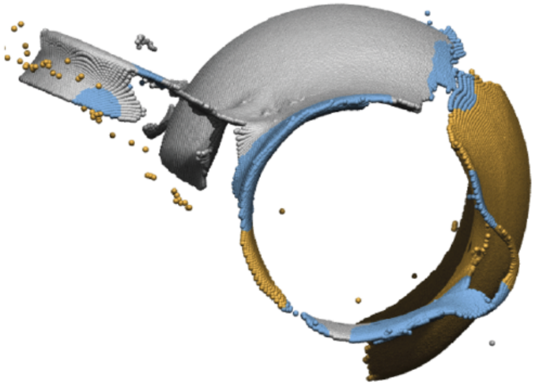
4-Points Congruent Sets for Robust Pairwise Surface Registration



Dror Aiger

Ben Gurion University

Niloy J. Mitra

IIT Delhi / TU Vienna

Daniel Cohen-Or

Tel Aviv University

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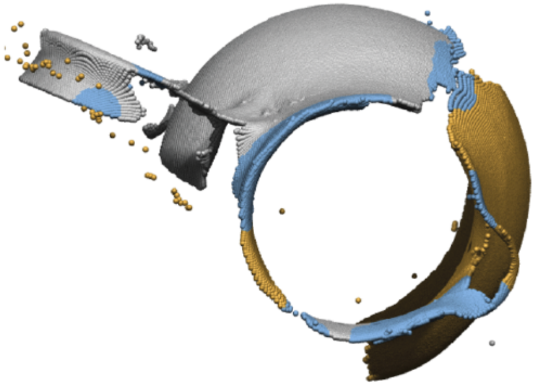

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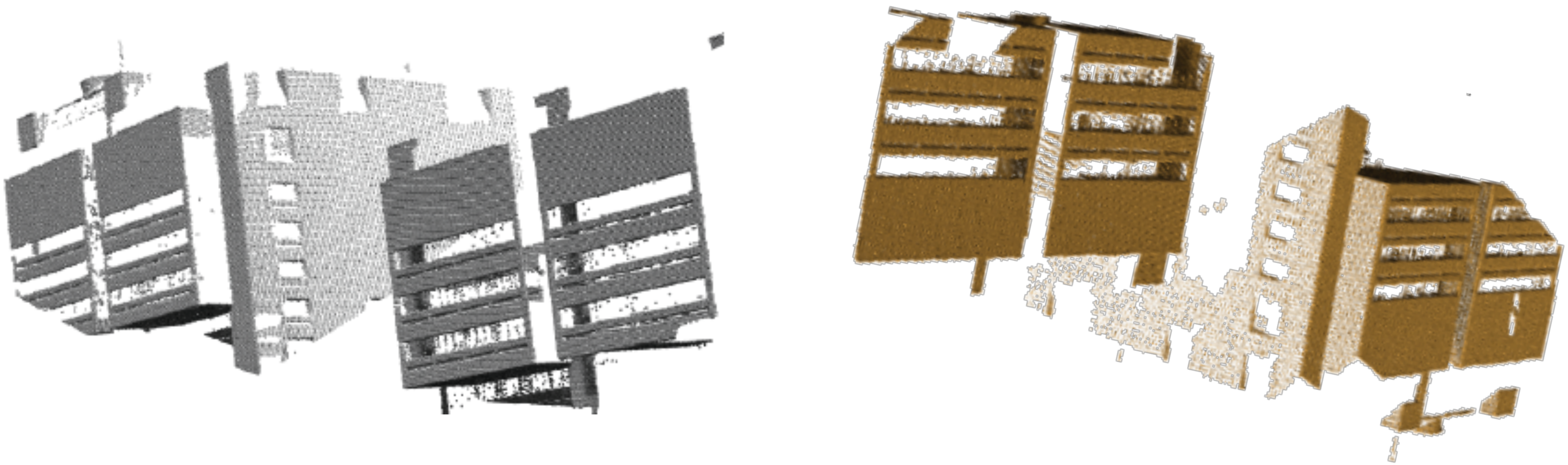
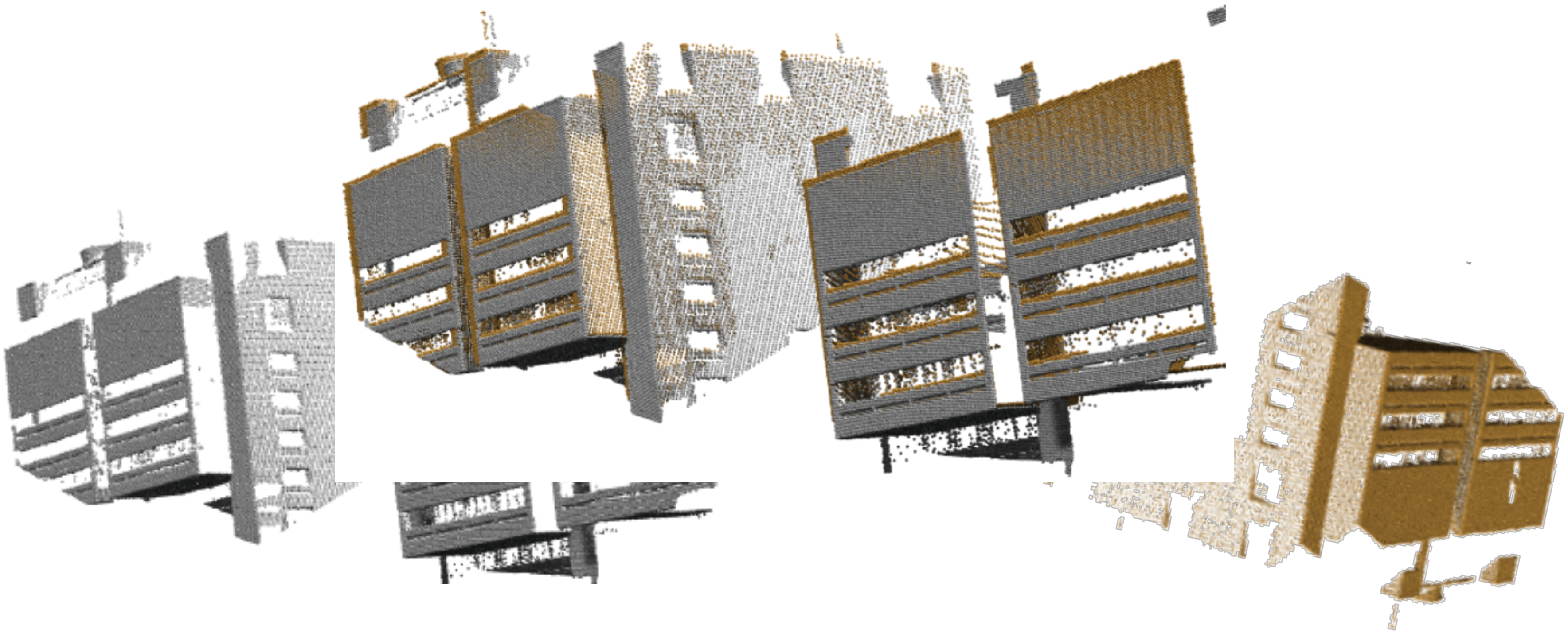
Problem Statement

Given: Two models



Problem Statement

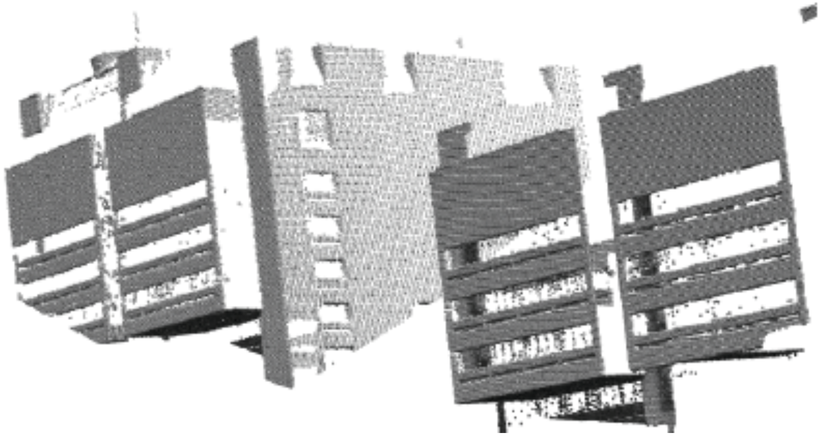
Goal: *Automatically* align the models



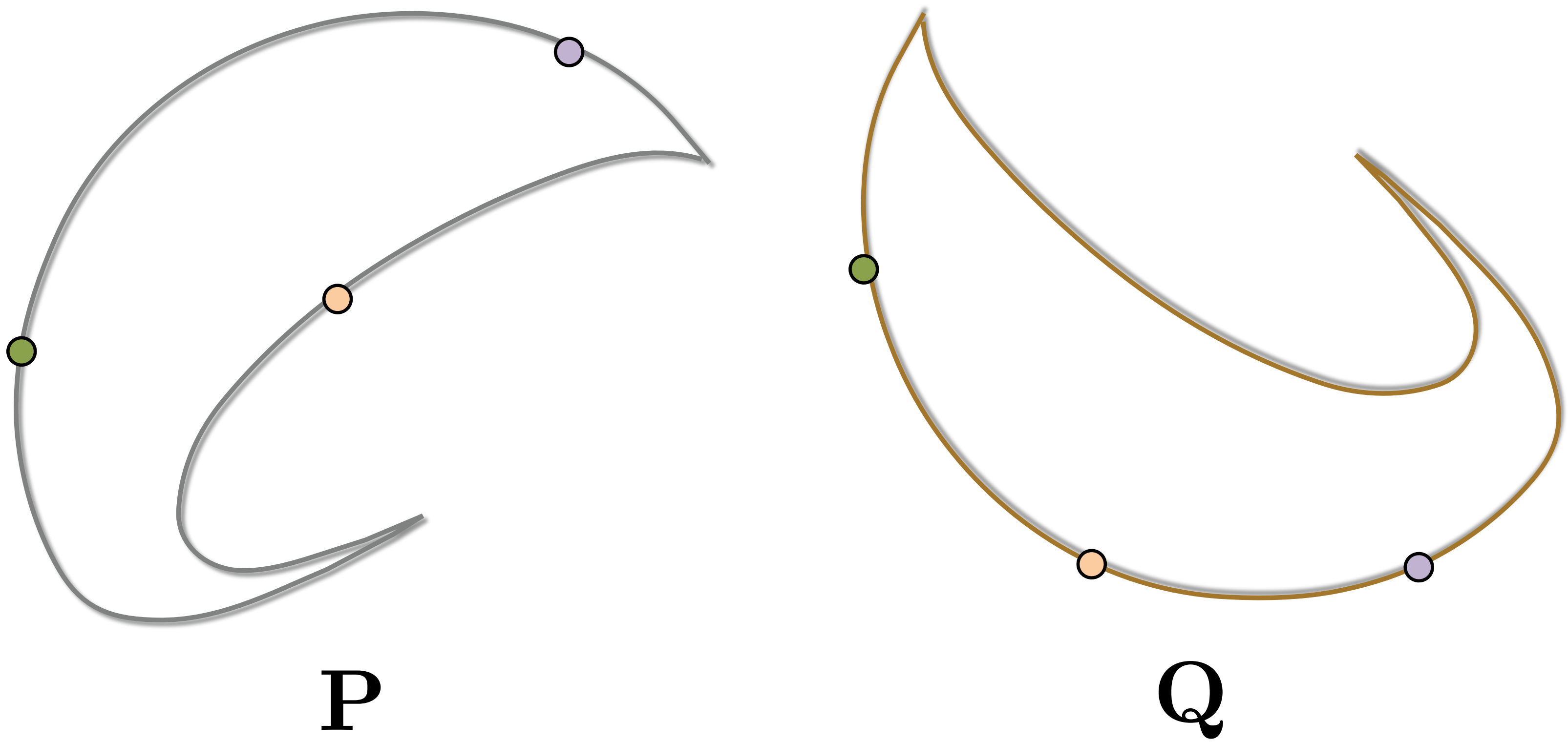
Why is this Hard?

Given: Two scans

- corrupted with ***noise*** and ***outliers***
- in **arbitrary initial poses** with ***unknown overlap***



Alignment Approach



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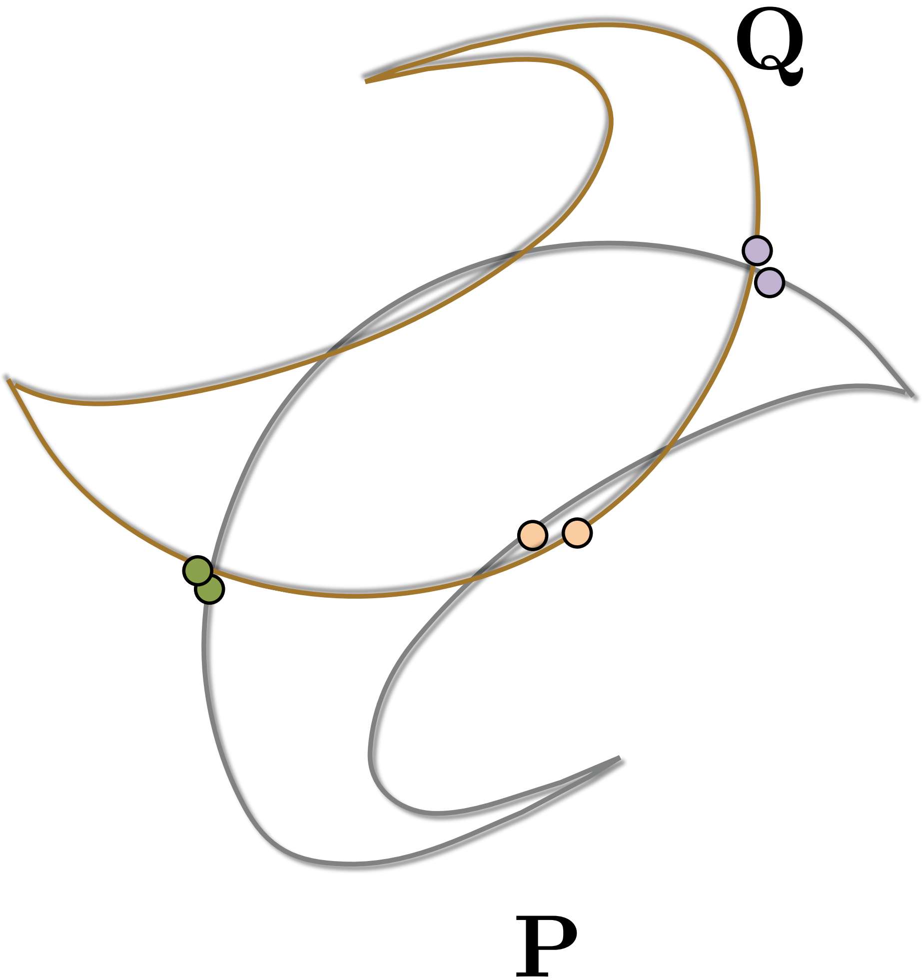

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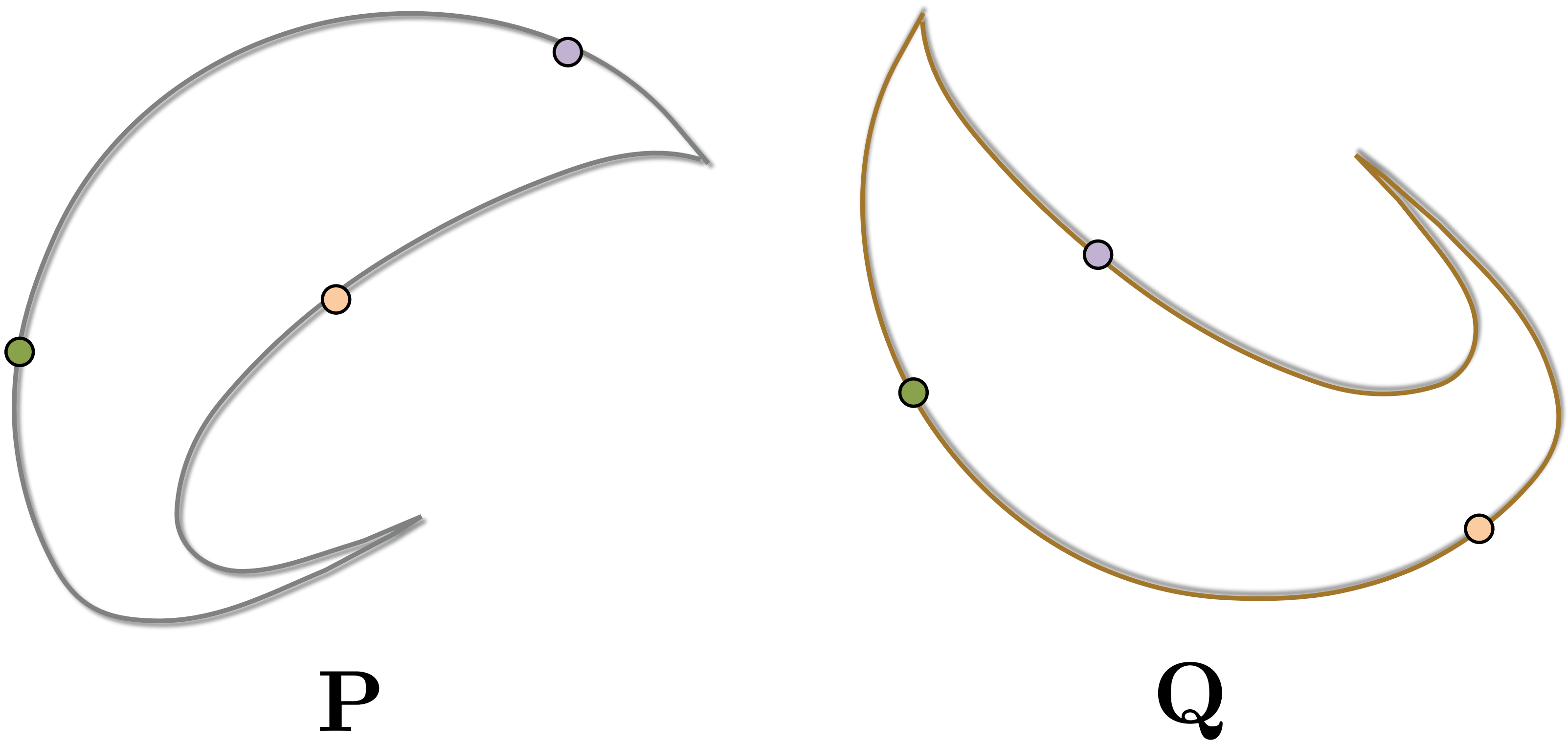

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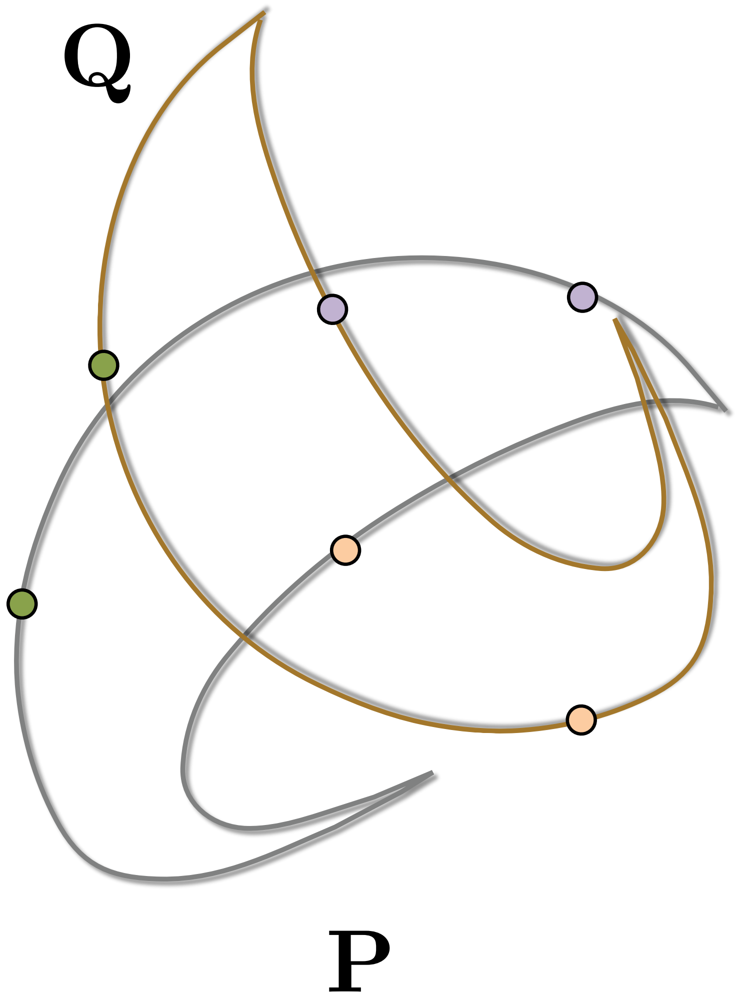
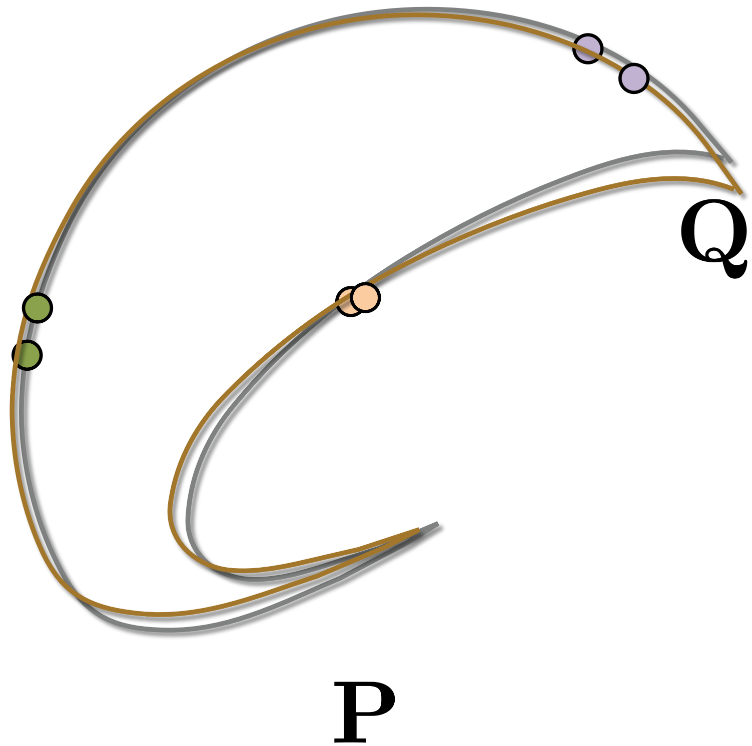


Alignment Approach

• • •

Alignment Approach



guess and verify

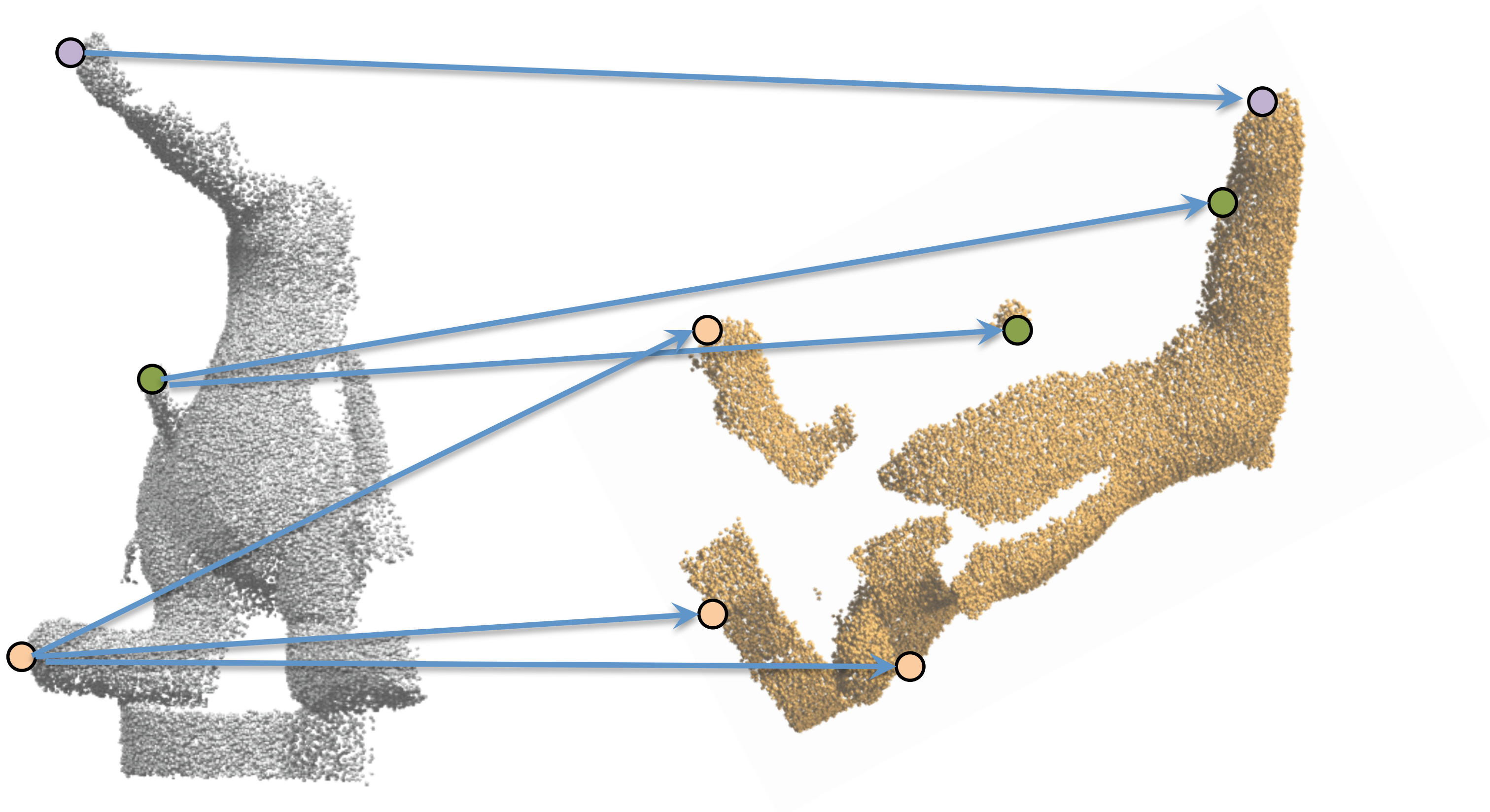
Partial Matching

Improvements

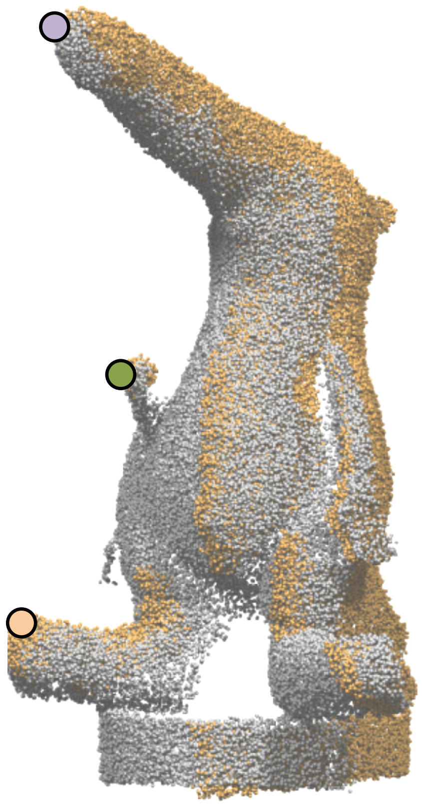
- Naïve approach $\longrightarrow$ too expensive
- RANSAC
  - test only a few (correspondences) ***guesses***
  - keep the ***best*** transform
- Look only at ***interesting*** points

Aligning with Feature Points (FP)

Aligning with Feature Points

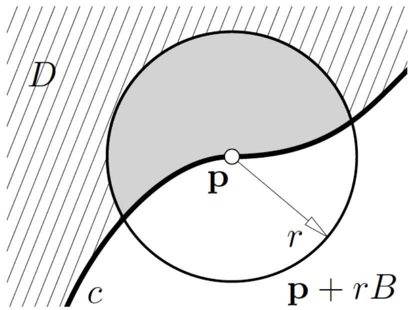


Aligning with Feature Points



Noise, Outliers, and Holes

- Feature points $\longrightarrow$ differential entities
  - *noise + outliers* unstable FPs
    wrong correspondence
  - integral invariants
    - overlapping regions
    - *missing* data

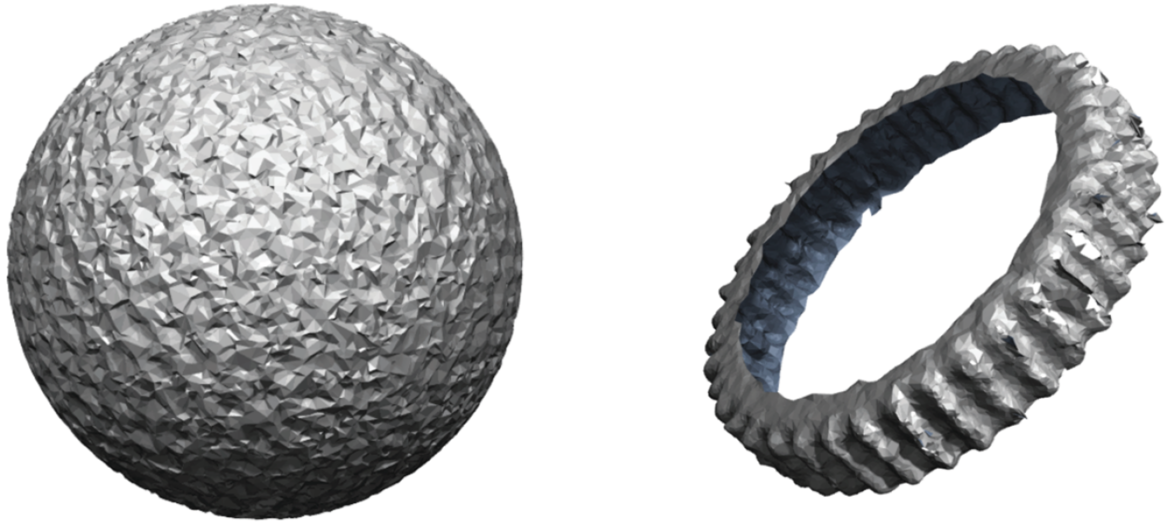


[Pottmann et al. `07]

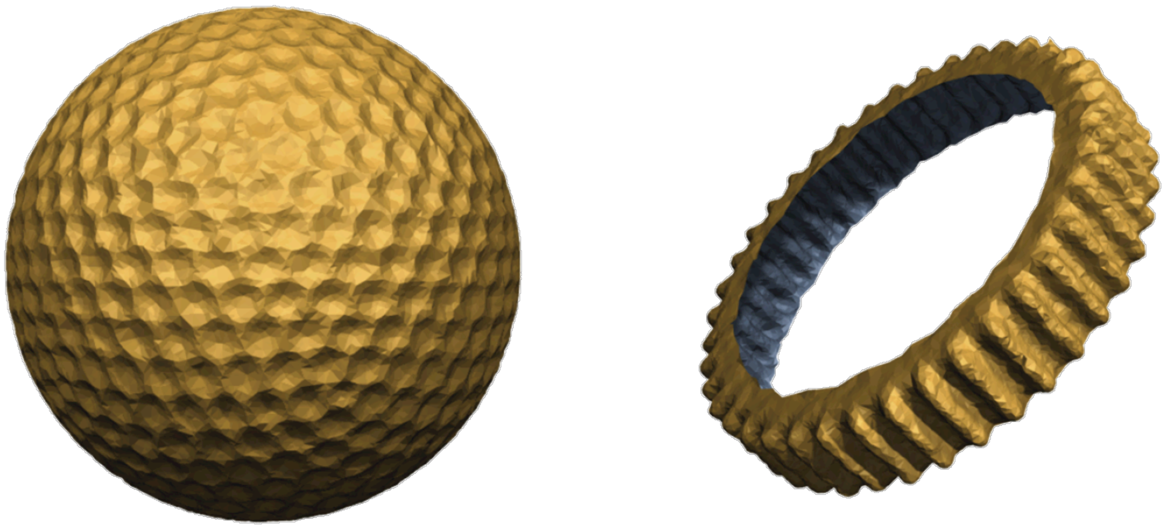
- Noise + outliers + holes $\longrightarrow$ FPs fail

Why not Denoise Scans?

de-noise,
compute FP-s,
align



align with 4PCS,
de-noise



Related Works

Random Sampling:

- [Fiscler and Bolles `81]
- [Ballard `87]
- [Wolfson and Rigoutsos `97]
- [Gal and Cohen-Or `06]
- [Mitra et al. `06]

Non-Rigid Alignment:

- [Pauly et al. `05]
- [Brown et al. `07]
- SGP `08

Feature Based Alignment:

- [Johnson `97]
- [Mori et al. `05]
- [Gelfand et al. `05]
- [Guskov et al. `05]
- [Pottmann et al. `07]

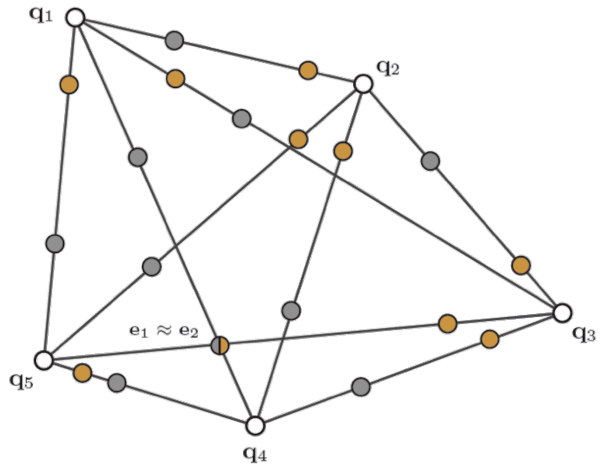
Computational Geometry:

- [Huttenlocher and Ullman `90]
- [Huttenlocher `91]
- [Goodrich et al. `94]
- [Irani and Raghavan `96]
- [Chen et al. `99]
- [Indyk et al. `99]
- [Agarwal and Sharir `02]

Key Observation

A pair of triples (from *P* and Q) is enough to uniquely define a ***rigid transform*** ⟶ $O(n^3)$

Surprisingly, a *special* set of 4-points, ***congruent sets***, makes the problem simpler ⟶ $O(n^2)$



4-Points Congruent Sets

- Few matches $\longrightarrow$ output sensitive algorithm

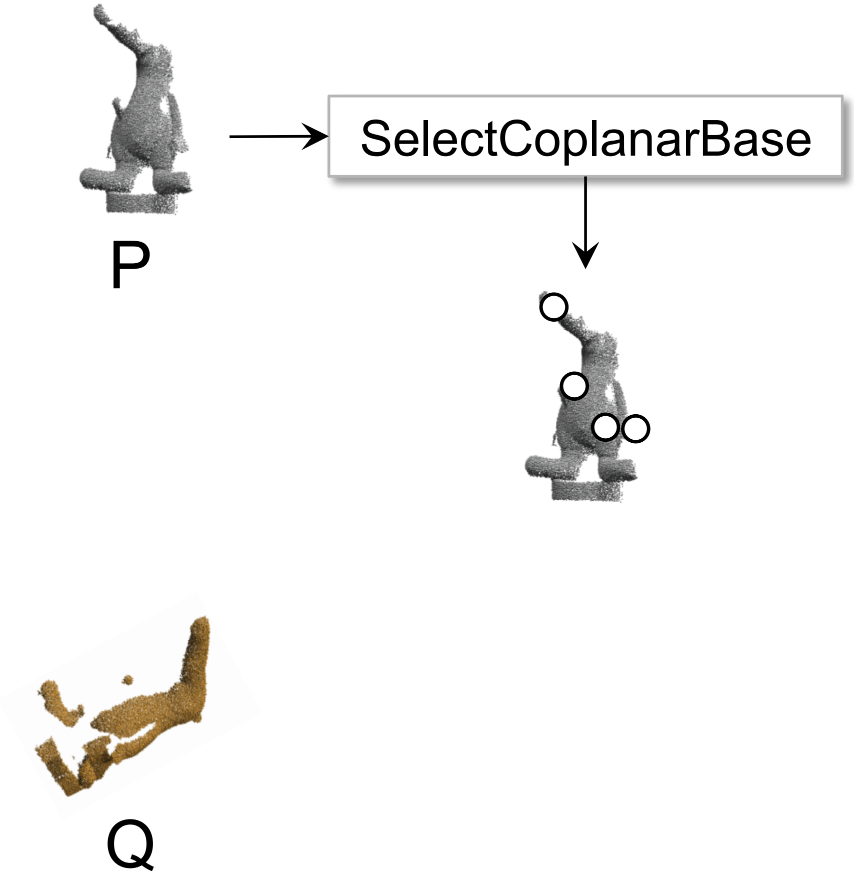
- Can be efficiently extracted

4PCS Algorithm

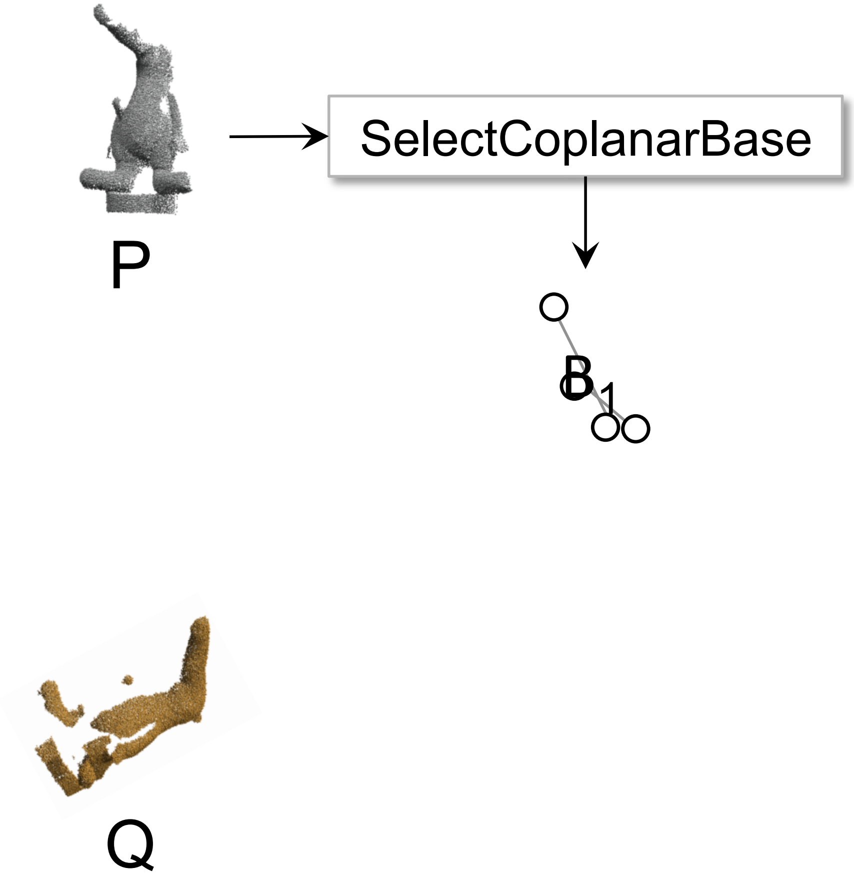
P

Q

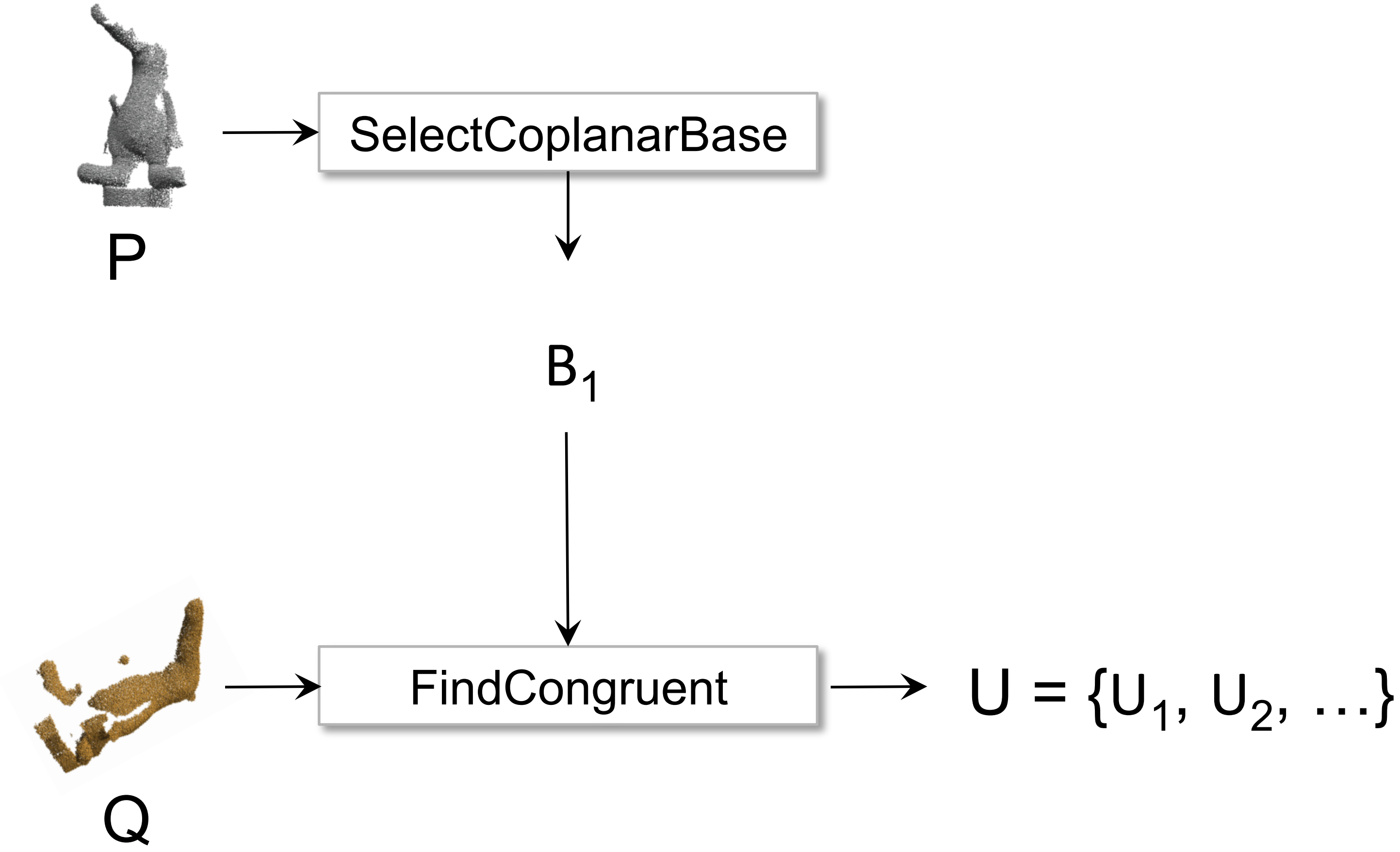
4PCS Algorithm



4PCS Algorithm



4PCS Algorithm



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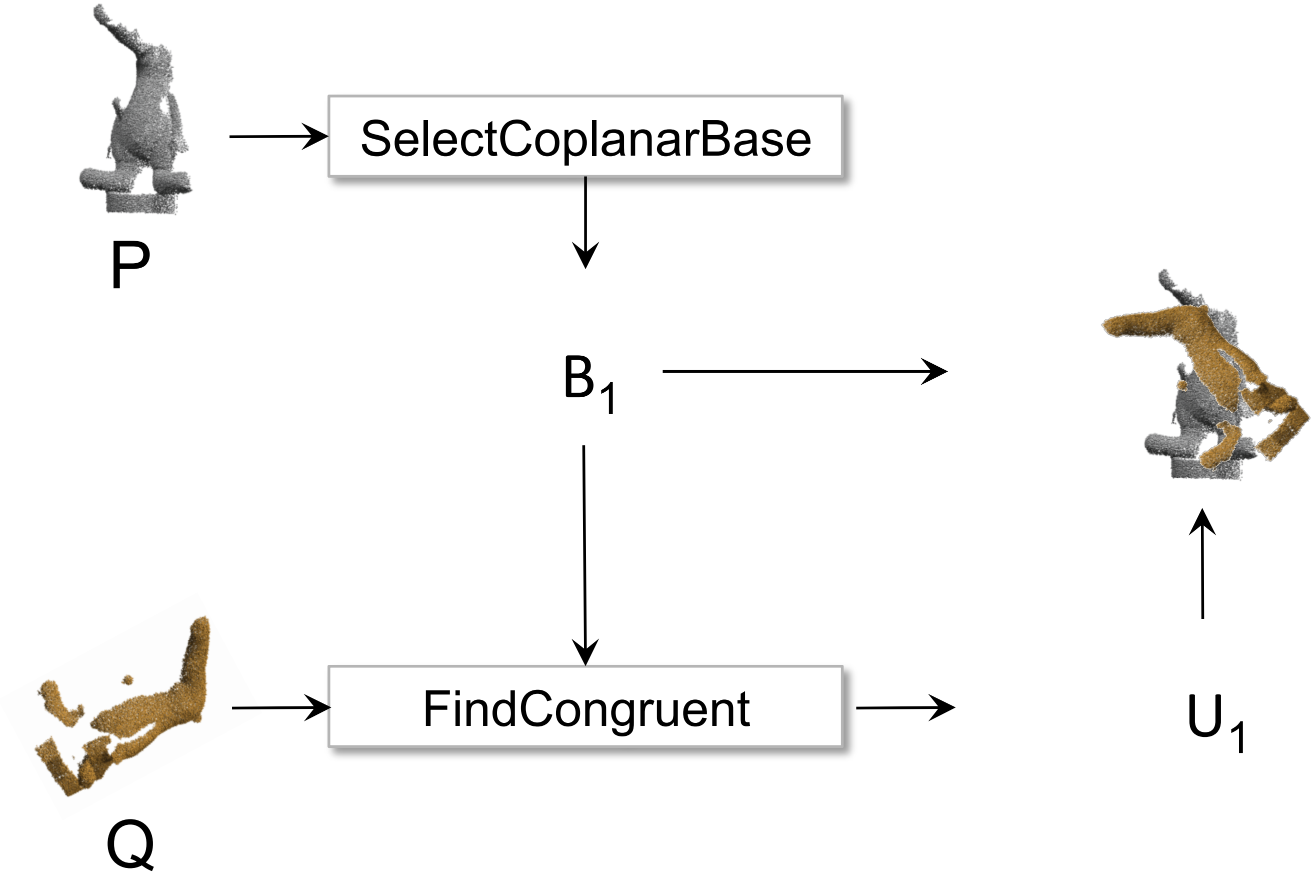

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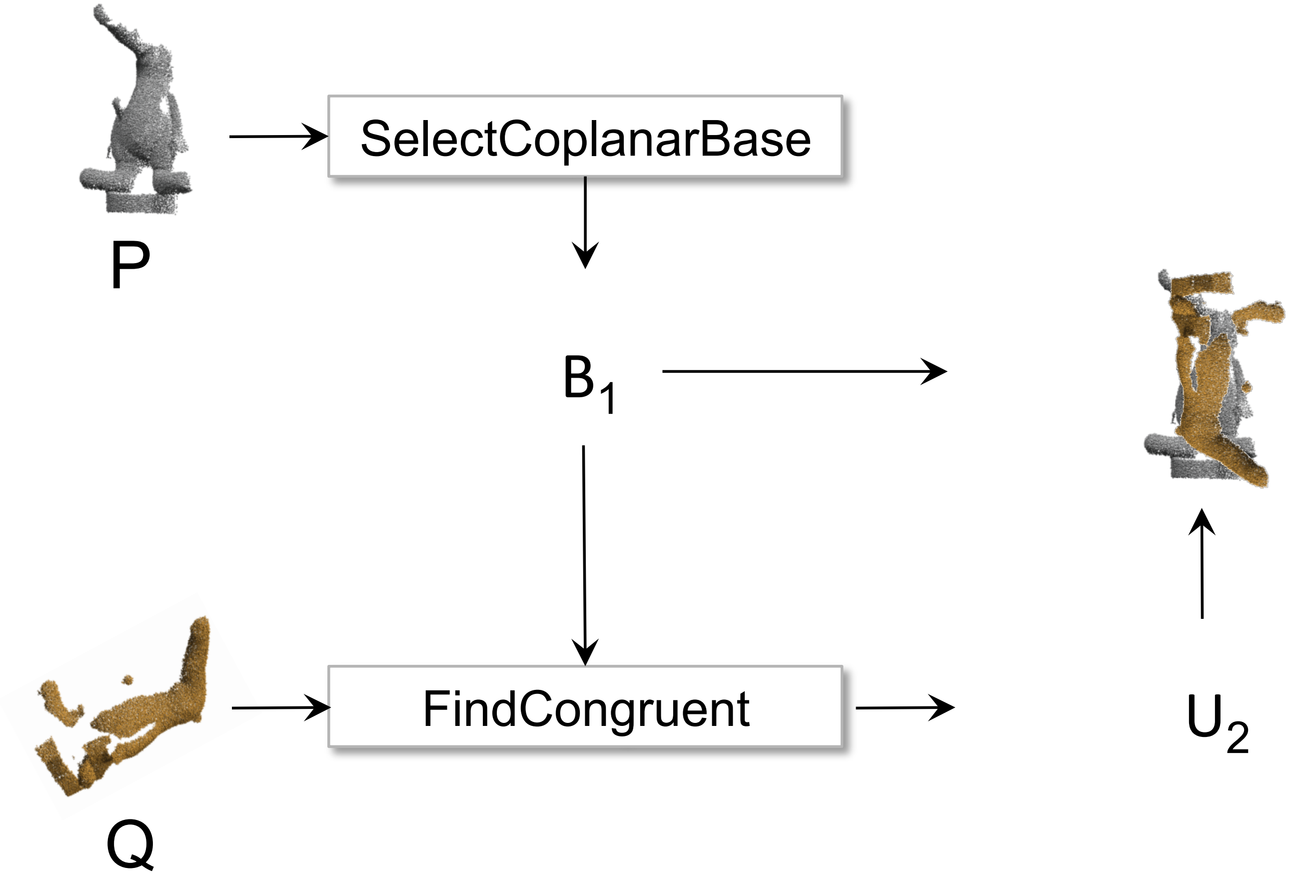

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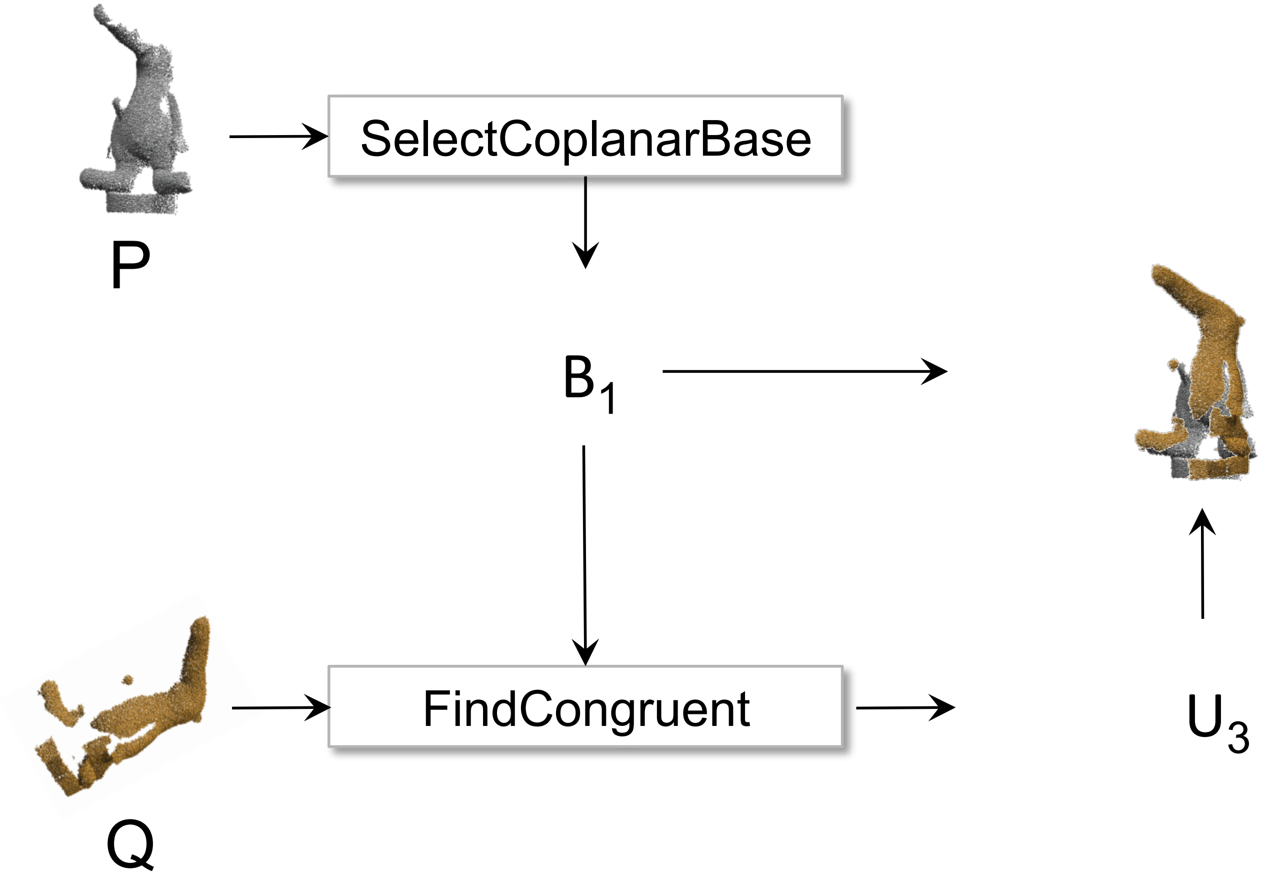

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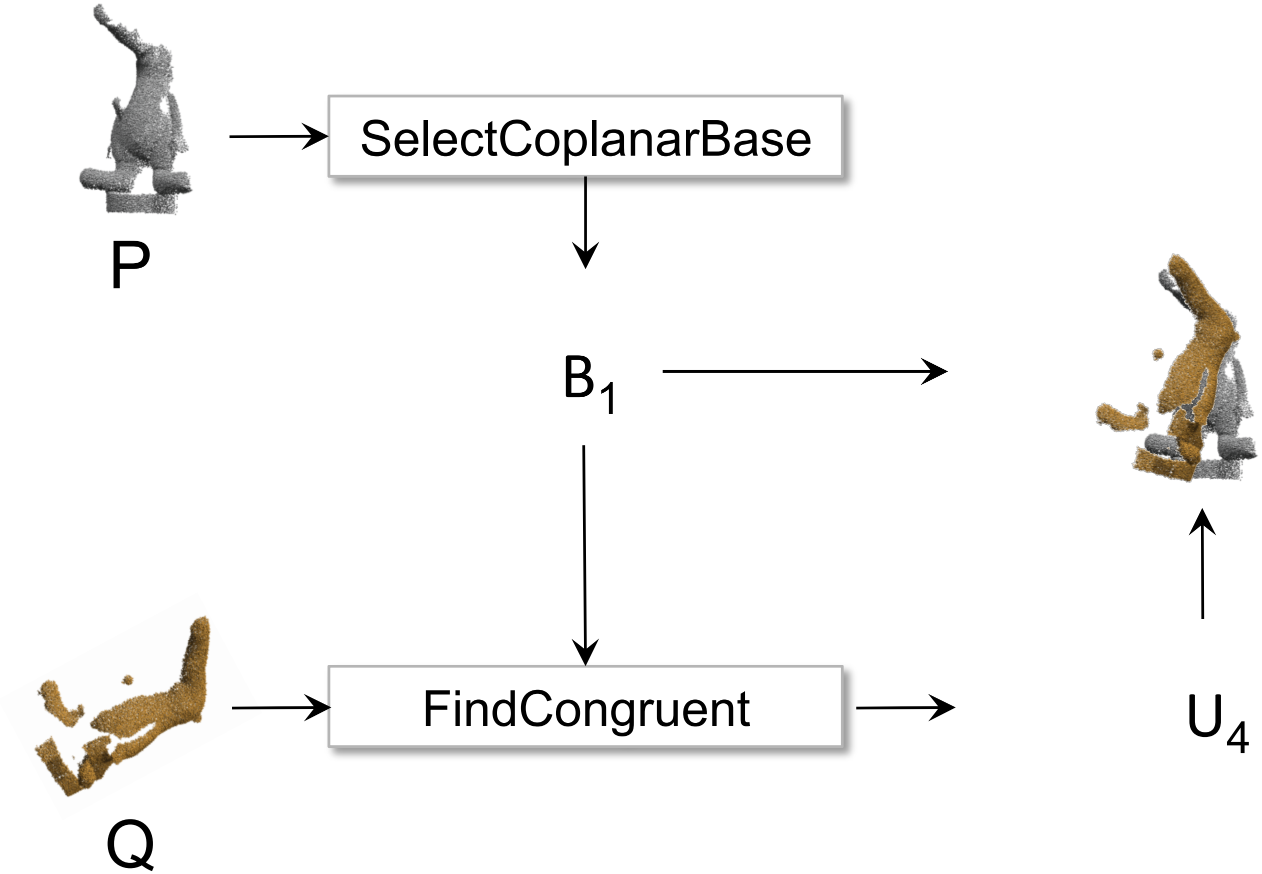

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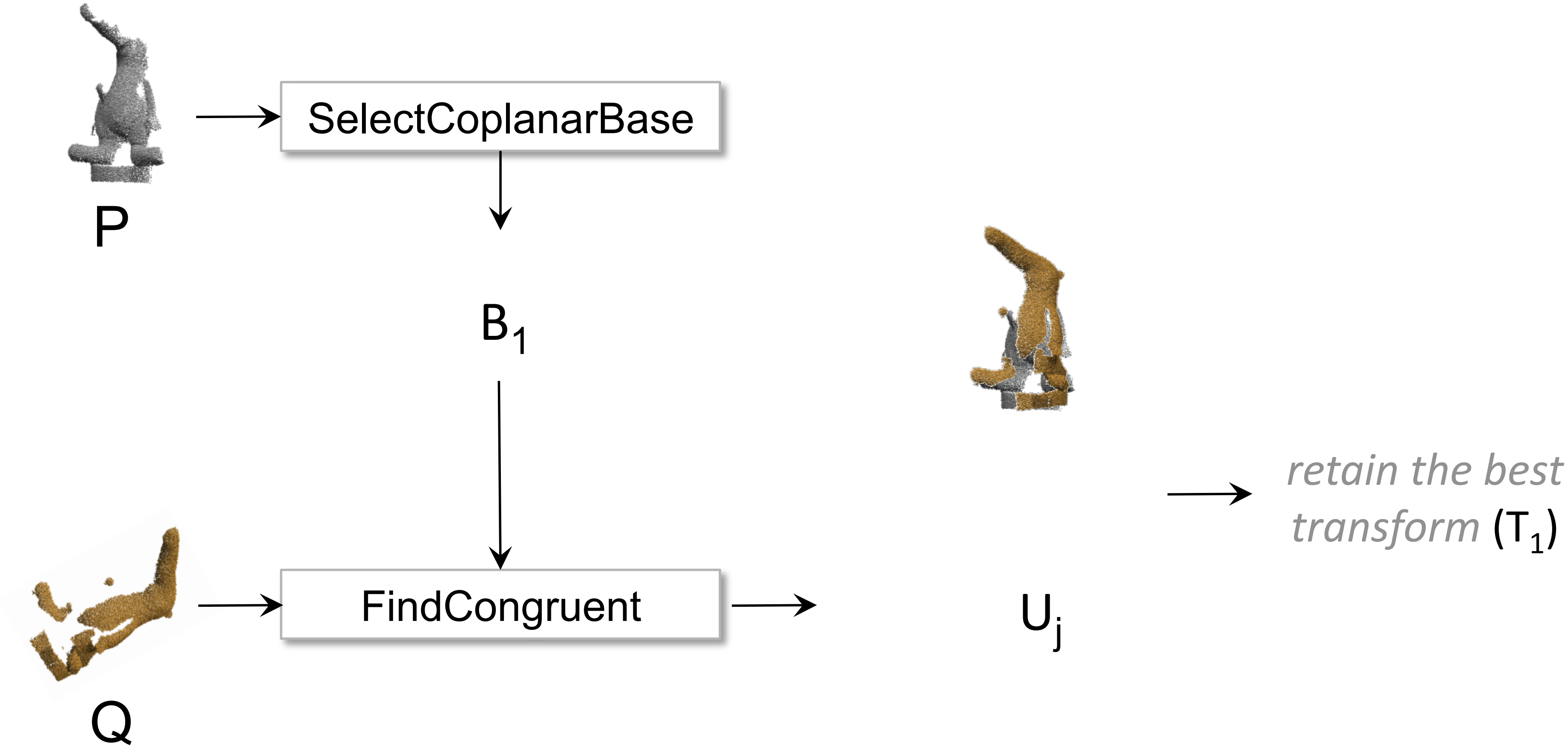

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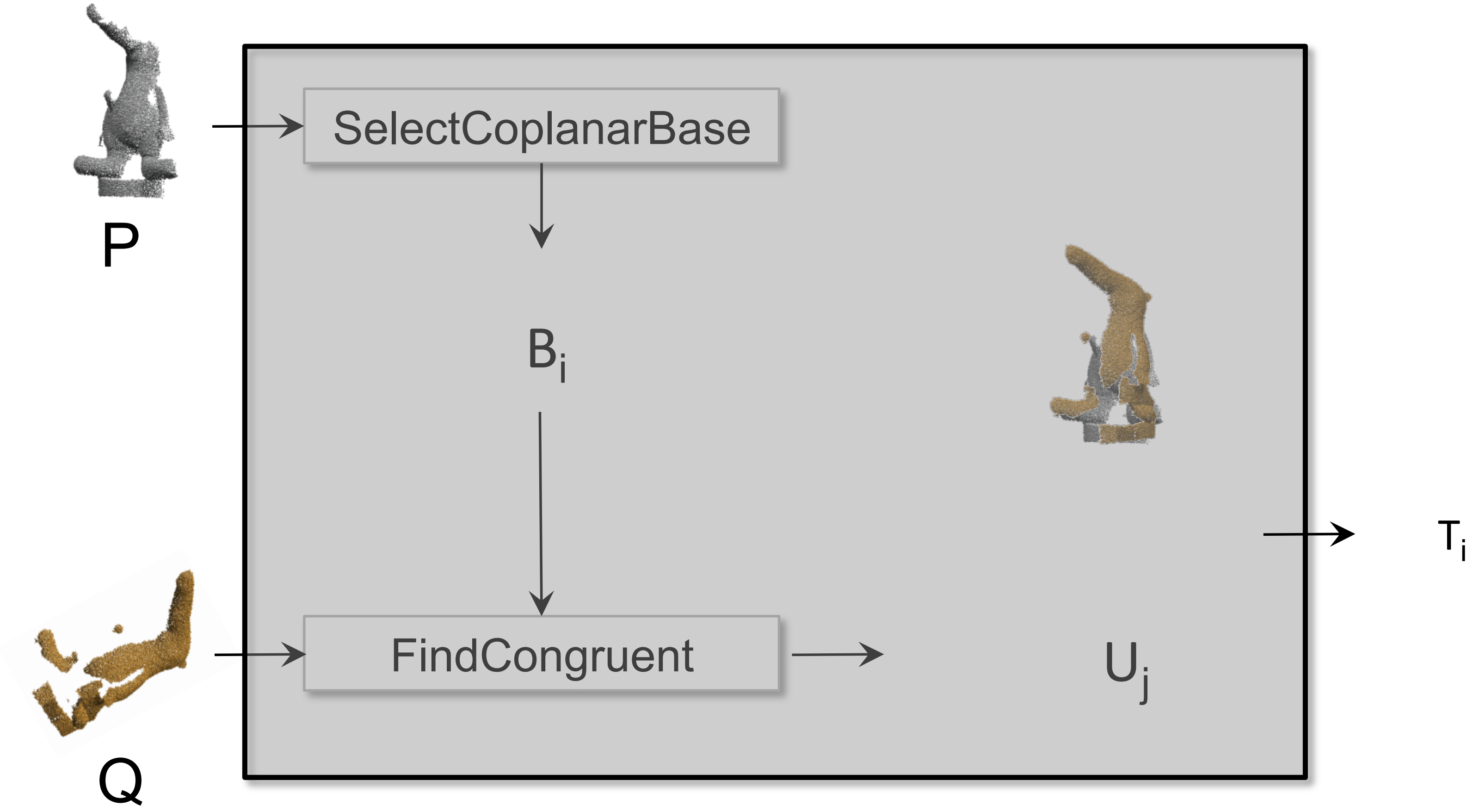
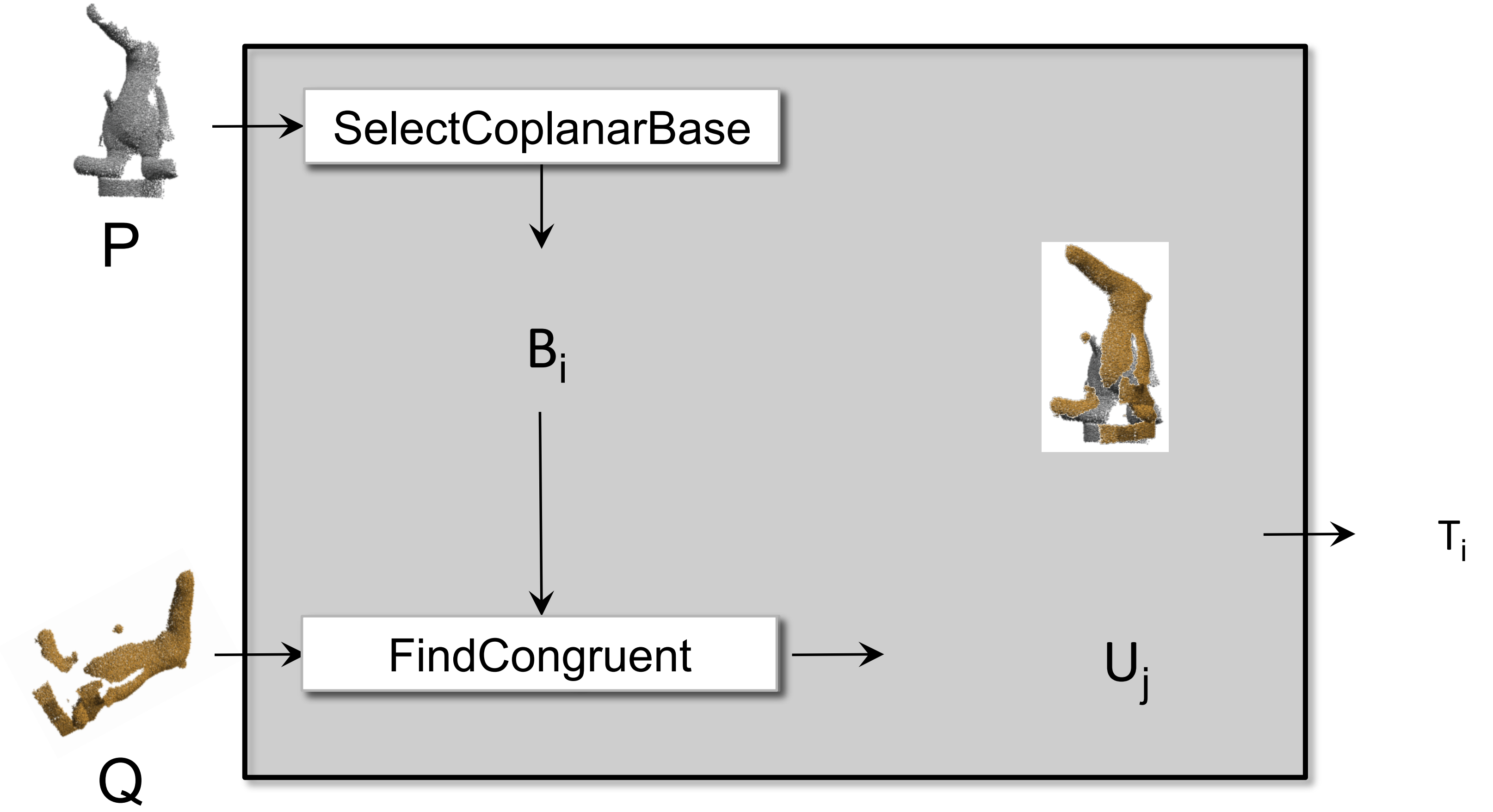


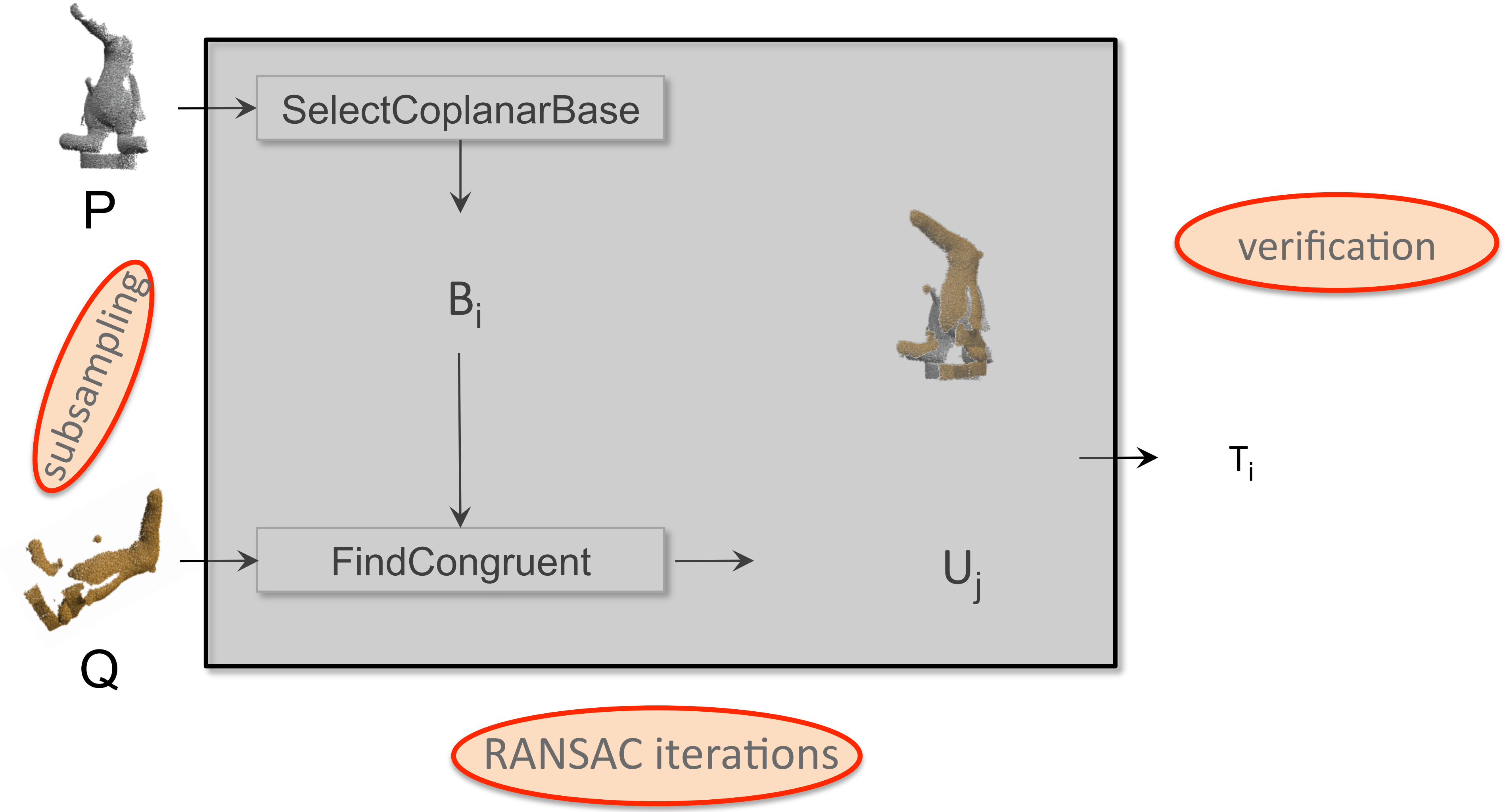
RANSAC iterations

4PCS Algorithm



RANSAC iterations

Random Sampling



FindCongruent

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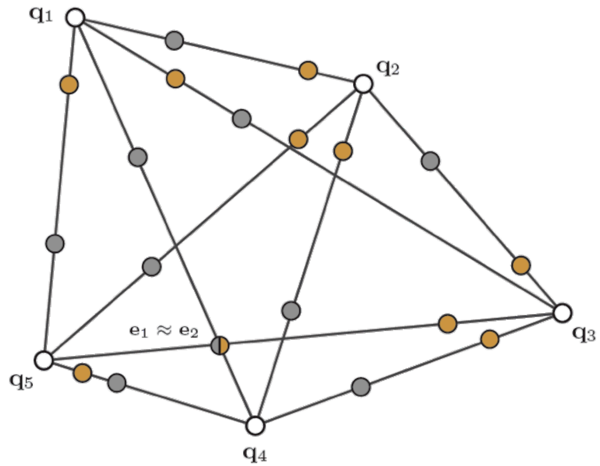
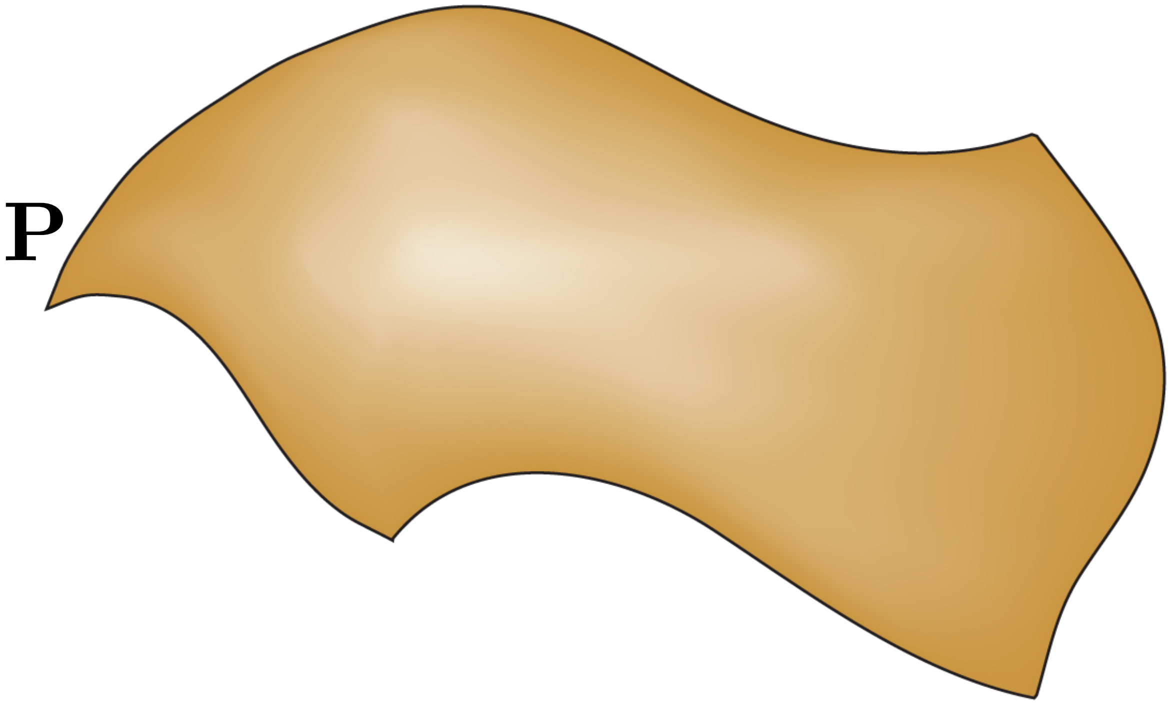
Surprisingly, a *special* set of 4-points, ***congruent sets***, makes the problem simpler ⟶ $O(n^2)$



Affine Invariance



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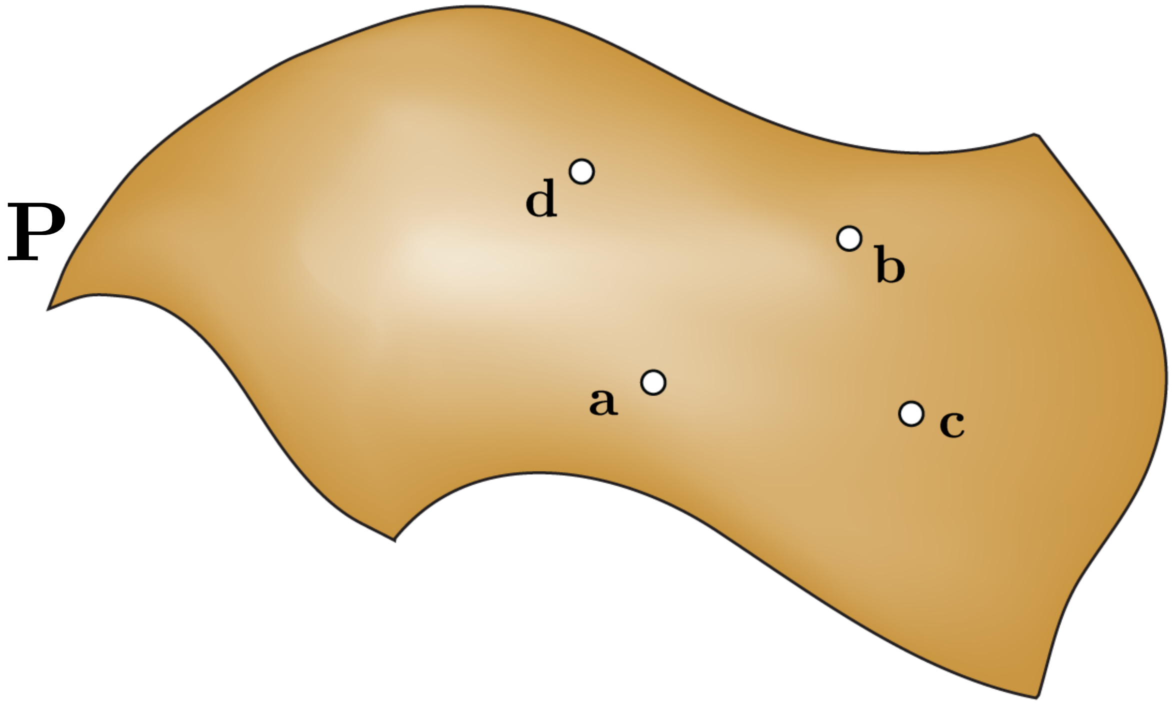

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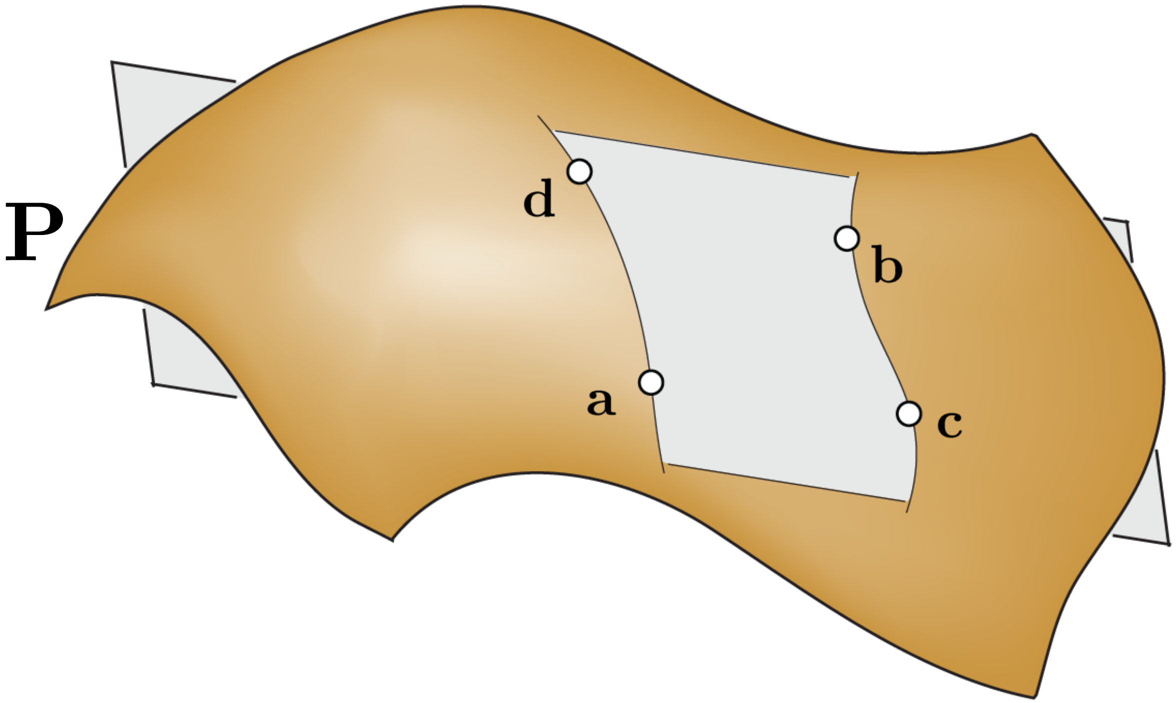
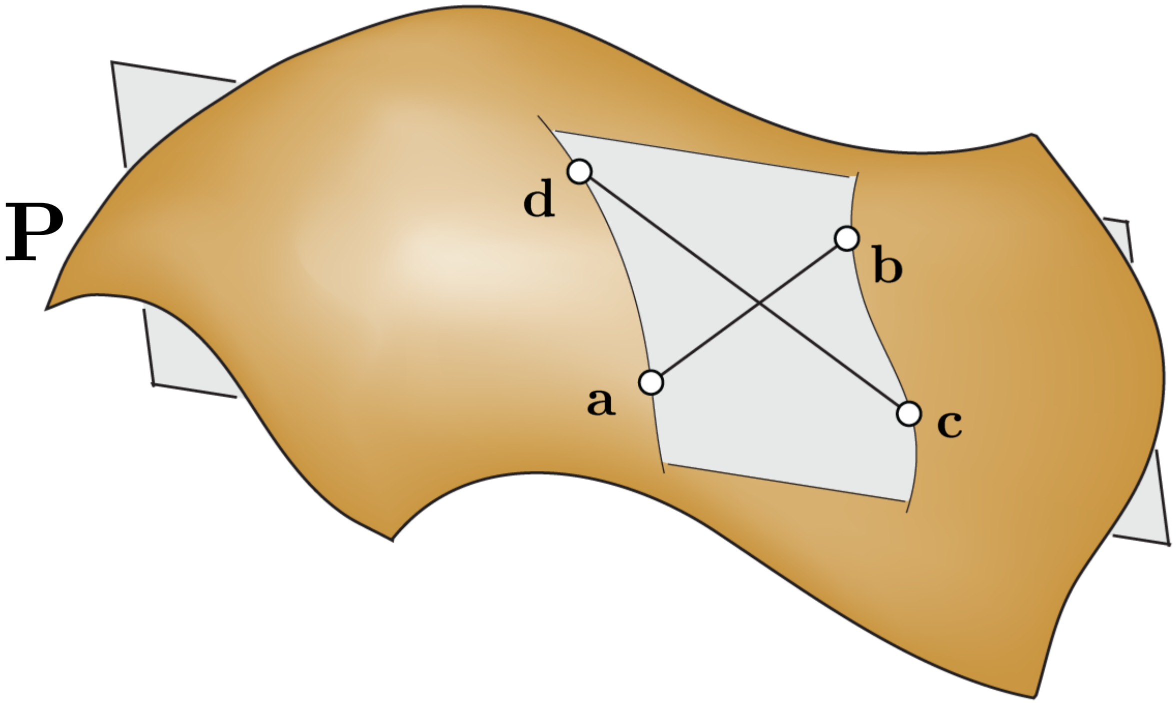
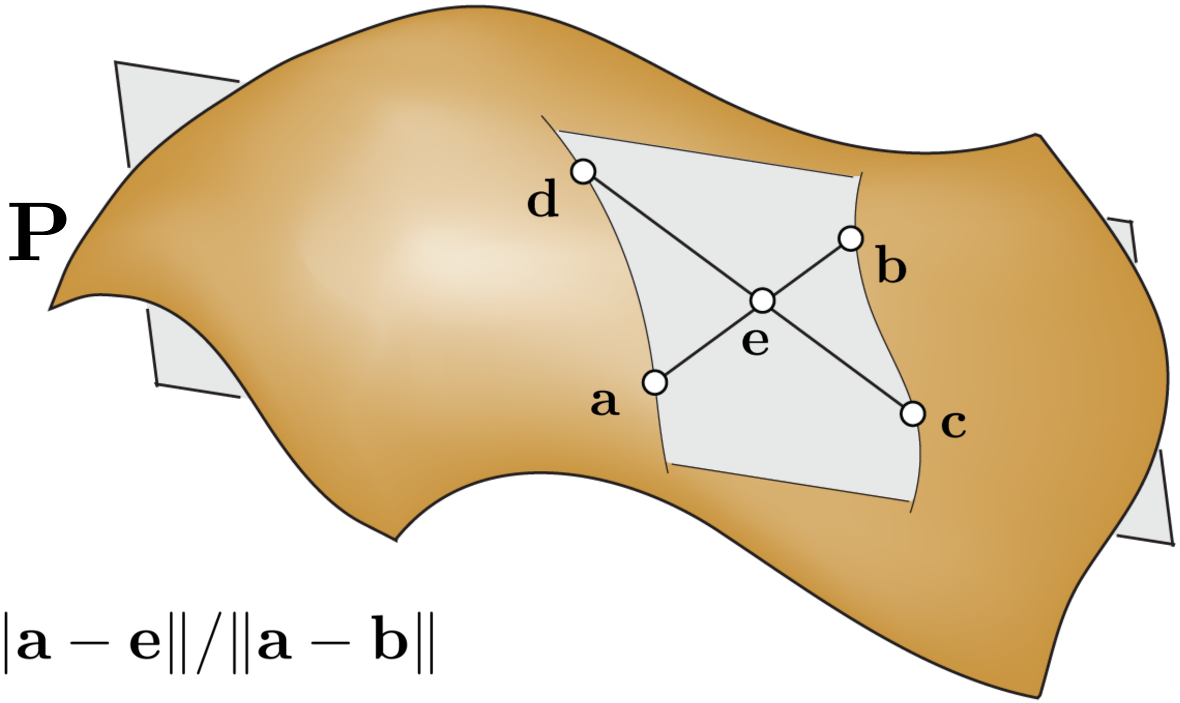


4 coplanar points

Affine Invariance



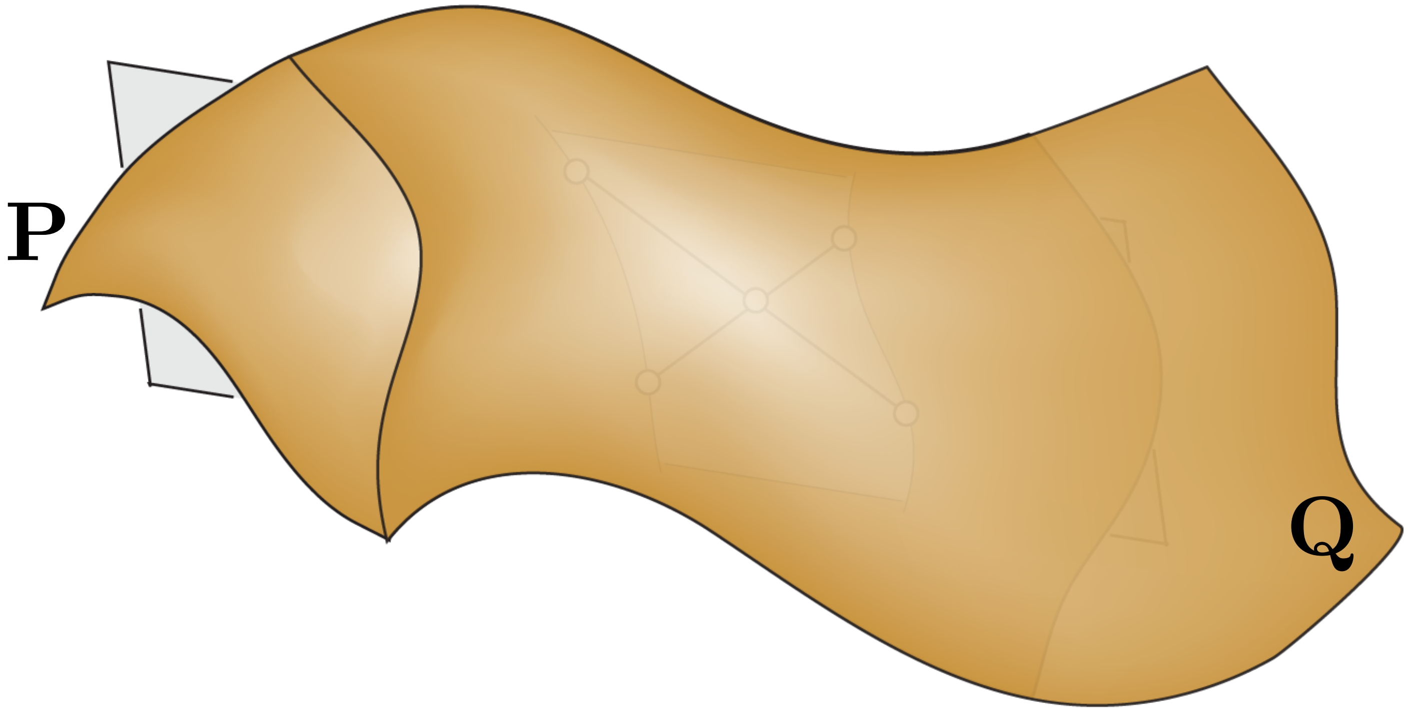
Affine Invariance



$$r_1 = \|\mathbf{a} - \mathbf{e}\| / \|\mathbf{a} - \mathbf{b}\|$$

$$r_2 = \|\mathbf{c} - \mathbf{e}\| / \|\mathbf{c} - \mathbf{d}\|$$

Affine Invariance



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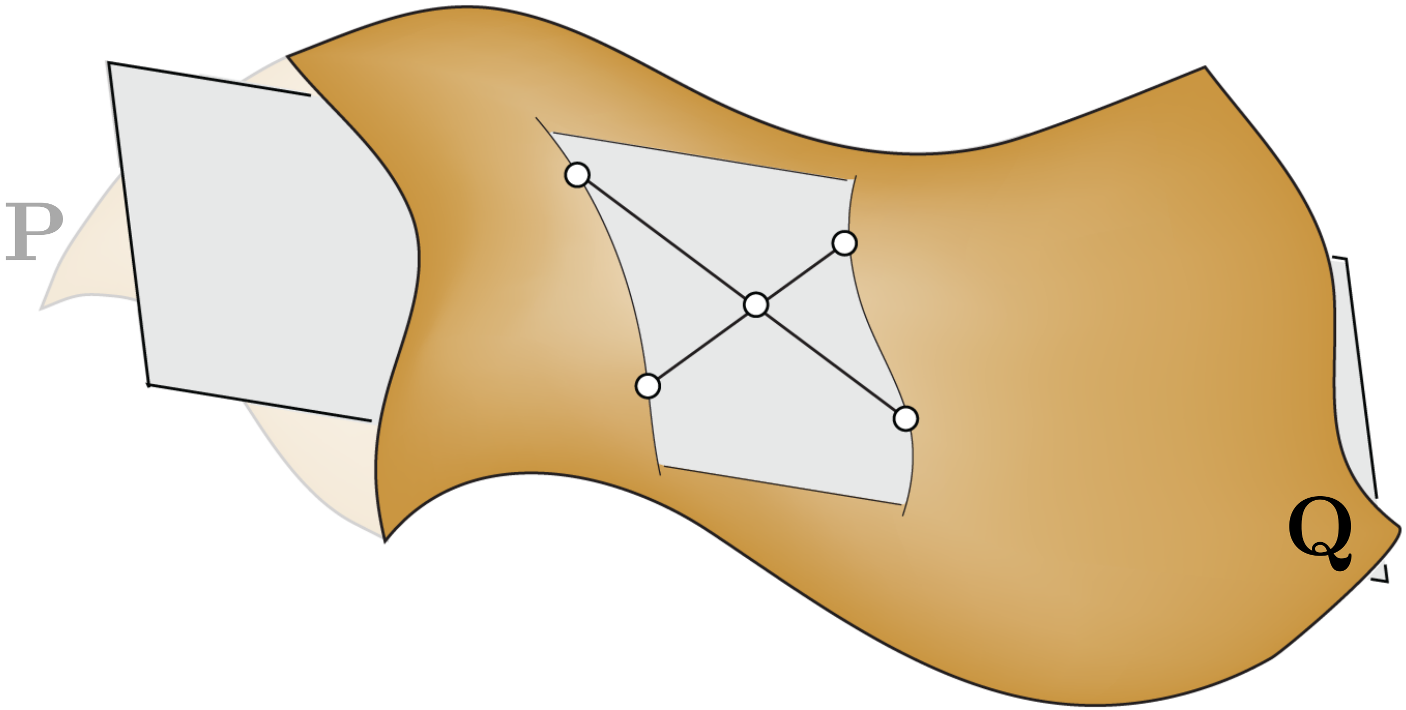
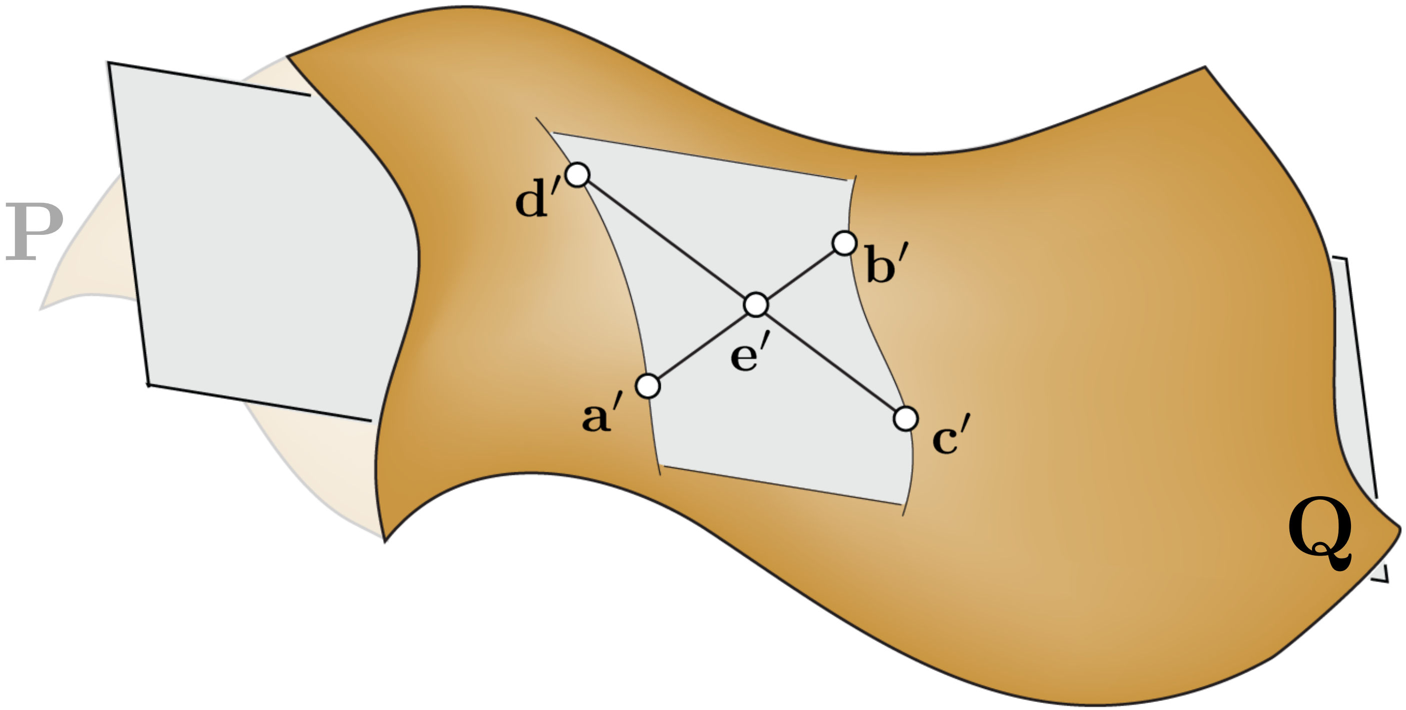


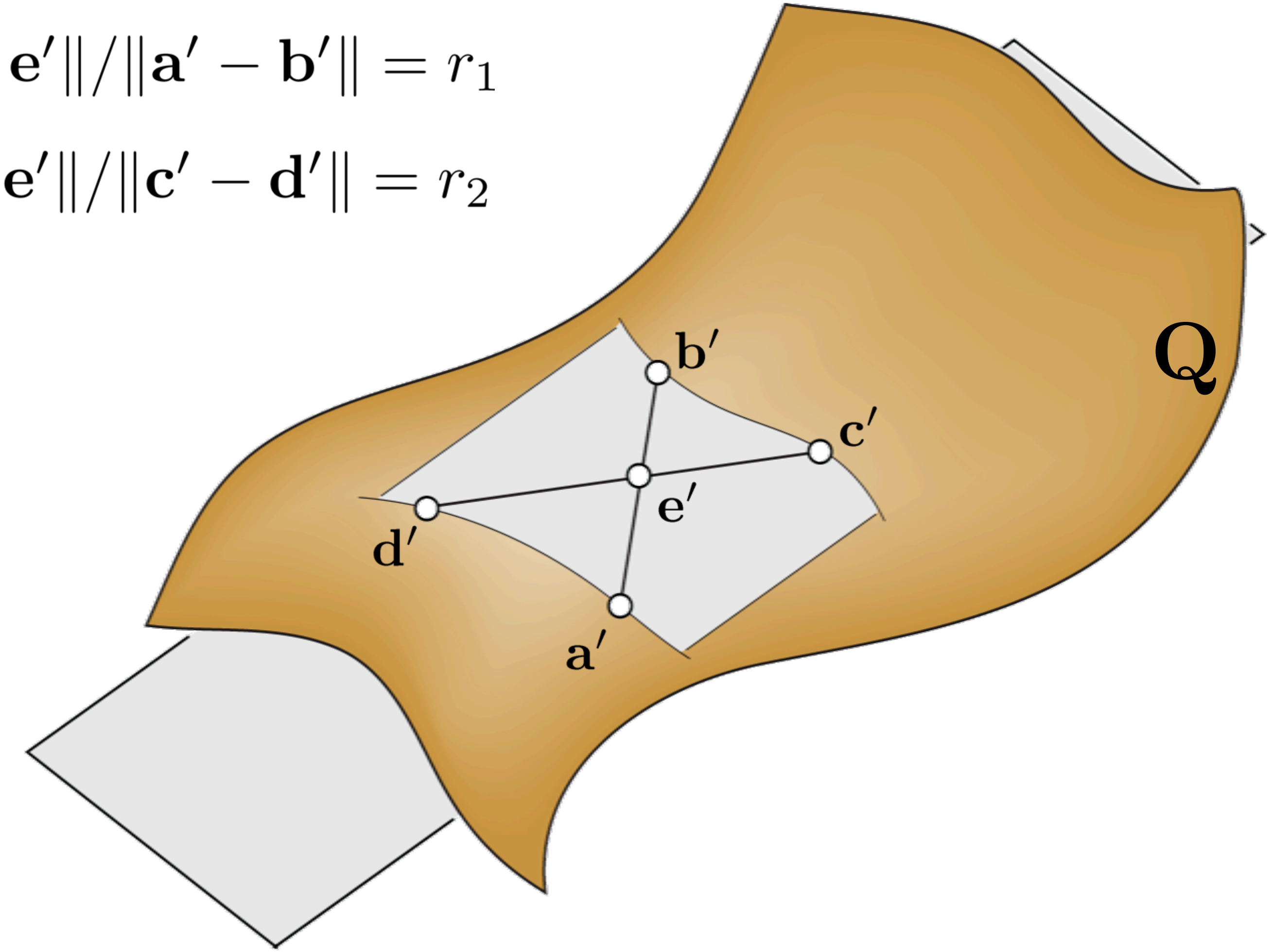
Affine Invariance



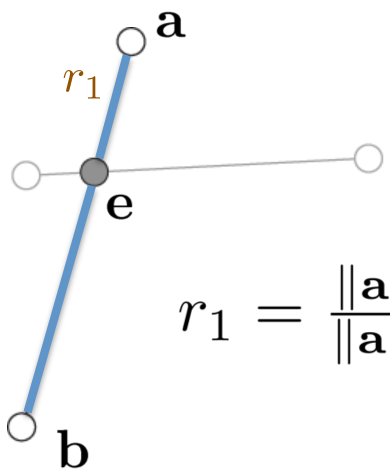
Affine Invariance

$$\|\mathbf{a}' - \mathbf{e}'\| / \|\mathbf{a}' - \mathbf{b}'\| = r_1$$

$$\|\mathbf{c}' - \mathbf{e}'\| / \|\mathbf{c}' - \mathbf{d}'\| = r_2$$



Extracting Congruent 4-points

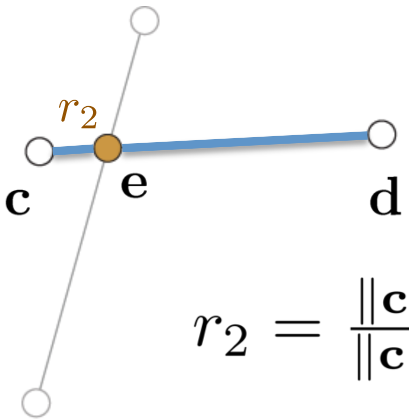


$$r_1 = \frac{\|\mathbf{a}-\mathbf{e}\|}{\|\mathbf{a}-\mathbf{b}\|}$$

$$\{a, b, c, d\}$$

$$\rightarrow \mathrm{e}, r_1, r_2$$

P



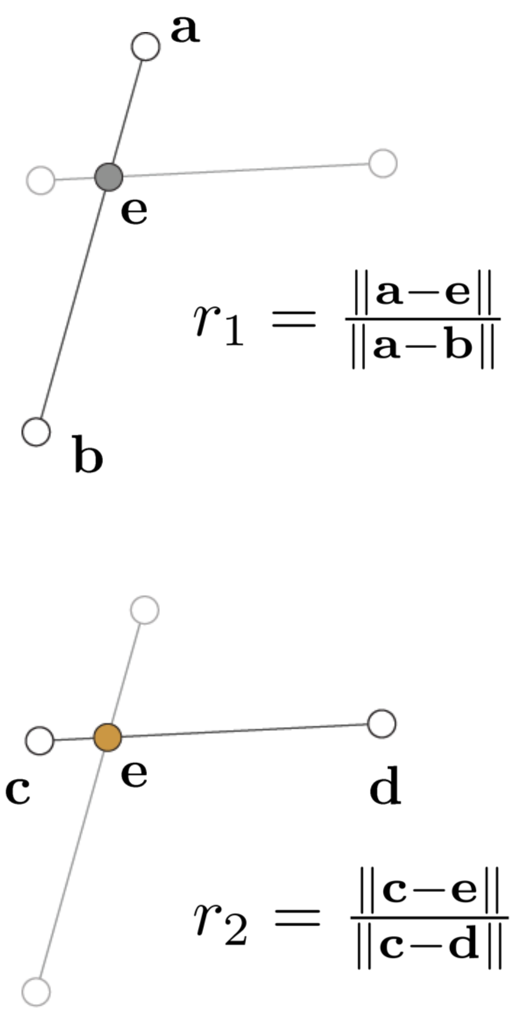
$$r_2 = \frac{\|\mathbf{c}-\mathbf{e}\|}{\|\mathbf{c}-\mathbf{d}\|}$$

$$\mathbf{a}', \mathbf{b}', r_1 \rightarrow \mathbf{e}'$$

$$r_1 = \frac{\|\mathbf{a}'-\mathbf{e}'\|}{\|\mathbf{a}'-\mathbf{b}'\|}$$

Q

Extracting Congruent 4-points



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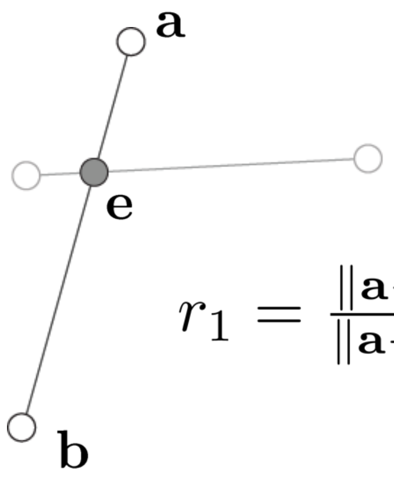

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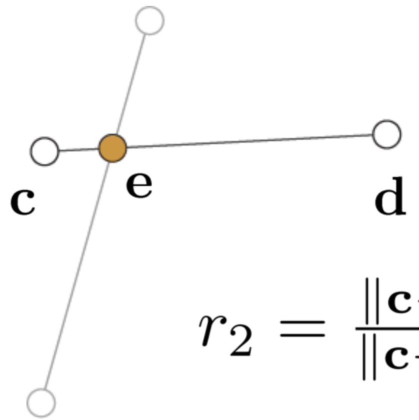

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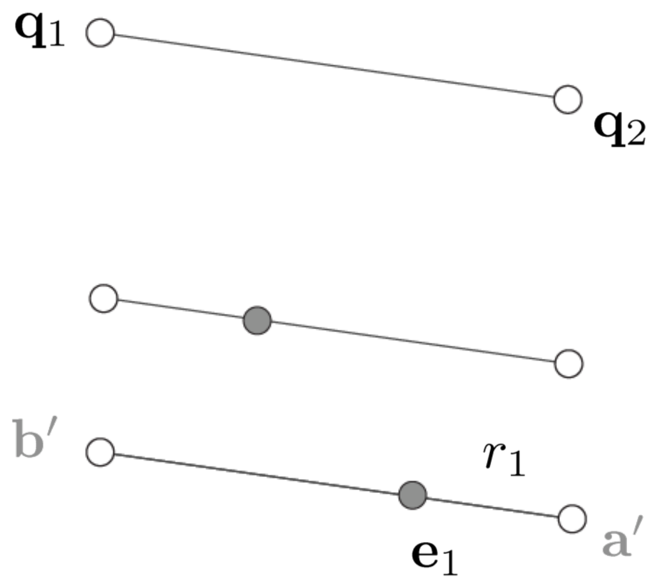

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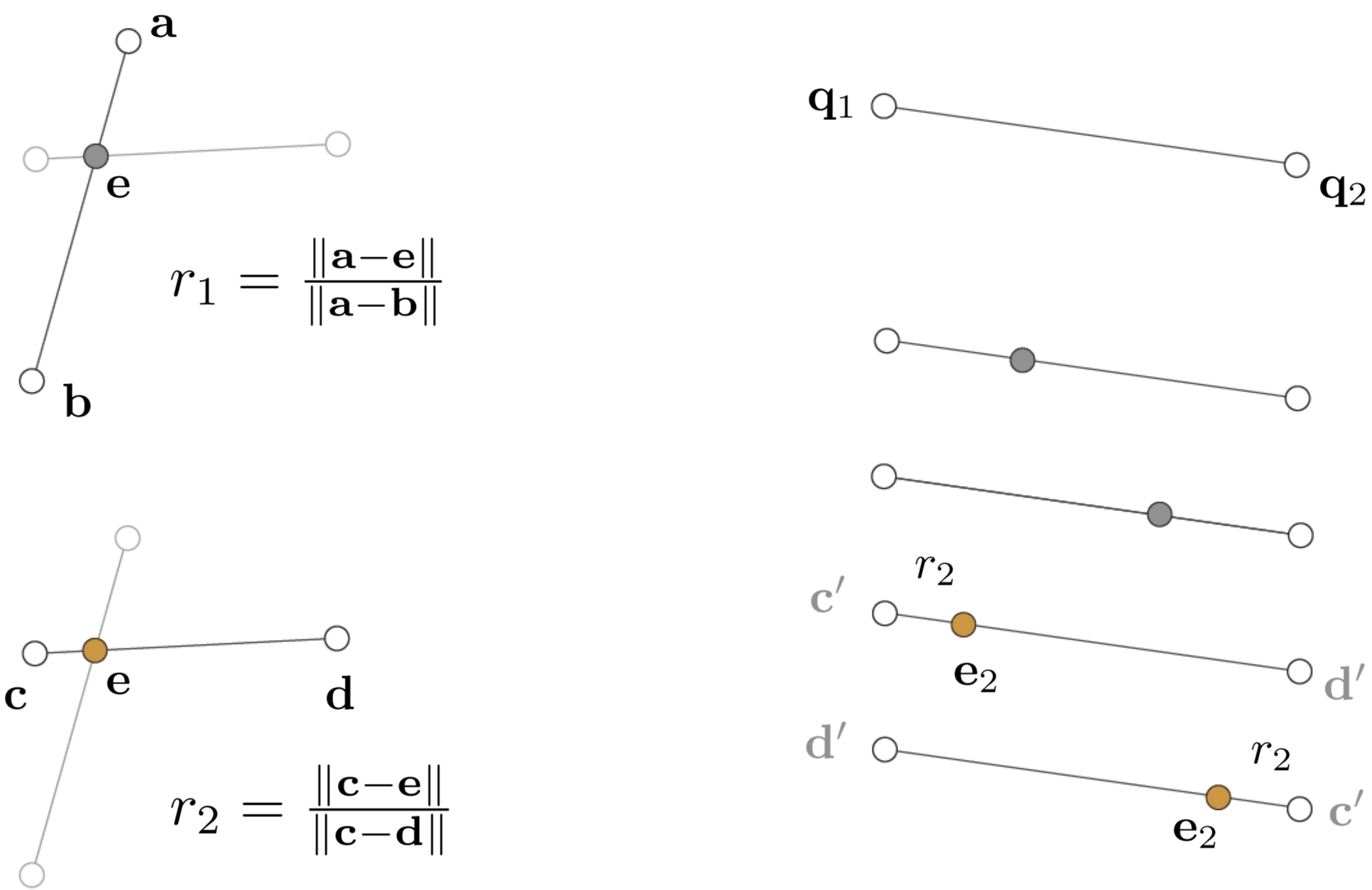

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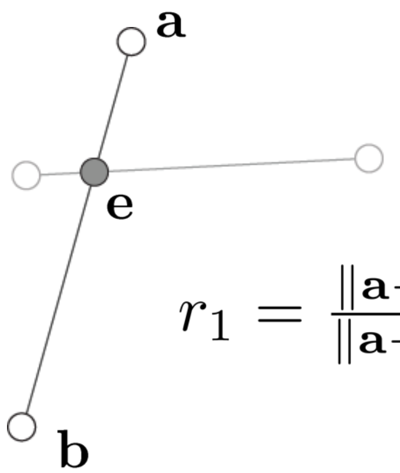

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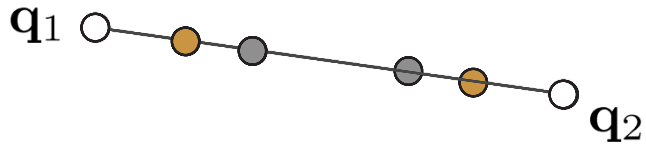
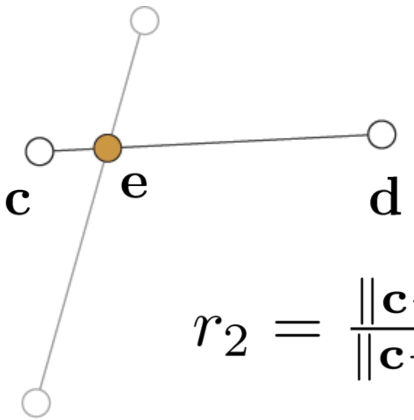
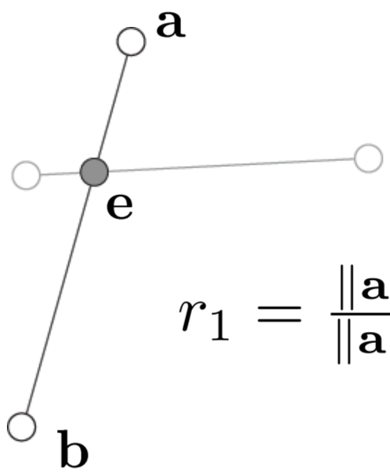




$$r_2 = \frac{\|\mathbf{c}-\mathbf{e}\|}{\|\mathbf{c}-\mathbf{d}\|}$$

What if $\mathbf{e}_1 \neq \mathbf{e}_2$?



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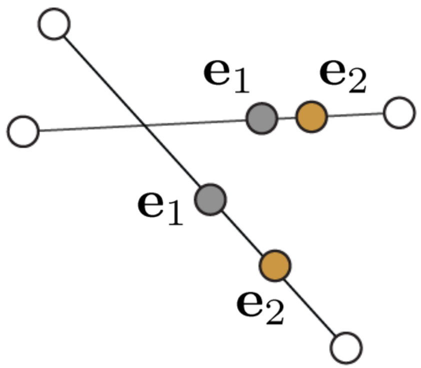
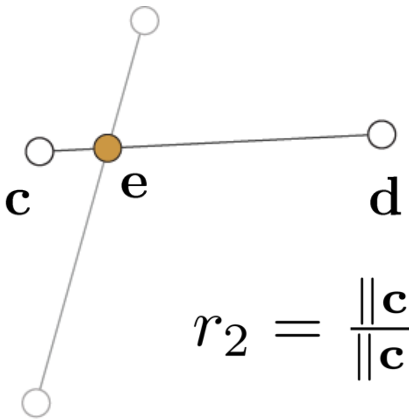
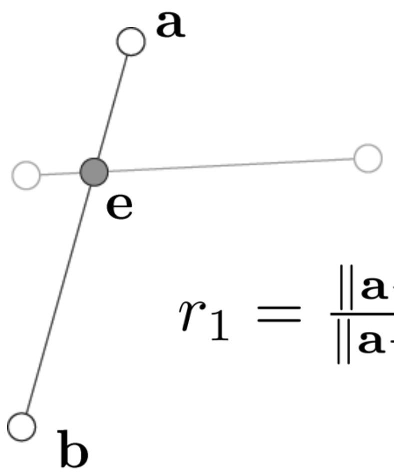


typical scenario



$$r_2 = \frac{\|\mathbf{c}-\mathbf{e}\|}{\|\mathbf{c}-\mathbf{d}\|}$$

What if $\mathbf{e}_1 = \mathbf{e}_2$?



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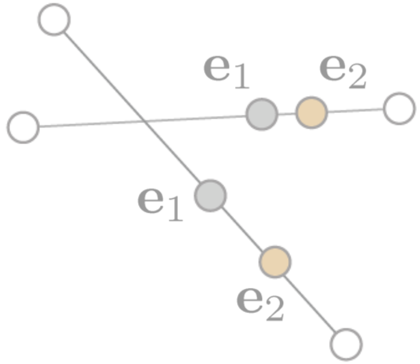

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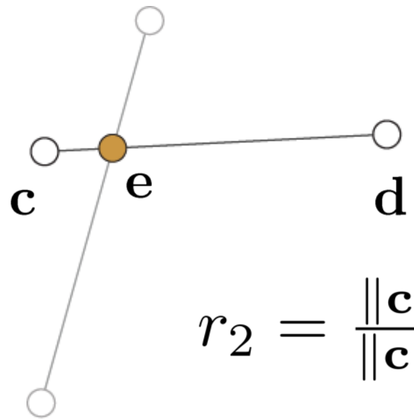
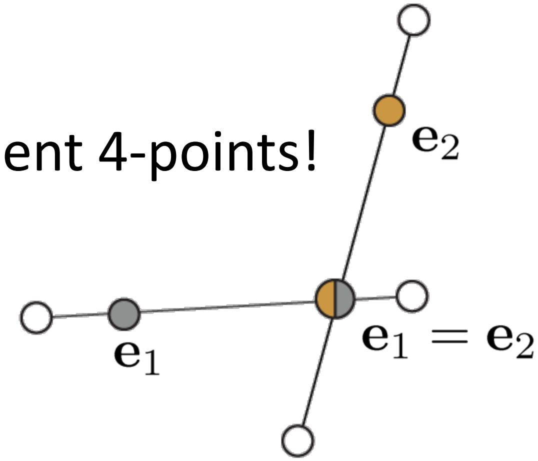
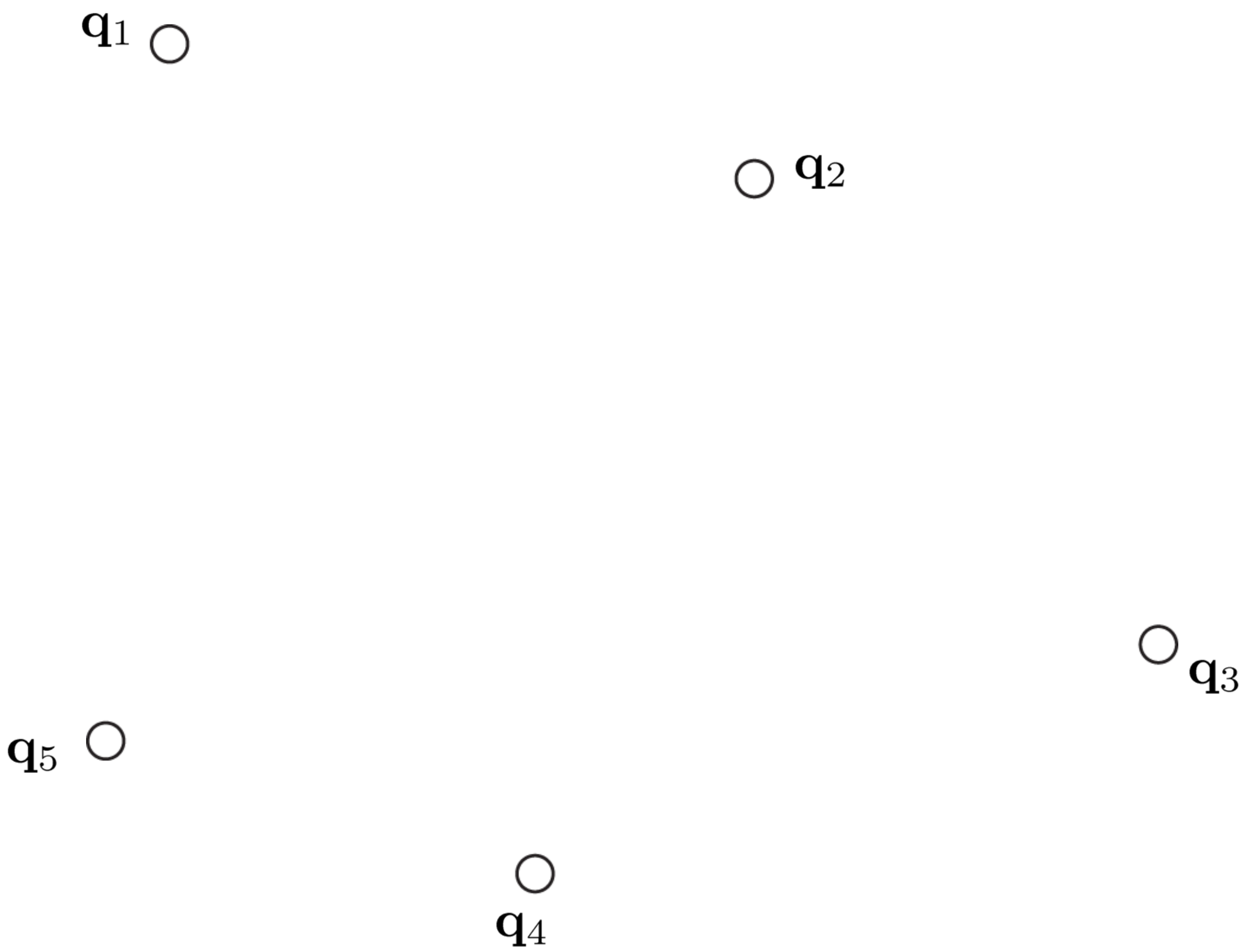


$$r_2 = \frac{\|\mathbf{c}-\mathbf{e}\|}{\|\mathbf{c}-\mathbf{d}\|}$$

congruent 4-points!



Extracting Congruent 4-points



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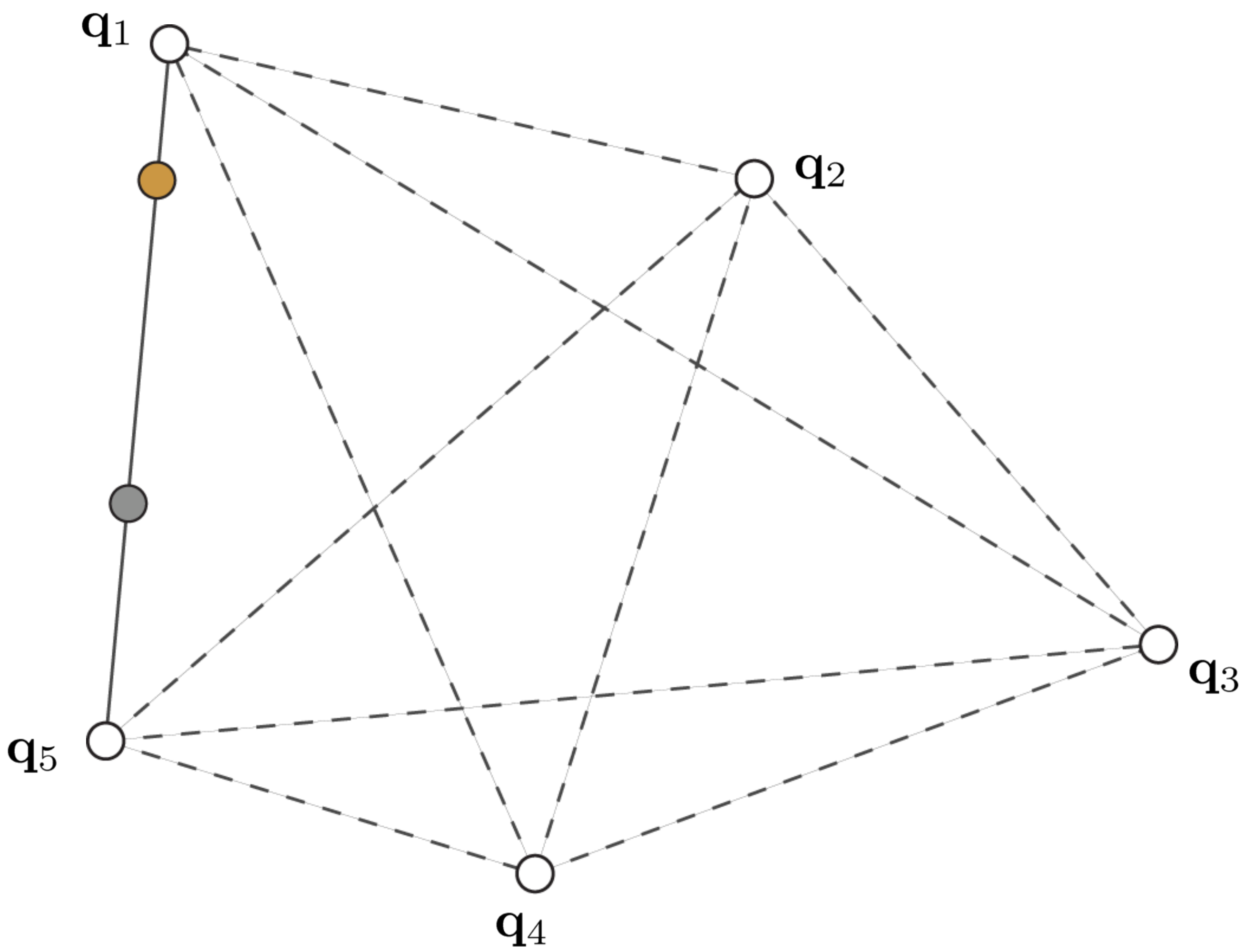

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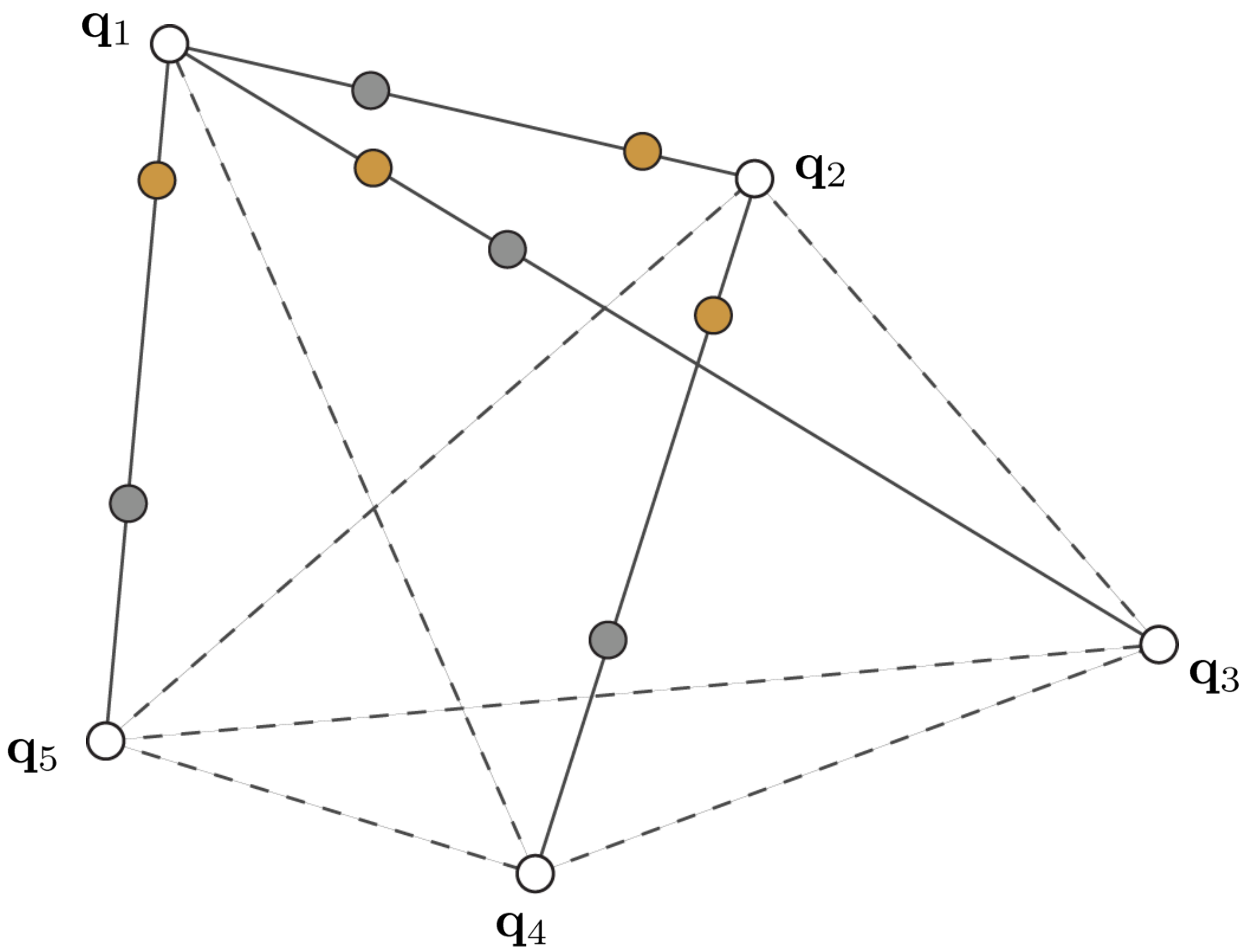

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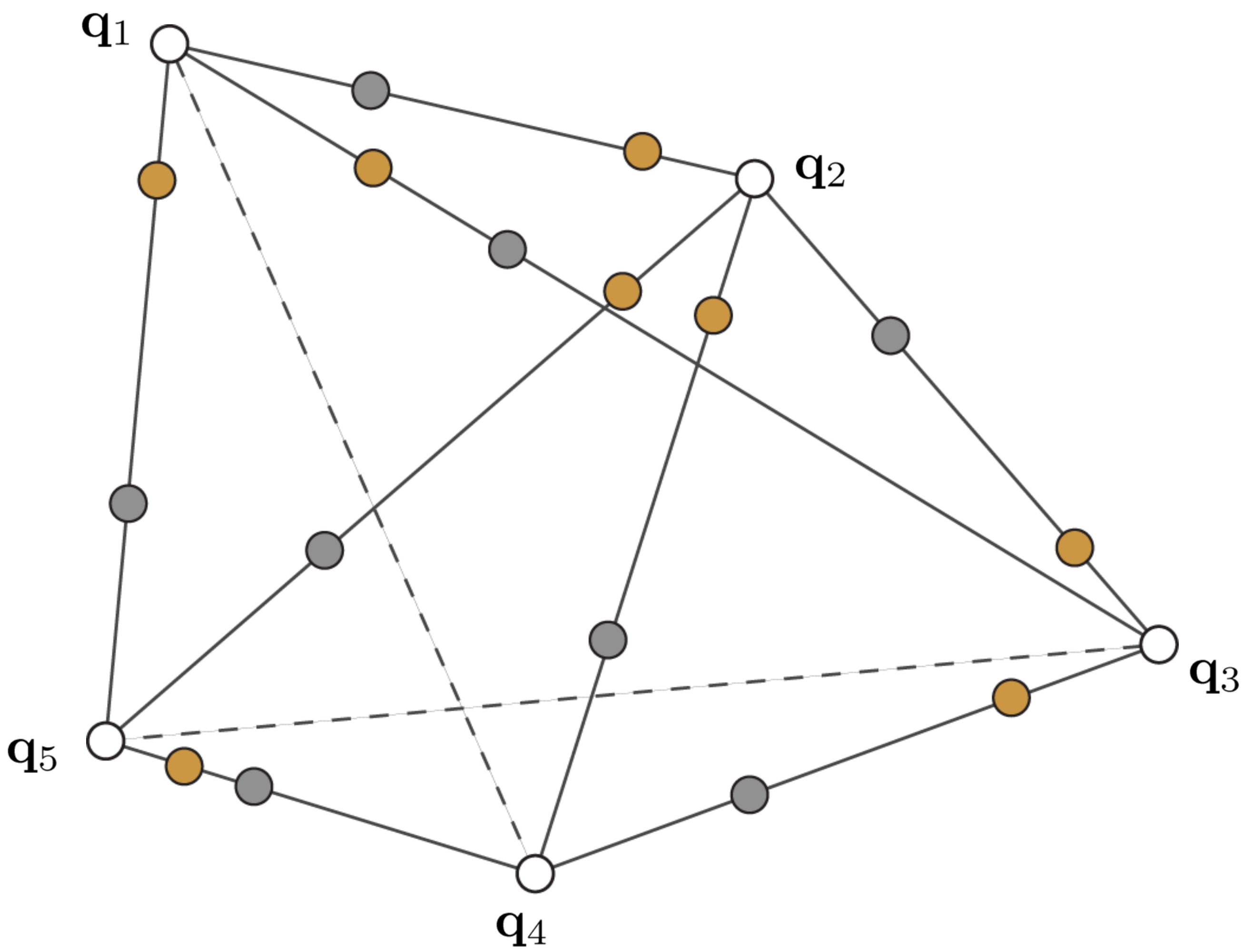

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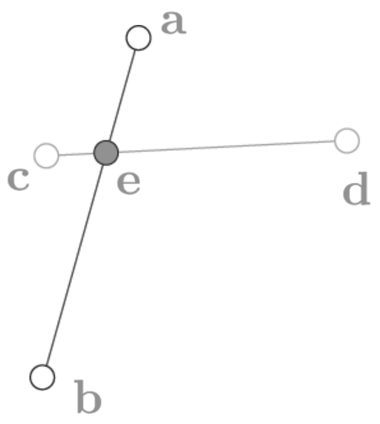
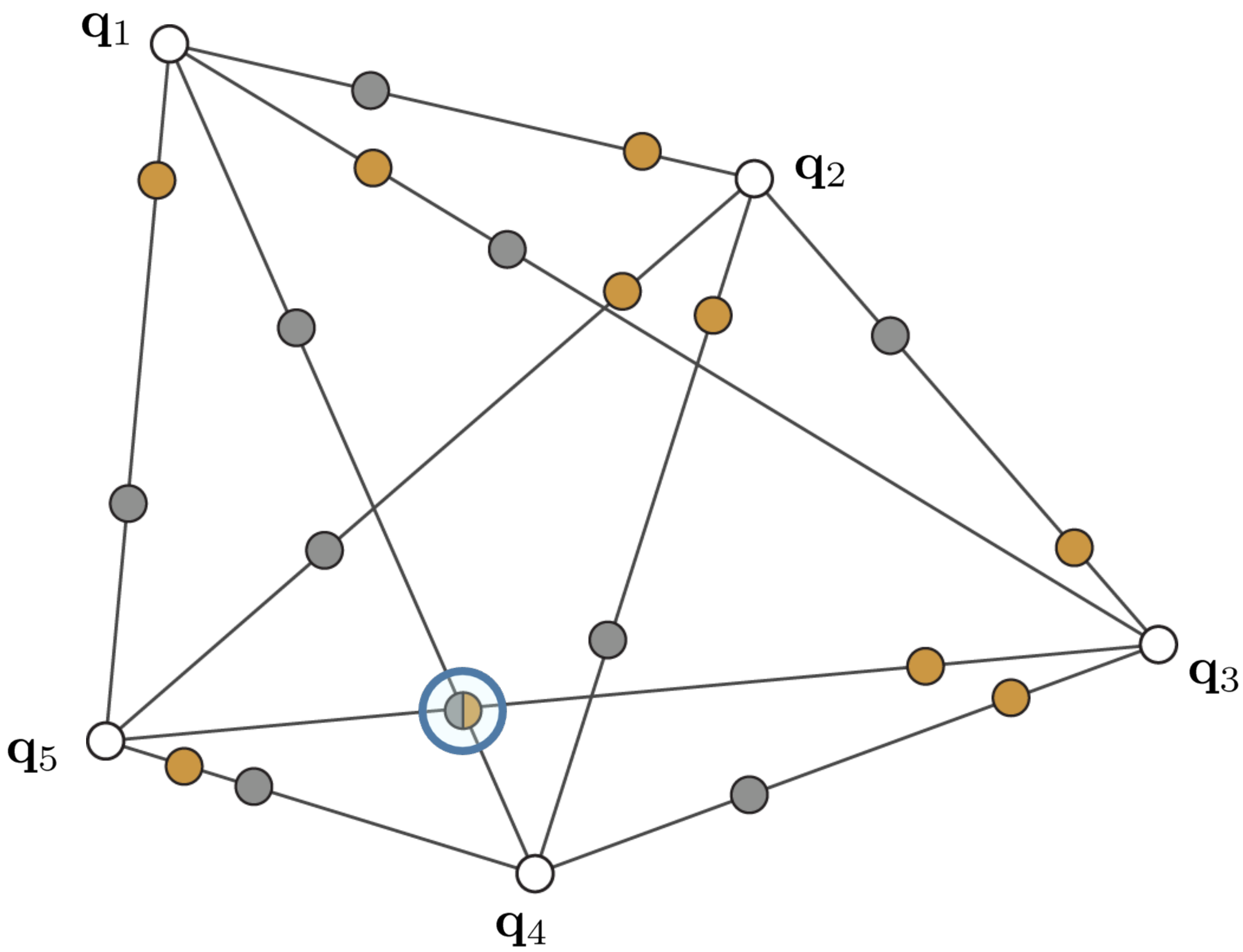


$$\{\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}\} \equiv \{\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3, \mathbf{q}_4\}$$



FindCongruent

- For all the points arising using r_1
  build an ***approximate range-tree*** (ANN)

- For all the points due to r_2,
  quickly lookup for neighbors using range-tree

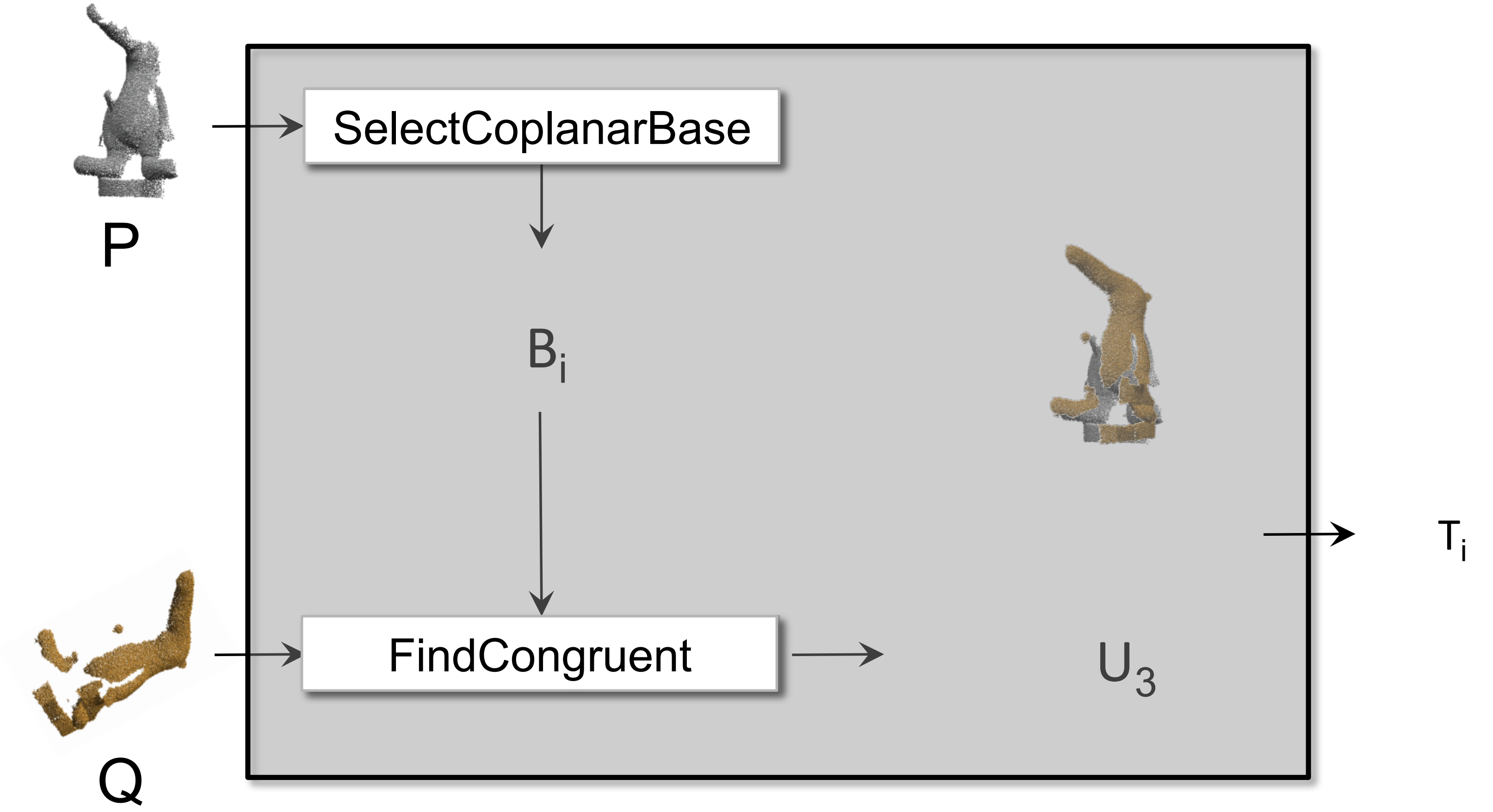
Rigid Transformation

- Euclidean distances are preserved

$$(\mathbf{q}_1, \mathbf{q}_2) \longrightarrow \|\mathbf{q}_1 - \mathbf{q}_2\| \approx \|\mathbf{a} - \mathbf{b}\|$$

SelectCoplanarBase

4PCS Algorithm



RANSAC iterations

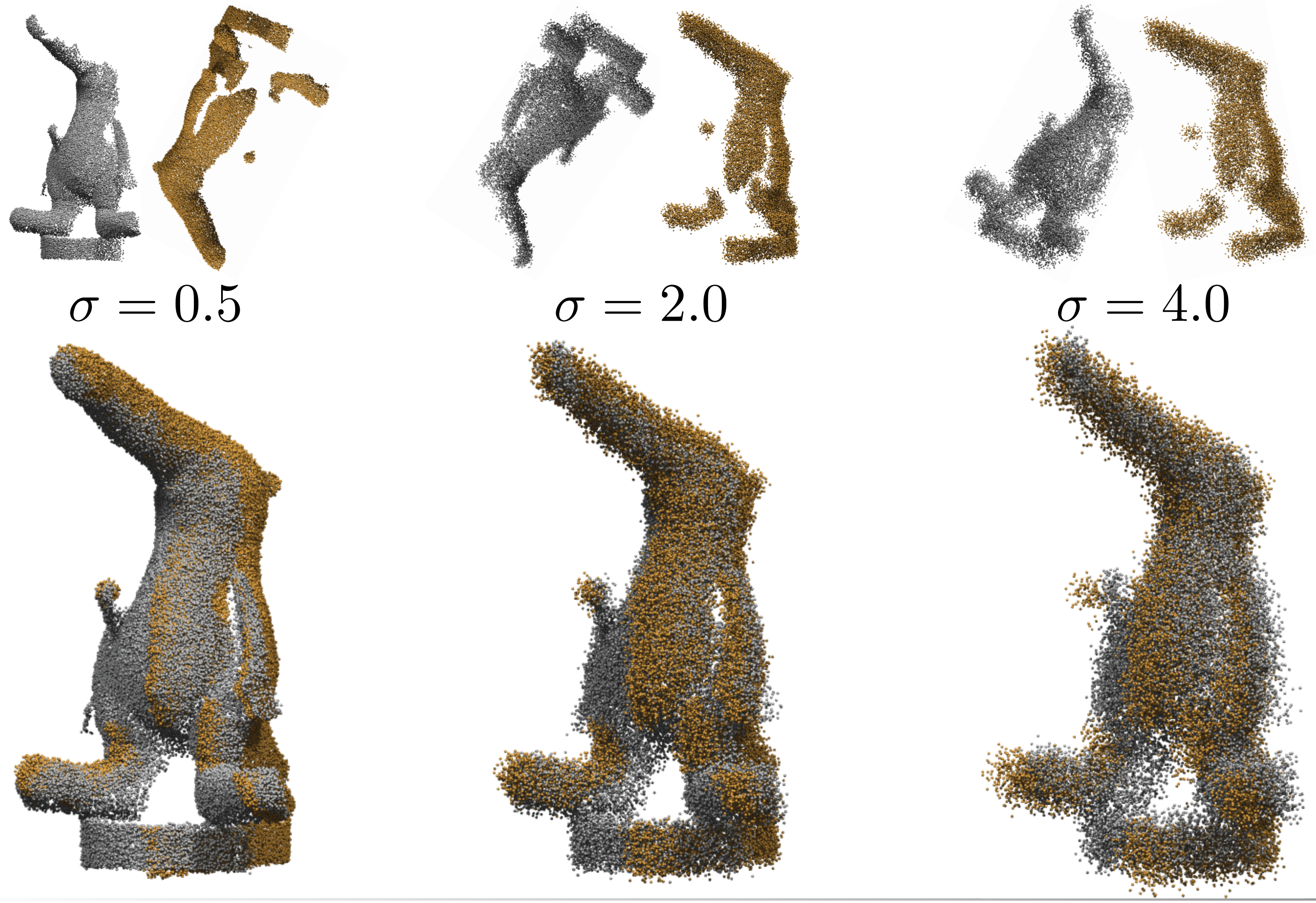
SelectCoplanarBase

- Select 3 points (from P) at random ⟶
  4th point to ensure (approx) ***coplanarity***

- Overlap amount ⟶
  decreasing guesses f = 1, 0.5, 0.25, ...

Results

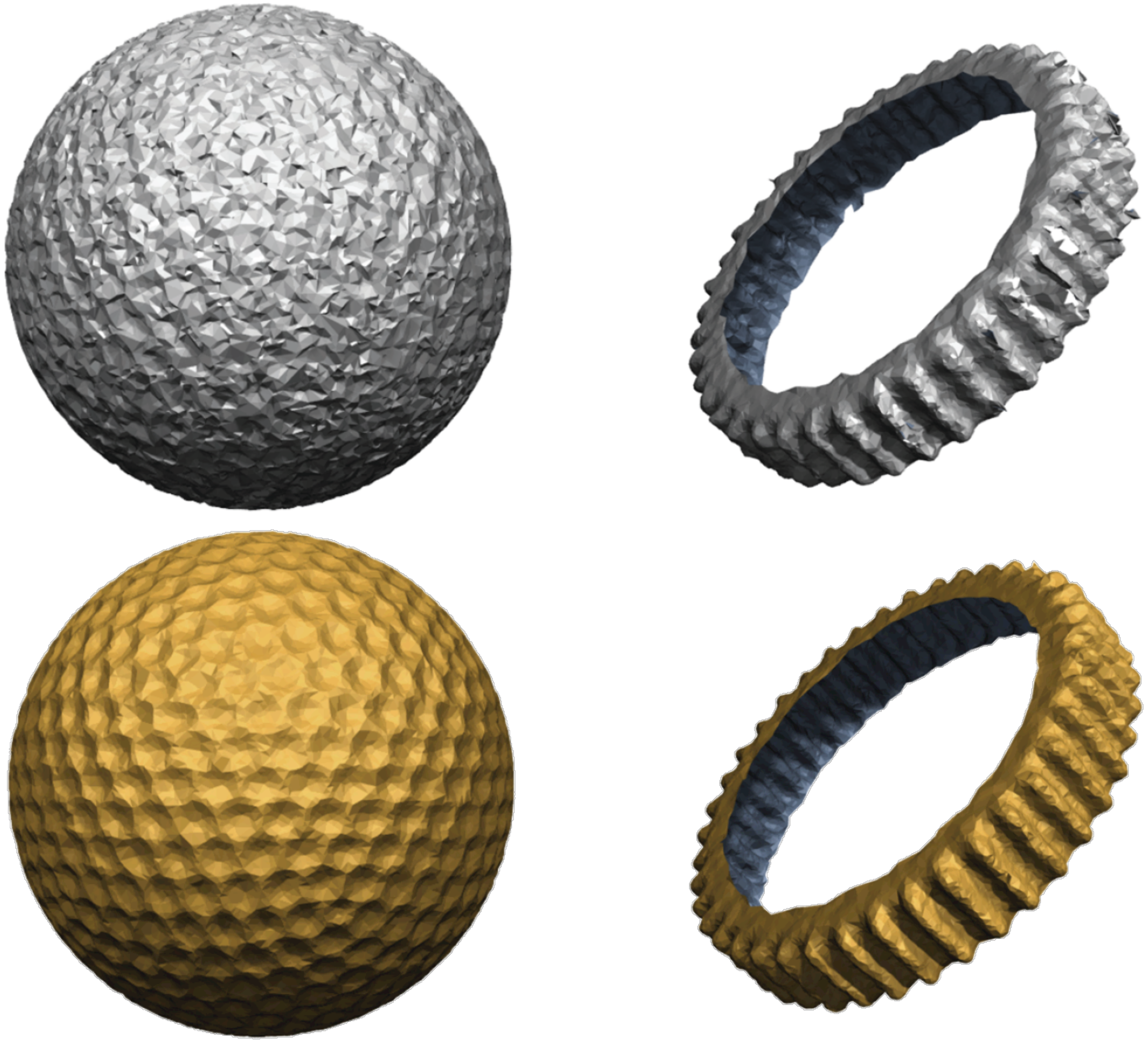
Effect of Noise



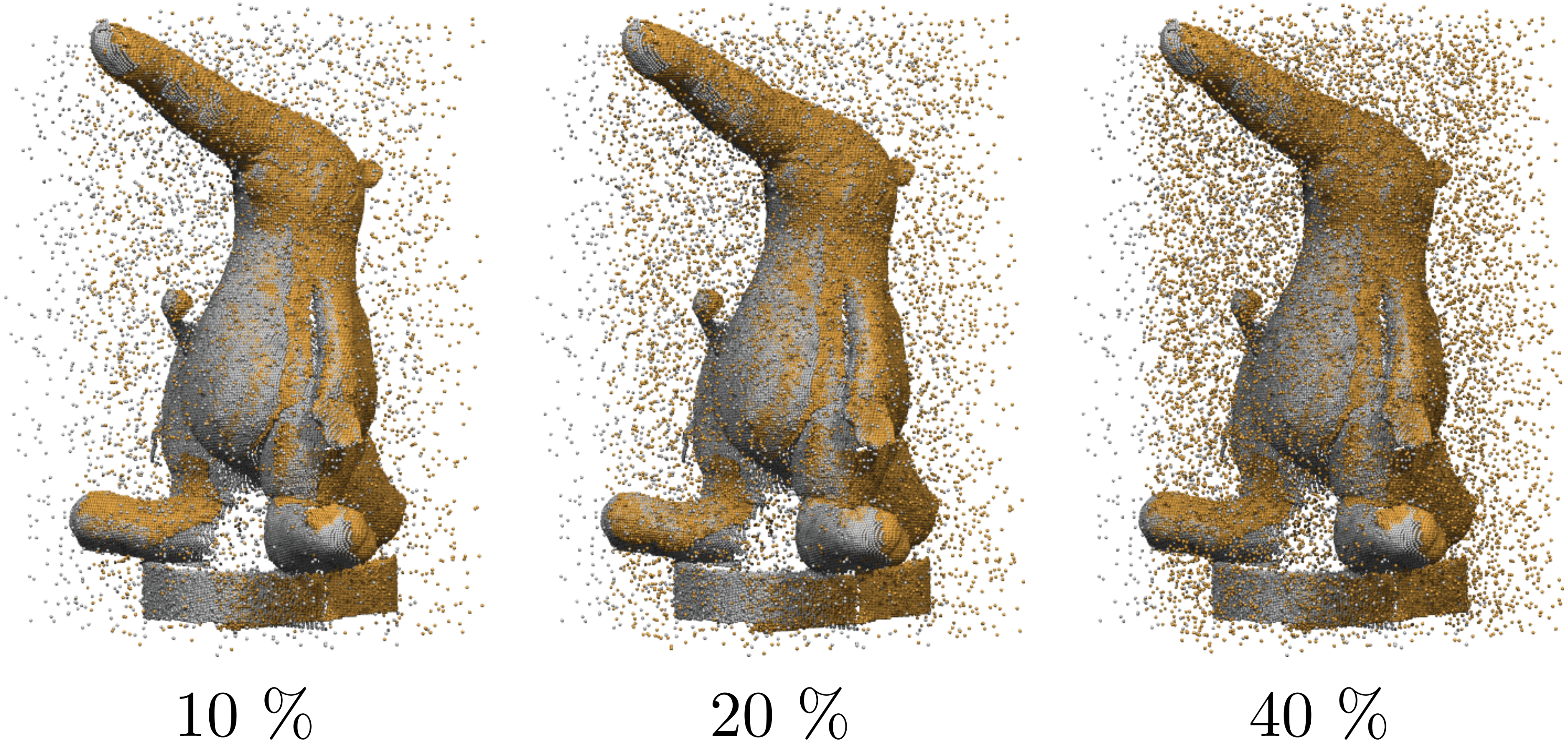
Why not Denoise Scans?

de-noise,
compute FP-s,
align

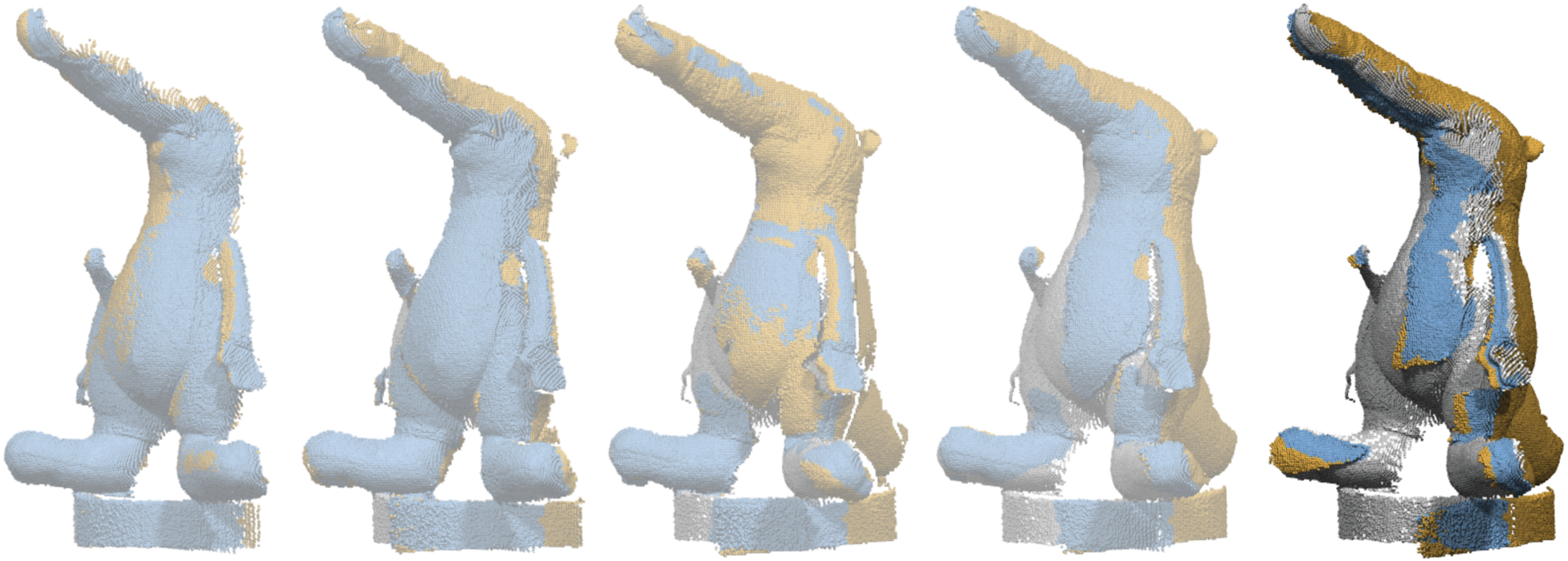
align with 4PCS,
de-noise



Effect of Outliers



Varying Overlap



80 %

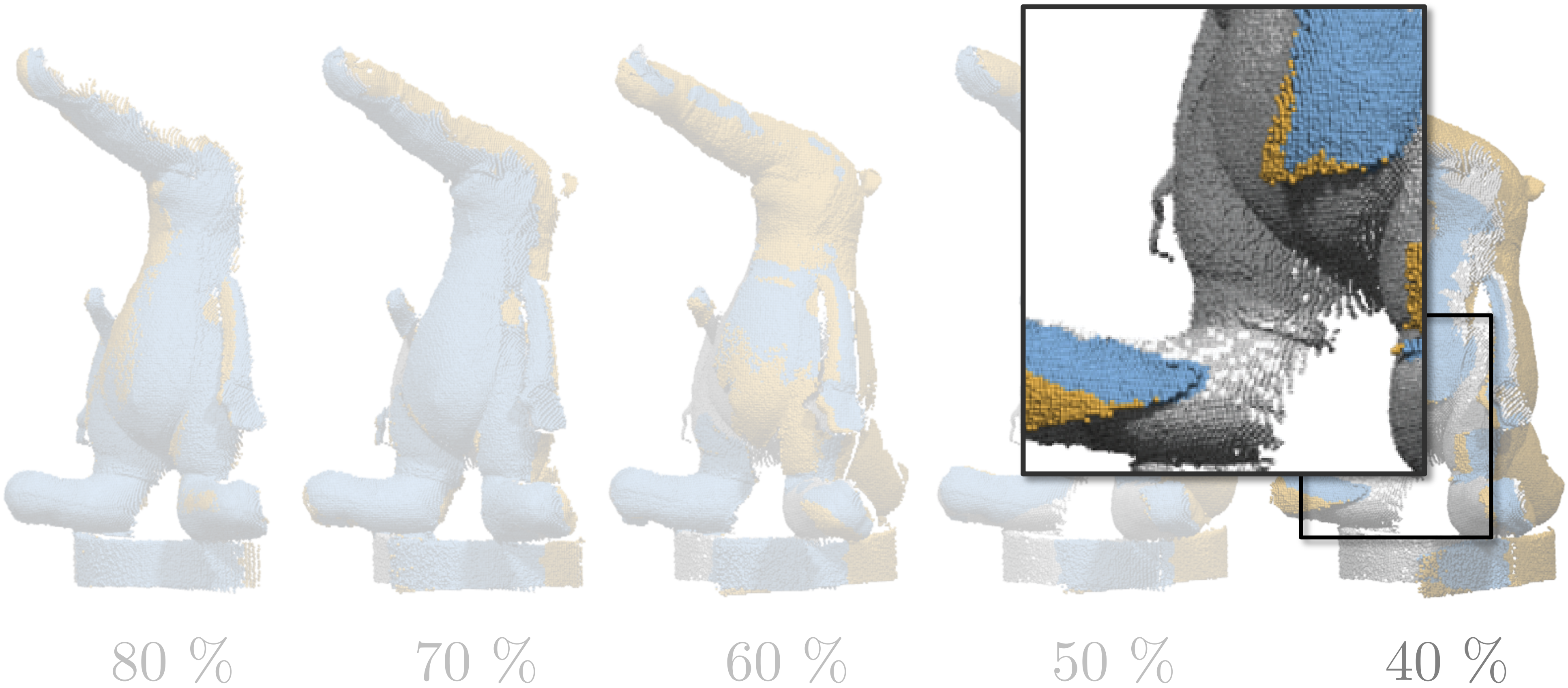
70 %

60 %

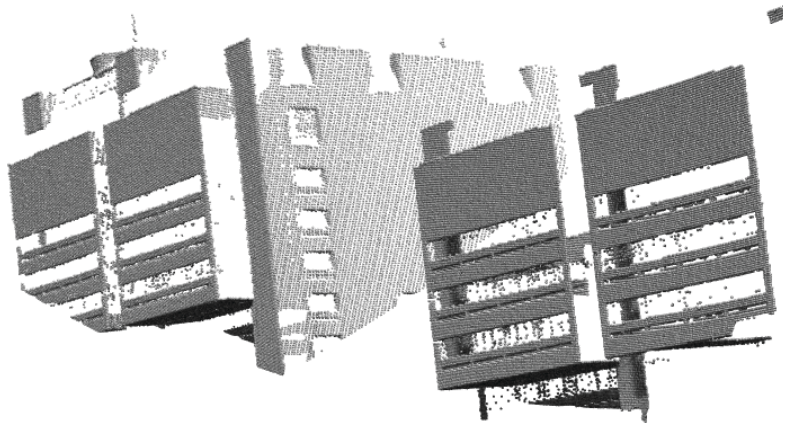
50 %

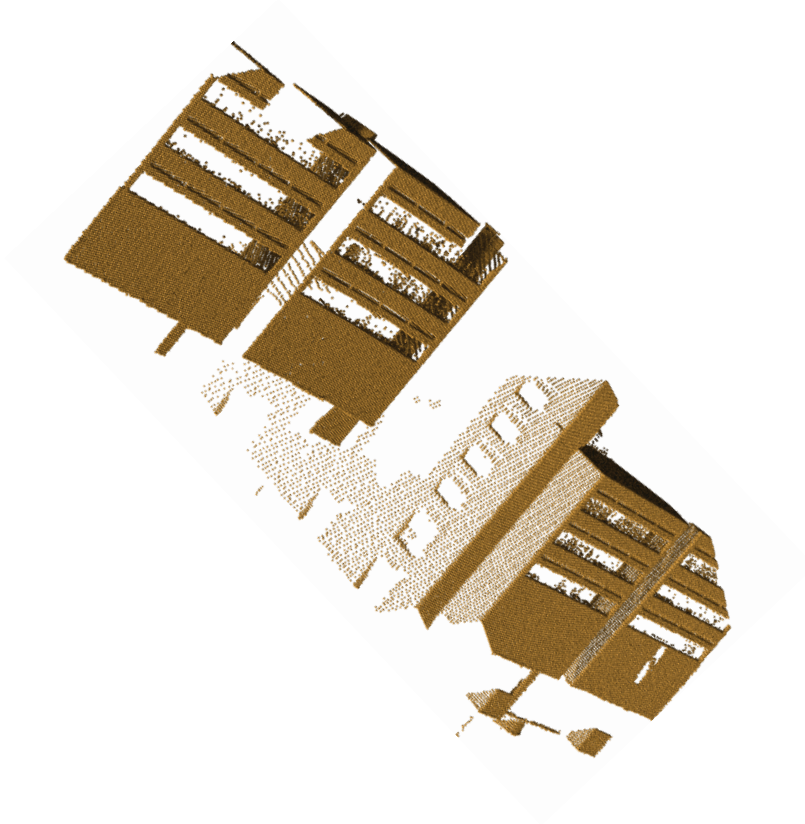
40 %

Varying Overlap

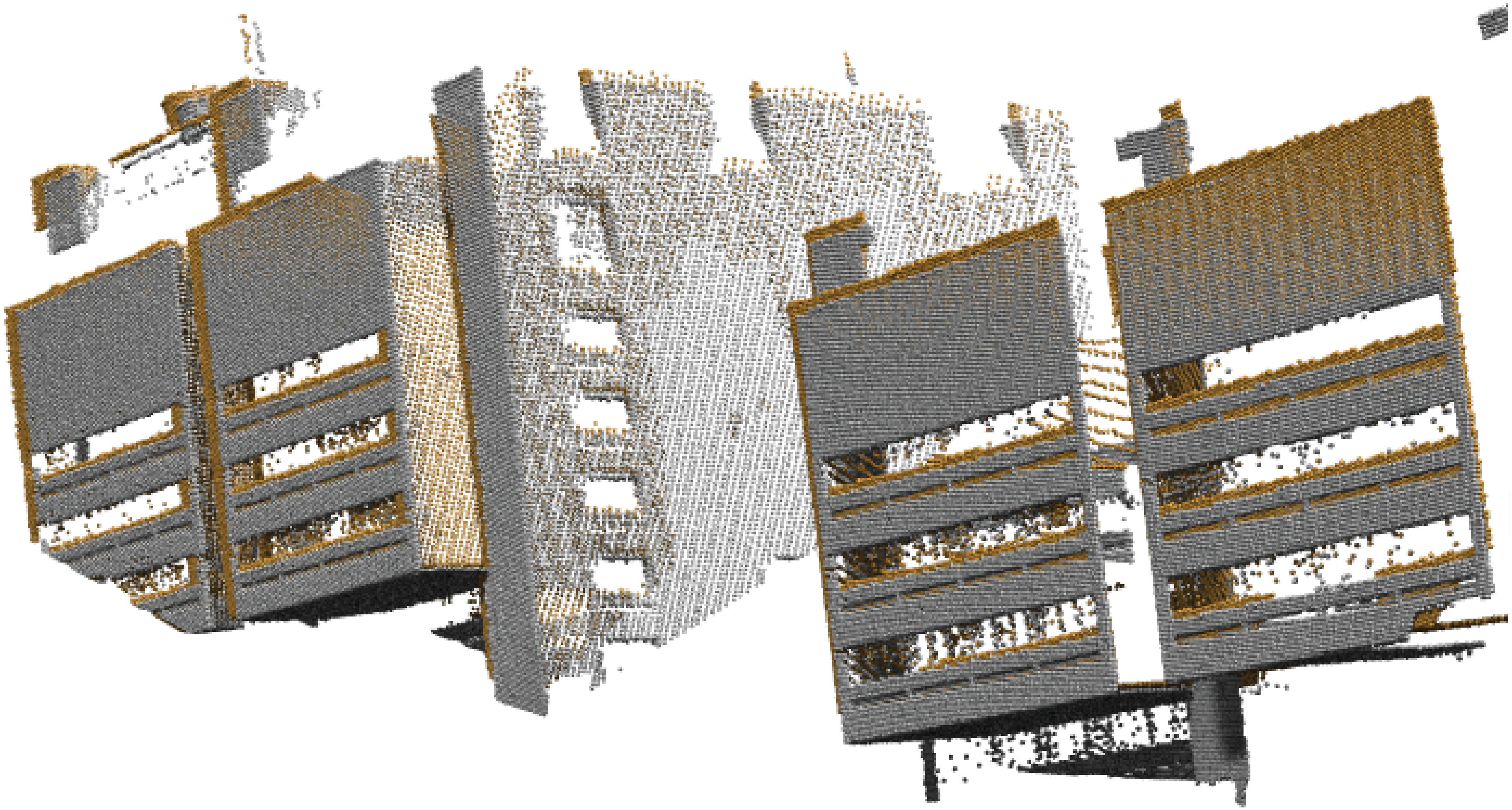


Building Facade

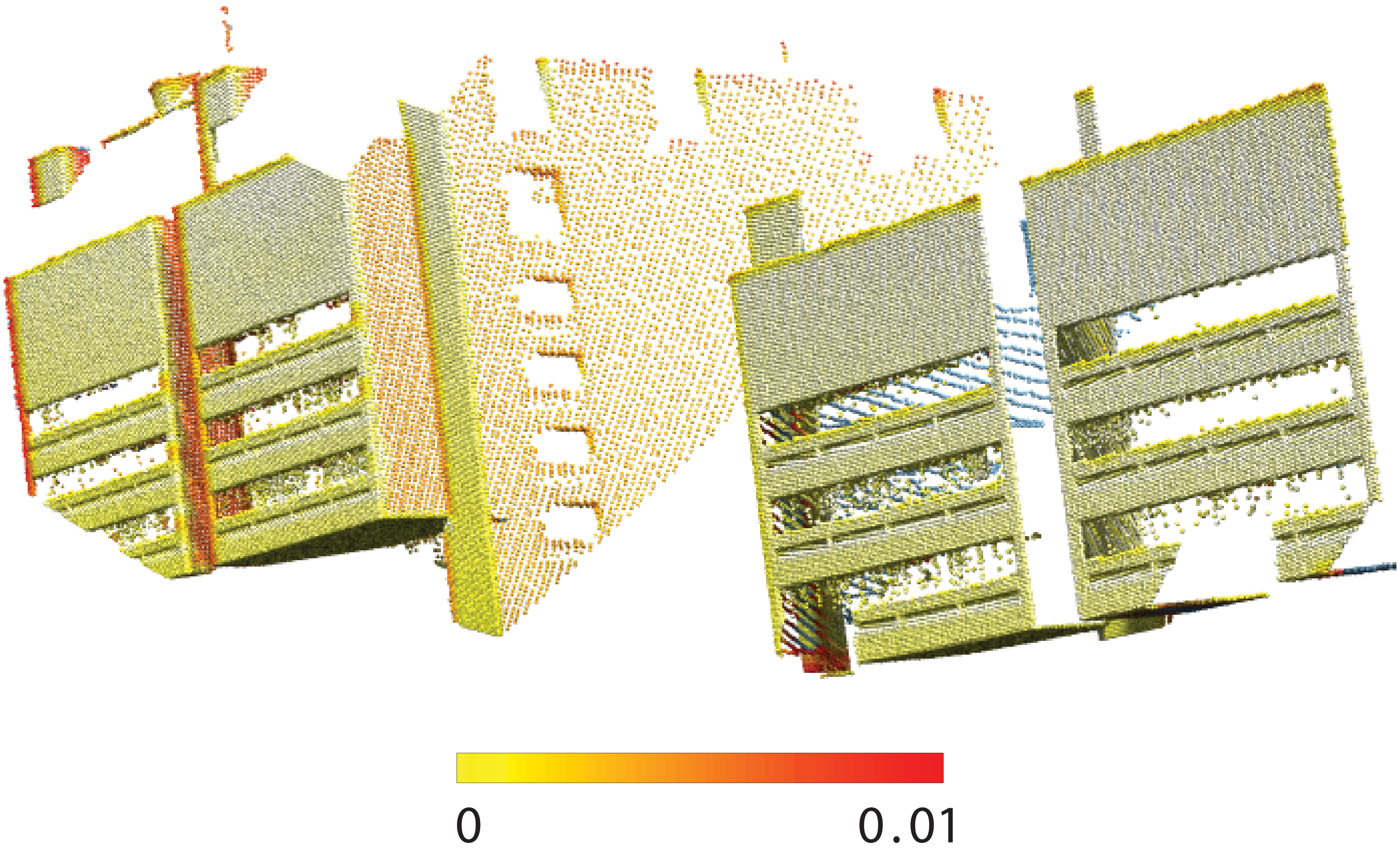




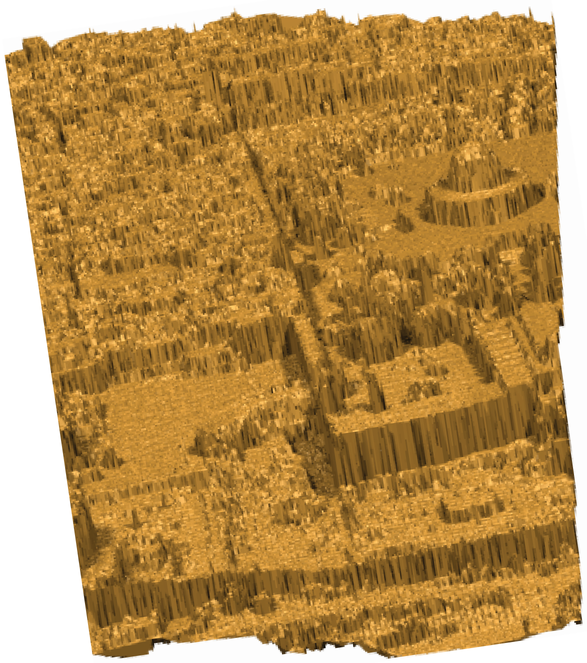
Building Facade

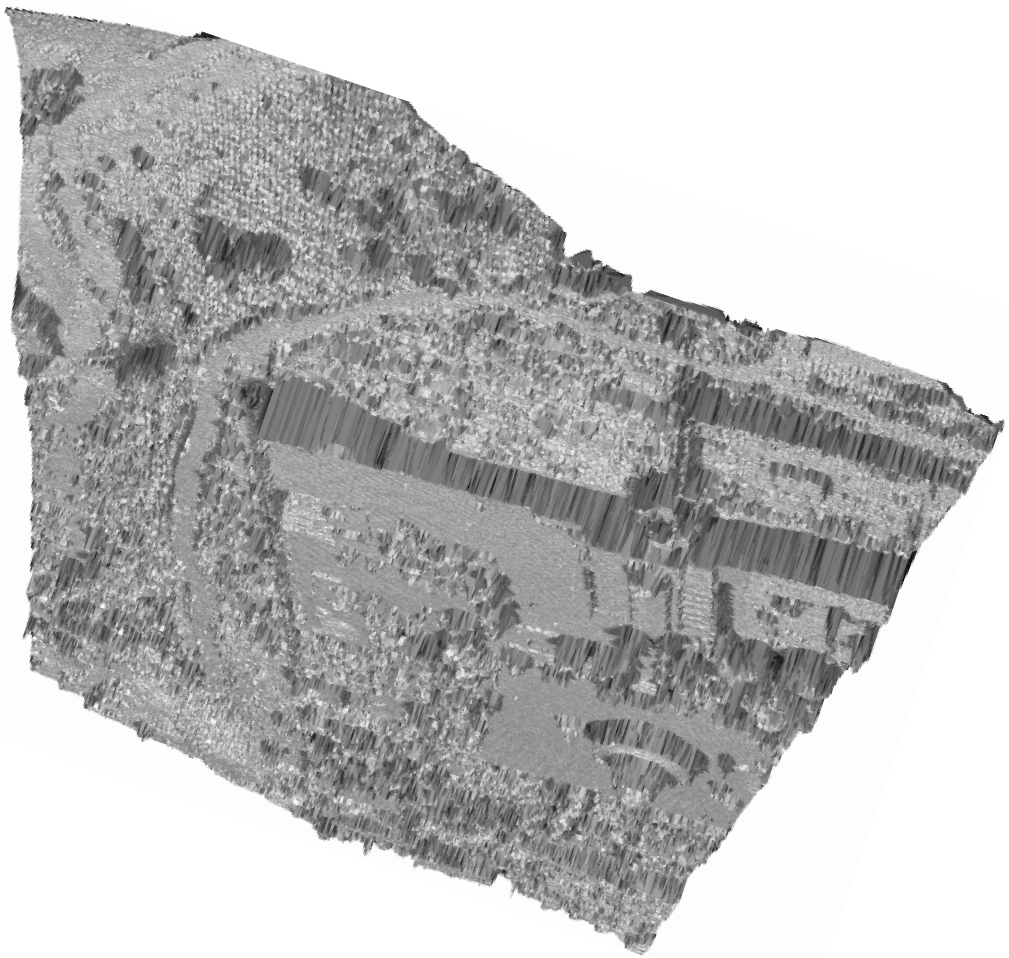


Building Facade

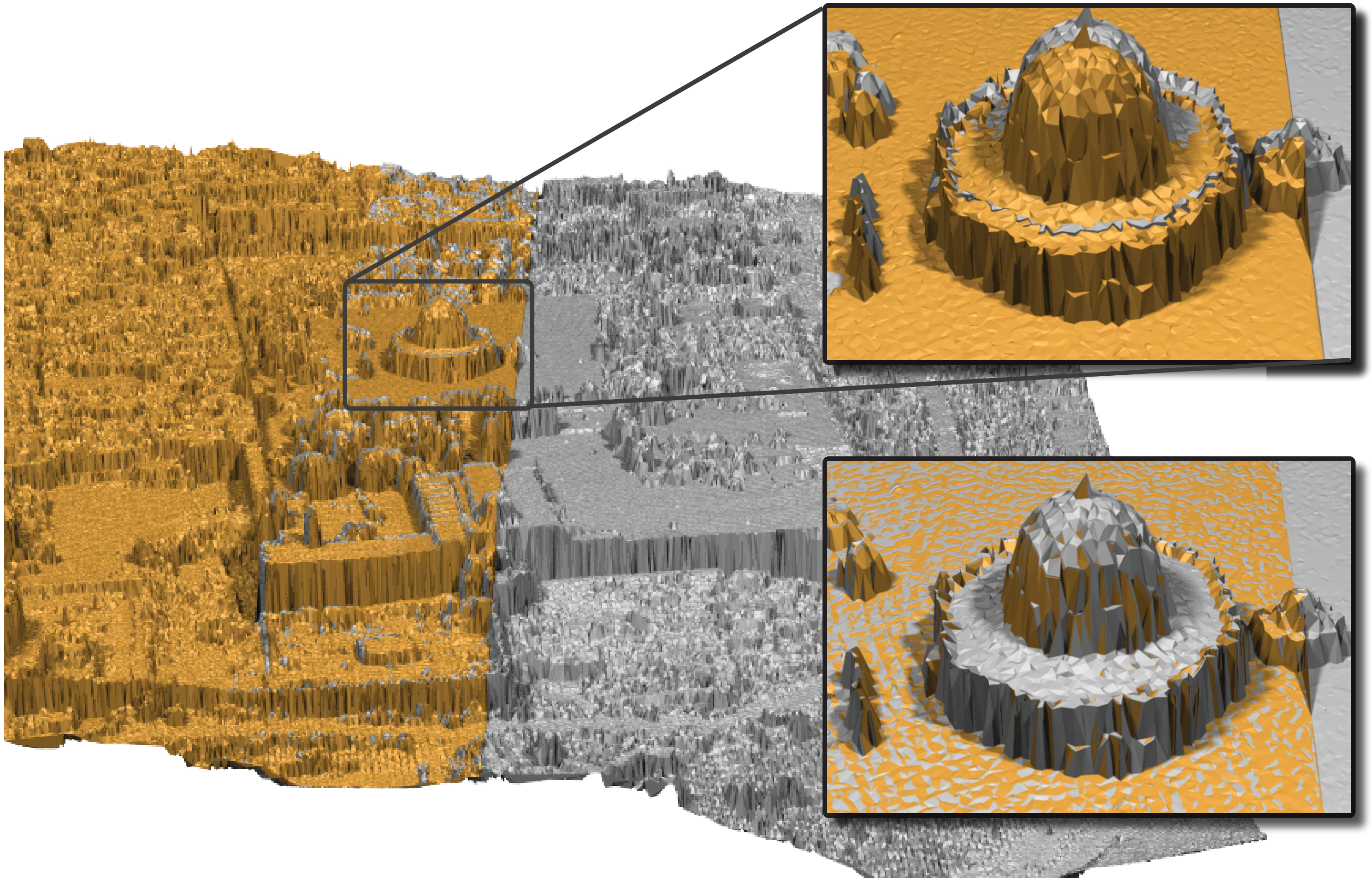


Jerusalem Scan

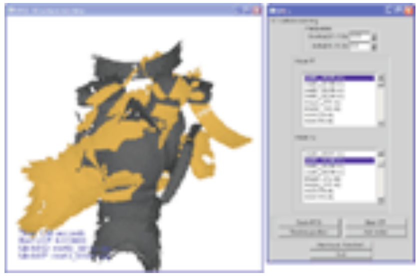




Jerusalem Scan



Try It



application

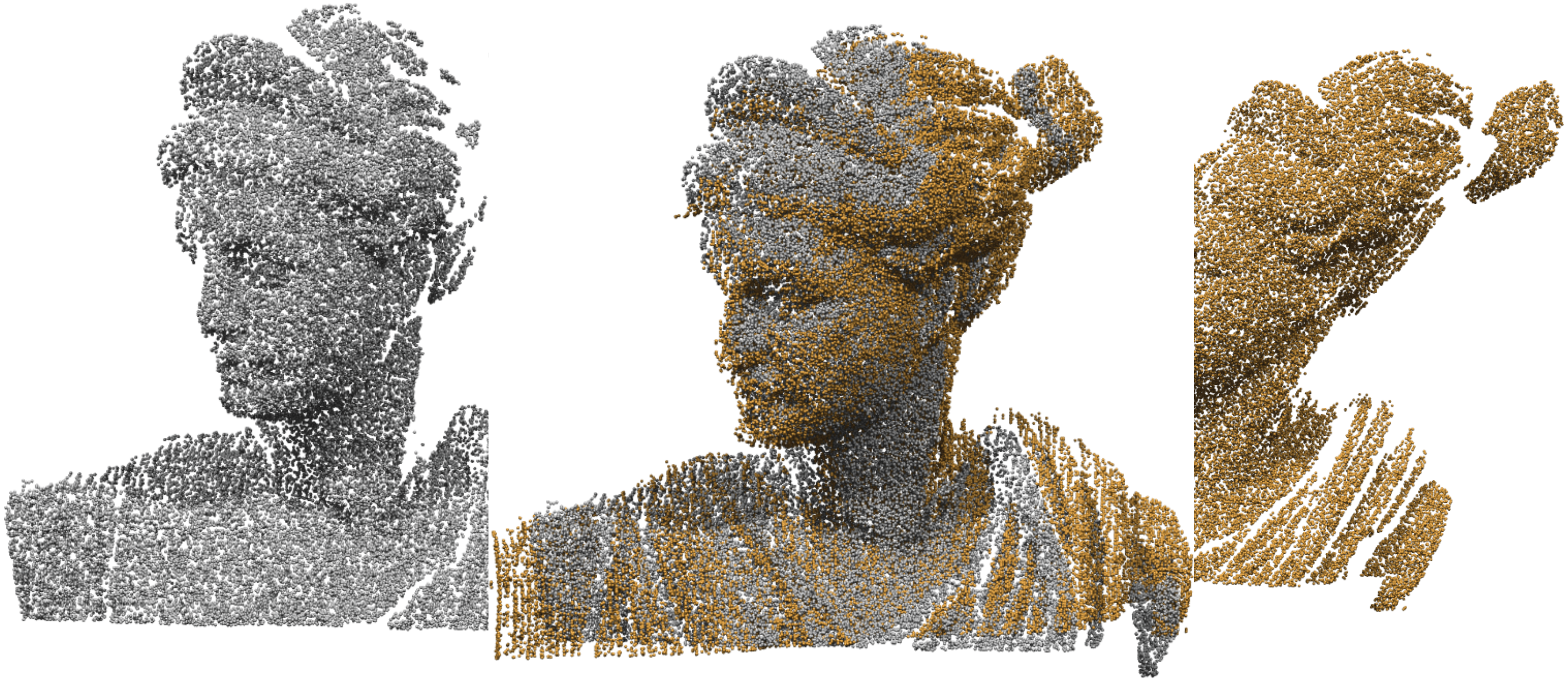
code

http://graphics.stanford.edu/~niloy/research/fpcs/fpcs_sig_08.html

Comments

- 4PCS orthogonal to:
  - Feature Points $\mathrm{O}(n^2) \rightarrow \mathrm{O}(n)$
  - Local Refinement (ICP)
- Extension to affine transforms

Affine Alignment



Limitations

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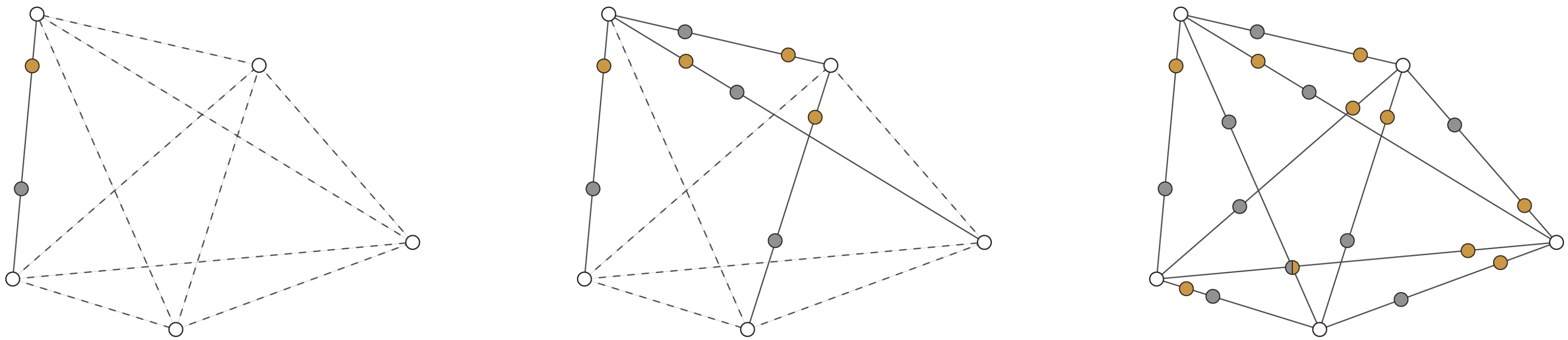
Thank You



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