# Point-Based Computer Graphics

Eurographics 2003 Tutorial T1

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## Tutorial Schedule

Introduction (Markus Gross)
Acquisition of Point-Sampled Geometry and Appearance (Jeroen van Baar)
Point-Based Surface Representations (Marc Alexa)
Point-Based Rendering (Matthias Zwicker)

*Lunch*

Sequential Point Trees (Carsten Dachsbacher)
Efficient Simplification of Point-Sampled Geometry (Mark Pauly)
Spectral Processing of Point-Sampled Geometry (Markus Gross)
Pointshop3D: A Framework for Interactive Editing of Point-Sampled Surfaces (Markus Gross)
Shape Modeling (Mark Pauly)
Pointshop3D Demo (Mark Pauly)
Discussion (all)

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M. Zwicker, H. Pfister, J. van Baar, M. Gross, EWA splatting. IEEE Transactions on Visualization and Computer Graphics.

M. Zwicker, M. Pauly, O. Knoll, M. Gross, Pointshop 3D: an interactive system for point-based surface editing. Proceedings of SIGGRAPH 2002, San Antonio, TX, July 2002

**Project Pages**

- Rendering
  http://graphics.ethz.ch/surfels
- Acquisition
  http://www.merl.com/projects/3Dimages/

- Sequential point trees
  http://www9.informatik.uni-erlangen.de/Persons/Stamminger/Research
- Modeling, processing, sampling and filtering
  http://graphics.ethz.ch/points
- Pointshop3D
  http://www.pointshop3d.com

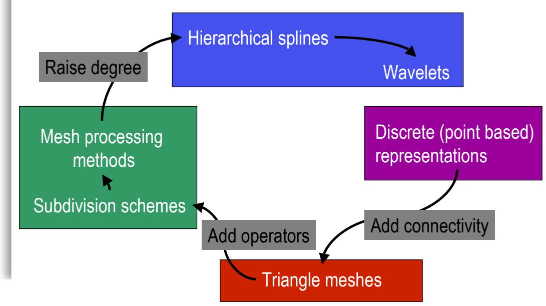
## Eurographics 2003

# Point-Based Computer Graphics

Tutorial T1

Marc Alexa, Carsten Dachsbacher,
Markus Gross, Mark Pauly,
Hanspeter Pfister, Marc Stamminger,
Jeroen Van Baar, Matthias Zwicker

## Surf. Reps. for Graphics






## Polynomials...

- ✓ Rigorous mathematical concept
- ✓ Robust evaluation of geometric entities
- ✓ Shape control for smooth shapes
- ✓ Advanced physically-based modeling

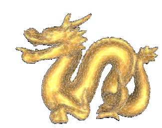
- × Require parameterization
- × Discontinuity modeling
- × Topological flexibility

***Refine h rather than p !***



## Polynomials -> Triangles

- Piecewise linear approximations
- Irregular sampling of the surface
- Forget about parameterization



Triangle meshes

- Multiresolution modeling
- Compression
- Geometric signal processing



## Triangles...

- ✓ Simple and efficient representation
- ✓ Hardware pipelines support Δ
- ✓ Advanced geometric processing is being in sight
- ✓ The widely accepted queen of graphics primitives

- × Sophisticated modeling is difficult
- × (Local) parameterizations still needed
- × Complex LOD management
- × Compression and streaming is highly non-trivial

***Remove connectivity !***



## Triangles -> Points

- From piecewise linear functions to Delta distributions
- Forget about connectivity

Point clouds

- Points are natural representations within 3D acquisition systems
- Meshes provide an articifical enhancement of the acquired point samples

## History of Points in Graphics

- Particle systems [Reeves 1983]
- Points as a display primitive [Whitted, Levoy 1985]
- Oriented particles [Szeliski, Tonnesen 1992]
- Particles and implicit surfaces [Witkin, Heckbert 1994]
- Digital Michelangelo [Levoy et al. 2000]
- Image based visual hulls [Matusik 2000]
- Surfels [Pfister et al. 2000]
- QSplat [Rusinkiewicz, Levoy 2000]
- Point set surfaces [Alexa et al. 2001]
- Radial basis functions [Carr et al. 2001]
- Surface splatting [Zwicker et al. 2001]
- Randomized z-buffer [Wand et al. 2001]
- Sampling [Stamminger, Drettakis 2001]
- Opacity hulls [Matusik et al. 2002]
- Pointshop3D [Zwicker, Pauly, Knoll, Gross 2002]...?



## The Purpose of our Course is …

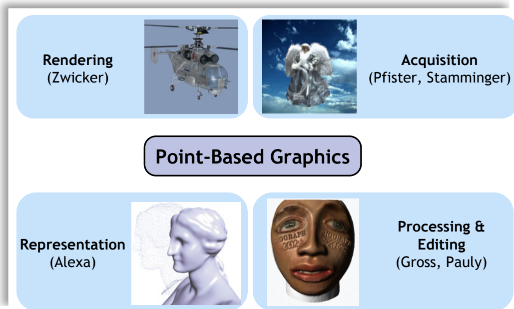
*I) …to introduce points as a versatile and powerful graphics primitive*

*II) …to present state of the art concepts for acquisition, representation, processing and rendering of point sampled geometry*

*III) …to stimulate YOU to help us to further develop Point Based Graphics*



## Taxonomy






## Morning Schedule

- Introduction (Markus Gross)
- Acquisition of Point-Sampled Geometry and Apprearance (Jeroen van Baar)
- Point-Based Surface Representations (Marc Alexa)
- Point-Based Rendering (Matthias Zwicker)



## Afternoon Schedule

- Sequential point trees (Carsten Dachsbacher)
- Efficient simplification of point-sampled geometry (Mark Pauly)
- Spectral processing of point-sampled geometry (Markus Gross)
- Pointshop3D: A framework for interactive editing of point-sampled surfaces (Markus Gross)
- Shape modeling (Mark Pauly)
- Pointshop3D demo (Mark Pauly)
- Discussion (all)

# Eurographics 2003

## Acquisition of Point-Sampled Geometry and Appearance


Jeroen van Baar and Hanspeter Pfister, MERL
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Wojciech Matusik, MIT
Addy Ngan, MIT
Paul Beardsley, MERL
Remo Ziegler, MERL
Leonard McMillan, MIT




## The Goal: To Capture Reality

- Fully-automated 3D model creation of real objects.
- Faithful representation of appearance for these objects.



## Image-Based 3D Photography

- An image-based 3D scanning system.
  - Handles fuzzy, refractive, transparent objects.
  - Robust, automatic
  - Point-sampled geometry based on the *visual hull*.
  - Objects can be rendered in novel environments.



## Previous Work

- Active and passive 3D scanners
  - Work best for diffuse materials.
  - Fuzzy, transparent, and refractive objects are difficult.
- BRDF estimation, inverse rendering
- Image based modeling and rendering
  - Reflectance fields [Debevec et al. 00]
    - Light Stage system to capture reflectance fields
    - Fixed viewpoint, no geometry
  - Environment matting [Zongker et al. 99, Chuang et al. 00]
    - Capture reflections and refractions
    - Fixed viewpoint, no geometry

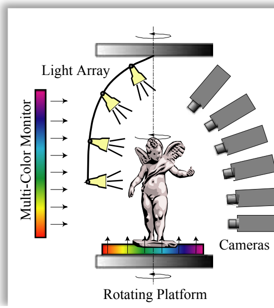


## Outline

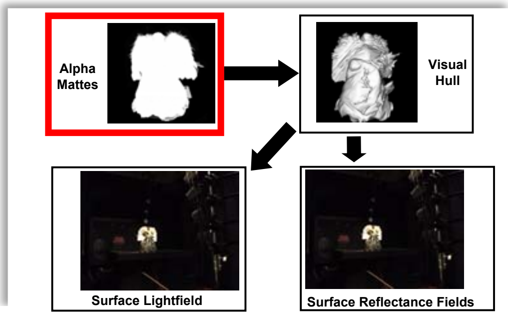
- Overview
- ➢ System
- Geometry
- Reflectance
- Refraction & Transparency



## Acquisition System

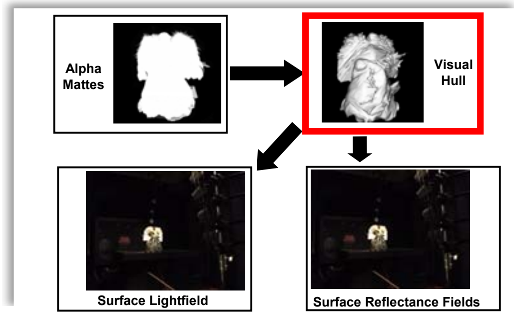



## Acquisition Process



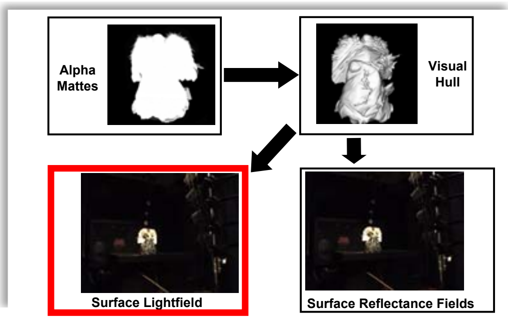



## Acquisition Process



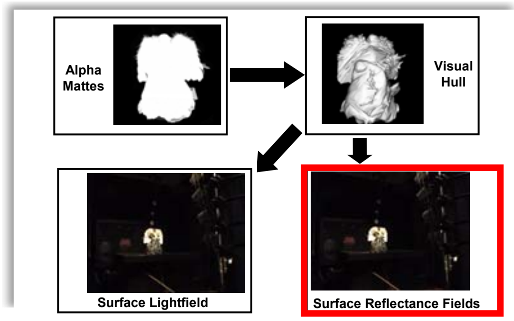



## Acquisition Process






## Acquisition Process






## Outline

- Overview
- System
- ➢ Geometry
- Reflectance
- Refraction & Transparency

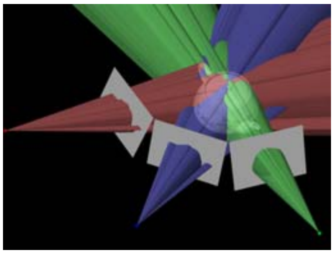


## Acquisition

- For each viewpoint ( 6 cameras x 72 positions )
  - Alpha mattes
    - Use multiple backgrounds [Smith and Blinn 96]
  - Reflectance images
    - Pictures of the object under different lighting (4 lights x 11 positions)
  - Environment mattes
    - Use similar techniques as [Chuang et al. 2000]

## Geometry – Opacity Hull

- Visual hull: The maximal object consistent with a given set of silhouettes.





## Geometry Example



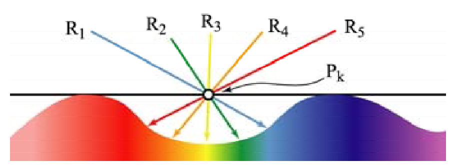
## Approximate Geometry

- The approximate visual hull is augmented by radiance data to render concavities, reflections, and transparency.



## Surface Light Fields

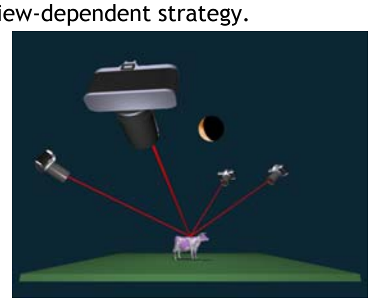
- A surface light field is a function that assigns a color to each ray originating on a surface. [Wood *et al.*, 2000]






## Shading Algorithm

- A view-dependent strategy.





## Color Blending

- Blend colors based on angle between virtual camera and stored colors.
- Unstructured Lumigraph Rendering
  [Buehler et al., SIGGRAPH 2001]
- View-Dependent Texture Mapping
  [Debevec, EGRW 98]

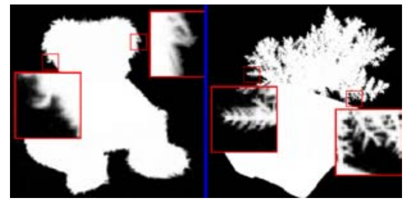
## Point-Based Rendering

- Point-based rendering using LDC tree, visibility splatting, and view-dependent shading.



## Geometry – Opacity Hull

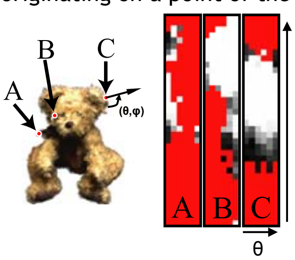
- Store the opacity of each observation at each point on the visual hull [Matusik et al. SIG2002].





## Geometry – Opacity Hull

- Assign view-dependent opacity to each ray originating on a point of the visual hull.




Red = invisible
White = opaque
Black = transparent



## Example

Photo



## Example

Photo

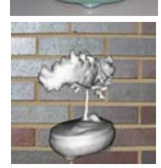
Visual Hull



## Example

Photo

Visual Hull



Opacity Hull

## Example

Photo

Surface Light Field

Visual Hull

Opacity Hull



## Results

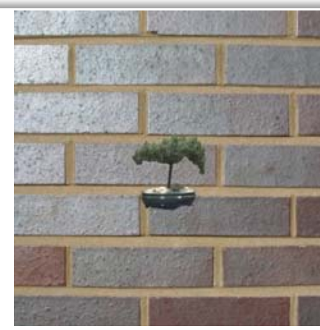
- Point-based rendering using EWA splatting, A-buffer blending, and edge antialiasing.



## Results Video



## Results Video

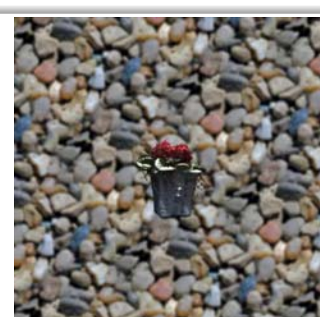




## Results Video



## Results Video



## Opacity Hull - Discussion

- View dependent opacity vs. geometry trade-off.
- Sometimes acquiring the geometry is not possible.
- Sometimes representing true geometry would be very inefficient.
- Opacity hull stores the "macro" effect.



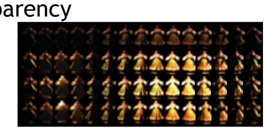
## Point-Based Models

- No need to establish topology or connectivity.
- No need for a consistent surface parameterization for texture mapping.
- Represent organic models (feather, tree) much more readily than polygon models.
- Easy to represent view-dependent opacity and radiance per surface point.



## Outline

- Overview
- Previous Works
- Geometry
- ➢ Reflectance
- Refraction & Transparency





## Light Transport Model

- Assume illumination originates from infinity.
- The light arriving at a camera pixel can be described as:

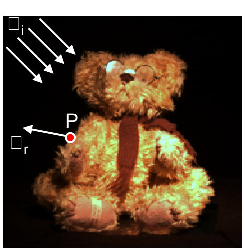
$$C(x,y) = \int_{\Omega} W(\omega) E(\omega) d\omega$$

*C(x,y)* - the pixel value
*E* - the environment
*W* - the *reflectance field*



## Surface Reflectance Fields

- 6D function: $W(P,\omega_i,\omega_r) = W(u_r,v_r;\theta_i,\Phi_i;\theta_r,\Phi_r)$

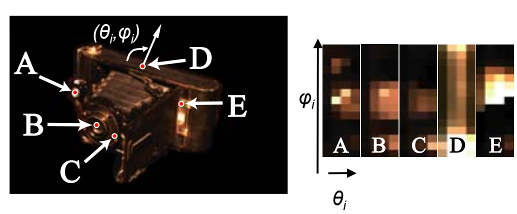




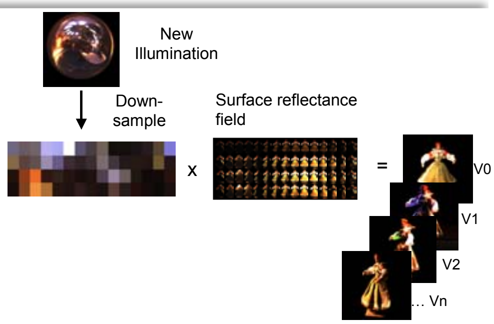

## Reflectance Functions

- For each viewpoint, 4D function:

$$W_{xy}(\omega_i) = W(x,y;\theta_i,\Phi_i)$$



## Relighting



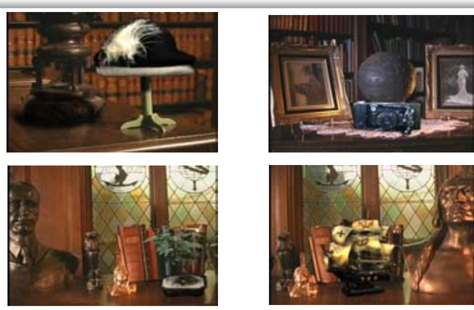



## Compression

- Subdivide images into 8 x 8 pixel blocks.
- Keep blocks containing the object (avg. compression 1:7)
- PCA compression (avg. compression 1:10)

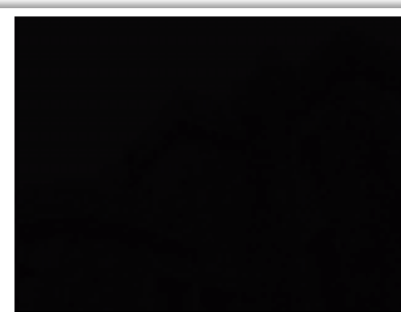



## Results





## The Library





## Surface Reflectance Fields

- Work without accurate geometry
- Surface normals are not necessary
- Capture more than reflectance
  - Inter-reflections
  - Subsurface scattering
  - Refraction
  - Dispersion
  - Non-uniform material variations
- Simplified version of the BSSRDF

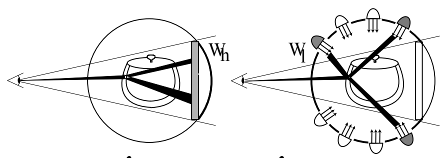


## Outline

- Overview
- Previous Works
- Geometry
- Reflectance
- ➢ Refraction & Transparency

## Acquisition

- We separate the hemisphere into high resolution □$_h$ and low resolution □$_l$.




$$C(x,y) = \int_{\Omega_h} W_h(\xi)T(\xi)d\xi + \int_{\Omega_l} W_l(\omega_i)L(\omega_i)d\omega$$



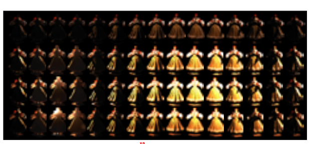
## Acquisition

- For each viewpoint ( 6 cameras x 72 positions )
  - Alpha mattes
    - Use multiple backgrounds [Smith and Blinn 96]
  - Reflectance images ← Low resolution
    - Pictures of the object under different lighting (4 lights x 11 positions)
  - Environment mattes ← High resolution
    - Use similar techniques as [Chuang et al. 2000]



## Low-Resolution Reflectance Field

$$C(x,y) = \int_{\Omega_h} W_h(\xi)T(\xi)d\xi + \int_{\Omega_l} W_l(\omega_i)L(\omega_i)d\omega$$



$$\int_{\Omega_l} W_l(\omega_i)L(\omega_i)d\omega \approx \sum_{i=1}^{n} W_i L_i$$ for n lights



## High-Resolution Reflectance Field

$$C(x,y) = \int_{\Omega_h} W_h(\xi)T(\xi)d\xi + \int_{\Omega_l} W_l(\omega_i)L(\omega_i)d\omega$$

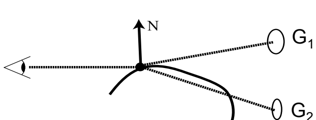
- Use techniques of environment matting [Chuang et al., SIGGRAPH 00].



## High-Resolution Reflectance Field

- Approximate $W_h$ by a sum of up to two Gaussians:
  - Reflective $G_1$.
  - Refractive $G_2$.

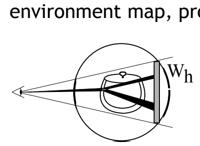
$$W_h(\xi) = a_1G_1 + a_2G_2$$





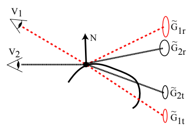

## Reproject □$_h$

- Project environment mattes onto the new environment.
  - Environment mattes acquired was parameterized on plane T (the plasma display).
  - We need to project the Gaussians to the new environment map, producing new Gaussians.




## View Interpolation

- Render low-resolution reflectance field.
- High-resolution reflectance field:
  - Match reflected and refracted Gaussians.




  - Interpolate *direction vectors*, not colors.
  - Determine new color along interpolated direction.



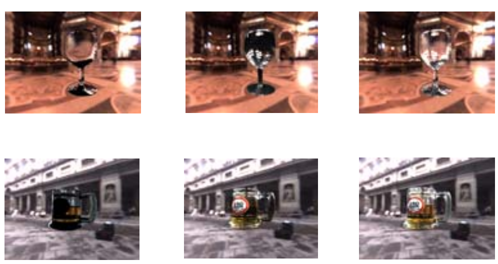
## Results

- Performance for 6x72 = 432 viewpoints
- 337,824 images taken in total !!
  - Acquisition (47 hours)
    - Alpha mattes – 1 hour
    - Environment mattes – 18 hours
    - Reflectance images – 28 hours
  - Processing
    - Opacity hull ~ 30 minutes
    - PCA Compression ~ 20 hours (MATLAB, unoptimized)
  - Rendering ~ 5 minutes per frame
  - Size
    - Opacity hull ~ 30 - 50 MB
    - Environment mattes ~ 0.5 - 2 GB
    - Reflectance images ~ Raw 370 GB / Compressed 2 - 4 GB



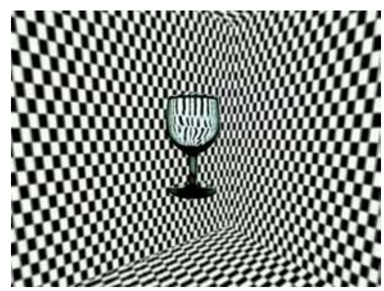
## Results

High-resolution $\Omega_h$ Low-resolution $\Omega_l$ Combined





## Results





## Results



## Results – $\Omega_h$

## Results – $\Omega_l$



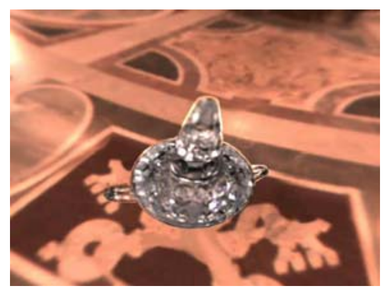
## Results – Combined



## Results



## Results





## Conclusions

- Data driven modeling is able to capture and render any type of object.
- Opacity hulls provide realistic 3D graphics models.
- Our models can be seamlessly inserted into new environments.
- Point-based rendering offers high image-quality for display of acquired models.



## Future Directions

- Real-time rendering
  - Done! [Vlasic et al., I3D 2003]
- Better environment matting
  - More than two Gaussians
- Better compression
  - MPEG-4 / JPEG 2000

## Acknowledgements

- Colleagues:
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- Papers available at:
  http://www.merl.com/people/pfister/

## Eurographics 2003

# Point-Based Computer Graphics

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## Point-based Surface Reps

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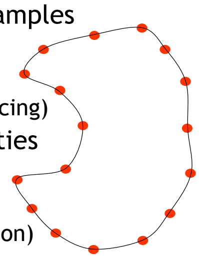
## Motivation

- Many applications need a definition of surface based on point samples
  - Reduction
  - Up-sampling
  - Interrogation (e.g. ray tracing)
- Desirable surface properties
  - Manifold
  - Smooth
  - Local (efficient computation)





## Overview

- Introduction & Basics
- Fitting Implicit Surfaces
- Projection-based Surfaces



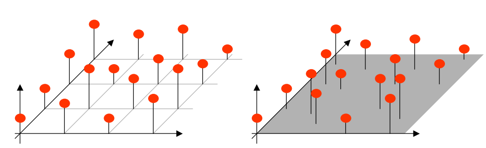
## Introduction & Basics

- Terms
  - Regular/Irregular, Approximation/Interpolation, Global/Local
- Standard interpolation/approximation techniques
  - Triangulation, Voronoi-Interpolation, Least Squares (LS), Radial Basis Functions (RBF), Moving LS
- Problems
  - Sharp edges, feature size/noise
- Functional -> Manifold



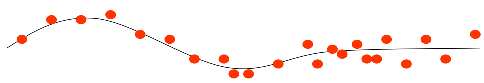
## Terms: Regular/Irregular

- Regular (on a grid) or irregular (scattered)
- Neighborhood (topology) is unclear for irregular data

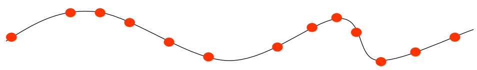


## Terms: Approximation/Interpolation

- Noisy data -> Approximation



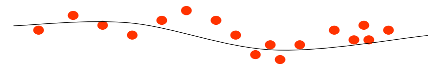
- Perfect data -> Interpolation



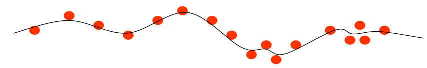


## Terms: Global/Local

- Global approximation



- Local approximation

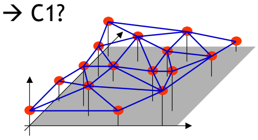


- Locality comes at the expense of smoothness



## Triangulation

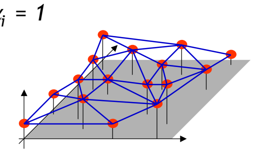
- Exploit the topology in a triangulation (e.g. Delaunay) of the data
- Interpolate the data points on the triangles
  - Piecewise linear → C0
  - Piecewise quadratic → C1?
  - …





## Triangulation: Piecewise linear

- Barycentric interpolation on simplices (triangles)
  - given $d+1$ points $x_i$ with values $f_i$ and a point $x$ inside the simplex defined by $x_i$
  - Compute $\alpha_i$ from $x = \Sigma_i\, \alpha_i \cdot x_i$ and $\Sigma_i\, \alpha_i = 1$
  - Then $f = \Sigma_i\, \alpha_i \cdot f_i$





## Voronoi Interpolation

- compute Voronoi diagram
- for any point $x$ in space
  - add $x$ to Voronoi diagram
  - Voronoi cell $\tau$ around $x$ intersects original cells $\tau_i$ of natural neighbors $n_i$
  - interpolate

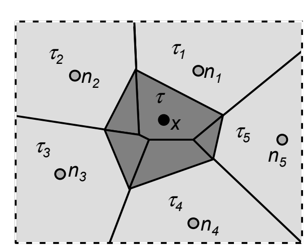
$$f(x) = \frac{\sum_i \lambda_i(x) \cdot \left(f_i + \nabla f_i^T \cdot (x - x_i)\right)}{\sum_i \lambda_i(x)}$$

with

$$\lambda_i(x) = \frac{|\tau \cap \tau_i|}{|\tau| \cdot \|x - x_i\|}$$



## Voronoi Interpolation



## Voronoi Interpolation

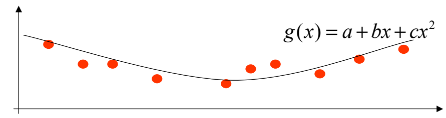
***Properties of Voronoi Interpolation:***

- linear Precision
- local
- for $d = 1 \rightarrow f(x)$ piecewise cubic
- $f(x) \in C^1$ on domain
- $f(x, x_1, \ldots, x_n)$ is continuous in $x_i$



## Least Squares

- Fits a primitive to the data
- Minimizes squared distances between the $p_i$'s and primitive $g$




$$\min_g \sum_i \left(p_{i_y} - g(p_{i_x})\right)^2$$



## Least Squares - Example

- Primitive is a polynomial

$$g(x) = (1, x, x^2, \ldots) \cdot \mathbf{c}^T$$

- $\min \sum_i \left(p_{i_y} - (1, p_{i_x}, p_{i_x}^2, \ldots)\mathbf{c}^T\right)^2 \Rightarrow$

$$0 = \sum_i 2 p_{i_x}^j \left(p_{i_y} - (1, p_{i_x}, p_{i_x}^2, \ldots)\mathbf{c}^T\right)$$

- Linear system of equations



## Least Squares - Example

- Resulting system

$$0 = \sum_i 2 p_{i_x}^j \left(p_{i_y} - (1, p_{i_x}, p_{i_x}^2, \ldots)\mathbf{c}^T\right) \Leftrightarrow$$

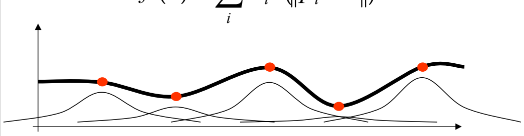
$$\begin{pmatrix} 1 & x & x^2 & \cdots \\ x & x^2 & x^3 & \\ x^2 & x^3 & x^4 & \\ \vdots & & & \ddots \end{pmatrix} \begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ \vdots \end{pmatrix} = \begin{pmatrix} y \\ yx \\ yx^2 \\ \vdots \end{pmatrix}$$



## Radial Basis Functions

- Represent interpolant as
  - Sum of radial functions $r$
  - Centered at the data points $p_i$

$$f(x) = \sum_i w_i r(\|p_i - x\|)$$





## Radial Basis Functions

- Solve $p_{j_y} = \sum_i w_i r(\|p_{i_x} - p_{j_x}\|)$

  to compute weights $w_i$
- Linear system of equations

$$\begin{pmatrix} r(0) & r(\|p_{0_x} - p_{1_x}\|) & r(\|p_{0_x} - p_{2_x}\|) & \cdots \\ r(\|p_{1_x} - p_{0_x}\|) & r(0) & r(\|p_{1_x} - p_{2_x}\|) & \\ r(\|p_{2_x} - p_{0_x}\|) & r(\|p_{2_x} - p_{1_x}\|) & r(0) & \\ \vdots & & & \ddots \end{pmatrix} \begin{pmatrix} w_0 \\ w_1 \\ w_2 \\ \vdots \end{pmatrix} = \begin{pmatrix} p_{0_y} \\ p_{1_y} \\ p_{2_y} \\ \vdots \end{pmatrix}$$

## Radial Basis Functions

- Solvability depends on radial function
- Several choices assure solvability
  - $r(d) = d^2 \log d$ (thin plate spline)
  - $r(d) = e^{-d^2/h^2}$ (Gaussian)
    - $h$ is a data parameter
    - $h$ reflects the feature size or anticipated spacing among points



## Function Spaces!

- Monomial, Lagrange, RBF share the same principle:
  - Choose basis of a function space
  - Find weight vector for base elements by solving linear system defined by data points
  - Compute values as linear combinations
- Properties
  - One costly preprocessing step
  - Simple evaluation of function in any point



## Function Spaces?

- Problems
  - Many points lead to large linear systems
  - Evaluation requires global solutions
- Solutions
  - RBF with compact support
    - Matrix is sparse
    - Still: solution depends on every data point, though drop-off is exponential with distance
  - Local approximation approaches



## Shepard Interpolation

- Approach for $\mathbf{R}^d$: $f(x) = \Sigma_i\, \phi_i(x)\, f_i$

  with basis functions $\phi_i(x) = \dfrac{\|x - x_i\|^{-p}}{\sum_j \|x - x_j\|^{-p}}$

- define $f(x_i) := f_i = \lim_{x \to x_i} f(x)$



## Shepard Interpolation

- $f(x)$ is a convex combination of $\phi_i$, because all $\phi_i(\mathbf{R}^d) \subseteq [0,1]$ and $\Sigma_i\, \phi_i(x) \equiv 1$.
  - ➔ $f(x)$ is contained in the convex hull of data points
- for $p>1$ $f(p) \in \mathbf{C}^\infty$ and $\nabla_x \phi_i(x_i) = 0$
  - ➔ Data points are saddles
- global interpolation
  - ➔ every $f(x)$ depends on all data points
- Only constant precision, i.e. only constant functions are reproduced exactly



## Shepard Interpolation

Localization:

- Set $f(x) = \Sigma_i\, \mu_i(x) \cdot \phi_i(x) \cdot f_i$

  with $\mu_i(x) = \begin{cases} \left(1 - \frac{\|x - x_i\|}{R_i}\right)^{\nu} & \text{für } \|x - x_i\| < R_i \\ 0 & \text{sonst} \end{cases}$

  for reasonable $R_i$ and $\nu > 1$

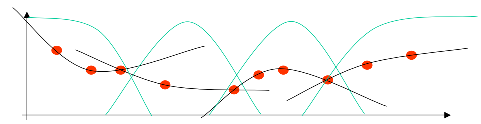
  ➔ no constant precision because of possible holes in the data

## Spatial subdivisions

- Subdivide parameter domain into overlapping cells $\tau_i$ with centroids $c_i$
- Compute Shepard weights $\phi_i(x) = \frac{\|x - c_i\|^{-p}}{\sum_j \|x - c_j\|^{-p}}$ and localize them using the radius of the cell
- Interpolate/approximate data points in each cell by an arbitrary function $f_i$
- The interpolant is given as $f(x) = \Sigma_i\ \mu_i(x) \cdot \phi_i(x) \cdot f_i$



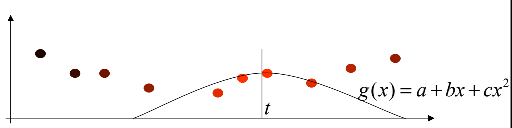
## Spatial subdivisions





## Moving Least Squares

- Compute a local LS approximation at $t$
- Weight data points based on distance to $t$




$$\min \sum_i \left(p_{i_y} - g\left(p_{i_x}\right)\right)^2 \theta\left(\left\|t - p_{i_x}\right\|\right)$$

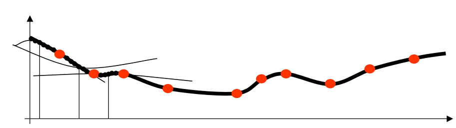


## Moving Least Squares

- The set

$$f(t) = g_t(t), g_t : \min_g \sum_i \left(p_{i_y} - g\left(p_{i_x}\right)\right)^2 \theta\left(\left\|t - p_{i_x}\right\|\right)$$

is a smooth curve, iff $\theta$ is smooth





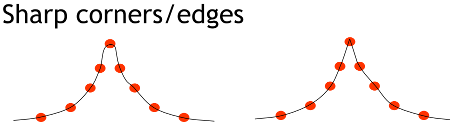
## Moving Least Squares

- Typical choices for $\theta$:
  - $\theta(d) = d^{-r}$
  - $\theta(d) = e^{-d^2/h^2}$
- Note: $\theta_i = \theta\left(\left\|t - p_{i_x}\right\|\right)$ is fixed
- For each $t$
  - Standard weighted LS problem
  - Linear iff corresponding LS is linear



## Typical Problems

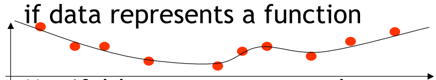
- Sharp corners/edges



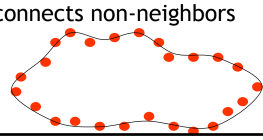
- Noise vs. feature size

## Functional -> Manifold

- Standard techniques are applicable if data represents a function



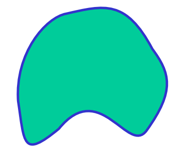
- Manifolds are more general
  - No parameter domain
  - No knowledge about neighbors, Delaunay triangulation connects non-neighbors





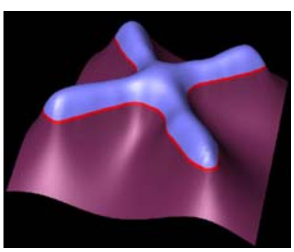
## Implicits

- Each orientable n-manifold can be embedded in n+1 - space
- Idea: Represent n-manifold as zero-set of a scalar function in n+1 - space
  - Inside: $f(\mathbf{x}) < 0$
  - On the manifold: $f(\mathbf{x}) = 0$
  - Outside: $f(\mathbf{x}) > 0$





## Implicits - Illustration

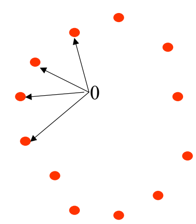


- Image courtesy Greg Turk



## Implicits from point samples

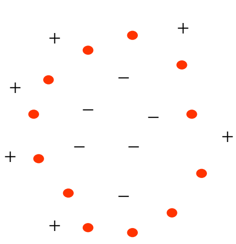
- Function should be zero in data points
  - $f(\mathbf{p}_i) = 0$
- Use standard approximation techniques to find $f$
- Trivial solution: $f = 0$
- Additional constraints are needed






## Implicits from point samples

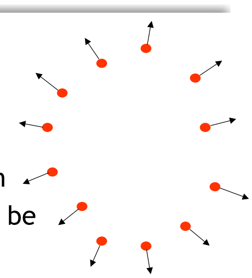
- Constraints define inside and outside
- Simple approach (Turk, O'Brien)
  - Sprinkle additional information manually
  - Make additional information soft constraints





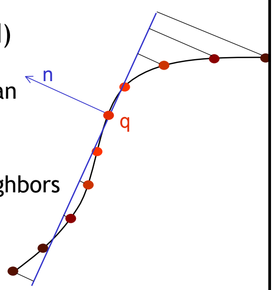
## Implicits from point samples

- Use normal information
- Normals could be computed from scan
- Or, normals have to be estimated



## Estimating normals

- Normal orientation (Implicits are signed)
  - Use inside/outside information from scan
- Normal direction by fitting a tangent
  - LS fit to nearest neighbors
  - Weighted LS fit
  - MLS fit



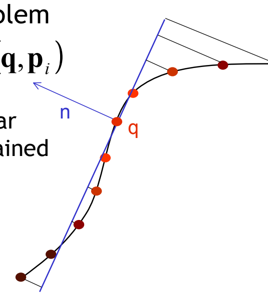



## Estimating normals

- General fitting problem

$$\min_{\|\mathbf{n}\|=1} \sum_i \langle \mathbf{q}-\mathbf{p}_i, \mathbf{n} \rangle^2 \theta(\mathbf{q}, \mathbf{p}_i)$$

  - Problem is non-linear because n is constrained to unit sphere






## Estimating normals

- The constrained minimization problem

$$\min_{\|\mathbf{n}\|=1} \sum_i \langle \mathbf{q}-\mathbf{p}_i, \mathbf{n} \rangle^2 \theta_i$$

is solved by the eigenvector corresponding to the smallest eigenvalue of

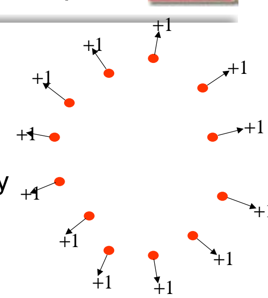
$$\begin{pmatrix} \sum_i (q_x - p_{i_x})^2 \theta_i & \sum_i (q_x - p_{i_y})^2 \theta_i & \sum_i (q_x - p_{i_z})^2 \theta_i \\ \sum_i (q_y - p_{i_x})^2 \theta_i & \sum_i (q_y - p_{i_y})^2 \theta_i & \sum_i (q_y - p_{i_z})^2 \theta_i \\ \sum_i (q_z - p_{i_x})^2 \theta_i & \sum_i (q_z - p_{i_y})^2 \theta_i & \sum_i (q_z - p_{i_z})^2 \theta_i \end{pmatrix}$$



## Implicits from point samples

- Compute non-zero anchors in the distance field
- Use normal information directly as constraints

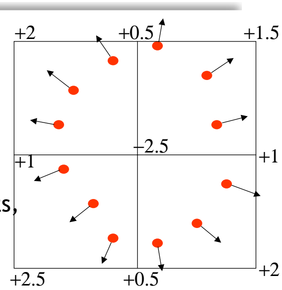
$$f(\mathbf{p}_i + \mathbf{n}_i) = 1$$






## Implicits from point samples

- Compute non-zero anchors in the distance field
- Compute distances at specific points
  - Vertices, mid-points, etc. in a spatial subdivision






## Computing Implicits

- Given N points and normals $p_i, n_i$ and constraints $f(\mathbf{p}_i) = 0, f(\mathbf{c}_i) = d_i$
- Let $\mathbf{p}_{i+N} = \mathbf{c}_i$
- An RBF approximation

$$f(\mathbf{x}) = \sum_i w_i r(\|\mathbf{p}_i - \mathbf{x}\|)$$

leads to a system of linear equations

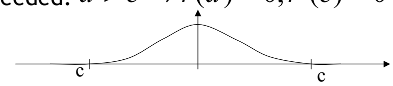
## Computing Implicits

- Practical problems: $N > 10000$
- Matrix solution becomes difficult
- Two solutions
  - Sparse matrices allow iterative solution
  - Smaller number of RBFs



## Computing Implicits

- Sparse matrices $\begin{pmatrix} r(0) & r(\|p_0-p_1\|) & r(\|p_0-p_2\|) & \Lambda \\ r(\|p_1-p_0\|) & r(0) & r(\|p_1-p_2\|) & \\ r(\|p-p_0\|) & r(\|p_2-p_1\|) & r(0) & \\ M & & & O \end{pmatrix}$
  - Needed: $d > c \rightarrow r(d) = 0, r'(c) = 0$

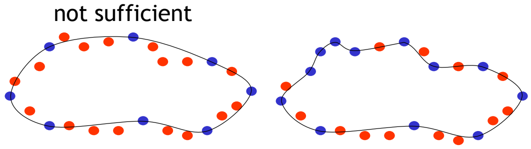



  - Compactly supported RBFs



## Computing Implicits

- Smaller number of RBFs
- Greedy approach (Carr et al.)
  - Start with random small subset
  - Add RBFs where approximation quality is not sufficient





## RBF Implicits - Results

- Images courtesy Greg Turk



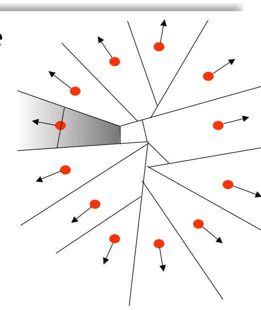

## RBF Implicits - Results

- Images courtesy Greg Turk



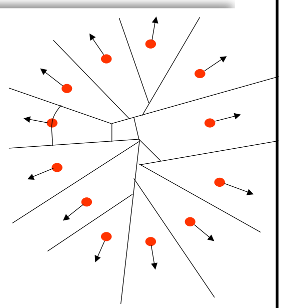

## Hoppe's approach

- Use linear distance field per point
  - Direction is defined by normal
- In every point in space use the distance field of the closest point



## Hoppe's approach - smoother

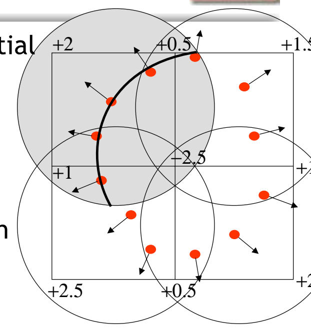
- Direction fields are interpolated using Voronoi interpolation





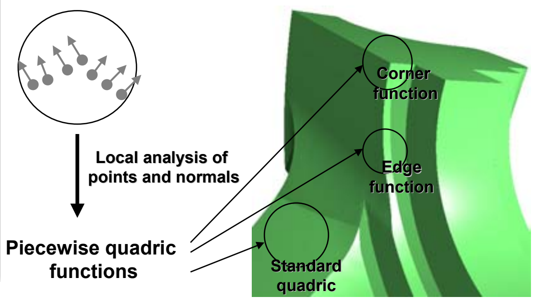
## PuO Implicits

- Construct a spatial subdivision
- Compute local distance field approximations
  - e.g. Quadrics
- Blend them with local Shepard weights





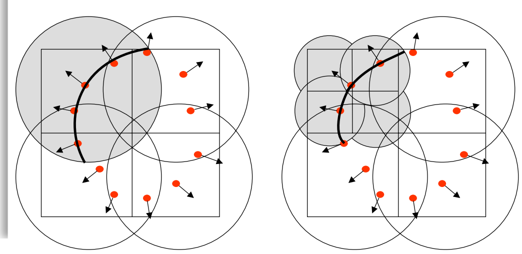

## PuO Implicits: Sharp features






## Multi-level PuO Implicits

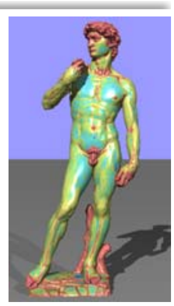
- Subdivide cells based on local error





## Multi-level PuO Implicits

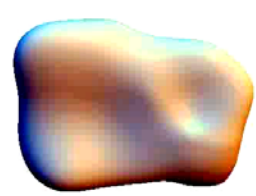
- Local computations
  - Insensitive to number of points
- Local adaptation to shape complexity
- Sensitive to output complexity





## Multi-level PuO Implicits

- Aproximation at arbitrary accuracy



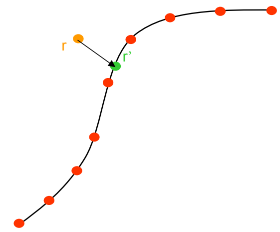
## Implicits - Conclusions

- Scalar field is underconstrained
  - Constraints only define where the field is zero, not where it is non-zero
  - Additional constraints are needed
- Signed fields restrict surfaces to be unbounded
  - All implicit surfaces define solids



## Projection

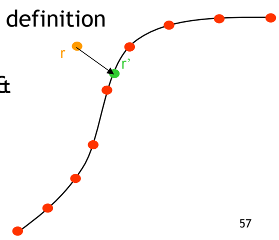
- Idea: Map space to surface
- Surface is defined as fixpoints of mapping






## Surface definition

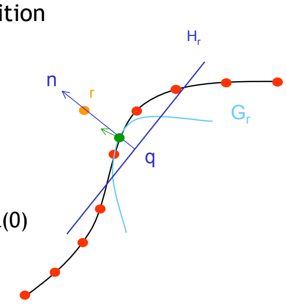
- Projection procedure (Levin)
  - Local polyonmial approximation
    - Inspired by differential geometry
  - "Implicit" surface definition
  - Infinitely smooth &
  - Manifold surface






## Surface Definition

- Constructive definition
  - Input point r
  - Compute a local reference plane $H_r$=<q,n>
  - Compute a local polynomial over the plane $G_r$
  - Project point $r'=G_r(0)$
  - Estimate normal

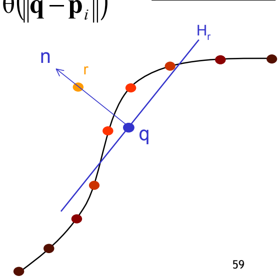





## Local Reference Plane

- Find plane $H_r = \langle \mathbf{q}, \mathbf{n} \rangle + D$
  - $\min_{\mathbf{q}, \|\mathbf{n}\|=1} \sum_i \langle \mathbf{q} - \mathbf{p}_i, \mathbf{n} \rangle^2 \theta(\|\mathbf{q} - \mathbf{p}_i\|)$
  - $\theta(\mathrm{d}) = e^{d^2/h^2}$
    - $h$ is feature size/ point spacing
  - $H_r$ is independent of $r$'s distance
  - Manifold property

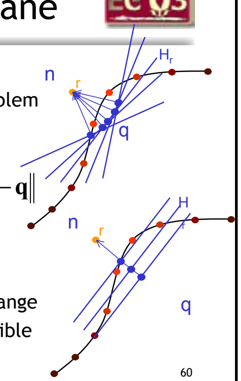
Weight function based on distance to q, not r





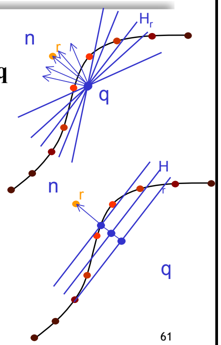

## Local Reference Plane

- Computing reference plane
  - Non-linear optimization problem
- Minimize independent variables:
  - Over $\mathbf{n}$ for fixed distance $\|\mathbf{r} - \mathbf{q}\|$
  - Along $\mathbf{n}$ for fixed direction $\mathbf{n}$
  - $\mathbf{q}$ changes -> the weights change
  - Only iterative solutions possible



## Local Reference Plane

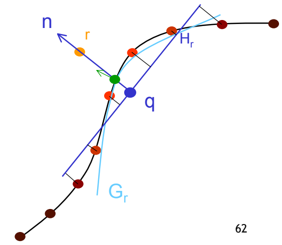
- Practical computation
  - Minimize over $\mathbf{n}$ for fixed $\mathbf{q}$
    - Eigenvalue problem
  - Translate q so that
    $$\mathbf{r} = \mathbf{q} + \|\mathbf{r} - \mathbf{q}\|\mathbf{n}$$
    - Effectively changes $\|\mathbf{r} - \mathbf{q}\|$
  - Minimize along $\mathbf{n}$ for fixed direction $\mathbf{n}$
    - Exploit partial derivative






## Projecting the Point

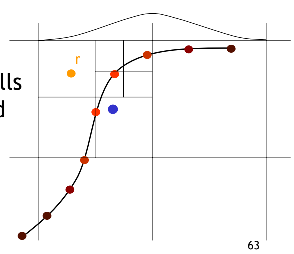
- MLS polyonomial over $H_r$
  - $\min_{G\in\Pi_d}\sum_i \left(\langle \mathbf{q}-\mathbf{p}_i, \mathbf{n}\rangle - G\left(\mathbf{p}_i|_{H_r}\right)\right)^2 \theta\left(\|\mathbf{q}-\mathbf{p}_i\|\right)$
  - LS problem
  - $r' = G_r(0)$
  - Estimate normal






## Spatial data structure

- Regular grid based on support of $\theta$
  - Each point influences only 8 cells
- Each cell is an octree
  - Distant octree cells are approximated by one point in center of mass






## Conclusions

- Projection-based surface definition
  - Surface is smooth and manifold
  - Surface may be bounded
  - Representation error mainly depends on point density
  - Adjustable feature size h allows to smooth out noise

Eurographics 2003

# Point-Based Rendering

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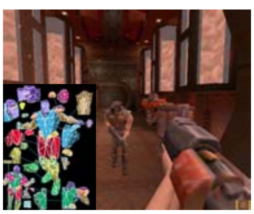
ETH Zürich

## Point-Based Rendering

- Introduction and motivation
- Surface elements
- Rendering
- Antialiasing
- Hardware Acceleration
- Conclusions



## Motivation 1



Quake 2, 1998
10k triangles

Nvidia, 2002
millions of triangles



## Motivation 1

- Performance of 3D hardware has exploded (e.g., GeForce4: 136 million vertices per second)
- Projected triangles are very small (i.e., cover only a few pixels)
- Overhead for triangle setup increases (initialization of texture filtering, rasterization)

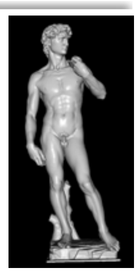
➡ A simpler, more efficient rendering primitive than triangles?



## Motivation 2

- Modern 3D scanning devices (e.g., laser range scanners) acquire huge point clouds
- Generating consistent triangle meshes is time consuming and difficult

➡ A rendering primitive for direct visualization of point clouds, without the need to generate triangle meshes?

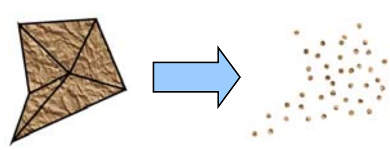


4 million pts.
[Levoy et al. 2000]



## Points as Rendering Primitives

- Point clouds instead of triangle meshes [Levoy and Whitted 1985]
- 2D vector versus pixel graphics



triangle mesh (with textures)

point cloud

## Point-Based Surface Representation

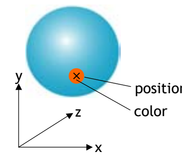
- Points are ***samples*** of the surface
- The point cloud describes:
  - 3D geometry of the surface
  - Surface reflectance properties (e.g., diffuse color, etc.)
- There is no additional information, such as
  - connectivity (i.e., explicit neighborhood information between points)
  - texture maps, bump maps, etc.



## Surface Elements - Surfels

- Each point corresponds to a surface element, or ***surfel***, describing the surface in a small neighborhood
- Basic surfels:

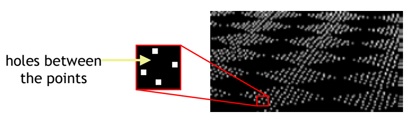
```
BasicSurfel {
  position;
  color;
}
```






## Surfels

- How to represent the surface between the points?



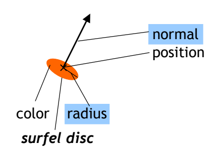

- Surfels need to ***interpolate*** the surface between the points
- A certain ***surface area*** is associated with each surfel



## Surfels

- Surfels can be extended by storing additional attributes
- This allows for higher quality rendering or advanced shading effects

```
ExtendedSurfel {
  position;
  color;
  normal;
  radius;
  etc...
}
```





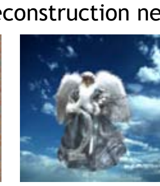

## Surfels

- Surfels store essential information for ***rendering***
- Surfels are primarily designed as a ***point rendering primitive***
- They do not provide a mathematically smooth surface definition (see [Alexa 2001], point set surfaces)



## Model Acquisition

- 3D scanning of physical objects
  - See Pfister, acquisition
  - Direct rendering of acquired point clouds
  - No mesh reconstruction necessary

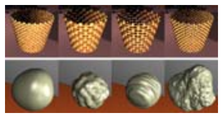


[Matusik et al. 2002]

## Model Acquisition

- Sampling synthetic objects
  - Efficient rendering of complex models
  - Dynamic sampling of procedural objects and animated scenes (see Stamminger, dynamic sampling)

[Zwicker et al. 2001]

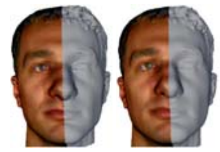


[Stamminger et al. 2001]



## Model Acquisition

- Processing and editing of point-sampled geometry



spectral processing
[Pauly, Gross 2002]
(see Gross, spectral processing)

point-based surface editing
[Zwicker et al. 2002]
(see Pauly, Pointshop3D)



## Point Rendering Pipeline

Point Cloud

Projection → Shading → Visibility → Image Reconstruction

Frame-buffer

- Simple, pure forward mapping pipeline
- Surfels carry all information through the pipeline („surfel stream")
- No texture look-ups
- Framebuffer stores RGB, alpha, and Z



## Point Rendering Pipeline

**Projection** → **Shading** → **Visibility** → **Image Reconstruction**

- Perspective projection of each point in the point cloud
- Analogous to projection of triangle vertices
  - homogeneous matrix-vector product
  - perspective division



## Point Rendering Pipeline

**Projection** → **Shading** → **Visibility** → **Image Reconstruction**

- Per-point shading
- Conventional models for shading (Phong, Torrance-Sparrow, reflections, etc.)

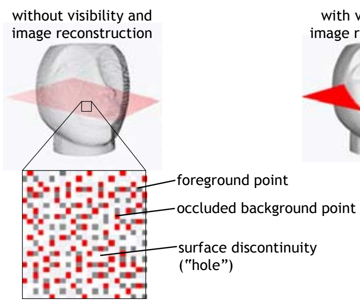


## Point Rendering Pipeline

**Projection** → **Shading** → **Visibility** → **Image Reconstruction**

- Visibility and image reconstruction is tightly coupled
  - Discard points that are occluded from the current viewpoint
  - Reconstruct continuous surfaces from projected points (antialiasing)

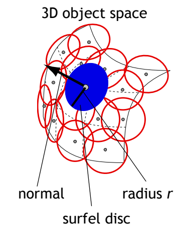
## Visibility and Image Reconstruction






## Visibility and Image Reconstruction

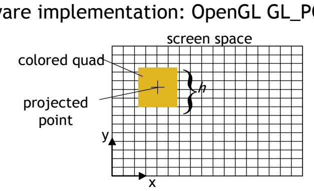
- Goal: avoid holes and discard occluded surfels
- Use surfel discs with radius *r* to cover surface completely
- Apply z-buffer to discard invisible surfels






## Quad Rendering Primitive

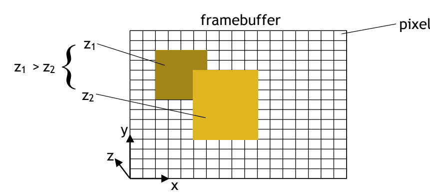
- Rasterize a colored quad centered at the projected point, use z-buffering
- The quad side length is *h*, where *h* = 2 * *r* * *s*
- The scaling factor *s* given by perspective projection and viewport transformation
- Hardware implementation: OpenGL GL_POINTS






## Visibility: Z-Buffering

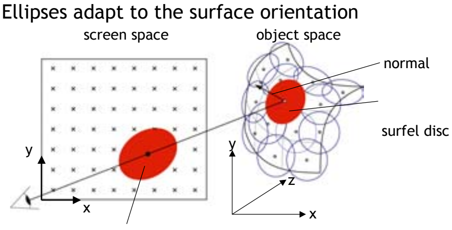
- ***No blending*** of rendering primitives






## Projected Disc Rendering Primitive

- Project surfel discs from object to screen space
- Projecting discs results in ellipses in screen space
- Ellipses adapt to the surface orientation



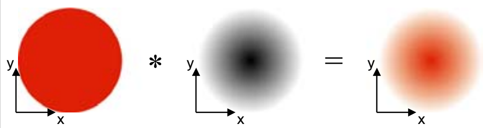



## Discussion

- Quad and projected disc primitive
  - Simple, efficient
  - Hardware support
  - Low image quality
  - Suitable for preview renderers (e.g. Qsplat [Rusinkiewicz et al. 2000] )
- Problem: no blending of primitives

## Splatting

- A splat primitive consists of a colored point primitive and an alpha mask




colored point primitive *c*

alpha mask *w(x,y)* (often a 2D Gauss function)

splat primitive *c * w(x,y)*



## Splatting

- The final color *c(x,y)* is computed by ***additive alpha blending***, i.e., by computing the weighted sum

color of splat *i* — alpha of splat *i* at position *(x,y)*

$$c(x,y) = \frac{\sum_i c_i w_i(x,y)}{\sum_i w_i(x,y)}$$

- Normalization is necessary, because the weights do not sum up to one with irregular point distributions

$$\sum_i w_i(x,y) \neq 1$$

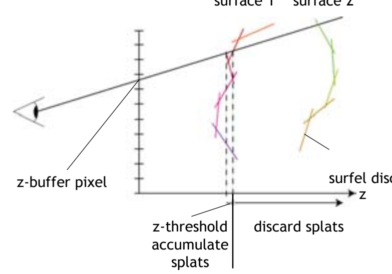


## Splatting




## Splatting

- Extended z-buffering





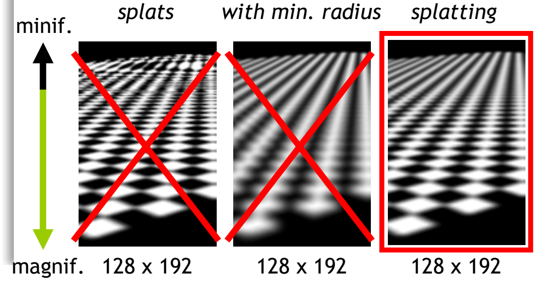

## Extended Z-Buffering

```
DepthTest(x,y) {
  if (abs(splat z – z(x,y)) < threshold) {
    c(x,y) = c(x,y) + splat color
    w(x,y) = w(x,y) + splat w(x,y)
  } else if (splat z < z(x,y)) {
    z(x,y) = splat z
    c(x,y) = splat color
    w(x,y) = splat w(x,y)
  }
}
```



## Splatting Comparison

*elliptical splats* | *circular splats with min. radius* | *surface splatting*



# High Quality Splatting

- High quality splatting requires careful analysis of ***aliasing*** issues
  - Review of signal processing theory
  - Application to point rendering
  - Surface splatting [Zwicker et al. 2001]



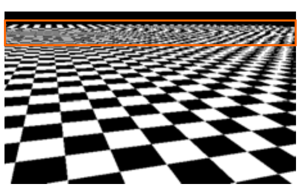
# Aliasing in Computer Graphics

- Aliasing = Sampling of continuous functions below the ***Nyquist frequency***
  - To avoid aliasing, sampling rate must be twice as high as the maximum frequency in the signal
- Aliasing effects:
  - Loss of detail
  - Moire patterns, jagged edges
  - Disintegration of objects or patterns
- Aliasing in Computer Graphics
  - Texture Mapping
  - Scan conversion of geometry



# Aliasing in Computer Graphics

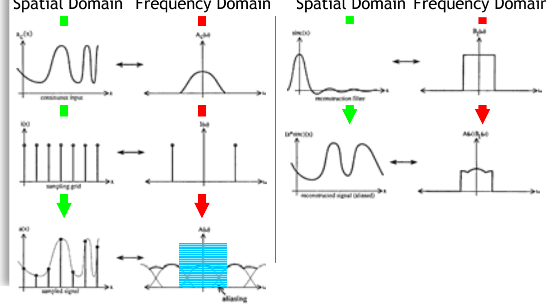
- Aliasing: high frequencies in the input signal appear as low frequencies in the reconstructed signal





# Occurrence of Aliasing

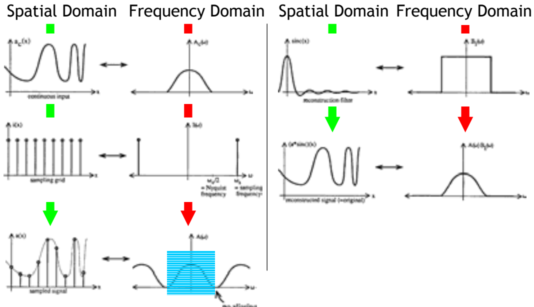
Spatial Domain Frequency Domain Spatial Domain Frequency Domain






# Aliasing-Free Reconstruction

Spatial Domain Frequency Domain Spatial Domain Frequency Domain

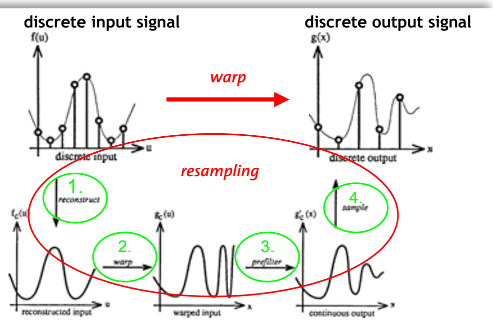





# Antialiasing

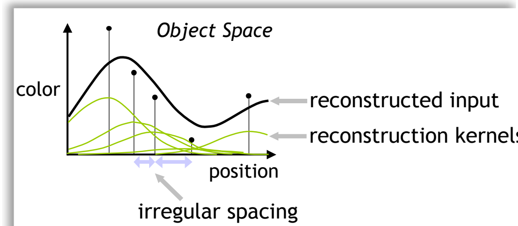
- Prefiltering
  - Band-limit the continuous signal before sampling
  - Eliminates all aliasing (with an ideal low-pass filter)
  - Closed form solution not available in general
- Supersampling
  - Raise sampling rate
  - Reduces, but does not eliminate all aliasing artifacts (in practice, many signals have infinite frequencies)
  - Simple implementation (hardware)

# Resampling



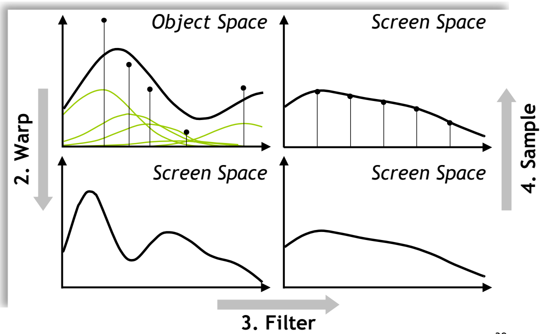



# Resampling Filters



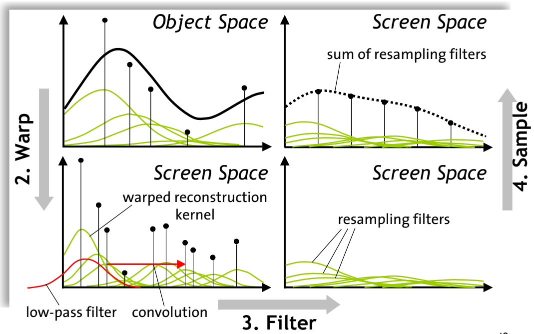



# Resampling Filters






# Resampling Filters





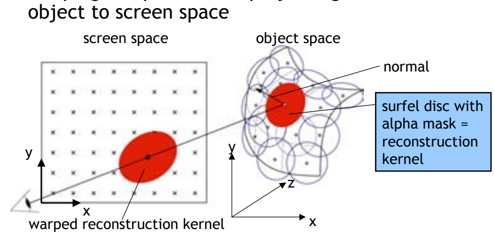

# Resampling

- Resampling in the context of surface rendering
  - Discrete input function = surface texture (discrete 2D function)
  - Warping = projecting surfaces to the image plane (2D to 2D projective mapping)



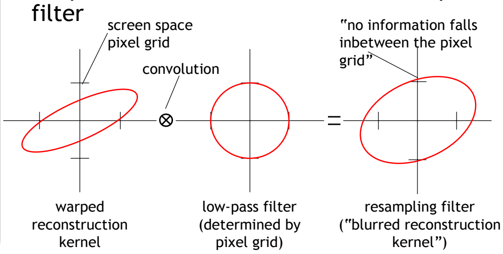
# 2D Reconstruction Kernels

- 2D reconstruction kernels are given by surfel discs with alpha masks
- Warping is equivalent to projecting the kernel from object to screen space



## Resampling Filters

- A resampling filter is a convolution of a warped reconstruction filter and a low-pass filter






## Mathematical Formulation

resampling filter

$$c(x,y)=\sum_k c_k r_k(m^{-1}(x,y))\otimes h(x,y)$$

pixel color

warping function

low pass filter

reconstruction kernel

reconstruction kernel color



## Gaussian Resampling Filters

- Gaussians are closed under linear warping and convolution
- With Gaussian reconstruction kernels and low-pass filters, the resampling filter is a Gaussian, too
- Efficient rendering algorithms (***surface splatting*** [Zwicker et al. 2001])

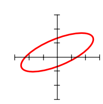


## Mathematical Formulation

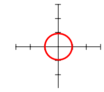
$$c(x,y)=\sum_k c_k r_k(m^{-1}(x,y))\otimes h(x,y)$$

Gaussian reconstruction kernel

Gaussian low-pass filter



*screen space*



*screen space*



## Mathematical Formulation

$$c(x,y)=\sum_k c_k r_k(m^{-1}(x,y))\otimes h(x,y)$$

$$=\sum_k c_k G_k(x,y)$$

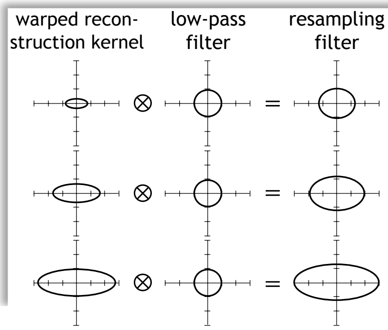
Gaussian resampling filter



## Algorithm

```
for each point P {
  project P to screen space;
  shade P;
  determine resampling kernel G;
  splat G;
}
for each pixel {
  normalize;
}
```

## Properties of 2D Resampling Filters



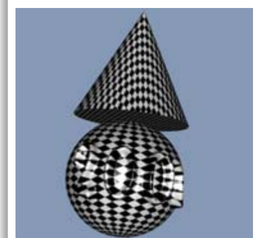

minification

magnification

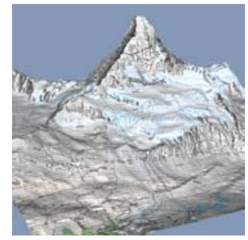


## Results

- High quality reconstruction and filtering



200k points



4783k points

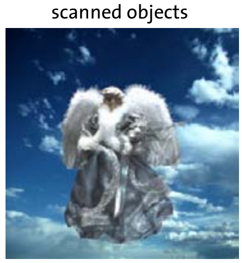


## Results

transparent surfaces

987k points

scanned objects



[MERL/MIT Matusik et al.]



## Hardware Implementation

- Based on the object space formulation of EWA filtering
- Implemented using textured triangles
- All calculations are performed in the programmable hardware (extensive use of vertex shaders)
- Presented at EG 2002 ([Ren et al. 2002])



## Surface Splatting Performance

- Software implementation
  - 500 000 splats/sec on 866 MHz PIII
  - 1 000 000 splats/sec on 2 GHz P4
- Hardware implementation [Ren et al. 2002]
  - Uses texture mapping and vertex shaders
  - 3 000 000 splats/sec on GeForce4 Ti 4400



## Conclusions

- Points are an efficient rendering primitive for highly complex surfaces
- Points allow the direct visualization of real world data acquired with 3D scanning devices
- High performance, low quality point rendering is supported by 3D hardware (tens of millions points per second)
- High quality point rendering with anisotropic texture filtering is available
  - 3 million points per second with hardware support
  - 1 million points per second in software
- Antialiasing technique has been extended to volume rendering

## Applications

- Direct visualization of point clouds
- Real-time 3D reconstruction and rendering for virtual reality applications
- Hybrid point and polygon rendering systems
- Rendering animated scenes
- Interactive display of huge meshes
- On the fly sampling and rendering of procedural objects



## Future Work

- Dedicated rendering hardware
- Efficient approximations of exact EWA splatting
- Rendering architecture for on the fly sampling and rendering



## Acknowledgments


- Hanspeter Pfister, Jeroen van Baar (MERL, Cambridge MA)
- Markus Gross, Mark Pauly, CGL
- Liu Ren

http://graphics.ethz.ch/surfels
http://graphics.ethz.ch/pointshop3d




## References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**Eurographics 2003**

# Point-Based Computer Graphics

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## Introduction

- point rendering
  - how adapt point densities?
    - *for a given viewing position, how can we get n points that suffice for that viewer?*
  - how render the points?
    - *given n points, how can we render an image from them ?*



## Introduction

- how render the points?
  - project point to pixel, set pixel color
  - hardware solution (Radeon 9700 Pro)
    - ~80 mio. points per second
    - no hole filling
  - software solution
    - ~8 mio. points per second
    - hole filling
- *hardware != software*



## Introduction

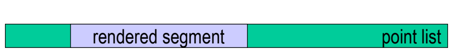
- even with hardware:
  - `for (int i = 0; i < N; i++) renderPointWithNormalAndColor (x[i],y[i],z[i],nx[i],ny[i],nz[i],…);`
    → 10 mio points per second
  - `for (int i = 0; i < N; i++) renderPoint(x[i],y[i],z[i]);`
    → 20 mio points per second
  - `float *p = {...} renderPoints(p);`
    → 80 mio points per second
- → *best performance with sequential processing of large chunks !*



## Introduction

- what we want:
  - sequential processing *and*
  - adaptive point densities

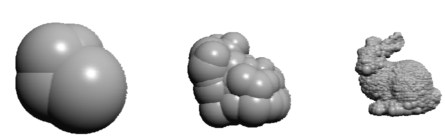
→ precomputed point lists
→ render continuous segments only






## Hierarchical Processing

- Q-Splat
  - Rusinkiewicz et al., Siggraph 2000
  - hierarchical point rendering based on Bounding Sphere Hierarchy



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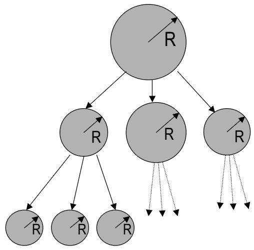
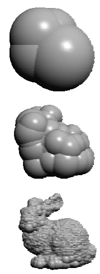
## Hierarchical Processing

- Q-Splat hierarchy





## Hierarchical Processing

- Q-Splat recursive rendering

```
render( Node n ) {
  // compute screen size of node
  s = n.R / distanceToCamera( n );
  // screen size too big?
  if ( s > threshold )
    // → render children
    forall children c
      render( c );
  else
    // else draw node
    renderPoint( n.xyz );
}
```



## Hierarchical Processing

- not sequential
- no array, but tree structure
- most work on CPU
- CPU is bottleneck: ~8 mio points per second

→ sequential version ?



## Sequential Point Trees

- store with node $d_{min}$ = n.R / 1 Pixel

```
• render( Node n ) {
    // node too close?
    if ( distanceToCamera( n ) < n.dmin )
      // → render children
      forall children c
        render( c );
    else
      // else draw node
      renderPoint( n.xyz );
  }
```



## Sequential Point Trees

- node `n` is rendered if:
  - n is not too close and
  - parent is not rendered
- or
  - `distToCam( n ) < n.dmin`
  - `distToCam( n.parent ) ≥ n.parent.dmin`
- parent is too close, but node is far enough

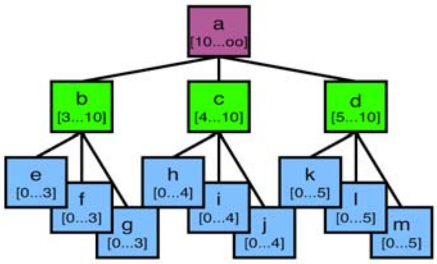


## Sequential Point Trees

- assume
  - `distToCam(n) ≈ distToCam(n.parent)`
- store with `n`
  - `n.dmax = n.parent.dmin`
- then a node is rendered if
  - `n.dmin ≤ distToCam(n) < n.dmax`

## Sequential Point Trees

- example tree






## Sequential Point Trees

- sequential version

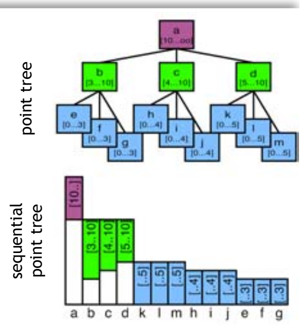
```
• foreach tree node n
    if ( n.dmin < distToCam(n) &&
         distToCam(n) < n.dmax )
       renderPoint(n);
```

- how enumerate nodes?



## Sequential Point Trees

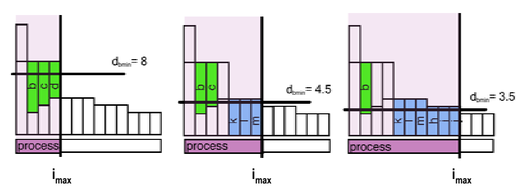
- sort nodes by $d_{max}$






## Sequential Point Trees

- compute lower bound $d_{bmin}$ on distToCam(n) with bounding volume
- all elements with $d_{max} < d_{bmin}$ can be skipped
- only prefix must be considered





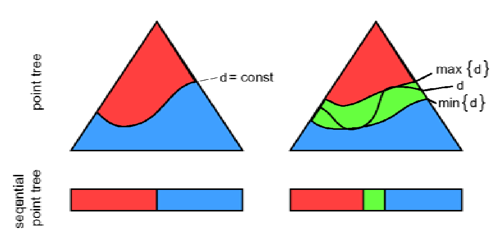

## Sequential Point Trees

- account for $d \neq d(\text{parent})$:
  - $d_{max} = d_{min}(\text{parent})$ + distance to parent
  - partially parent and some children selected
  - no visible artifacts from this



## Sequential Point Trees

- culling by GPU necessary, because d is not constant over object



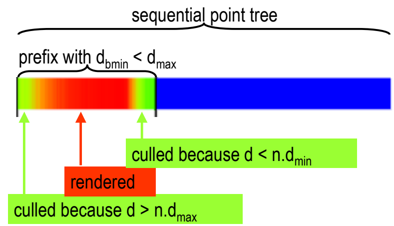
## Sequential Point Trees

- CPU does per frame:
  - compute $d_{bmin}$
  - search last node $i_{max}$ with $d_{max} > d_{bmin}$
  - send first $i_{max}$ points to GPU
- GPU then does for every node n
  - compute d = distToCam(n)
  - if $n.d_{min} \leq d \leq n.d_{max}$
    - render node



## Sequential Point Trees

- CPU does first interval selection by $d_{bmin}$
- GPU does fine granularity selection






## Sequential Point Trees

- Result
  - culling by GPU: only 10 - 40%
  - on a 2,4 GHz Pentium with Radeon 9700:
  - CPU-Load < 20% (usually much less)
  - > 50 Mio points *after* culling



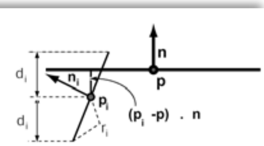
## Sequential Point Trees

- better error measurement
  - in flat regions
    - increase $d_{min}$, $d_{max}$
    - render larger points

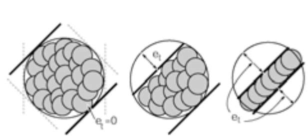


## Sequential Point Trees

- geometric
  - perpendicular error



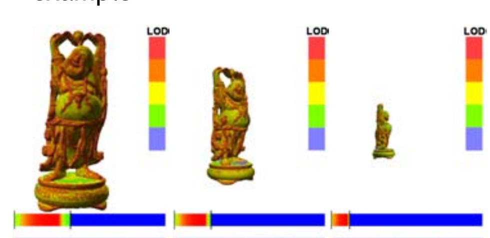

  - tangential error





## Sequential Point Trees

- example



## Sequential Point Trees

- also add texture criterion
- necessary for flat textured regions




## Sequential Point Trees

- if significant color variation in child nodes:
  - modify tangential error
  - increase error to node diameter
- prevents washed out colors in flat regions



## Sequential Point Trees

- perpendicular, tangential, texture error
- scale with 1/(view distance)
- fits into sequential point trees



## Sequential Point Trees

- combine errors
  - perpendicular $e_p$
  - tangential $e_t$
  - texture $e_{tex}$
- $e_{com} = \begin{cases} r & \text{if texture variation} \\ \sqrt{e_p^2 + e_t^2} & \text{else} \end{cases}$
- => screen error = $e_{com}$ / viewDistance



## Sequential Point Trees

- can be combined with polygons



## Sequential Point Trees

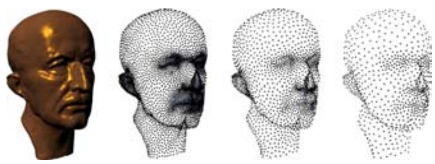
- combine with polygonal rendering
  - for every triangle
    - compute $d_{max}$ (longest side / $d_{max}$ = ε)
    - remove all points from triangle with smaller $d_{max}$
  - sort triangles for $d_{max}$
  - during rendering
    - for every object, compute upper bound $d_{bmax}$ on distance
    - send triangles with $d_{max} < d_{bmax}$ to GPU
    - on the GPU (vertex program)
      - test $d < d_{max}$
      - cull by alpha-test

## Sequential Point Trees

- pros
  - very simple!
  - CPU-load low
  - most work moved to GPU
  - GPU runs at maximum efficiency
- cons
  - no view frustum culling
  - currently: bad splatting support by GPU

## Eurographics 2003

# Efficient Simplification of Point-sampled Surfaces





## Overview

- Introduction
- Local surface analysis
- Simplification methods
- Error measurement
- Comparison



## Introduction

- Point-based models are often sampled very densely
- Many applications require coarser approximations, e.g. for efficient
  - Storage
  - Transmission
  - Processing
  - Rendering

⇨ We need simplification methods for reducing the complexity of point-based surfaces



## Introduction

- Example: Level-of-detail (LOD) rendering

10k 20k 60k 200k 2000k



## Introduction

- We transfer different simplification methods from triangle meshes to point clouds:
  - Hierarchical clustering
  - Iterative simplification
  - Particle simulation
- Depending on the intended use, each method has its pros and cons (see comparison)



## Local Surface Analysis

- Cloud of point samples describes underlying (manifold) surface
- We need:
  - Mechanisms for locally approximating the surface ⇨ MLS approach
  - Fast estimation of tangent plane and curvature ⇨ principal component analysis of local neighborhood

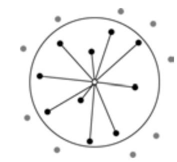
# Neighborhood

- No explicit connectivity between samples (as with triangle meshes)
- Replace geodesic proximity with spatial proximity (requires sufficiently high sampling density!)
- Compute neighborhood according to Euclidean distance



# Neighborhood

- K-nearest neighbors

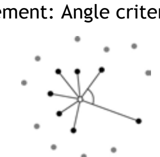


  - Can be quickly computed using spatial data-structures (e.g. kd-tree, octree, bsp-tree)
  - Requires isotropic point distribution



# Neighborhood

- Improvement: Angle criterion (Linsen)



  - Project points onto tangent plane
  - Sort neighbors according to angle
  - Include more points if angle between subsequent points is above some threshold



# Neighborhood

- Local Delaunay triangulation (Floater)

  - Project points into tangent plane
  - Compute local Voronoi diagram



# Covariance Analysis

- Covariance matrix of local neighborhood N:

$$\mathbf{C} = \begin{bmatrix} \mathbf{p}_{i_1} - \bar{\mathbf{p}} \\ \Lambda \\ \mathbf{p}_{i_n} - \bar{\mathbf{p}} \end{bmatrix}^T \cdot \begin{bmatrix} \mathbf{p}_{i_1} - \bar{\mathbf{p}} \\ \Lambda \\ \mathbf{p}_{i_n} - \bar{\mathbf{p}} \end{bmatrix}, \qquad i_j \in N$$

- with centroid $\bar{\mathbf{p}} = \frac{1}{|N|} \sum_{i \in N} \mathbf{p}_i$



# Covariance Analysis

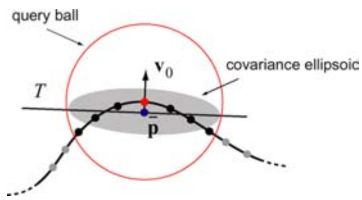
- Consider the eigenproblem:

$$\mathbf{C} \cdot \mathbf{v}_l = \lambda_l \cdot \mathbf{v}_l, \qquad l \in \{0,1,2\}$$

- C is a 3x3, positive semi-definite matrix
  - ⇨ All eigenvalues are real-valued
  - ⇨ The eigenvector with smallest eigenvalue defines the least-squares plane through the points in the neighborhood, i.e. approximates the surface normal

## Covariance Analysis

- Covariance ellipsoid spanned by the eigenvectors scaled with corresponding eigenvalue






## Covariance Analysis

- The total variation is given as:

$$\sum_{i \in N} \left| \mathbf{p}_i - \overline{\mathbf{p}} \right|^2 = \lambda_0 + \lambda_1 + \lambda_2$$

- We define surface variation as:

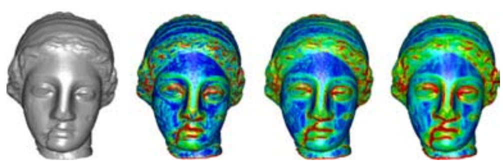
$$\sigma_n(\mathbf{p}) = \frac{\lambda_0}{\lambda_0 + \lambda_1 + \lambda_2}, \qquad \lambda_0 \le \lambda_1 \le \lambda_2$$

  - Measures the fraction of variation along the surface normal, i.e. quantifies how strong the surface deviates from the tangent plane ⇨ estimate for curvature



## Covariance Analysis

- Comparison with curvature:



original mean curvature variation n=20 variation n=50



## Surface Simplification

- Hierarchical clustering
- Iterative simplification
- Particle simulation



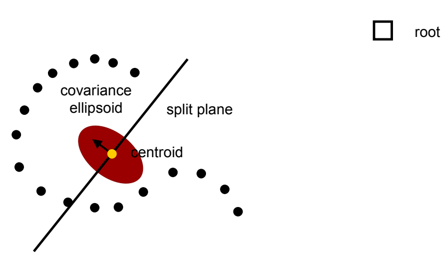
## Hierarchical Clustering

- Top-down approach using binary space partition:
- Split the point cloud if:
  - Size is larger than user-specified maximum or
  - Surface variation is above maximum threshold
- Split plane defined by centroid and axis of greatest variation (= eigenvector of covariance matrix with largest associated eigenvector)
- Leaf nodes of the tree correspond to clusters
- Replace clusters by centroid



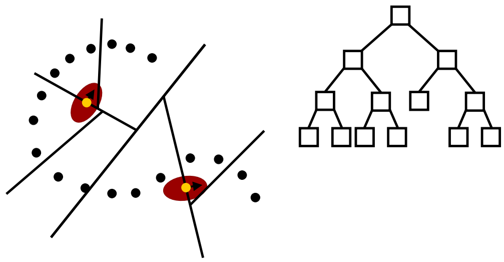
## Hierarchical Clustering

- 2D example



## Hierarchical Clustering

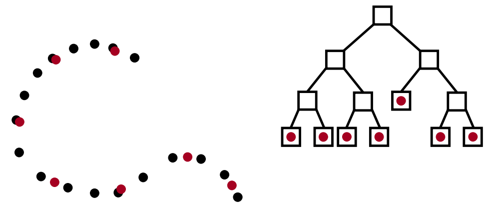
- 2D example





## Hierarchical Clustering

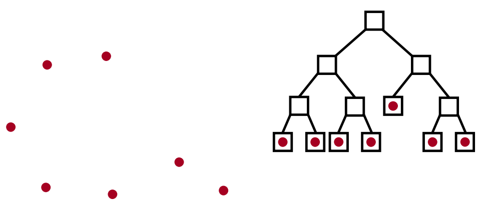
- 2D example





## Hierarchical Clustering

- 2D example





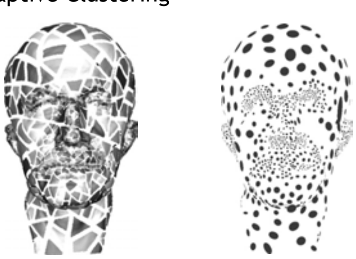
## Hierarchical Clustering

43 Clusters 436 Clusters 4,280 Clusters



## Hierarchical Clustering

- Adaptive Clustering



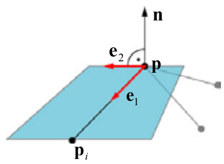


## Iterative Simplification

- Iteratively contracts point pairs
  - ⇨ Each contraction reduces the number of points by one
- Contractions are arranged in priority queue according to quadric error metric (Garland and Heckbert)
- Quadric measures cost of contraction and determines optimal position for contracted sample
- Equivalent to QSlim except for definition of approximating planes

## Iterative Simplification

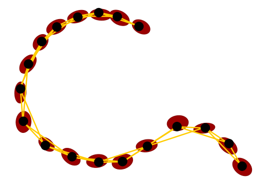
- Quadric measures the squared distance to a set of planes defined over *edges* of neighborhood
  - plane spanned by vectors $\mathbf{e}_1 = \mathbf{p}_i - \mathbf{p}$ and $\mathbf{e}_2 = \mathbf{e}_1 \times \mathbf{n}$






## Iterative Simplification

- 2D example



- Compute initial point-pair contraction candidates
- Compute fundamental quadrics
- Compute edge costs



## Iterative Simplification

- 2D example

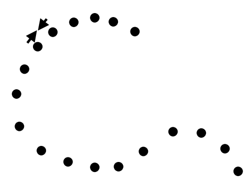
priority queue

| edge | cost |
|---|---|
| 6 | 0.02 |
| 2 | 0.03 |
| 14 | 0.04 |
| 5 | 0.04 |
| 9 | 0.09 |
| 1 | 0.11 |
| 13 | 0.13 |
| 3 | 0.22 |
| 11 | 0.27 |
| 10 | 0.36 |
| 7 | 0.44 |
| 4 | 0.56 |



## Iterative Simplification

- 2D example



priority queue

| edge | cost |
|---|---|
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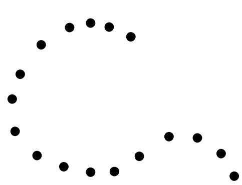
## Iterative Simplification

- 2D example



priority queue

| edge | cost |
|---|---|
| 6 | 0.02 |
| 2 | 0.03 |
| 14 | 0.04 |
| 5 | 0.06 |
| 9 | 0.09 |
| 1 | 0.11 |
| 13 | 0.13 |
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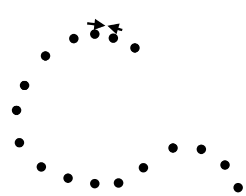
## Iterative Simplification

- 2D example



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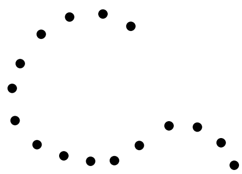
## Iterative Simplification

- 2D example



priority queue

| edge | cost |
|---|---|
| 2 | 0.03 |
| 14 | 0.04 |
| 5 | 0.06 |
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| 13 | 0.13 |
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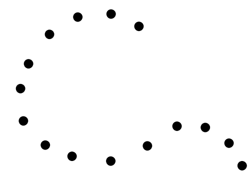
## Iterative Simplification

- 2D example



priority queue

| edge | cost |
|---|---|
| 14 | 0.04 |
| 5 | 0.06 |
| 9 | 0.09 |
| 1 | 0.11 |
| 13 | 0.13 |
| 3 | 0.23 |
| 11 | 0.27 |
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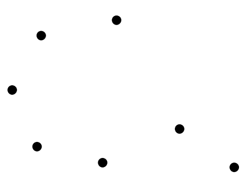
## Iterative Simplification

- 2D example



priority queue

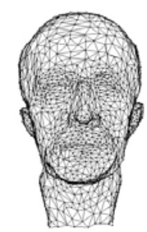
| edge | cost |
|---|---|
| 11 | 0.27 |
| 10 | 0.36 |
| 7 | 0.49 |
| 4 | 0.56 |



## Iterative Simplification

original model (296,850 points)

simplified model (2,000 points)



remaining point pair contraction candidates



## Particle Simulation

- Resample surface by distributing particles on the surface
- Particles move on surface according to inter-particle repelling forces
- Particle relaxation terminates when equilibrium is reached (requires damping)
- Can also be used for up-sampling!



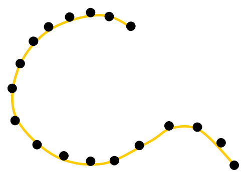
## Particle Simulation

- Initialization
  - Randomly spread particles
- Repulsion
  - Linear repulsion force $F_i(\mathbf{p}) = k(r - \|\mathbf{p}-\mathbf{p}_i\|)\cdot(\mathbf{p}-\mathbf{p}_i)$

  ⇨ only need to consider neighborhood of radius r
- Projection
  - Keep particles on surface by projecting onto tangent plane of closest point
  - Apply full MLS projection at end of simulation

## Particle Simulation

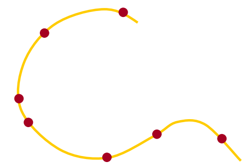
- 2D example





## Particle Simulation

- 2D example
  - Initialization
    - randomly spread particles

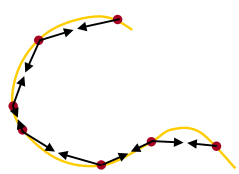




## Particle Simulation

- 2D example
  - Initialization
    - randomly spread particles
  - Repulsion
    - linear repulsion force

$$F_i(\mathbf{p}) = k(r - \|\mathbf{p}-\mathbf{p}_i\|)\cdot(\mathbf{p}-\mathbf{p}_i)$$

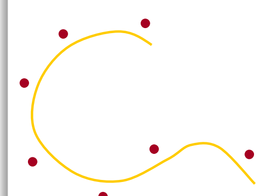




## Particle Simulation

- 2D example
  - Initialization
    - randomly spread particles
  - Repulsion
    - linear repulsion force

$$F_i(\mathbf{p}) = k(r - \|\mathbf{p}-\mathbf{p}_i\|)\cdot(\mathbf{p}-\mathbf{p}_i)$$



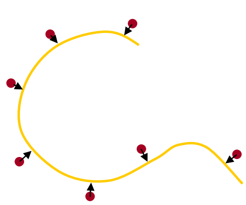


## Particle Simulation

- 2D example
  - Initialization
    - randomly spread particles
  - Repulsion
    - linear repulsion force

$$F_i(\mathbf{p}) = k(r - \|\mathbf{p}-\mathbf{p}_i\|)\cdot(\mathbf{p}-\mathbf{p}_i)$$

  - Projection
    - project particles onto surface





## Particle Simulation

- 2D example
  - Initialization
    - randomly spread particles
  - Repulsion
    - linear repulsion force

$$F_i(\mathbf{p}) = k(r - \|\mathbf{p}-\mathbf{p}_i\|)\cdot(\mathbf{p}-\mathbf{p}_i)$$

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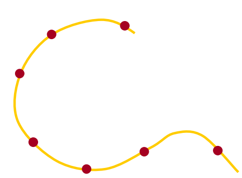


## Particle Simulation

- Adaptive simulation
  - Adjust repulsion radius according to surface variation
    ⇨ more samples in regions of high variation

variation estimation

simplified model (3,000 points)



## Particle Simulation

- User-controlled simulation
  - Adjust repulsion radius according to user input

uniform

original

selective



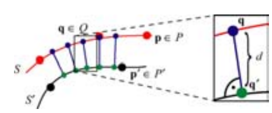
## Measuring Error

- Measure the distance between two point-sampled surfaces using a sampling approach
- Maximum error: $\Delta_{\max}(S, S') = \max_{\mathbf{q} \in Q} d(\mathbf{q}, S')$
  ⇨ Two-sided Hausdorff distance
- Mean error: $\Delta_{\text{avg}}(S, S') = \frac{1}{|Q|} \sum_{\mathbf{q} \in Q} d(\mathbf{q}, S')$
  ⇨ Area-weighted integral of point-to-surface distances
- $Q$ is an up-sampled version of the point cloud that describes the surface $S$



## Measuring Error

- $d(\mathbf{q}, S')$ measures the distance of point $\mathbf{q}$ to surface $S'$ using the MLS projection operator with linear basis functions






## Measuring Error

original

simplified

upsampled

error



## Comparison

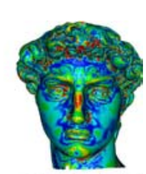
- Error estimate for Michelangelo's David simplified from 2,000,000 points to 5,000 points

$\Delta_{avg} = 6.14 \cdot 10^{-4}$ $\Delta_{max} = 0.0046$

adaptive hierarchical clustering

$\Delta_{avg} = 5.43 \cdot 10^{-4}$ $\Delta_{max} = 0.0052$

iterative simplification

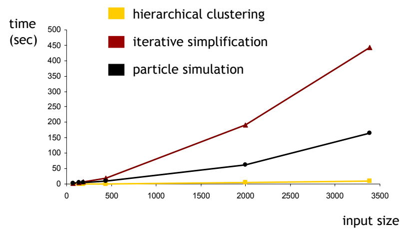


$\Delta_{avg} = 5.69 \cdot 10^{-4}$ $\Delta_{max} = 0.0061$

particle simulation

## Comparison

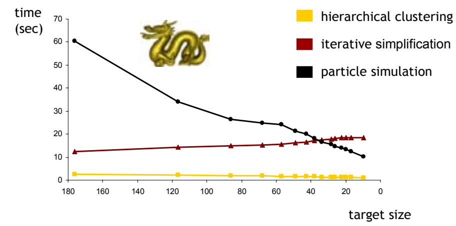
- Execution time as a function of input model size (reduction to 1%)






## Comparison

- Execution time as a function of target model size (input: dragon, 535,545 points)





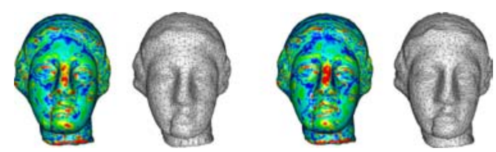

## Comparison

- Summary

| | Efficiency | Surface Error | Control | Implementation |
|---|---|---|---|---|
| Hierarchical Clustering | + | - | - | + |
| Iterative Simplification | - | + | o | o |
| Particle Simulation | o | + | + | - |



## Point-based vs. Mesh Simplification



point-based simplification with subsequent mesh reconstruction

mesh reconstruction with subsequent mesh simplification (QSlim)

⇨ point-based simplification saves an expensive surface reconstruction on the dense point cloud!



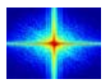
## References

- Pauly, Gross: *Efficient Simplification of Point-sampled Surfaces*, IEEE Visualization 2002
- Shaffer, Garland: *Efficient Adaptive Simplification of Massive Meshes*, IEEE Visualization 2001
- Garland, Heckbert: *Surface Simplification using Quadric Error Metrics*, SIGGRAPH 1997
- Turk: *Re-Tiling Polygonal Surfaces*, SIGGRAPH 1992
- Alexa et al. *Point Set Surfaces*, IEEE Visualization 2001

# Eurographics 2003

## Spectral Processing of Point-Sampled Geometry

Markus Gross



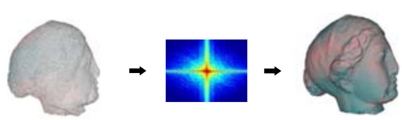
## Overview

- Introduction
- Fourier transform
- Spectral processing pipeline
- Applications
  - Spectral filtering
  - Adaptive subsampling
- Summary



## Introduction

- Idea: Extend the Fourier transform to manifold geometry



⇨ Spectral representation of point-based objects

⇨ Powerful methods for digital geometry processing



## Introduction

- Applications:
  - Spectral filtering:
    - Noise removal
    - Microstructure analysis
    - Enhancement
  - Adaptive resampling:
    - Complexity reduction
    - Continuous LOD



## Fourier Transform

- 1D example:

$$\underbrace{X_n}_{\text{output signal}} = \sum_{k=1}^{N} \underbrace{x_k}_{\text{input signal}} \underbrace{e^{-j2\pi\frac{nk}{N}}}_{\text{spectral basis function}}$$

- Benefits:
  - Sound concept of frequency
  - Extensive theory
  - Fast algorithms



## Fourier Transform

- Requirements:
  - Fourier transform defined on Euclidean domain
    - ⇨we need a global parameterization
  - Basis functions are eigenfunctions of Laplacian operator
    - ⇨requires regular sampling pattern so that basis functions can be expressed in analytical form (fast evaluation)
- Limitations:
  - Basis functions are globally defined
    - ⇨Lack of local control

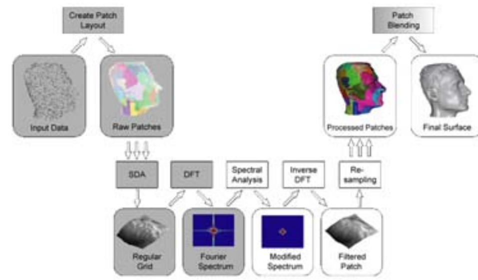
# Approach

- Split model into patches that:
  - are parameterized over the unit-square
    - ⇨ mapping must be continuous and should minimize distortion
  - are re-sampled onto a regular grid
    - ⇨ adjust sampling rate to minimize information loss
  - provide sufficient granularity for intended application (local analysis)

⇨ process each patch individually and blend processed patches



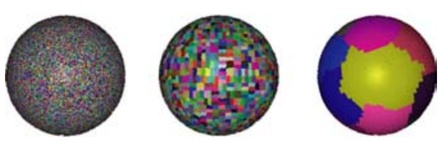
# Spectral Pipeline






# Patch Layout Creation

Clustering ⇨ Optimization



Samples ⇨ Clusters ⇨ Patches



# Patch Layout Creation

- Iterative, local optimization method
- Merge patches according to quality metric:

$$\Phi = \Phi_S \cdot \Phi_{NC} \cdot \Phi_B \cdot \Phi_{Reg}$$

$\Phi_S$ ⇨ patch Size

$\Phi_{NC}$ ⇨ curvature

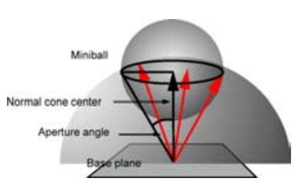
$\Phi_B$ ⇨ patch boundary

$\Phi_{Reg}$ ⇨ spring energy regularization



# Patch Layout Creation

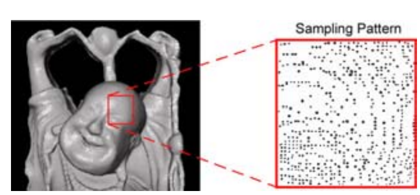
- Parameterize patches by orthogonal projection onto base plane
- Bound normal cone to control distortion of mapping using smallest enclosing sphere





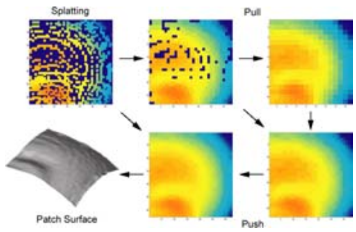

# Patch Resampling

- Patches are irregularly sampled:



## Patch Resampling

- Resample patch onto regular grid using hierarchical push-pull filter (scattered data approximation)






## Spectral Analysis

- 2D discrete Fourier transform (DFT)
  - ⇨ Direct manipulation of spectral coefficients
- Filtering as convolution:

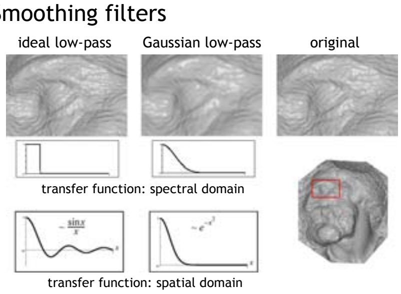
$$F(x \otimes y) = F(x) \cdot F(y)$$

  - ⇨ Convolution: $O(N^2)$ ⇨ multiplication: $O(N)$
- Inverse Fourier transform
  - ⇨ Filtered patch surface



## Spectral Filters

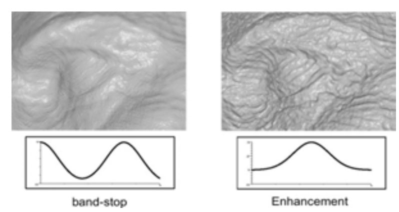
- Smoothing filters






## Spectral Filters

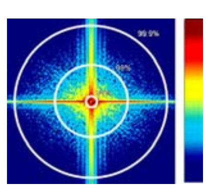
- Microstructure analysis and enhancement






## Spectral Resampling

- Low-pass filtering
  - ⇨ Band-limitation
- Regular Resampling
  - ⇨ Optimal sampling rate (sampling theorem)
  - ⇨ Error control (Parseval's theorem)

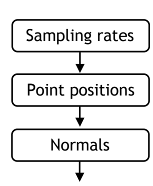


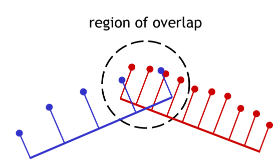
Power Spectrum



## Reconstruction

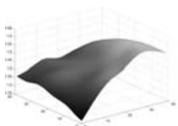
- Filtering can lead to discontinuities at patch boundaries
  - ⇨ Create patch overlap, blend adjacent patches



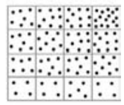
# Reconstruction

- Blending the sampling rate



blended sampling rate in region of patch overlap →

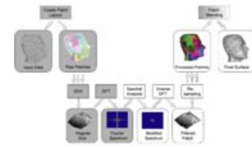
discretized sampling rate on regular grid →

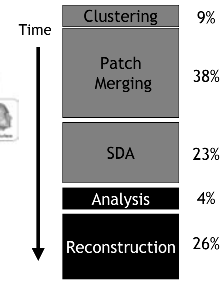


pre-computed sampling patterns



# Timings

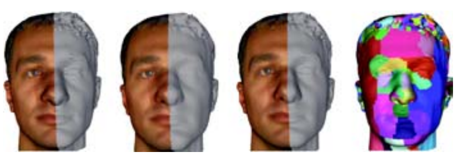







# Applications

- Surface Restoration

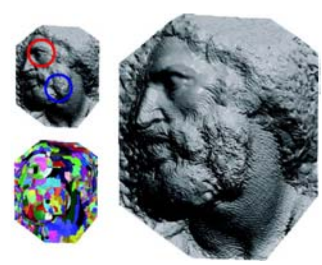


Original Gaussian low-pass Wiener filter Patch layout



# Applications

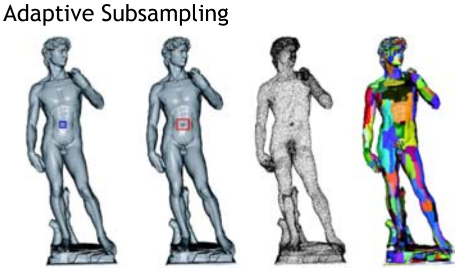
- Interactive filtering





# Applications

- Adaptive Subsampling



4,128,614 pts. = 100% 287,163 pts. = 6.9%



# Summary

- Versatile spectral decomposition of point-based models
- Effective filtering
- Adaptive resampling
- Efficient processing of large point-sampled models

# Reference

- Pauly, Gross: *Spectral Processing of Point-sampled Geometry*, SIGGRAPH 2001

## Eurographics 2003

pointshop

An Interactive System for Point-based Surface Editing

## Overview

- Introduction
- Pointshop3D System Components
  - Point Cloud Parameterization
  - Resampling Scheme
  - Editing Operators
- Summary

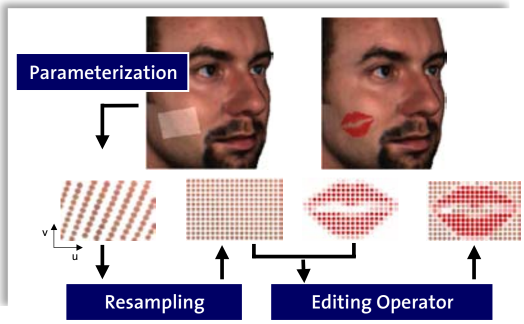


## PointShop3D

- Interactive system for point-based surface editing
- Generalizes 2D photo editing concepts and functionality to 3D point-sampled surfaces
- Uses 3D surface pixels (*surfels*) as versatile display and modeling primitive



## Concept






## Key Components

- Point cloud parameterization $\Phi$
  - brings surface and brush into common reference frame
- Dynamic resampling $\Psi$
  - creates one-to-one correspondence of surface and brush samples
- Editing operator $\Omega$
  - combines surface and brush samples

$$\underbrace{S'}_{\text{modified surface}} = \Omega(\Psi(\Phi(\underbrace{S}_{\text{original surface}})), \Psi(\underbrace{B}_{\text{brush}}))$$



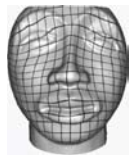
## Parameterization

- Constrained minimum distortion parameterization of point clouds

$$\mathbf{u} \in [0,1]^2 \Rightarrow X(\mathbf{u}) = \begin{bmatrix} x(\mathbf{u}) \\ y(\mathbf{u}) \\ z(\mathbf{u}) \end{bmatrix} = \mathbf{x} \in P \subset R^3$$

## Parameterization

contraints = matching
of feature points



minimum distortion =
maximum smoothness



## Parameterization

- Find mapping X that minimizes objective function:

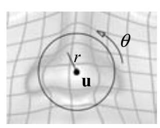
$$C(X) = \underbrace{\sum_{j \in M} (X(\overset{\text{brush points}}{\mathbf{p}_j}) - \overset{\text{surface points}}{\mathbf{x}_j})^2}_{\text{fitting constraints}} + \varepsilon \underbrace{\int_P \gamma(\mathbf{u})\, d\mathbf{u}}_{\text{distortion}}$$



## Parameterization

- Measuring distortion

$$\gamma(\mathbf{u}) = \int_\theta \left( \frac{\partial^2}{\partial r^2} X_{\mathbf{u}}(\theta, r) \right)^2 d\theta$$




- Integrates squared curvature using local polar re-parameterization

$$X_{\mathbf{u}}(\theta, r) = X\left( \mathbf{u} + r \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix} \right)$$



## Parameterization

- Discrete formulation:

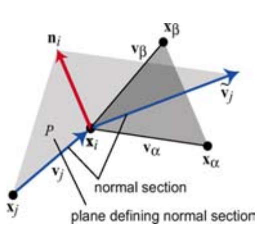
$$\tilde{C}(U) = \sum_{j \in M} (\mathbf{p}_j - \mathbf{u}_j)^2 + \varepsilon \sum_{i=1}^{n} \sum_{j \in N_i} \left( \frac{\partial U(\mathbf{x}_i)}{\partial \mathbf{v}_j} - \frac{\partial U(\mathbf{x}_i)}{\partial \tilde{\mathbf{v}}_j} \right)^2$$

- Approximation: mapping is piecewise linear



## Parameterization

- Directional derivatives as extension of divided differences based on k-nearest neighbors

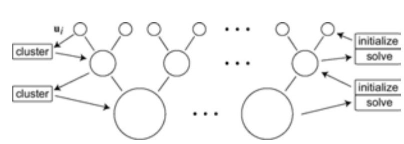





## Parameterization

- Multigrid solver for efficient computation of resulting sparse linear least squares problem

$$\tilde{C}(U) = \sum_j \left( \mathbf{b}_j - \sum_{i=1}^{n} a_{j,i} \mathbf{u}_i \right)^2 = \|\mathbf{b} - A\mathbf{u}\|^2$$



# Reconstruction

- Parameterized scattered data approximation

fitting functions

weight functions

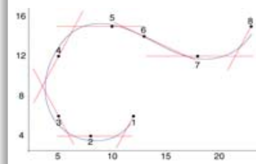
$$X(\mathbf{u}) = \frac{\sum_i \Phi_i(\mathbf{u}) r_i(\mathbf{u})}{\sum_i r_i(\mathbf{u})}$$

normalization factor

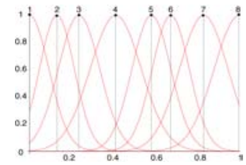
- Fitting functions
  - Compute local fitting functions using local parameterizations
  - Map to global parameterization using global parameter coordinates of neighboring points



# Reconstruction




reconstruction with linear fitting functions




weight functions in parameter space



# Reconstruction

- Reconstruction with linear fitting functions is equivalent to surface splatting!
  - ⇨ we can use the surface splatting renderer to reconstruct our surface function (see chapter on rendering)
  - This provides:
    - Fast evaluation
    - Anti-aliasing (Band-limit the weight functions before sampling using Gaussian low-pass filter)
  - Distortions of splats due to parameterization can be computed efficiently using local affine mappings



# Sampling

- Three sampling strategies:
  - Resample the brush, i.e., sample at the original surface points
  - Resample the surface, i.e., sample at the brush points
  - Adaptive resampling, i.e., sample at surface or brush points depending on the respective sampling density



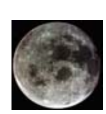
# Editing Operators

- Painting
  - Texture, material properties, transparency

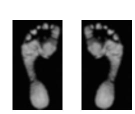


# Editing Operators

- Sculpting
  - Carving, normal displacement



texture map

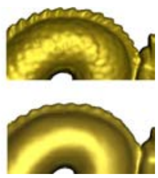


displacement maps

carved and texture mapped point-sampled surface

# Editing Operators

- Filtering
  - Scalar attributes, geometry





# Summary

- Pointshop3D provides sophisticated editing operations on point-sampled surfaces
  ⇨ points are a versatile and powerful modeling primitive
- Limitation: only works on “clean” models
  - sufficiently high sampling density
  - no outliers
  - little noise
  ⇨ requires model cleaning (integrated or as pre-process)



# Reference

- Zwicker, Pauly, Knoll, Gross: *Pointshop3D: An interactive system for Point-based Surface Editing*, SIGGRAPH 2002

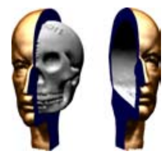
pointshop

- check out:

**www.pointshop3D.com**

# Eurographics 2003

## Shape Modeling



Mark Pauly

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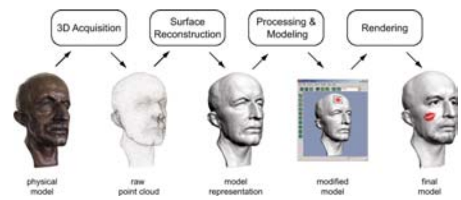
## Motivation

- 3D content creation pipeline



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## Motivation

- Surface representations
  - Implicit surfaces
    - Level sets
    - Radial basis functions → + Extreme deformations, + Changes of topology
    - Algebraic surfaces
  - Parametric surfaces
    - Polygonal meshes
    - Subdivision surfaces → + Sharp features, + Efficient rendering, + Intuitive Editing
    - NURBS



---

## Motivation

- Surface representations
  - Implicit surfaces
    - Level sets
    - Radial basis functions
    - Algebraic surfaces
  - Parametric surfaces
    - Polygonal meshes
    - Subdivision surfaces
    - Nurbs

Hybrid Representation

- Explicit cloud of point samples
- Implicit dynamic surface model



---

## Motivation

- Point cloud representation
  - Minimal consistency requirements for extreme deformations (dynamic re-sampling)
  - Fast inside/outside classification for boolean operations and collision detection
  - Explicit modeling and rendering of sharp feature curves
  - Integrated, intuitive editing of shape and appearance

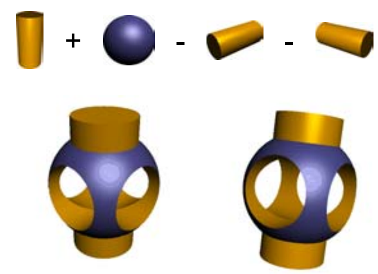


---

## Interactive Modeling

- Interactive design and editing of point-sampled models
  - Shape Modeling
    - Boolean operations
    - Free-form deformation
  - Appearance Modeling
    - Painting & texturing
    - Embossing & engraving

## Boolean Operations






## Boolean Operations

- Create new shapes by combining existing models using union, intersection, or difference operations
- Powerful and flexible editing paradigm mostly used in industrial design applications (CAD/CAM)



## Boolean Operations

- Easily performed on implicit representations
  - Requires simple computations on the distance function
- Difficult for parametric surfaces
  - Requires surface-surface intersection
- Topological complexity of resulting surface depends on geometric complexity of input models



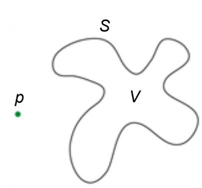
## Boolean Operations

- Point-Sampled Geometry
  - Classification
    - Inside-outside test using signed distance function induced by MLS projection
  - Sampling
    - Compute exact intersection of two MLS surfaces to sample the intersection curve
  - Rendering
    - Accurate depiction of sharp corners and creases using point-based rendering



## Boolean Operations

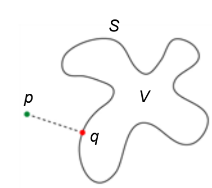
- Classification:
  - given a smooth, closed surface $S$ and point $p$. Is $p$ inside or outside of the volume $V$ bounded by $S$?





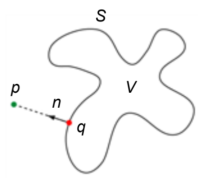

## Boolean Operations

- Classification:
  - given a smooth, closed surface $S$ and point $p$. Is $p$ inside or outside of the volume $V$ bounded by $S$?
  1. find closest point $q$ on $S$



## Boolean Operations

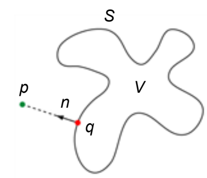
- Classification:
  - given a smooth, closed surface $S$ and point $p$. Is $p$ inside or outside of the volume $V$ bounded by $S$?
  1. find closest point $q$ on $S$
  2. $d=(p-q)\cdot n$ defines signed distance of $p$ to $S$






## Boolean Operations

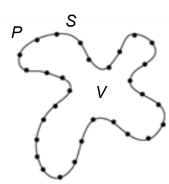
- Classification:
  - given a smooth, closed surface $S$ and point $p$. Is $p$ inside or outside of the volume $V$ bounded by $S$?
  1. find closest point $q$ on $S$
  2. $d=(p-q)\cdot n$ defines signed distance of $p$ to $S$
  3. classify $p$ as
     - inside $V$, if $d < 0$
     - outside $V$, if $d > 0$






## Boolean Operations

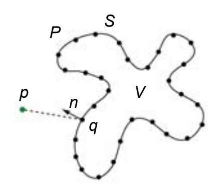
- Classification:
  - represent smooth surface $S$ by point cloud $P$






## Boolean Operations

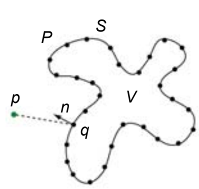
- Classification:
  - represent smooth surface $S$ by point cloud $P$
  1. find closest point $q$ in $P$






## Boolean Operations

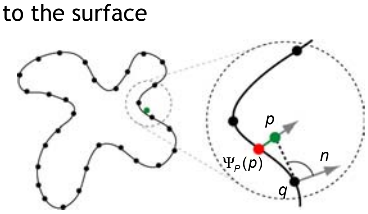
- Classification:
  - represent smooth surface $S$ by point cloud $P$
  1. find closest point $q$ in $P$
  2. classify $p$ as
     - inside $V$, if $(p-q)\cdot n < 0$
     - outside $V$, if $(p-q)\cdot n > 0$





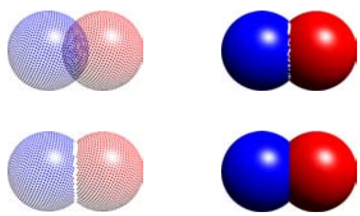

## Boolean Operations

- Classification:
  - apply full MLS projection for points close to the surface



# Boolean Operations

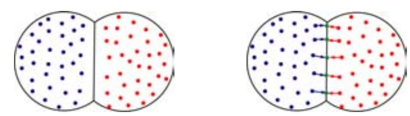
- Sampling the intersection curve





# Boolean Operations

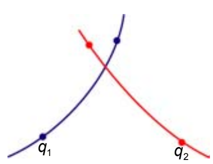
- Newton scheme:
  1. identify pairs of closest points





# Boolean Operations

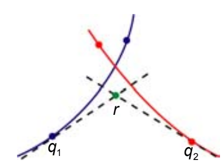
- Newton scheme:
  1. identify pairs of closest points






# Boolean Operations

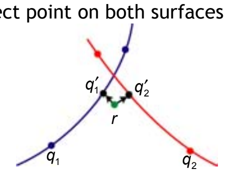
- Newton scheme:
  1. identify pairs of closest points
  2. compute closest point on intersection of tangent spaces






# Boolean Operations

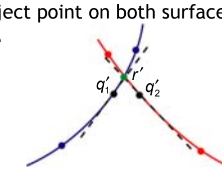
- Newton scheme:
  1. identify pairs of closest points
  2. compute closest point on intersection of tangent spaces
  3. re-project point on both surfaces





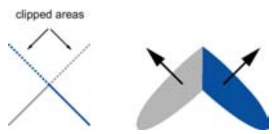

# Boolean Operations

- Newton scheme:
  1. identify pairs of closest points
  2. compute closest point on intersection of tangent spaces
  3. re-project point on both surfaces
  4. iterate



## Boolean Operations

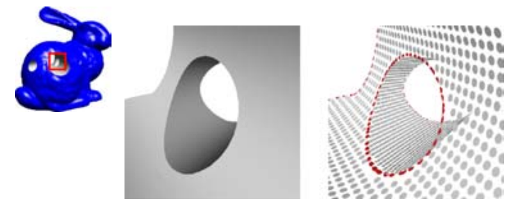
- Rendering sharp creases
  - represent points on intersection curve with two surfels that mutually clip each other






## Boolean Operations

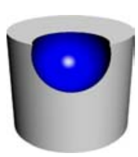
- Rendering sharp creases





## Boolean Operations

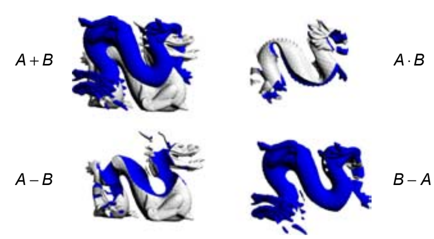
- Rendering sharp creases
  - easily extended to handle corners by allowing multiple clipping





## Boolean Operations

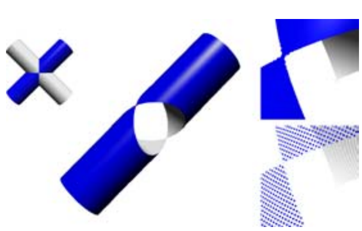
- Boolean operations can create intricate shapes with complex topology






## Boolean Operations

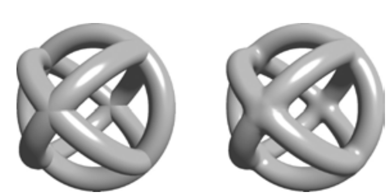
- Singularities lead to numerical instabilities (intersection of almost parallel planes)





## Boolean Operations

- Sharp creases can be blended using oriented particles (Szeliski, Tonnesen)

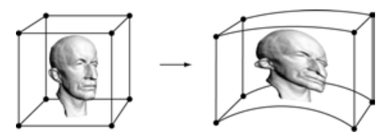


## Free-form Deformation



## Free-form Deformation

- Smooth deformation field $F: \mathbf{R}^3 \rightarrow \mathbf{R}^3$ that warps 3D space
- Can be applied directly to point samples





## Free-form Deformation

- How to define the deformation field?
  - ⇨ Painting metaphor
- How to detect and handle self-intersections?
  - ⇨ Point-based collision detection, boolean union, particle-based blending
- How the handle strong distortions?
  - ⇨ Dynamic re-sampling

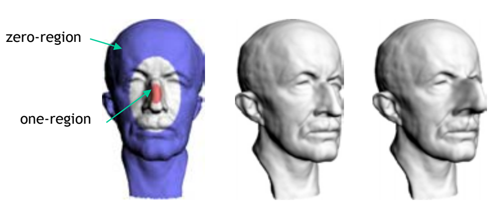


## Free-form Deformation

- Intuitive editing paradigm using painting metaphor
  - Define rigid surface part (zero-region) and handle (one-region) using interactive painting tool
  - Displace handle using combination of translation and rotation
  - Create smooth blend towards zero-region



## Free-form Deformation



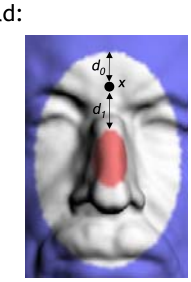

original surface

deformed surface



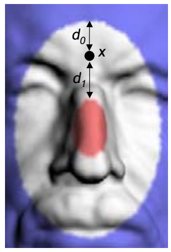
## Free-form Deformation

- Definition of deformation field:
  - Continuous scale parameter $t_x$
    - $t_x = \beta(d_0 / (d_0 + d_1))$
    - $d_0$ : distance of $x$ to zero-region
    - $d_1$ : distance of $x$ to one-region
  - Blending function
    - $\beta : [0,1] \rightarrow [0,1]$
    - $\beta \in C^0$, $\beta(0) = 0$, $\beta(1) = 1$
  - $t_x = 0$ if $x$ in zero-region
  - $t_x = 1$ if $x$ in one-region



# Free-form Deformation

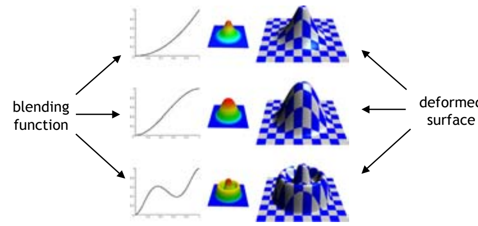
- Definition of deformation field:
  - Deformation function
    - $F(x) = F_T(x) + F_R(x)$
  - Translation
    - $F_T(x) = x + t_x \cdot v$
  - Rotation
    - $F_R(x) = M(t_x) \cdot x$






# Free-form Deformation

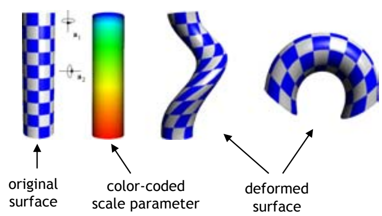
- Translation for three different blending functions






# Free-form Deformation

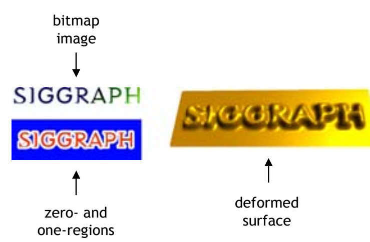
- Rotational deformation along two different rotation axes






# Free-form Deformation

- Embossing effect





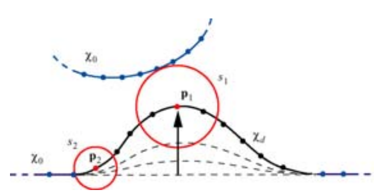

# Collision Detection

- Deformations can lead to self-intersections
- Apply boolean inside/outside classification to detect collisions
- Restricted to collisions between deformable region and zero-region to ensure efficient computations



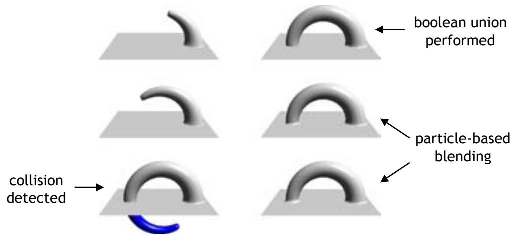
# Collision Detection

- Exploiting temporal coherence



## Collision Detection

- Interactive modeling session





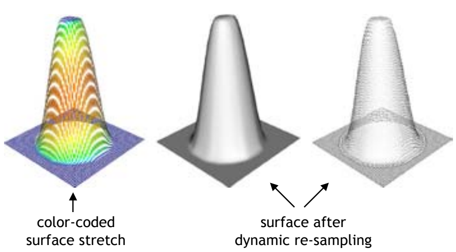

## Dynamic Sampling

- Large model deformations can lead to strong surface distortions
- Requires adaptation of the sampling density
- Dynamic insertion and deletion of point samples



## Dynamic Sampling

- Surface distortion varies locally





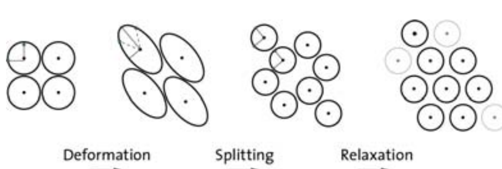

## Dynamic Sampling

1. Measure local surface stretch from first fundamental form
2. Split samples that exceed stretch threshold
3. Regularize distribution by relaxation
4. Interpolate scalar attributes



## Dynamic Sampling

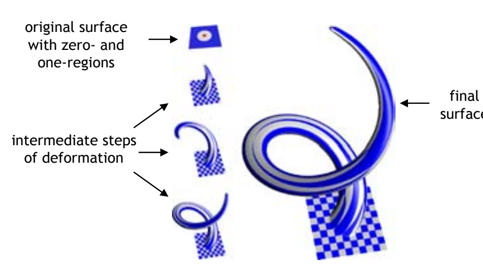
- 2D illustration






## Free-form Deformation

- Interactive modeling session with dynamic sampling



# Results

- 3D shape modeling functionality has been integrated into Pointshop3D to create a complete system for point-based shape and appearance modeling
  - Boolean operations
  - Free-form deformation
  - Painting & texturing
  - Sculpting
  - Filtering
  - Etc.



# Results

- Ab-initio design of an Octopus
  - Free-form deformation with dynamic sampling from 69,706 to 295,222 points



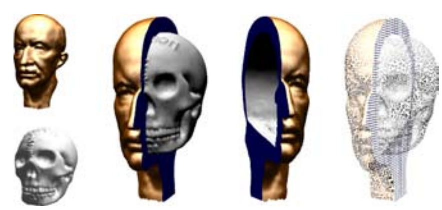
# Results

- Modeling with synthetic and scanned data
  - Combination of free-form deformation with collision detection, boolean operations, particle-based blending, embossing and texturing



# Results

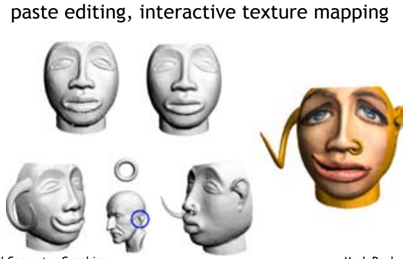
- Boolean operations on scanned data
  - Irregular sampling pattern, low resolution models





# Results

- Interactive modeling with scanned data
  - noise removal, free-form deformation, cut-and-paste editing, interactive texture mapping





# Conclusion

- Points are a versatile shape modeling primitive
  - Combines advantages of implicit and parametric surfaces
  - Integrates boolean operations and free-form deformation
  - Dynamic restructuring
  - Time and space efficient implementations

## Conclusion

- Complete and versatile point-based 3D shape and appearance modeling system
  - Directly applicable to scanned data
  - Suitable for low-cost 3D content creation and rapid proto-typing



## References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- www.pointshop3d.com