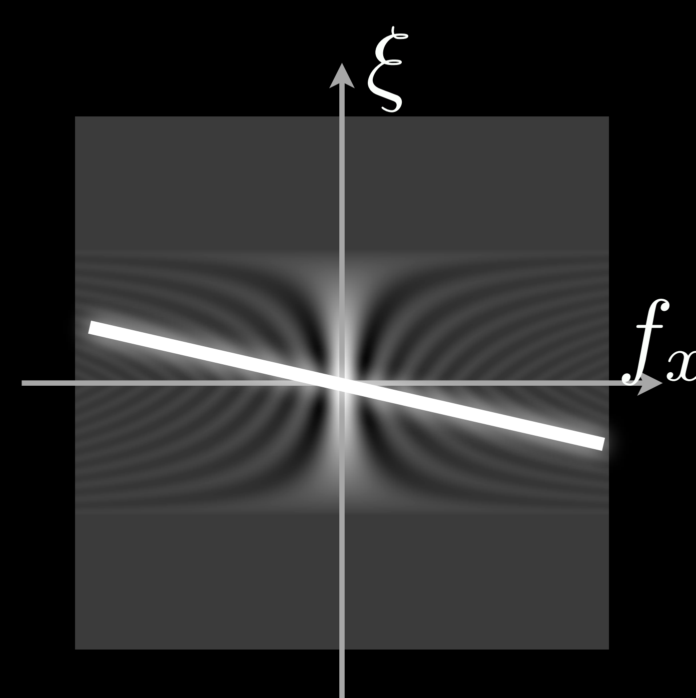


Ng
2005

light
field
spectrum

Wigner
distribution



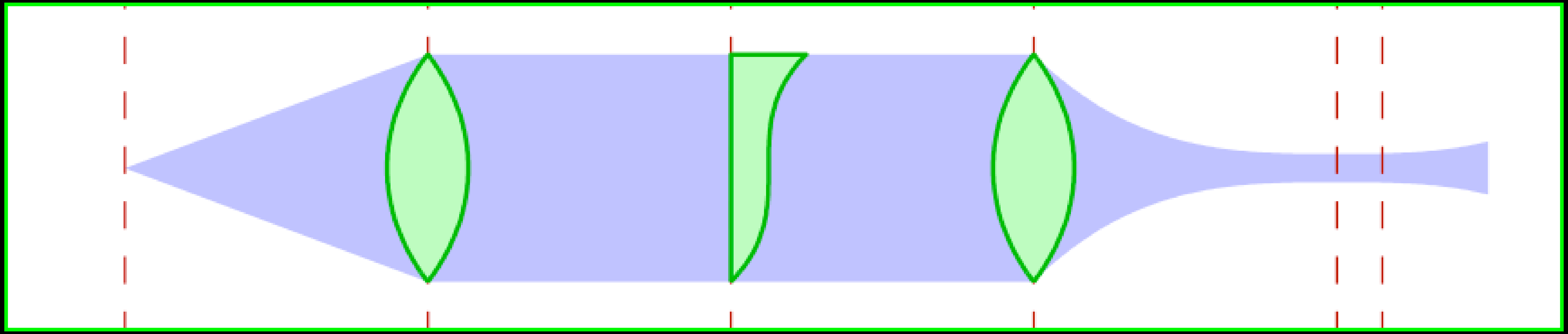


Papoulis
1974

ambiguity
function

Application - Wavefront Coding

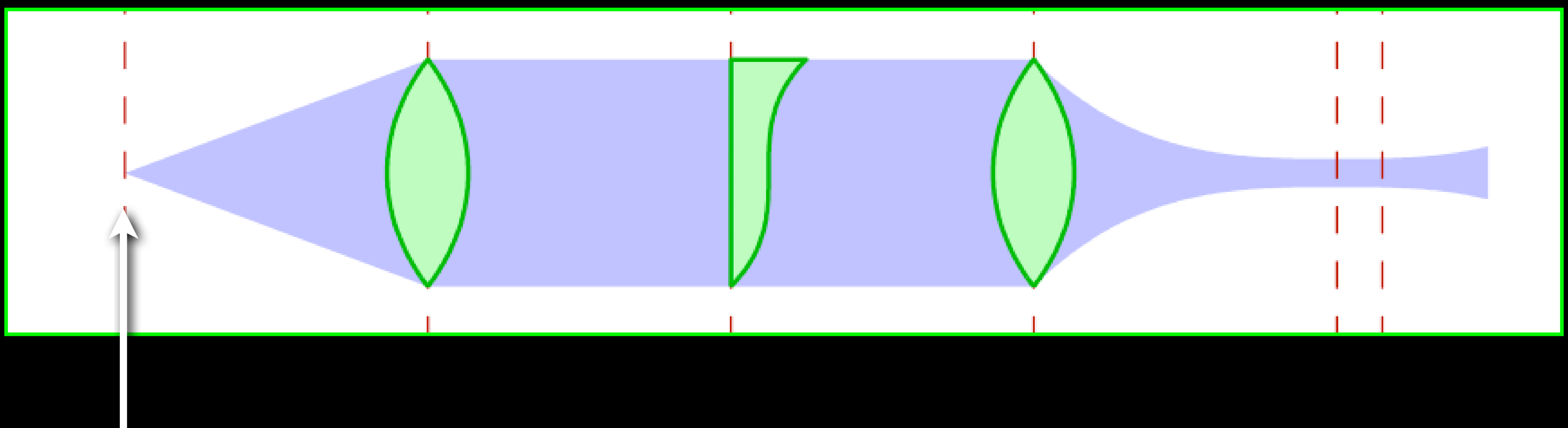
Dowski and Cathey 1995



same aberrant blur regardless of depth of focus

Application - Wavefront Coding

Dowski and Cathey 1995

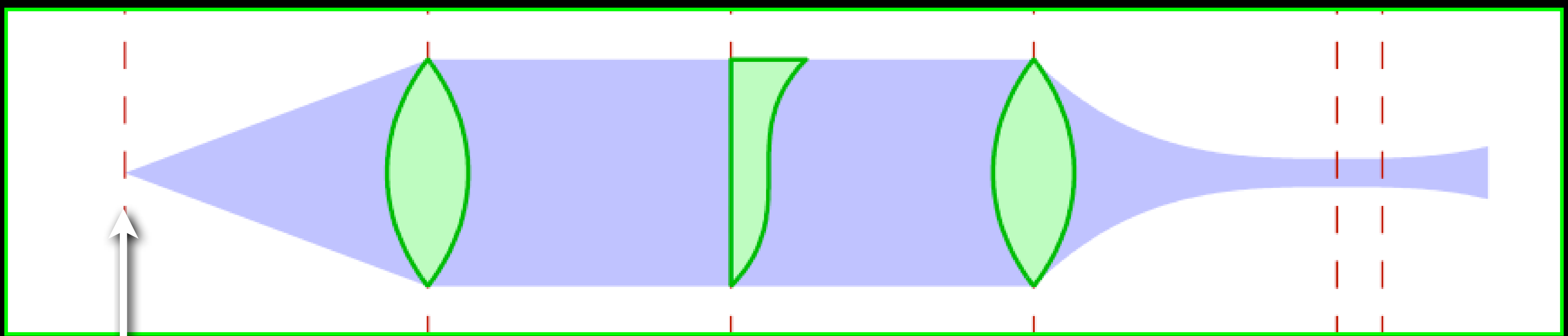


point
in scene

same aberrant blur regardless of depth of focus

Application - Wavefront Coding

Dowski and Cathey 1995



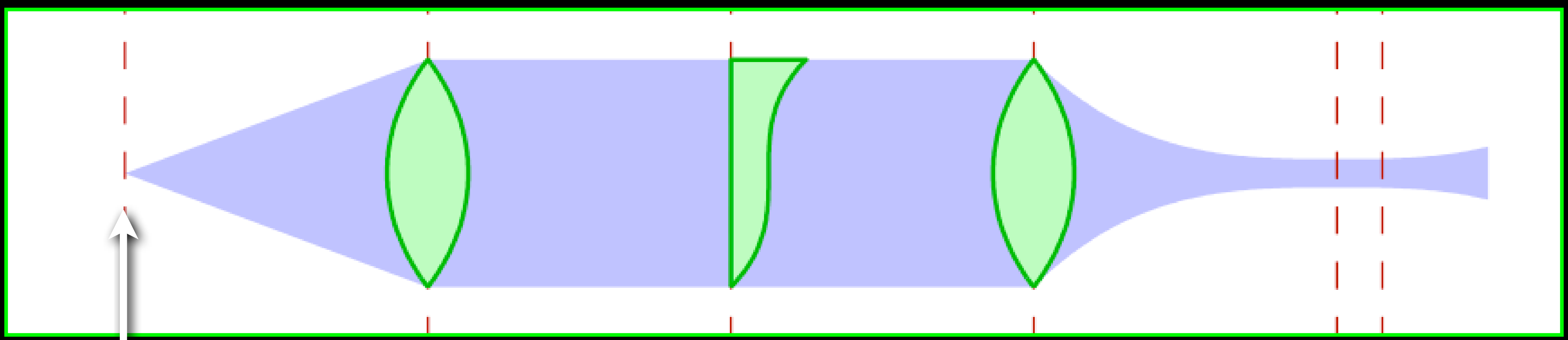
point
in scene

cubic
phase plate

same aberrant blur regardless of depth of focus

Application - Wavefront Coding

Dowski and Cathey 1995



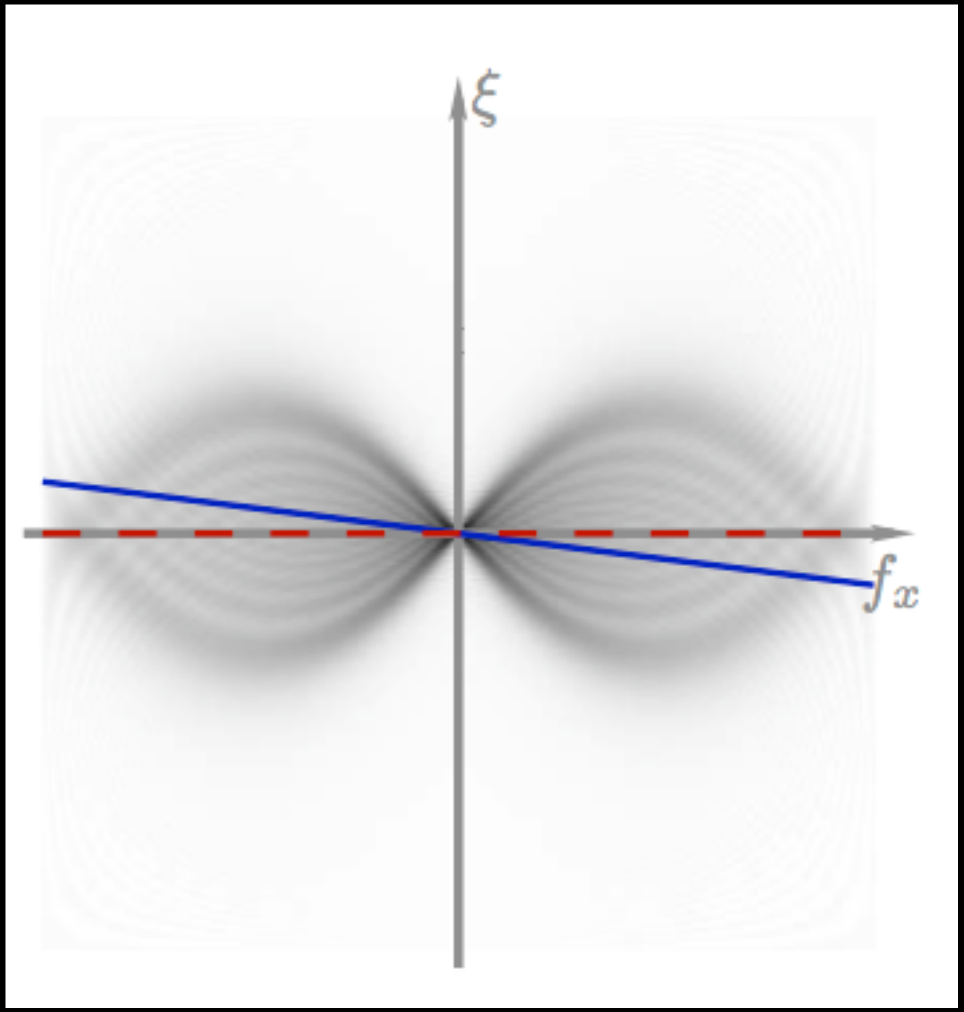
point in scene

cubic phase plate

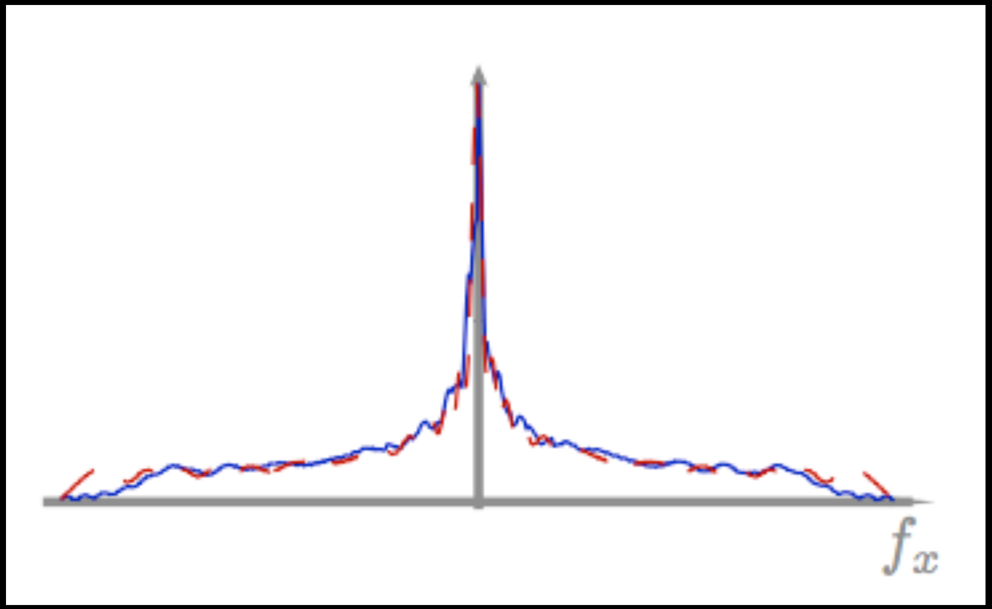
small change in blur shape

same aberrant blur regardless of depth of focus

Application - Wavefront Coding

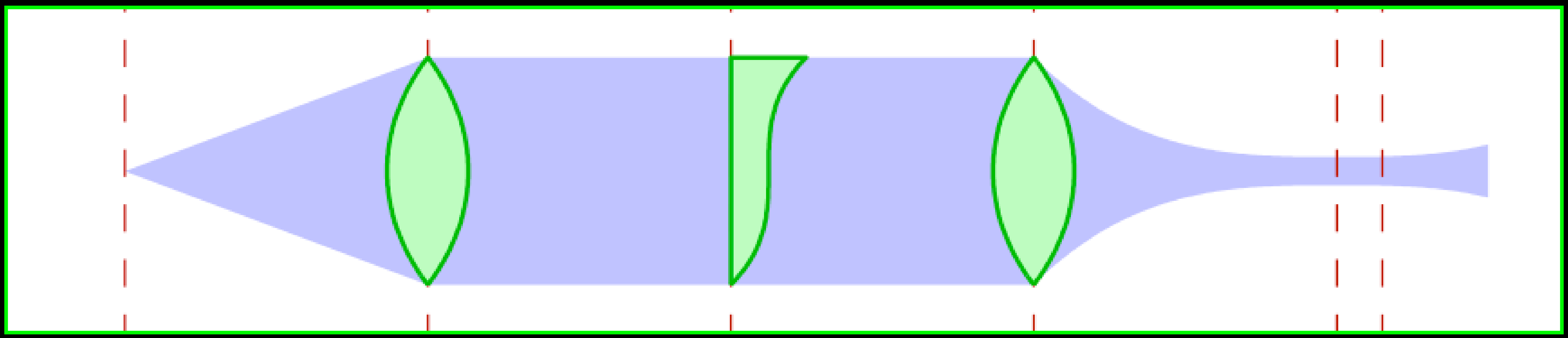


ambiguity
function

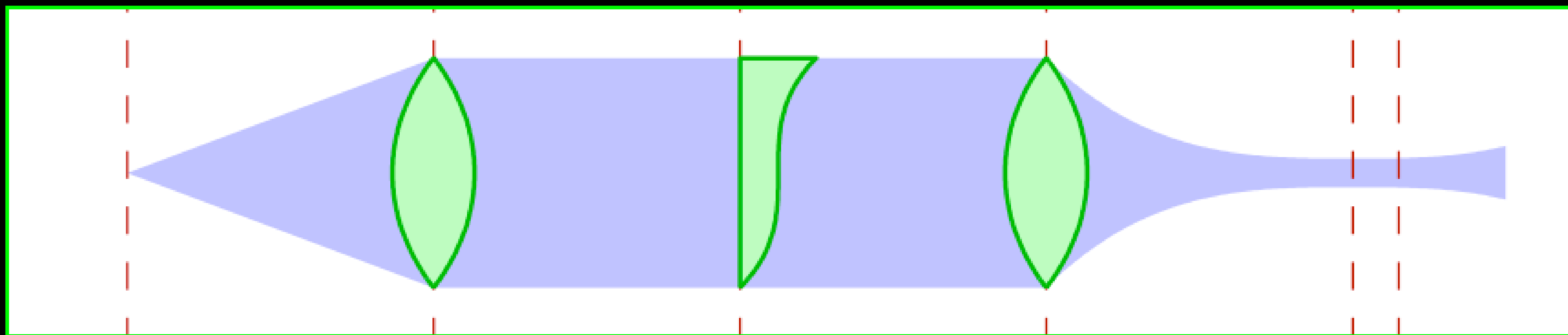


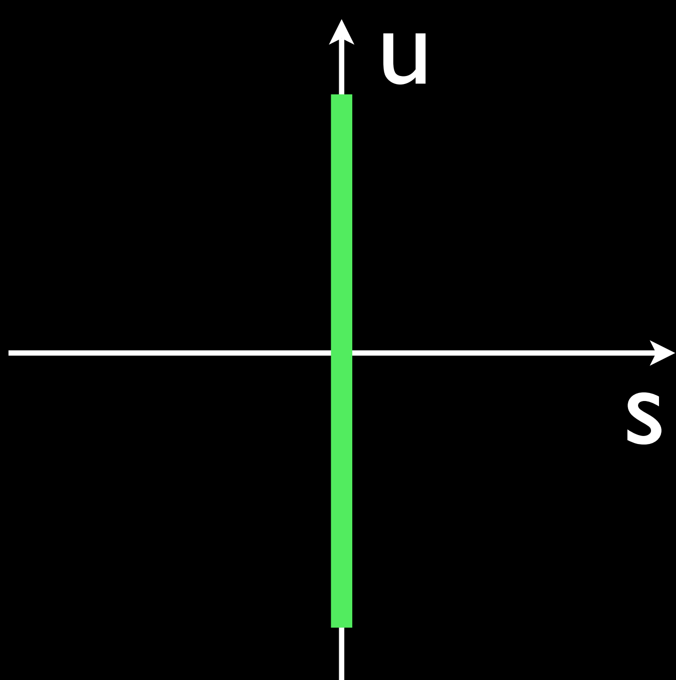
slices corresponding
to various depths

Application - Wavefront Coding



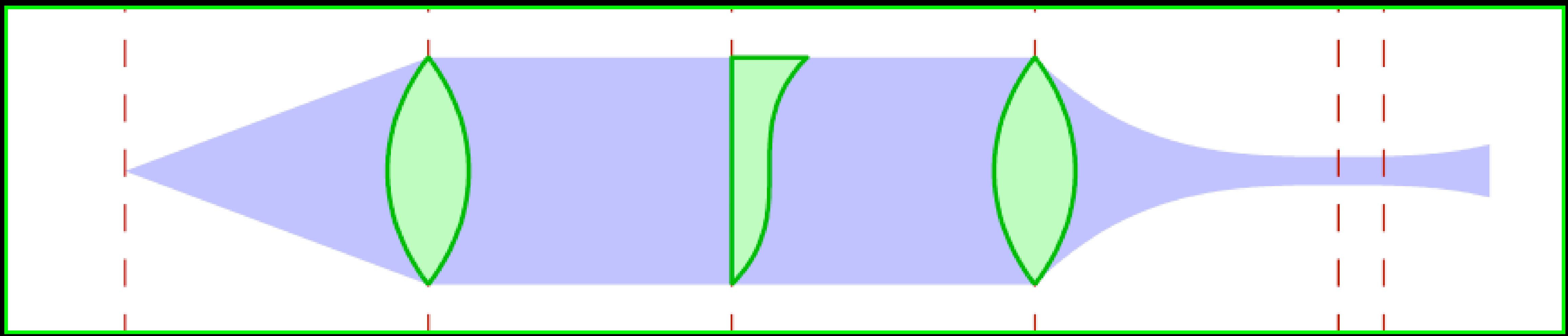
Application - Wavefront Coding

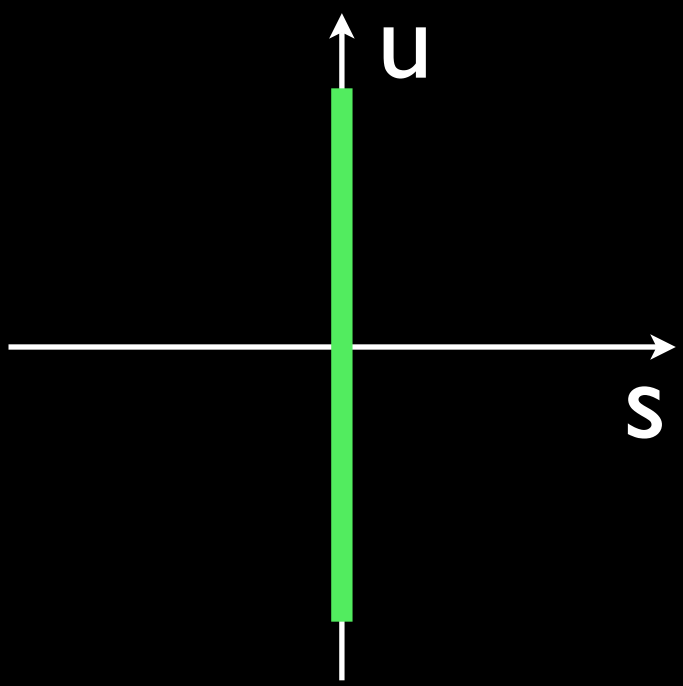




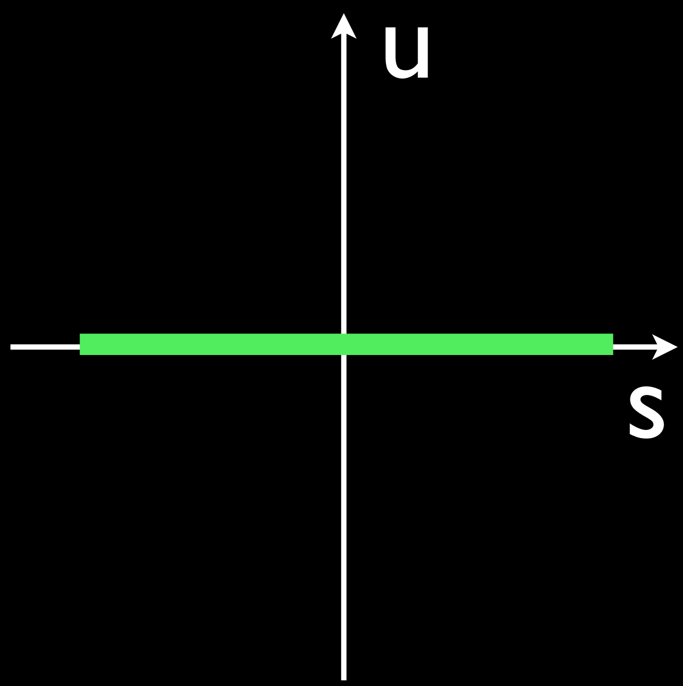
point

Application - Wavefront Coding



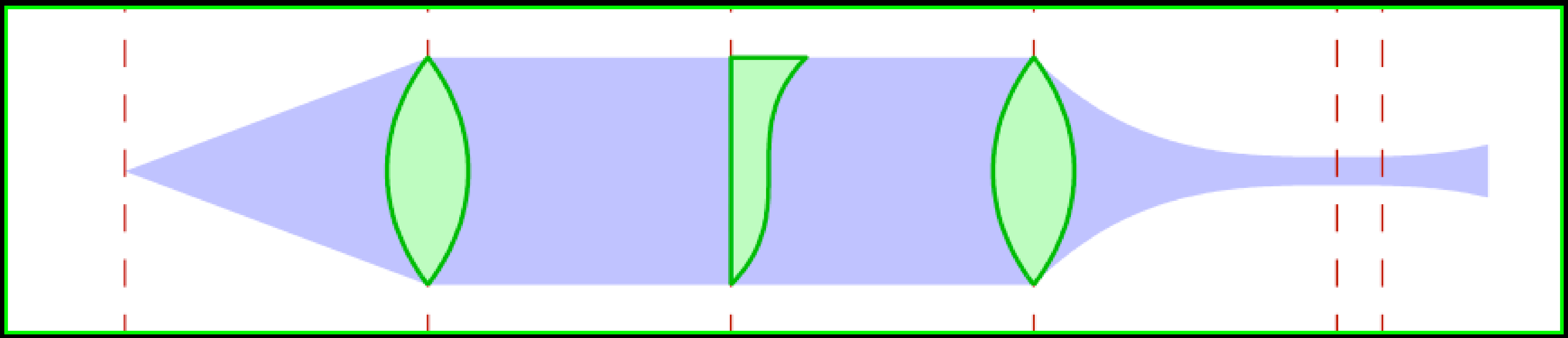


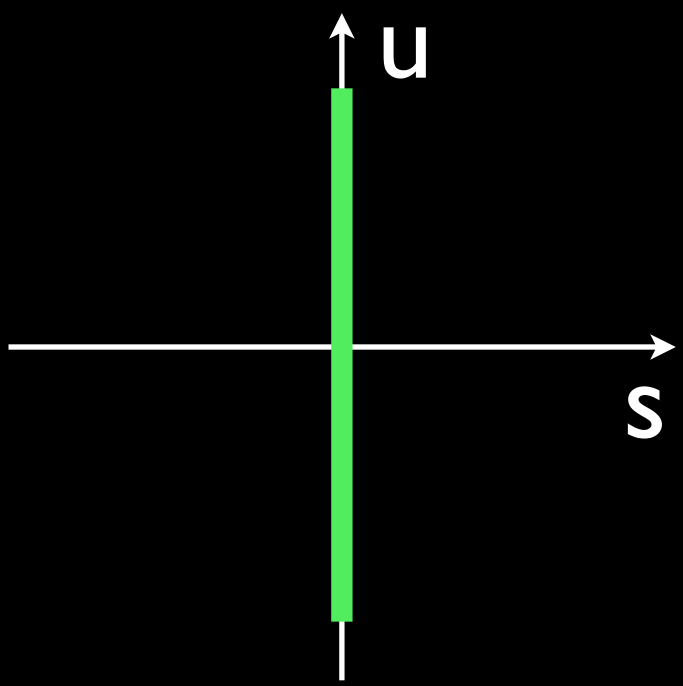
point



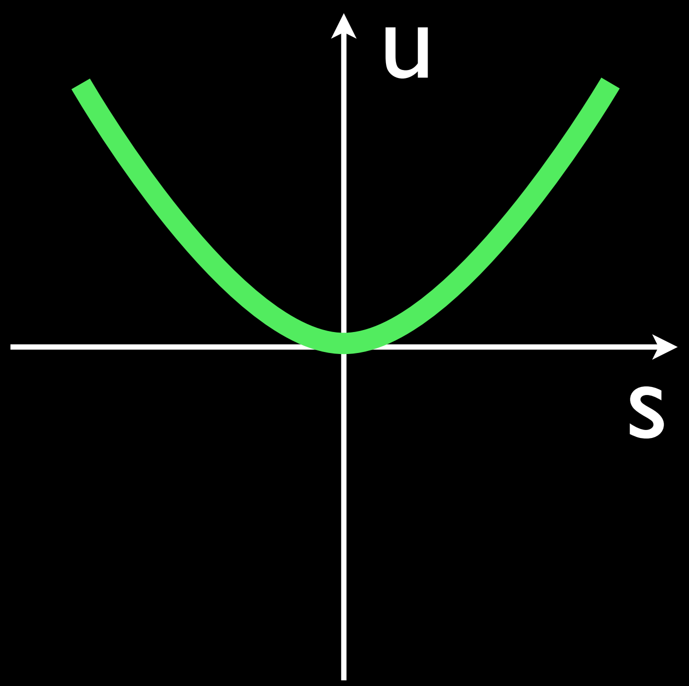
before phase plate

Application - Wavefront Coding



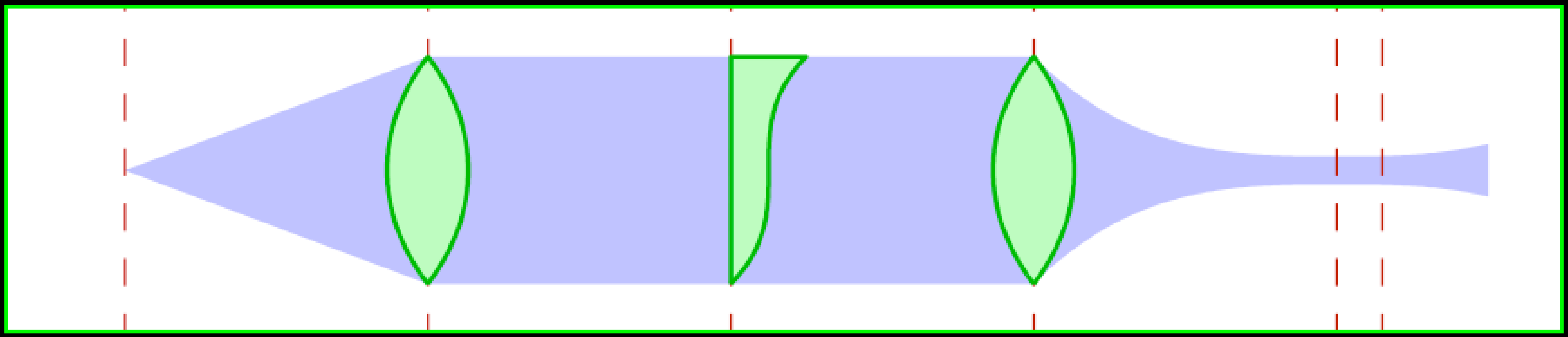


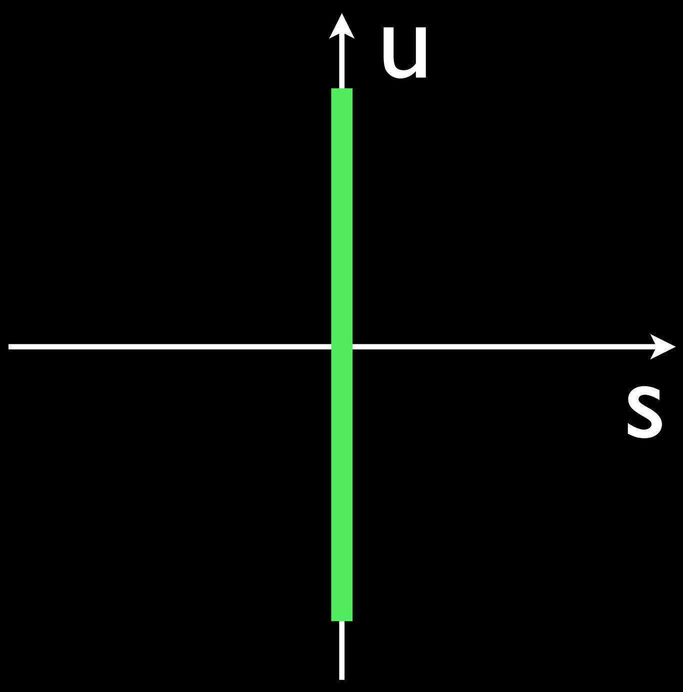
point



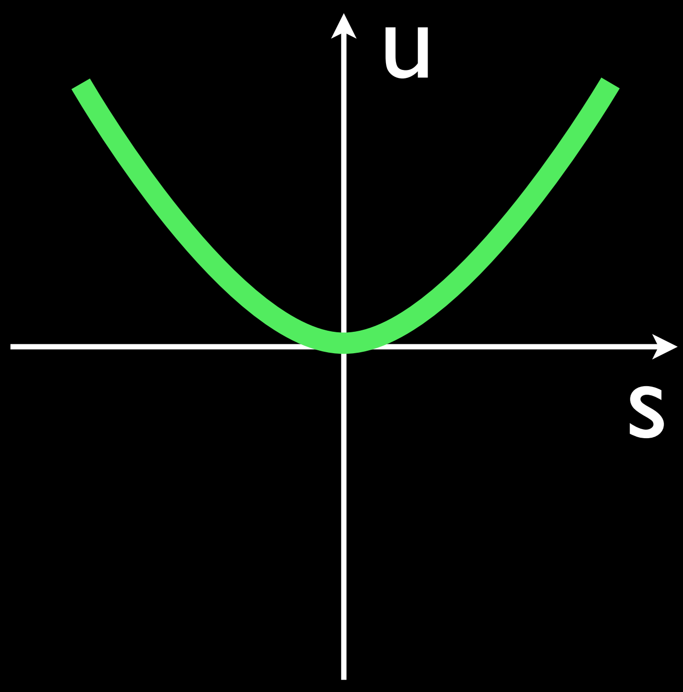
after phase plate

Application - Wavefront Coding

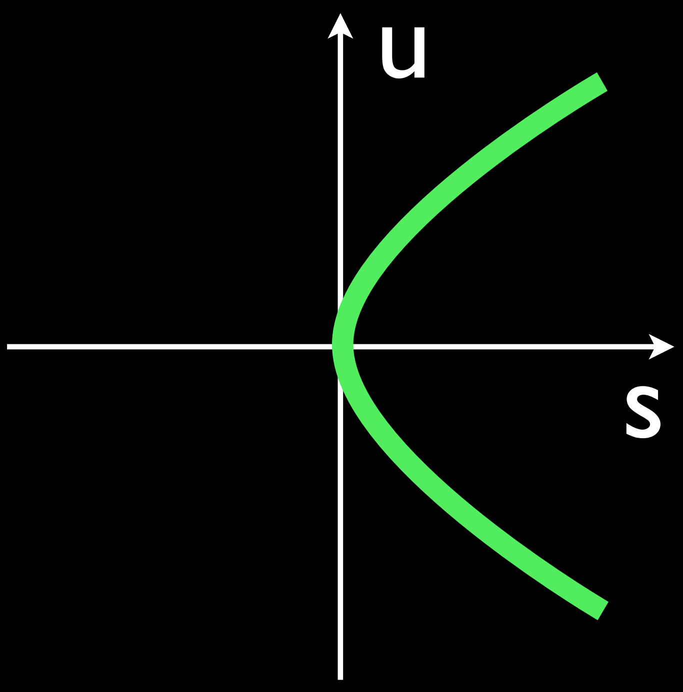




point

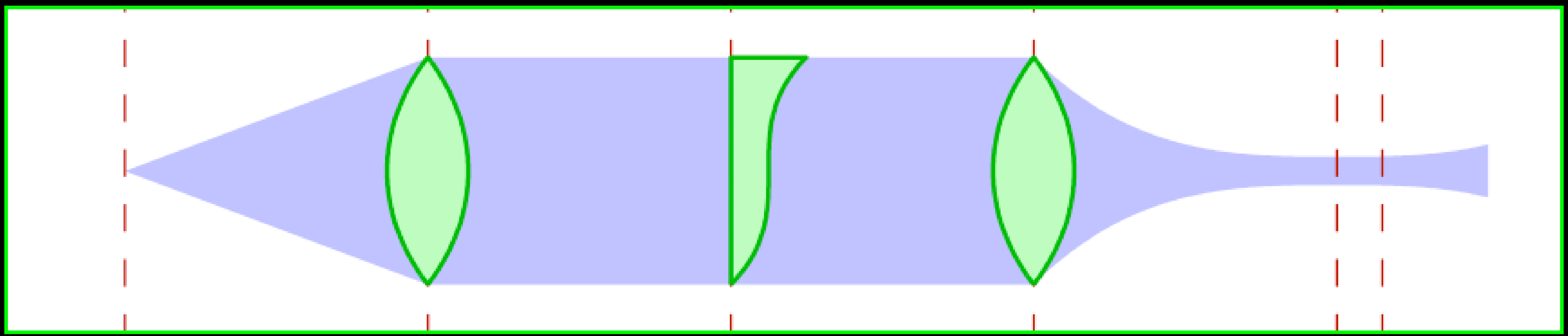


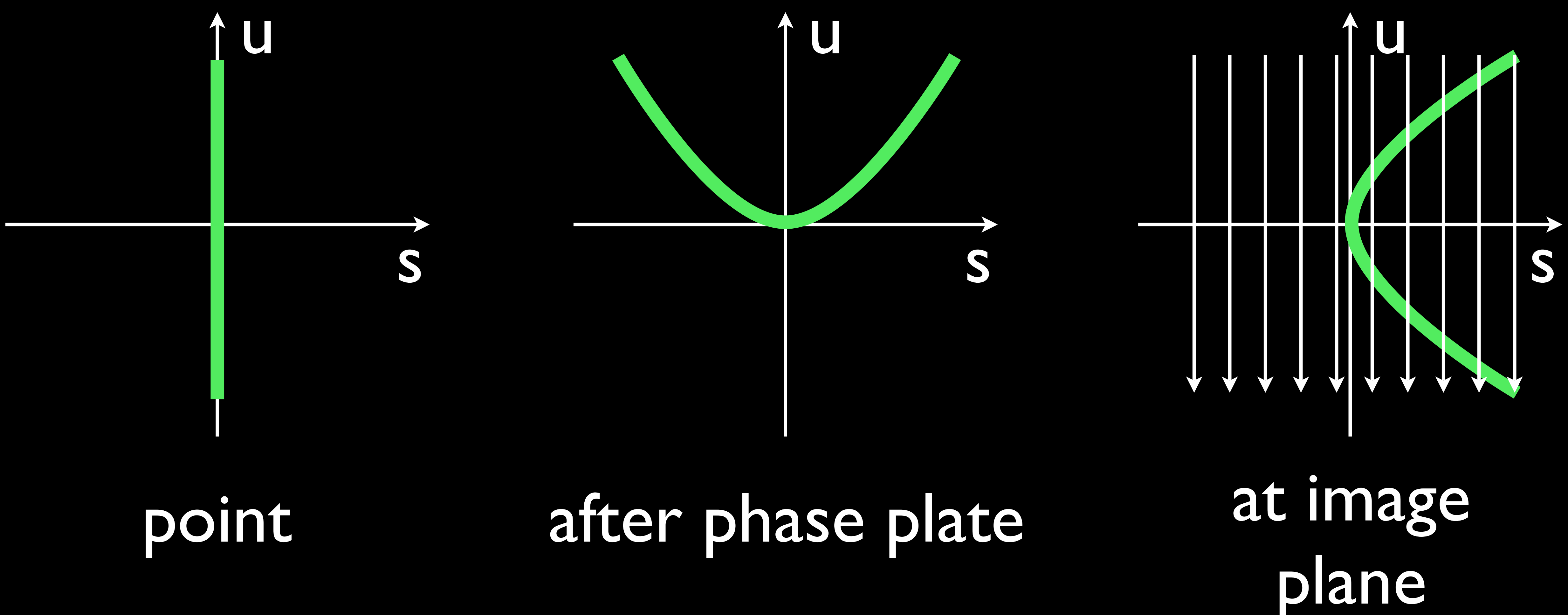
after phase plate



at image plane

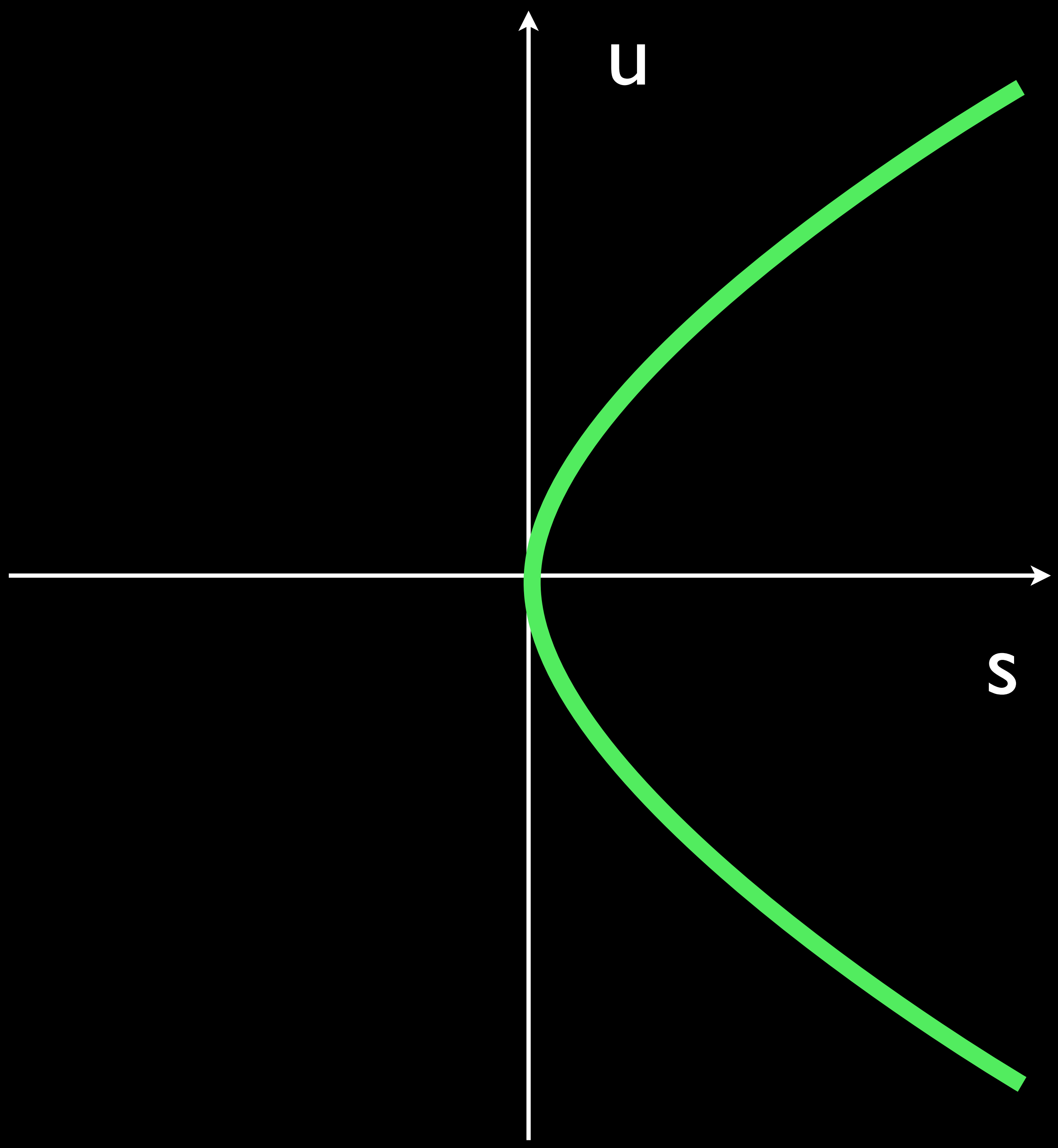
Application - Wavefront Coding





Application - Wavefront Coding

- refocusing in ray space is shearing
- shearing of a parabola results in translation
- blur shape invariant to refocusing



Application - Wavefront Coding

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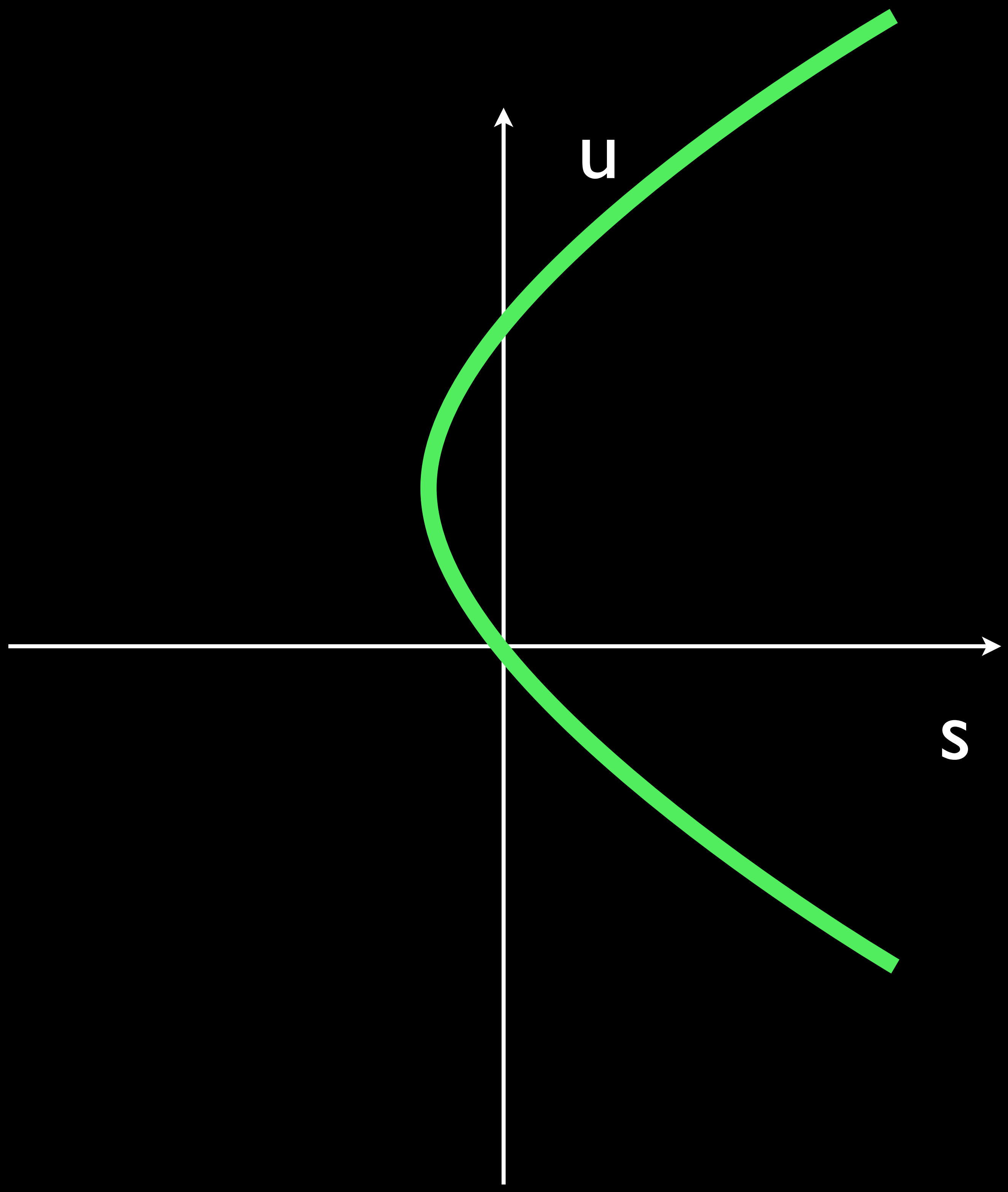
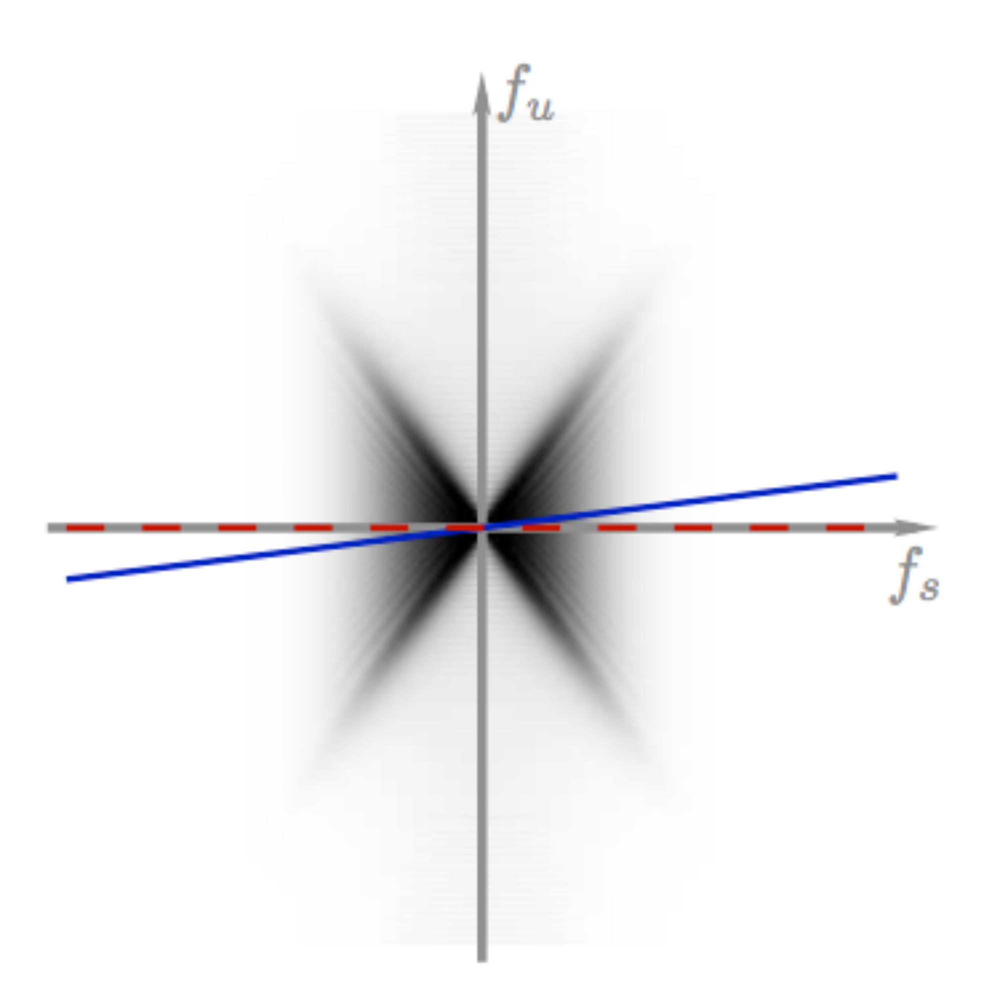
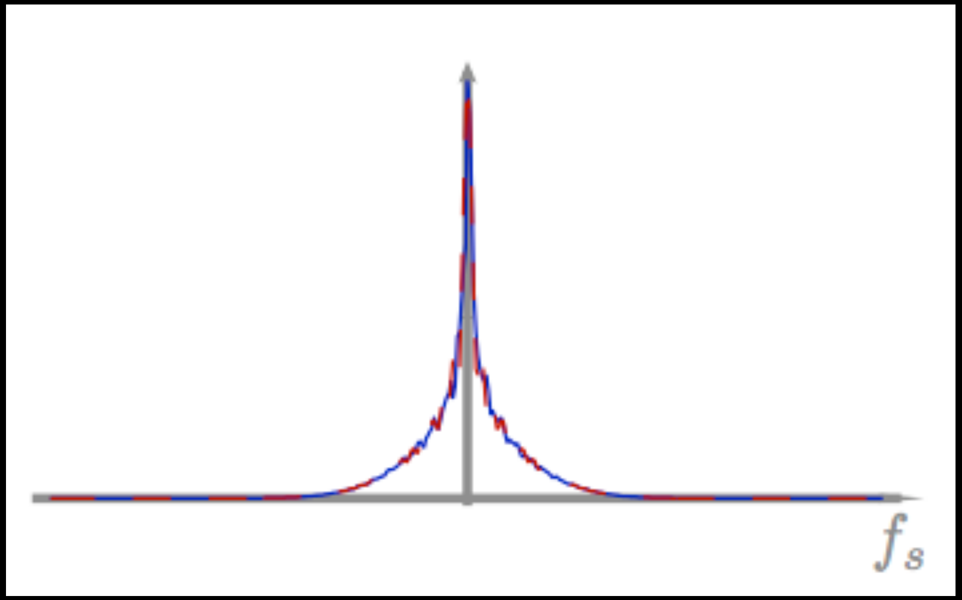


Application - Wavefront Coding

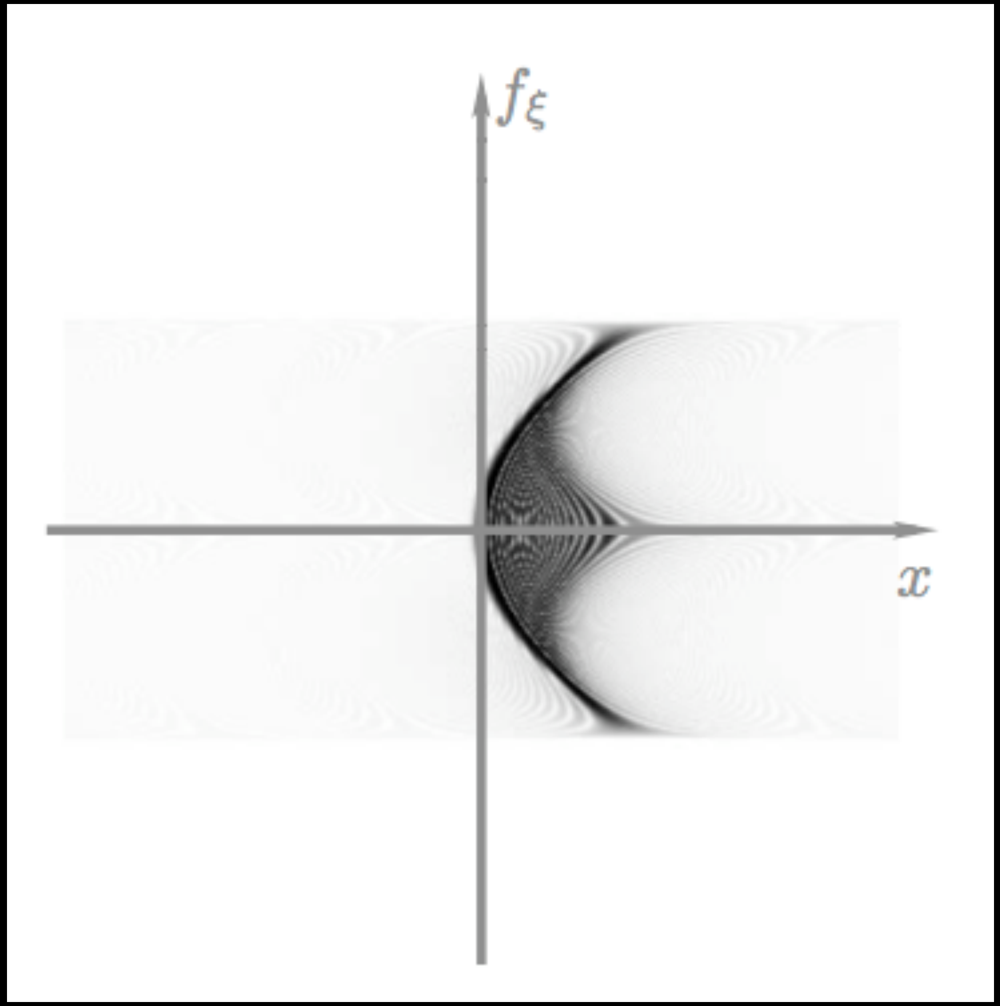


Fourier transform
of light field



slices corresponding
to various depths

Application - Wavefront Coding



Wigner distribution
for cubic phase plate system

Conclusions

- light field’s position and direction = wave optics’s position and frequency
- observable light field = blurred Wigner distribution (equal at zero wavelength limit)
- analysis using light fields and Wigner distribution interchangeable

Future Work

- analyze various light field capture and generation systems using wave optics
- rendering wave optics phenomena
- **adapt more ideas from optics community and vice versa!**

Acknowledgements

- Anat Levin, Fredo Durand and Bill Freeman
- Stanford Graduate Fellowship from Texas Instruments and NSF Grant CCF-0540872