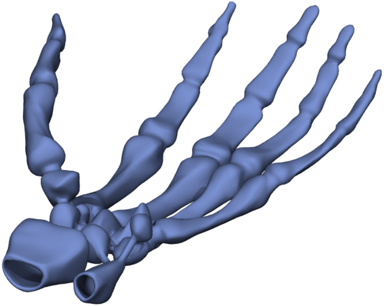
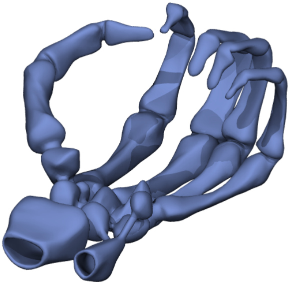
Geometric Modeling in Shape Space

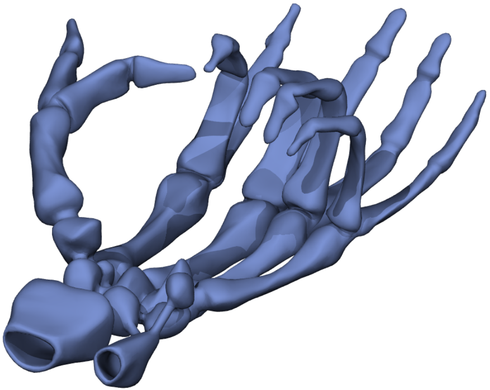
Martin Kilian Niloy J. Mitra Helmut Pottmann
Vienna University of Technology

Motivation

Morphing - An Example







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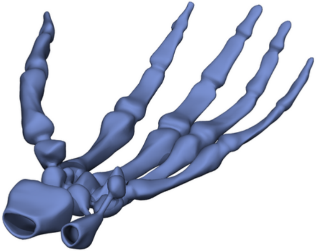

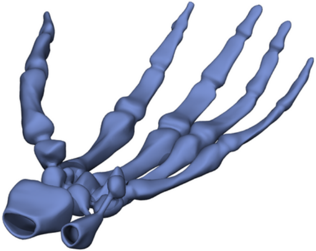

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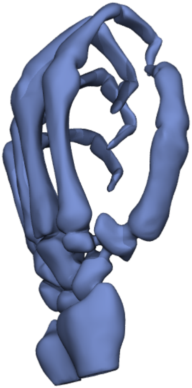

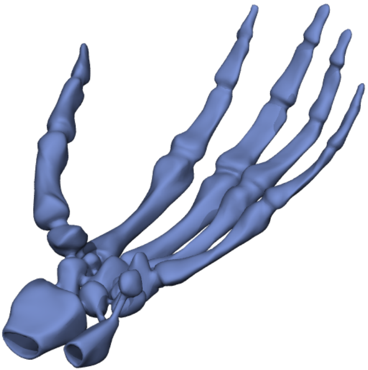

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Geometric reasoning
+ Riemannian geometry

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vertex path problem

A Unified Approach

Applications

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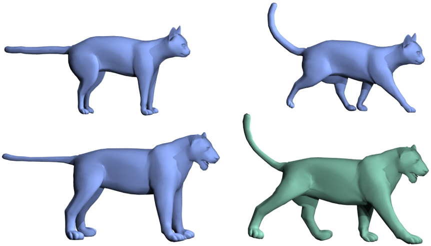

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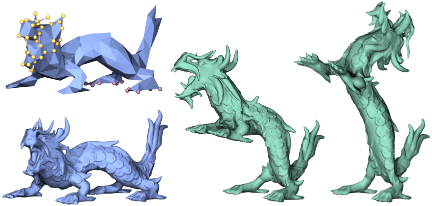

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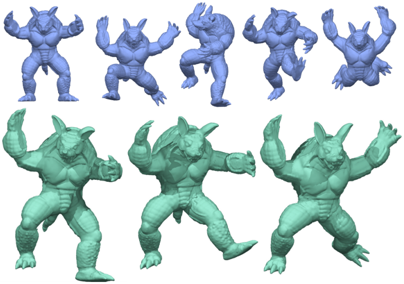

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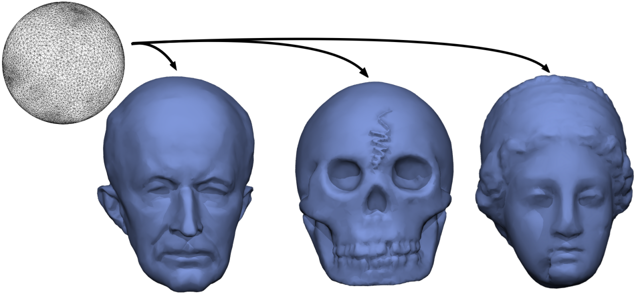
vertex path problem
deformation transfer
shape deformation
shape exploration

We consider isomorphic meshes given by a template connectivity.



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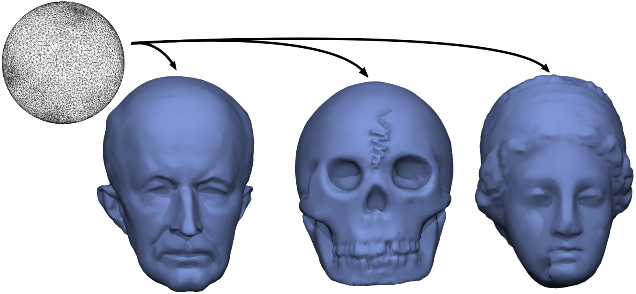

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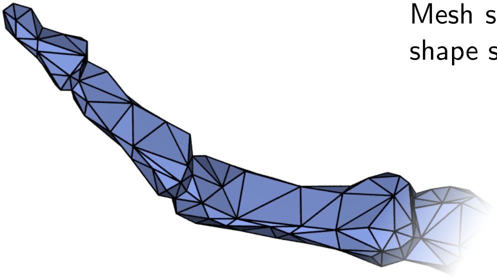
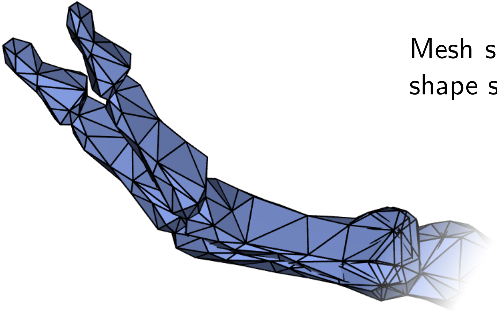
Schreiner et al. 04, Kraevoy and Sheffer 04



Mesh sequences as curves in shape space



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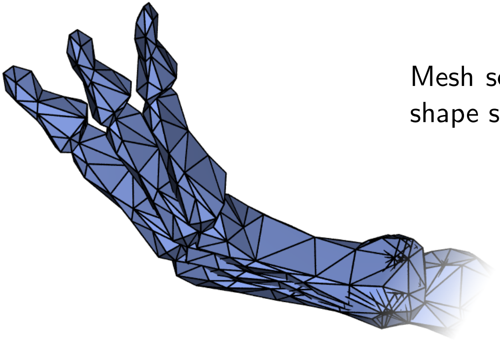

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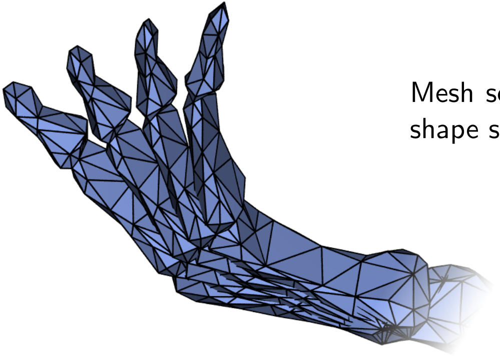

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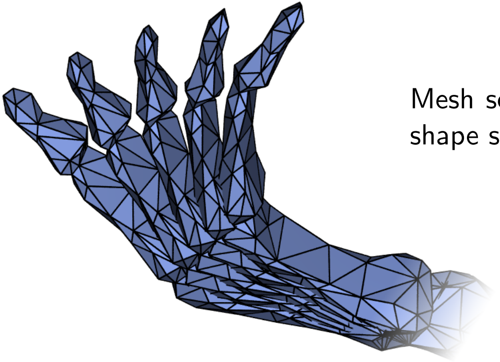

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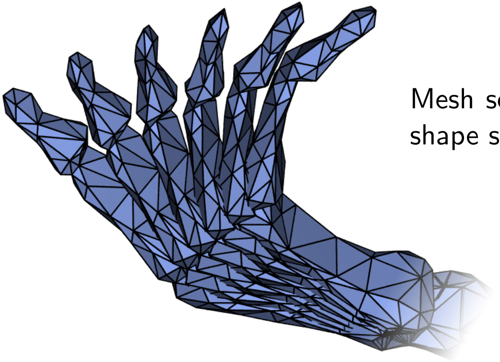

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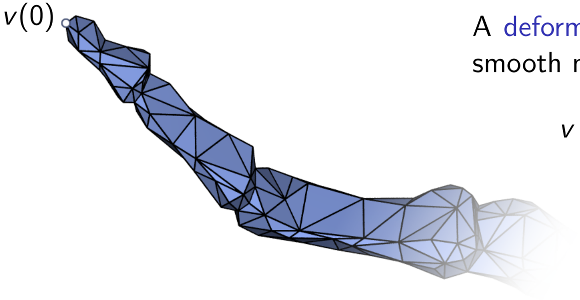
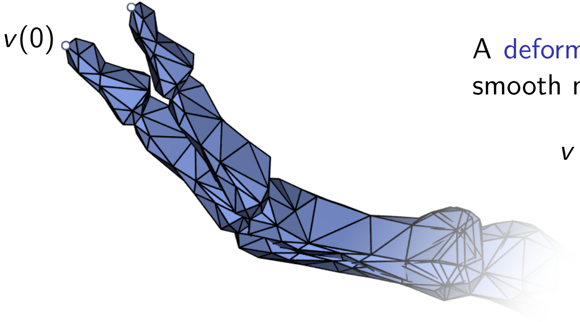


A deformation is a family of smooth mappings

$$v : [0,1] \to \mathbb{R}^3$$



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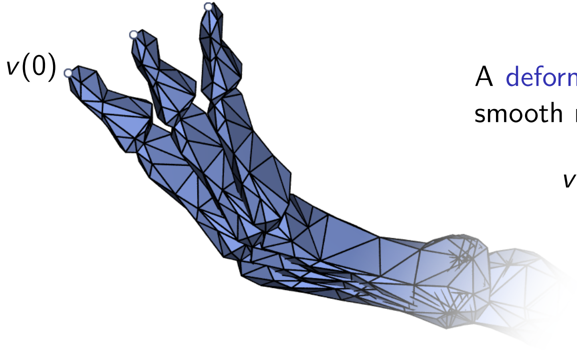

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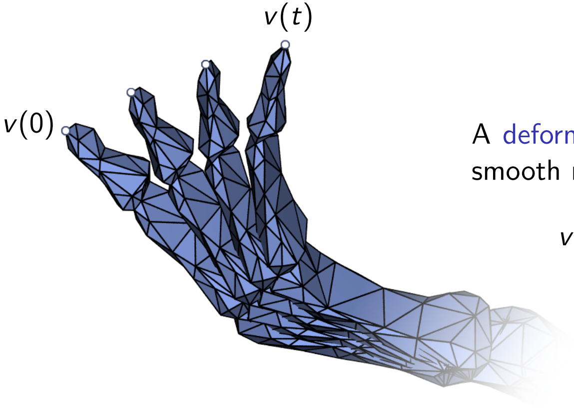

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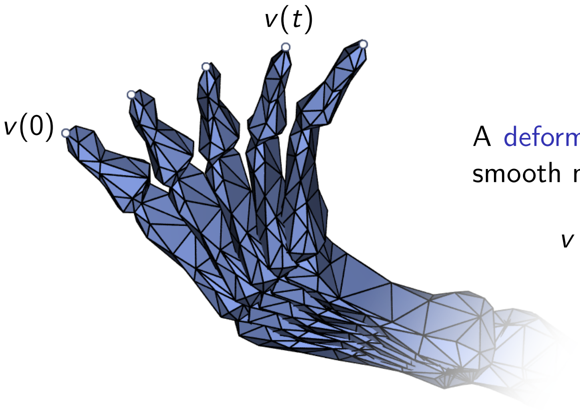
Basic Notions

Deformations



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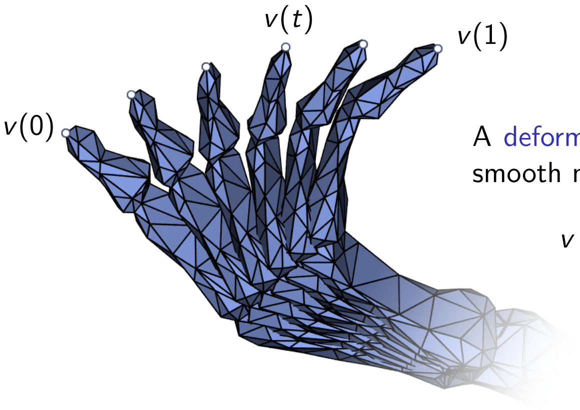

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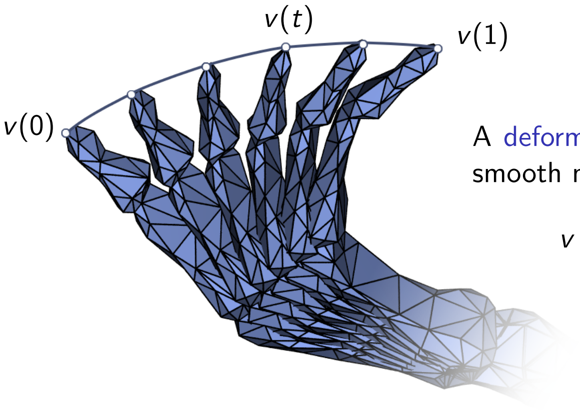

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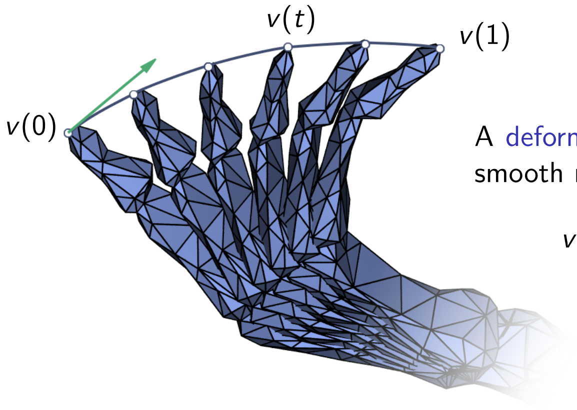

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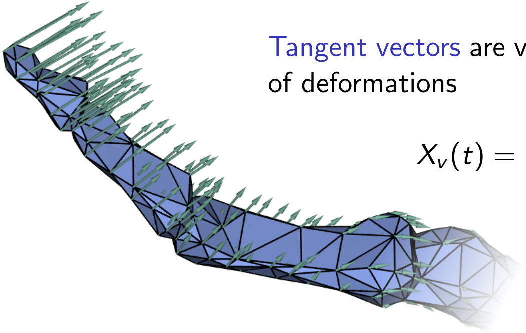

Tangent vectors are velocity vector fields of deformations

$$X_v(t) = \frac{d}{dt} v(t)$$

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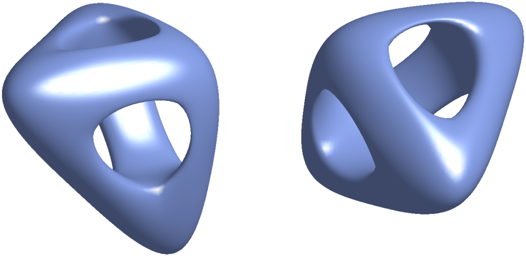
$$\int \|\dot{c}(t)\|^2 dt = \int \langle \dot{c}(t), \dot{c}(t) \rangle dt \quad \longrightarrow \int \langle\!\langle \dot{\varphi}(t), \dot{\varphi}(t) \rangle\!\rangle_{c(t)} dt$$

of a curve c in Euclidean space. For more general spaces we need a so called Riemannian metric:

- inner product for each tangent space
- smoothly depending on the points of the manifold

To get useful metrics on shape space we translate properties of a shape to conditions on tangent vectors. Useful properties include

- **Rigidity**
- Isometry



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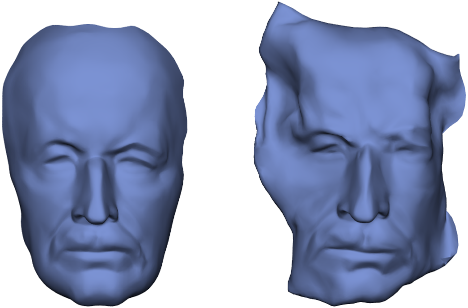

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$$\|p(t) - q(t)\|^2$$

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$$\langle\langle X, Y \rangle\rangle_M := \sum_{(p,q) \in E} \langle X_p - X_q, p - q \rangle \langle Y_p - Y_q, p - q \rangle$$

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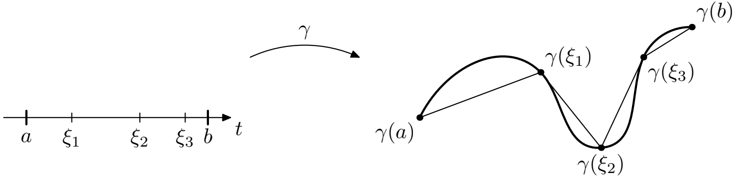
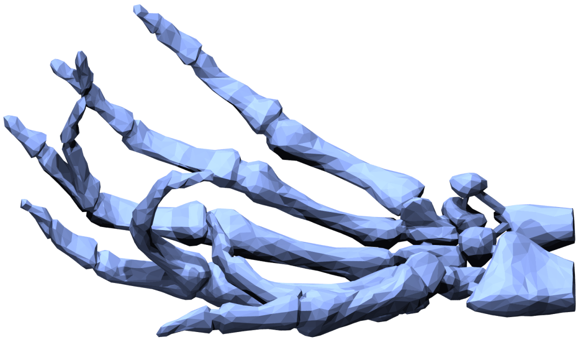
Given a deformation we inscribe a polyline and measure the squared norm of each segment – just as one would do with a curve in the Euclidean setting shown below:

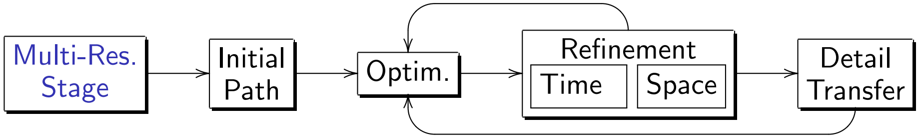


$$E = \sum \langle\langle X_i, X_i \rangle\rangle_{M_i} + \langle\langle X_i, X_i \rangle\rangle_{M_{i+1}}$$

Geodesics

Solving the Boundary Value Problem





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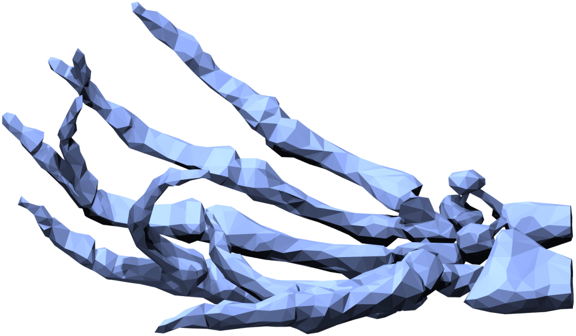

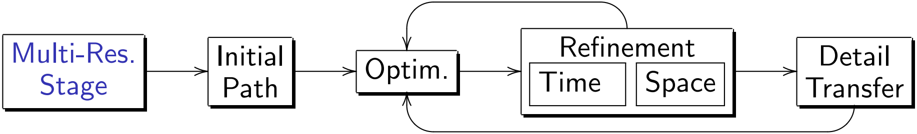

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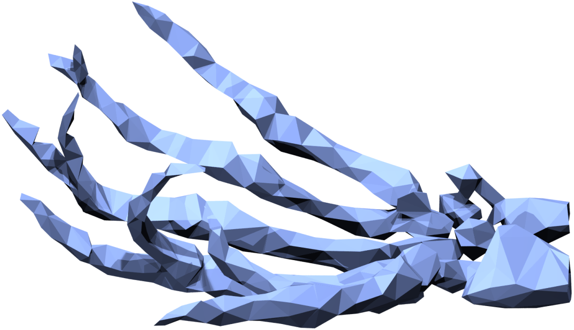

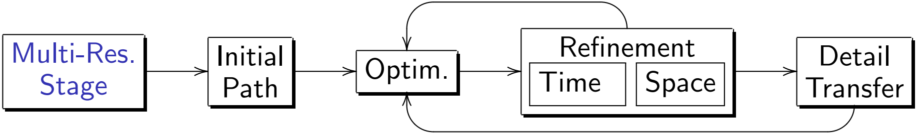

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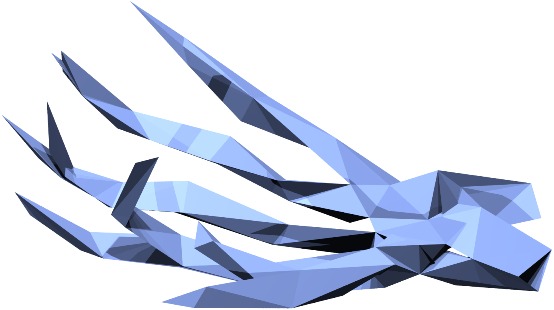

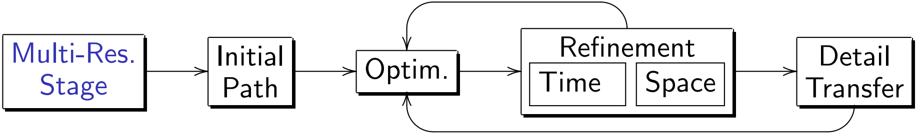

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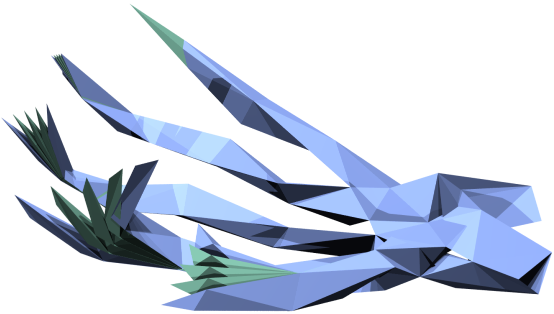

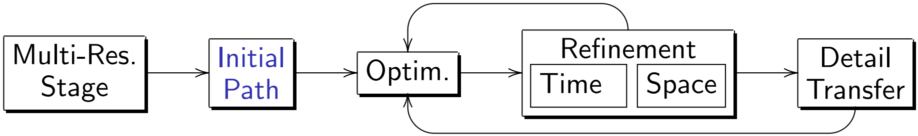

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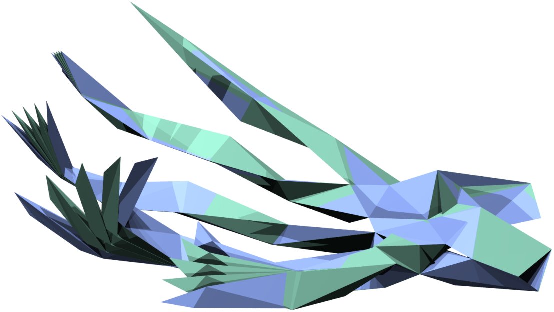

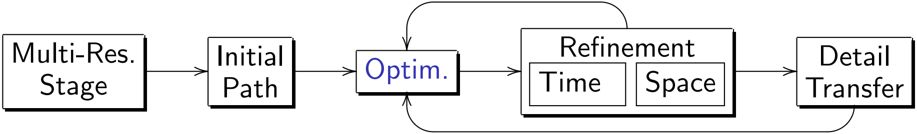

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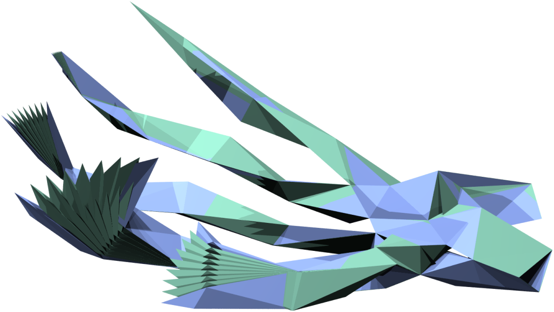

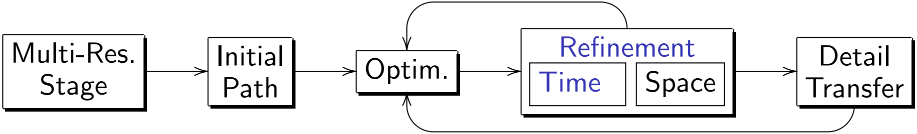

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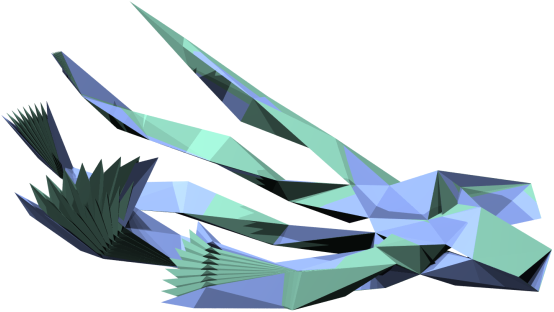

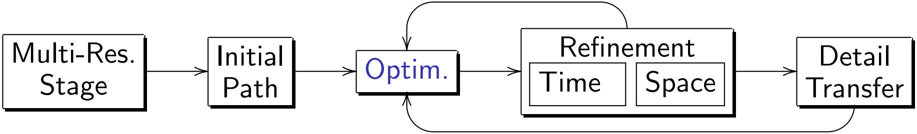

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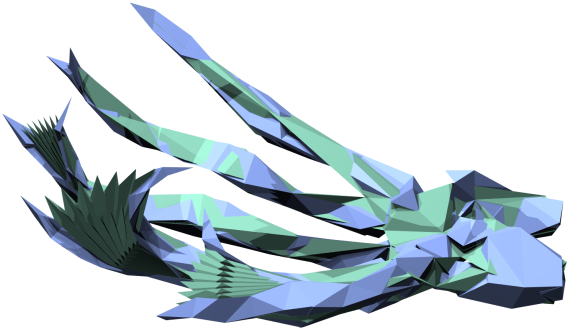

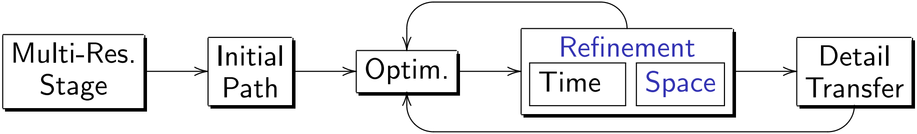

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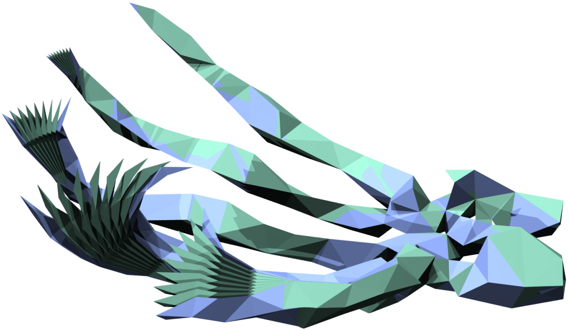

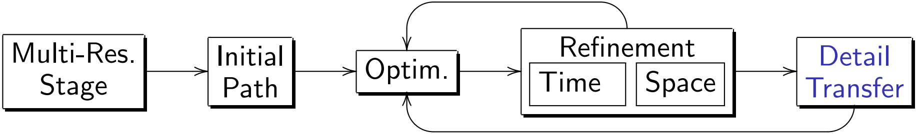

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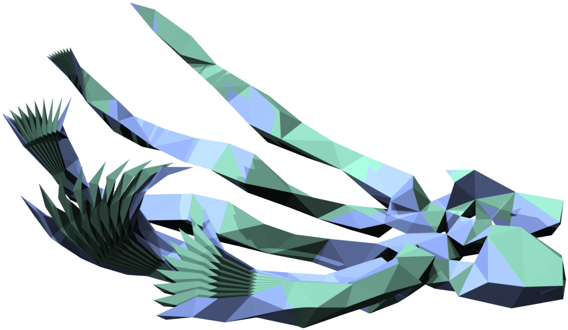

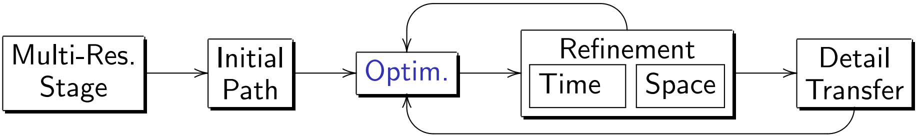

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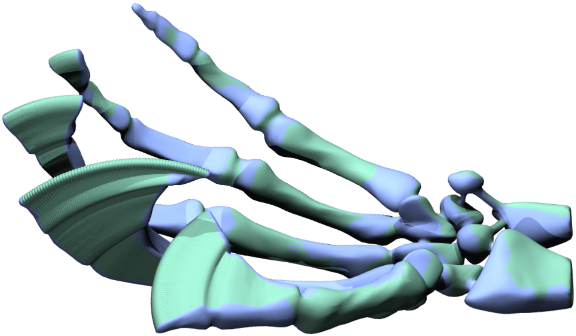
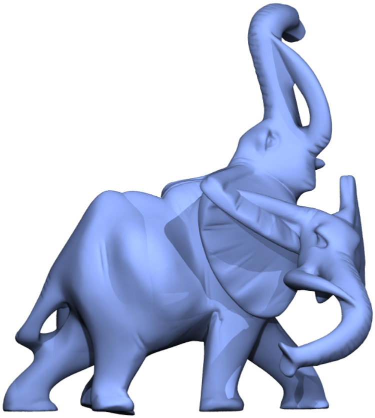


Morphing

Interpolation and Extrapolation



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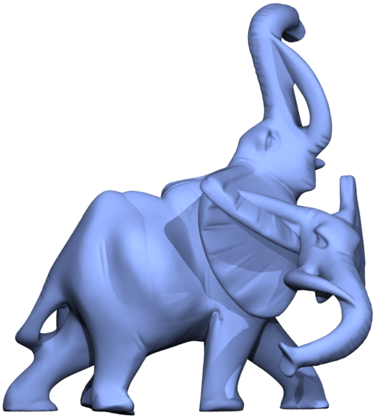
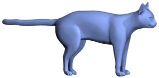


Deformation Transfer

Problem Statement



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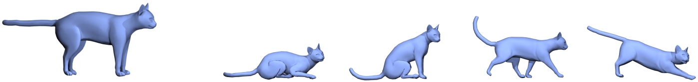

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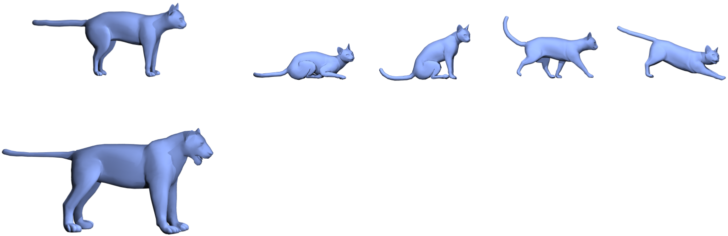

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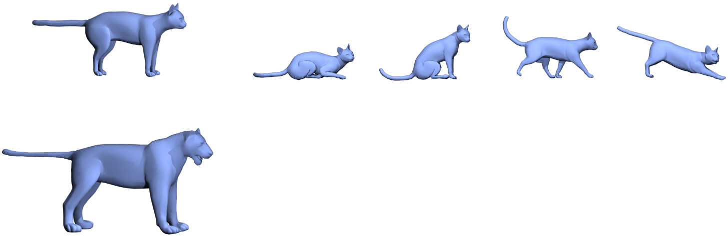
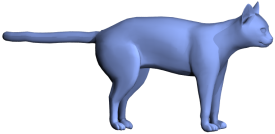
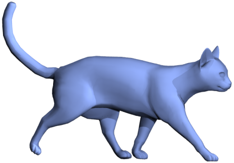


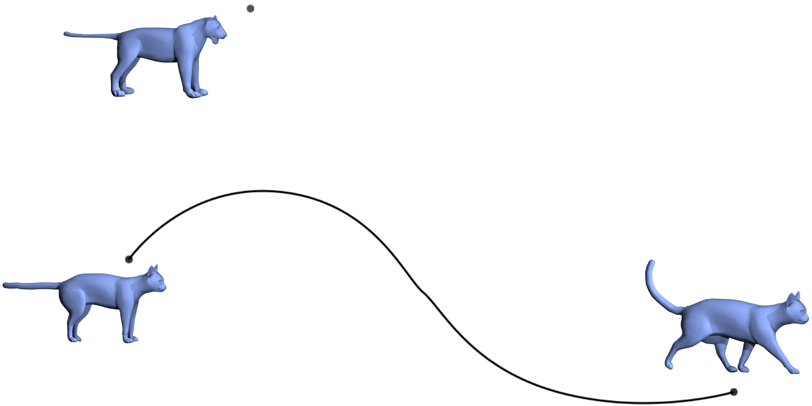
Sumner and Popović 04

We are given two input shapes and a deformation joining them – this does not need to be a geodesic!





We use the concept of parallel transport in a Riemannian manifold to attach the given deformation at a new starting point.



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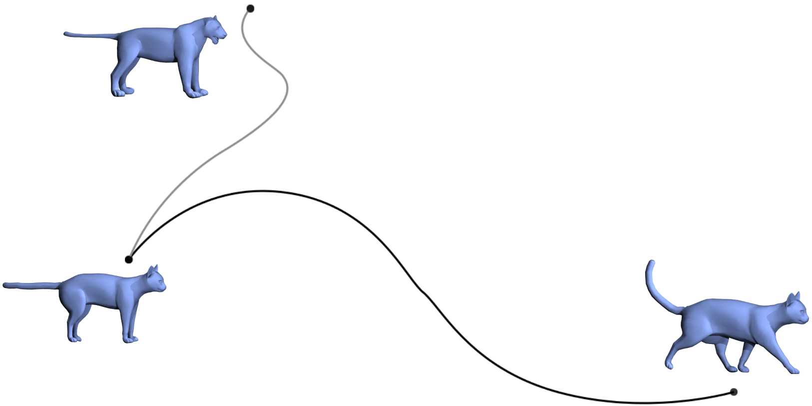

Deformation Transfer

Parallel Transport

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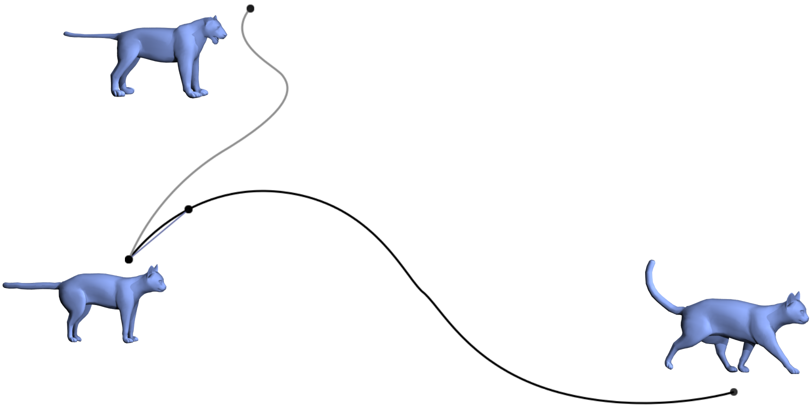

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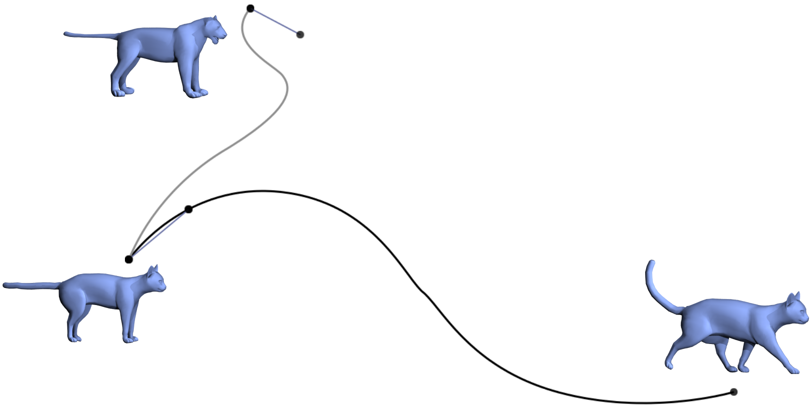

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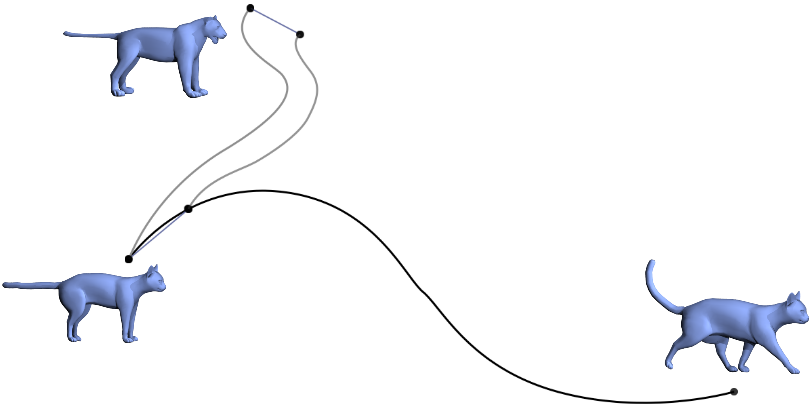

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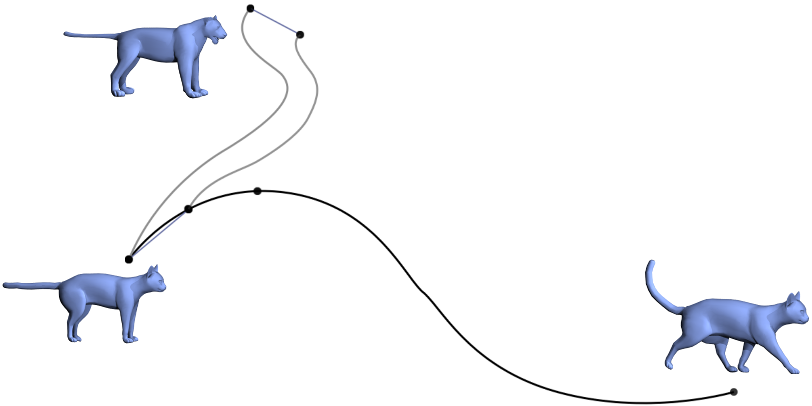

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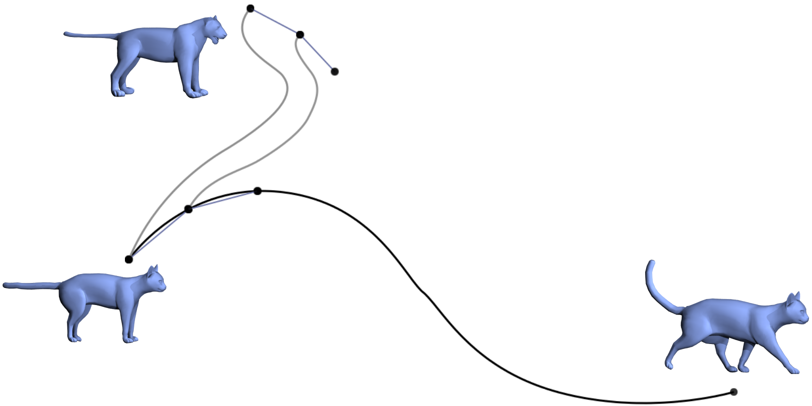

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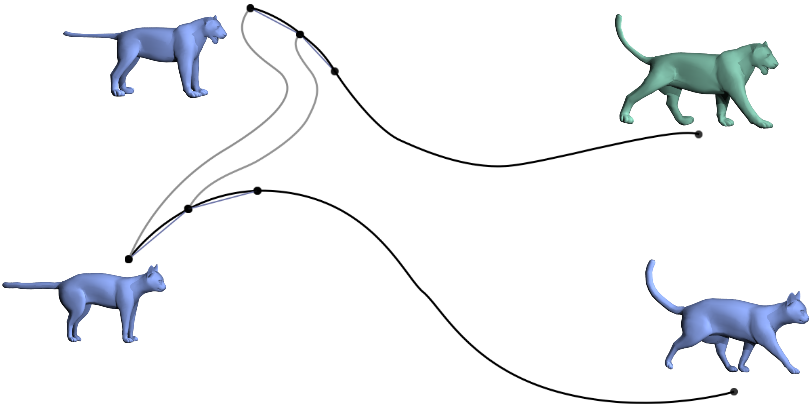
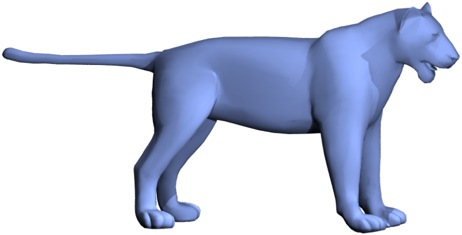


Several interesting deformations are computed during deformation transfer:



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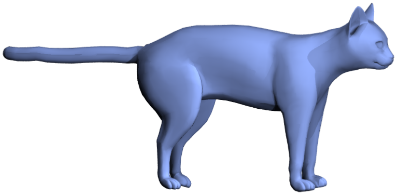

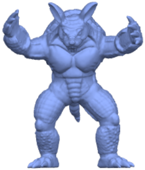

Shape exploration provides the user a real time interface to explore a certain subspace of shape space spanned by given input poses.

Der, Sumner and Popović 06

Shape Exploration

Subdivision in Shape Space

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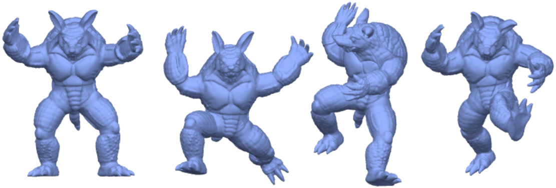

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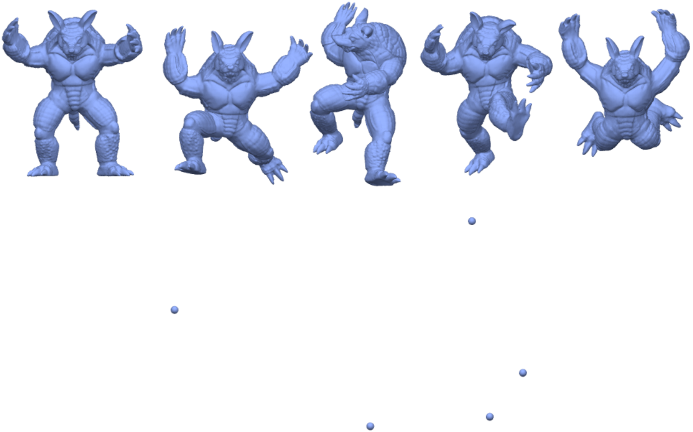

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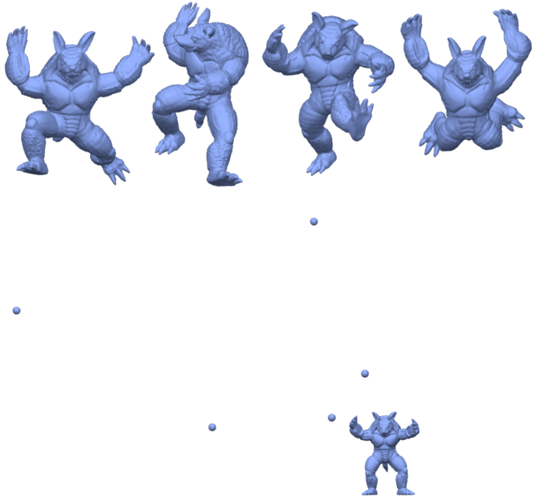

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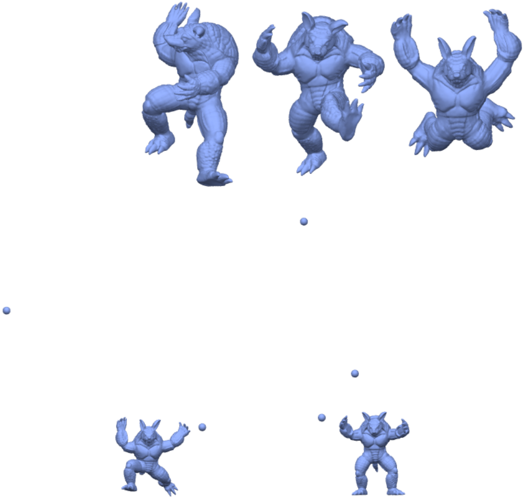

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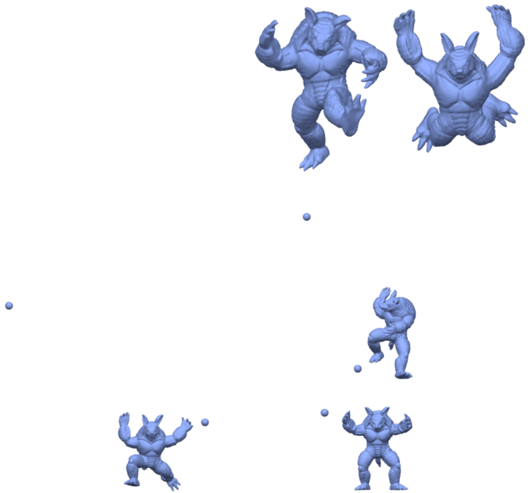

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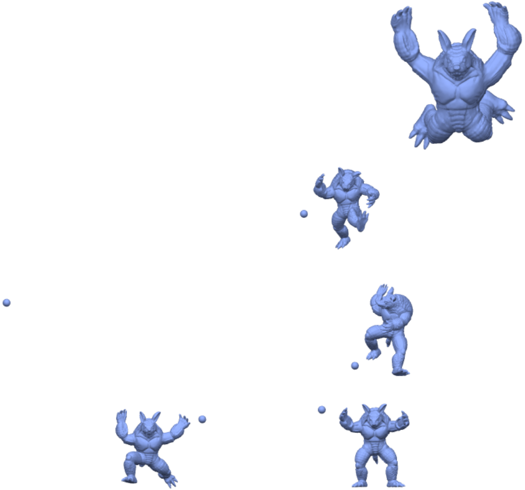

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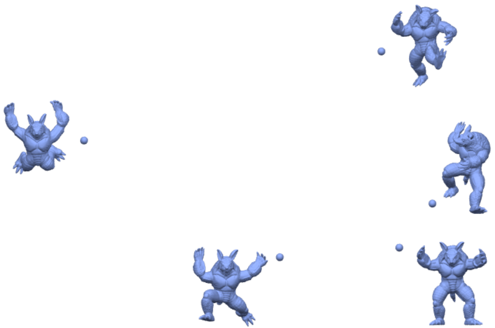

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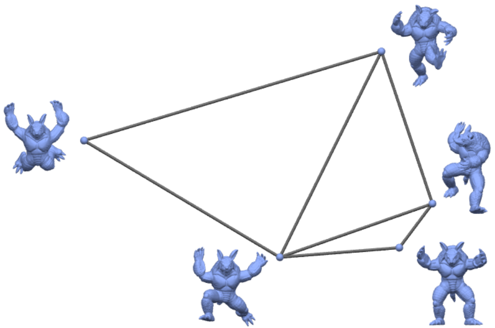

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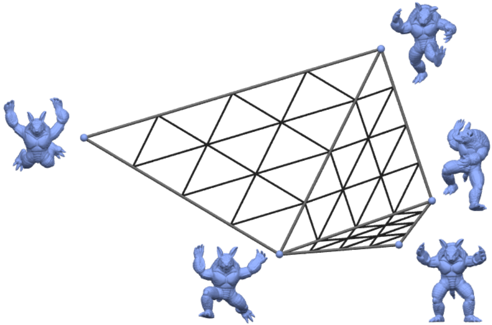

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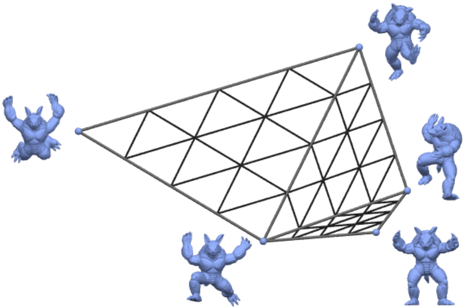

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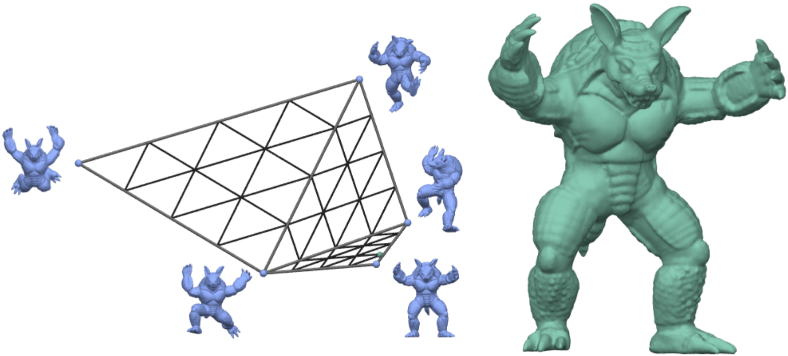
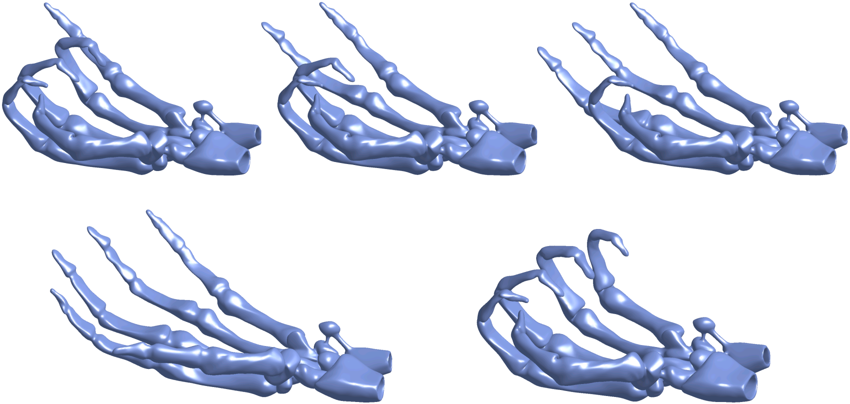


Shape Exploration

Demo



Conclusion and Future Work

A unified geometric approach

- Shapes are points of a smooth manifold.
- Shape properties are captured by the metric.
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Challenges

- Collision free geodesics
- Automatic correspondences

Acknowledgements

We thank

Johannes Wallner, Leonidas J. Guibas, Bob Sumner, Mario Botsch and Mark Pauly.

Meshes

are taken from the AIM@SHAPE shape repository, the Stanford 3D Scanning Repository and the Large Geometric Models Archive from the Georgia Institute of Technology.

This work is supported by the Austrian Science Fund (FWF) under grant S9206.

Der Wissenschaftsfonds.

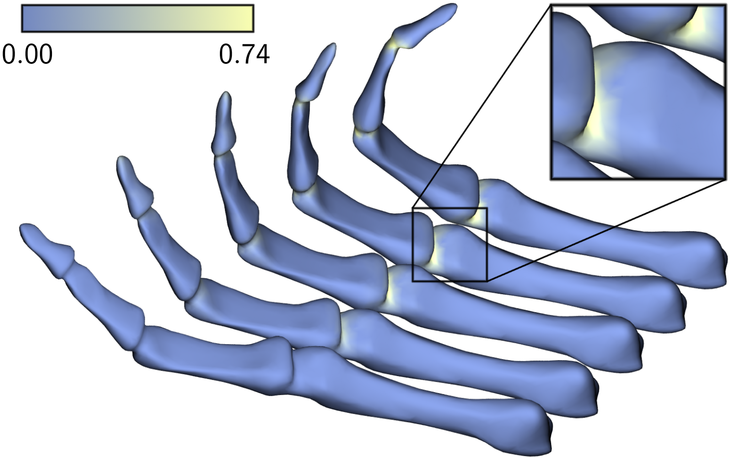
A Comparison

As-Rigid-As-Possible vs As-Isometric-As-Possible

Timings

Name	Resolution	Time
Grasp	26K	20 sec
Elephant	85K	274 sec
Armadillo (precomp. step)	100K	1 hour

Error Analysis



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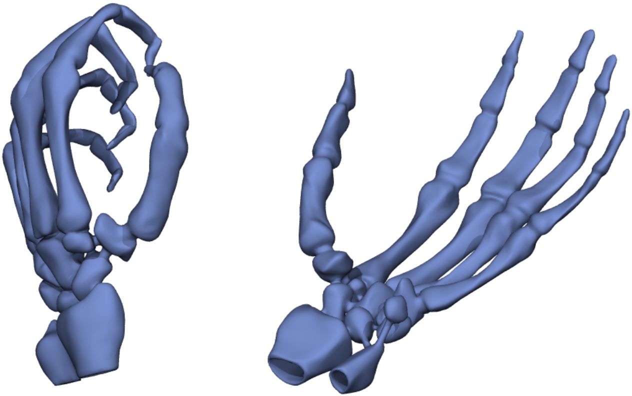

More Metrics

Rigidity and Volume