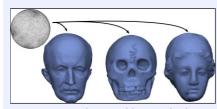
Geometric Modeling in Shape Space

Martin Kilian, Niloy J. Mitra, Helmut Pottmann

A discrete model of shape space



We model geometric objects – shapes – by mapping a fixed connectivity (a simplicial complex) to each object under consideration. The

vertex correspondence problem is solved in an independent preprocessing step.

Shapes and metrics

In order to solve geometic problems in shape space one needs algorithms to handle basic constructions of Riemannian geometry like the computation of geodesics, the exponential map, and parallel transport.

Straight lines, i.e., geodesics should correspond to pleasing deformations of a shape. While this may be hard to capture mathematically it shows that the preservation of intrinsic and extrinsic

Isometric deformations

In our discrete model a deformation is isometric if the length of each edge is preserved during deformation. This is equivalent to the following property of tangent vectors:

$$X_{p} - X_{q}, p - q? = 0$$
, (p,q) ? E.

The symmetric bilinear form

$$??X, Y ?? := \bigwedge_{(p,q) \in E} ?X_p - X_q, p - q??Y_p - Y_q, p - q?$$

gives rise to a Riemannian metric whose geodesics correspond to isometric deformations.

Constrained deformation

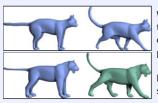


Instead of prescribing locations for all vertices of a shape we can fix only a certain subset. Minimizing the energy

$$E = ??X_{i}, X_{i}??_{M_{i}} + ??X_{i}, X_{i}??_{M_{i+1}}$$

of a curve for this more general kind of boundary value problem results in a natural placement off ree vertices.

Deformation transfer via parallel transport

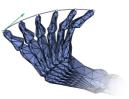


We use the concept of parallel transport to attach a given deformation at a new starting point.

Blue: Input poses. Green: Result of deformation transfer.

Deformations - curves in shape space

During deformation of a shape each vertex travels along a smooth curve. A curve in shape space – a deformation – is a smooth family of such vertex paths.



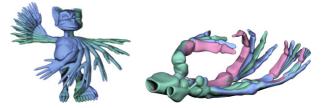
Differentiating each curve at a

common instant of time gives a collection of vectors attached to a shape. Such vector fields are tangent vectors to shape space.

characteristics of the surface of a shape during deformation is a good approximation to this notion.

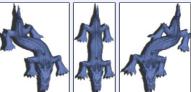
The goal is to define Riemannian metrics whose geodesic correspond to isometric, rigid, or volume preserving mappings, respectively. Of course this goal can only be reached exactly if the input shapes are isometric. Otherwise we try to achieve this goal in an as-good-as-possible way.

Geodesic interpolation and extrapolation



Left: Geodesic interpolation using the as-isometric-as-possible shape metric. Right: Geodesic interpolation with subsequent extrapolation (purple).

More constrained deformations



Geodesic interpolation can be used to cope with notions such as symmetry. Symmetric poses correspond to geodesic midpoints.

Fixing vertices in undistorted areas, constrained deformation can reduce overall distortion introduced during modeling.



Bibliography

 Martin Kilian, N. J. Mitra, and H. Pottmann, shape space, ACM Trans. Graphics (Proc. SIGGRAPH' 07), vol. 26, no. 3, 2007.





Der Wissenschaftsfonds.