Modeling

A Volkswagan Beetle becomes the subject of a 1970 simulation project. Ivan Sutherland(left) and assistants plot coordinates for digitizing the car.

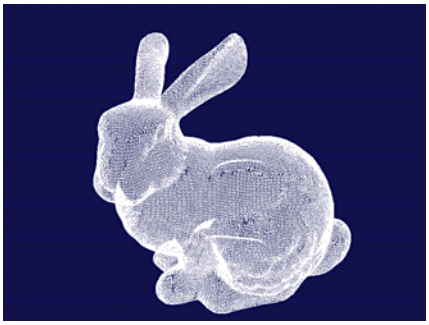
Modeling the Everyday World

Three broad areas:

- **Modeling (Geometric) = Shape**
- **Animation = Motion/Behavior**
- **Rendering = Appearance**

Geometric Modeling

1. How to represent 3d shapes
   - Polygonal meshes

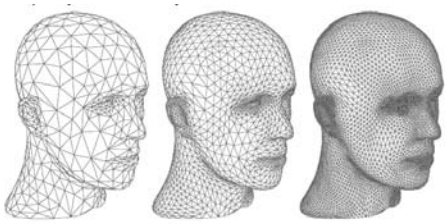


Stanford Bunny
69451 triangles

David, Digital Michelangelo Project
28,184,526 vertices, 56,230,343 triangles

Geometric Modeling

1. How to represent 3d shapes
   - Smooth surfaces
     - Bicubic spline surfaces
     - Subdivision surfaces

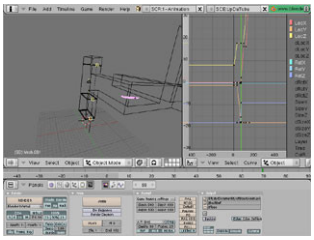


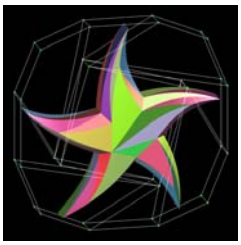
Caltech Head

Utah Teapot

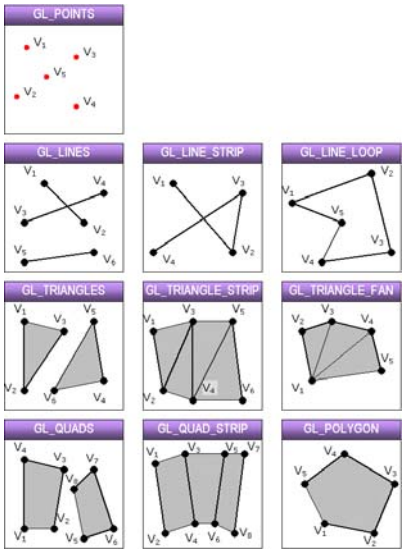
Geometric Modeling

1. How to represent 3D shapes
2. How to create 3D shapes
   1. CAD tools
   2. Scanners
   3. Procedurally
3. How to manipulate 3D shapes
   1. Deform/skin/morph/animate
   2. Smooth/compress
   3. Set operations, ...





OpenGL Primitives



Primitive API

```
glBegin(GL_POLYGON);
    glVertex3f(-1.0,-1.0,0.0);
    glVertex3f(1.0,-1.0,0.0);
    glVertex3f(1.0,1.0,0.0);
    glVertex3f(-1.0,1.0,0.0);
glEnd();
```

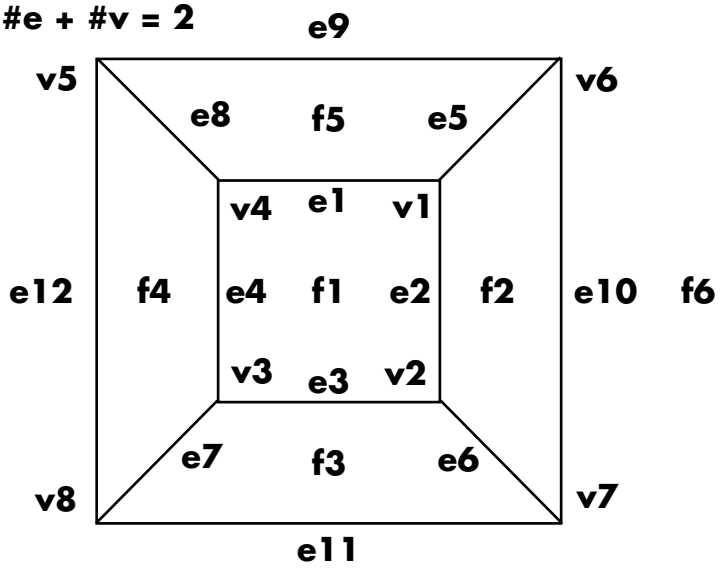

Polygons

```
float v1[3] = {-1.0,-1.0,0.0};
float v2[3] = { 1.0,-1.0,0.0};
float v3[3] = { 1.0, 1.0,0.0};
float v4[3] = {-1.0, 1.0,0.0};

glBegin(GL_POLYGON);
    glVertex3fv(v1);
    glVertex3fv(v2);
    glVertex3fv(v3);
    glVertex3fv(v4);
glEnd();
```

Topology

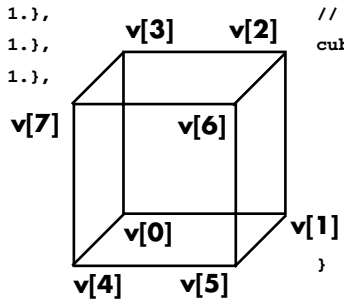
#f - #e + #v = 2



Points/Polygons

```
typedef float Point[3];

Point verts[6] = {
 {-1.,-1.,-1.},
 { 1.,-1.,-1.},
 { 1., 1.,-1.},
 {-1., 1.,-1.},
 {-1.,-1., 1.},
 { 1.,-1., 1.},
 { 1., 1., 1.},
 {-1., 1., 1.},
};
```




```
face(int a, int b, int c, int d) {
    glBegin(GL_POLYGON);
      glVertex3fv(verts[a]);
      glVertex3fv(verts[b]);
      glVertex3fv(verts[c]);
      glVertex3fv(verts[d]);
    glEnd();
}
// Note consistent ccw orientation!
cube() {
    face(0,3,2,1);
    face(2,3,7,6);
    face(0,4,7,3);
    face(1,2,6,5);
    face(4,5,6,7);
    face(0,1,5,4);
}
```

Points/Polygons

```
typedef float Point[3];

Point verts[6] = {
 {-1.,-1.,-1.},
 { 1.,-1.,-1.},
 { 1., 1.,-1.},
 {-1., 1.,-1.},
 {-1.,-1., 1.},
 { 1.,-1., 1.},
 { 1., 1., 1.},
 {-1., 1., 1.},
};
int polys[6][4] = {
 {0,3,2,1},
 {2,3,7,6},
 {0,4,7,3},
 {1,2,6,5),
 {4,5,6,7},
 {0,1,5,4}
}
```

```
face(int a, int b, int c, int d) {
    glBegin(GL_POLYGON);
      glVertex3fv(verts[a]);
      glVertex3fv(verts[b]);
      glVertex3fv(verts[c]);
      glVertex3fv(verts[d]);
    glEnd();
}

cube() {
 for( int i = 0; i < n; i++ )
    face(polys[i][0],
         polys[i][1],
         polys[i][2],
         polys[i][3]);
}
```


Representations

Polygons

- + Simple
- - Redundant information

Points/Polygons

- + Share vertices (compress/consistency)

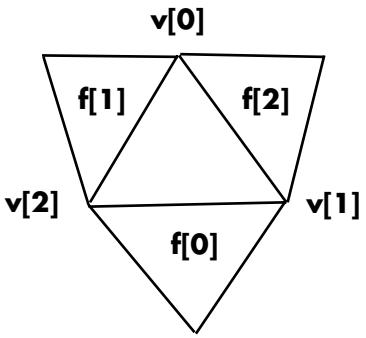
Additional topological information

- + Constant time access to neighbors
  - More advanced algorithms such as
  - surface normal calculation, subdivision ...
- - Additional storage for topology
- - More complicated data structures

Triangle Adjacency

```
struct Vert {
   Point pt;
   Face *f;
}

struct Face {
   Vert *v[3];
   Face *f[3];
}
```



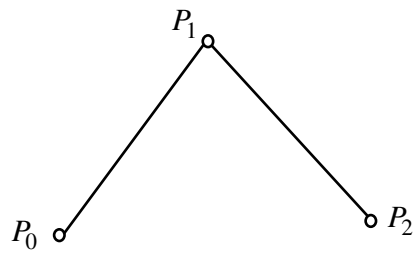
Recursive Subdivision

Bezier curves

Subdivision surfaces

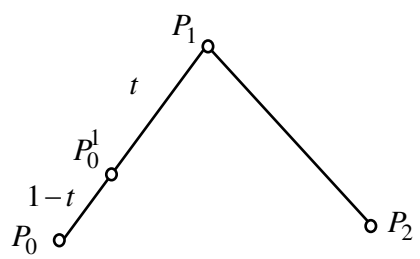
Fractals

Chaiken's Algorithm



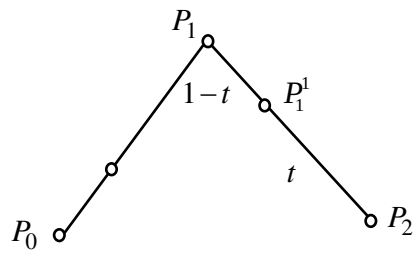
Chaiken's Algorithm

$$P_0^1 = (1-t)P_0 + tP_1$$



Chaiken's Algorithm

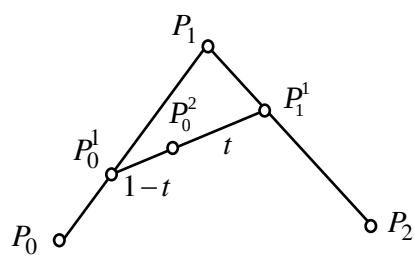
$$P_0^1 = (1-t)P_0 + tP_1$$
$$P_1^1 = (1-t)P_1 + tP_2$$



Chaiken's Algorithm

$$P_0^1 = (1-t)P_0 + tP_1$$
$$P_1^1 = (1-t)P_1 + tP_2$$

$$P_0^2 = (1-t)P_0^1 + tP_1^1$$

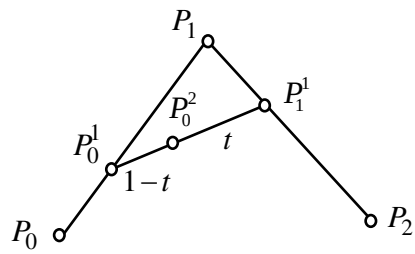


Chaiken's Algorithm

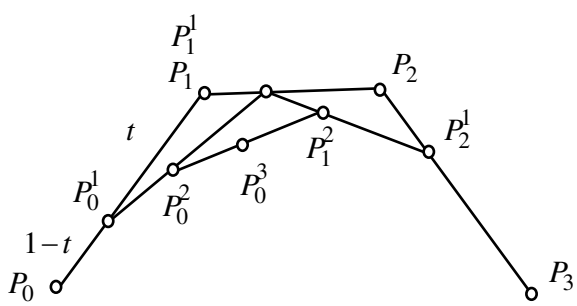
$$P_0^1 = (1-t)P_0 + tP_1$$
$$P_1^1 = (1-t)P_1 + tP_2$$

$$P_0^2 = (1-t)P_0^1 + tP_1^1$$

$$P(t) = P_0^2$$

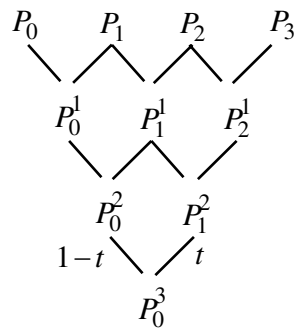


Bezier Curves



Generalize last algorithm to 4 points

Bezier Curve = Bernstein Polynomials

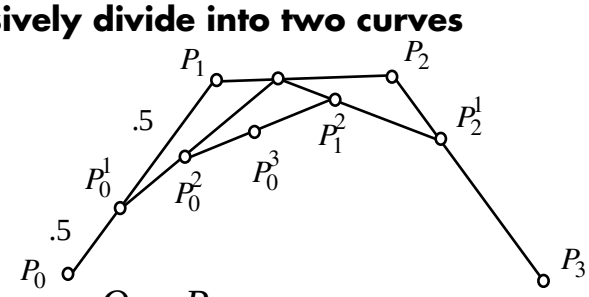


$$P(t) = \sum_{i=0}^{3} P_i B_i^3(t)$$

$$B_i^3(t) = \binom{3}{i} t^i (1-t)^{3-i}$$

Bezier Curves – Midpoint Subdivision

Recursively divide into two curves

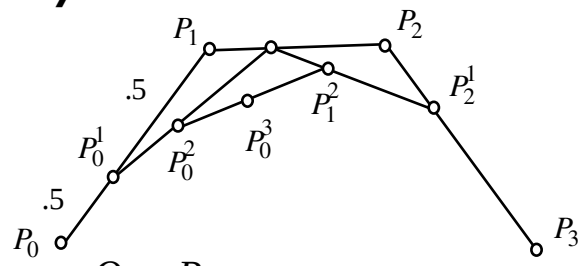


Left side

$$Q_0 = P_0$$
$$Q_1 = P_0^1$$
$$Q_2 = P_0^2$$
$$Q_3 = P_0^3$$

Bezier Curves – Midpoint Subdivision

Recursively divide into two curves



Left side

$$Q_0 = P_0$$

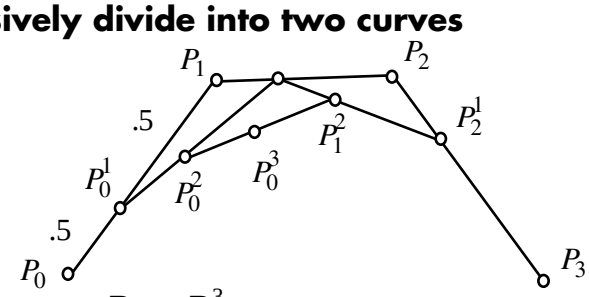
$$Q_1 = P_0^1 = \tfrac{1}{2}P_0 + \tfrac{1}{2}P_1$$

$$Q_2 = P_0^2 = \tfrac{1}{4}P_0 + \tfrac{1}{2}P_1 + \tfrac{1}{4}P_2$$

$$Q_3 = P_0^3 = \tfrac{1}{8}P_0 + \tfrac{3}{8}P_1 + \tfrac{3}{8}P_2 + \tfrac{1}{8}P_3$$

Bezier Curves – Midpoint Subdivision

Recursively divide into two curves



Right side

$$R_0 = P_0^3$$

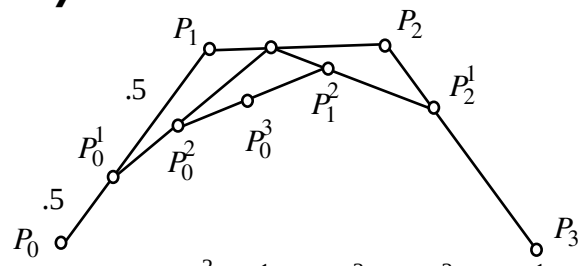
$$R_1 = P_1^2$$

$$R_2 = P_2^1$$

$$R_3 = P_3$$

Bezier Curves – Midpoint Subdivision

Recursively divide into two curves



Right side

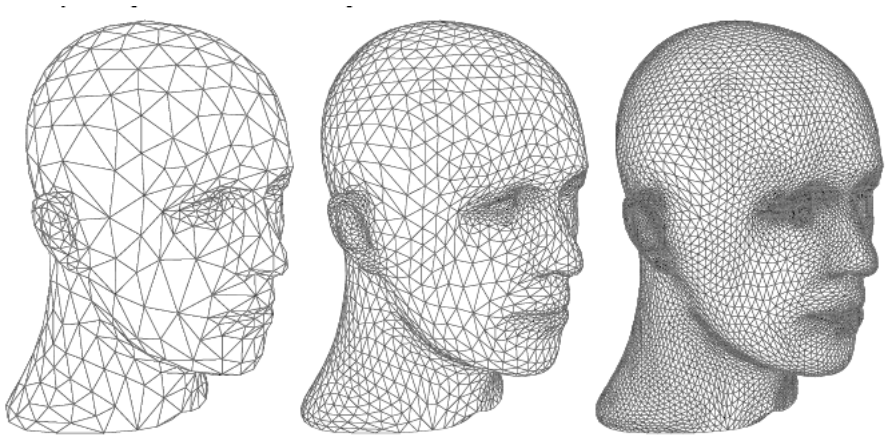
$$R_0 = P_0^3 = \tfrac{1}{8}P_0 + \tfrac{3}{8}P_1 + \tfrac{3}{8}P_2 + \tfrac{1}{8}P_3$$

$$R_1 = P_1^2 = \tfrac{1}{4}P_1 + \tfrac{1}{2}P_2 + \tfrac{1}{4}P_3$$

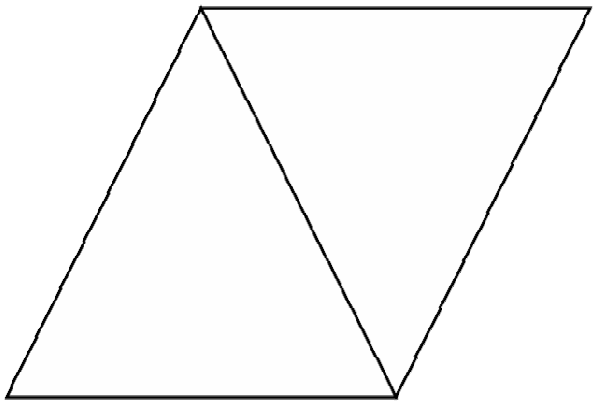
$$R_2 = P_2^1 = \tfrac{1}{2}P_2 + \tfrac{1}{2}P_3$$

$$R_3 = P_3$$

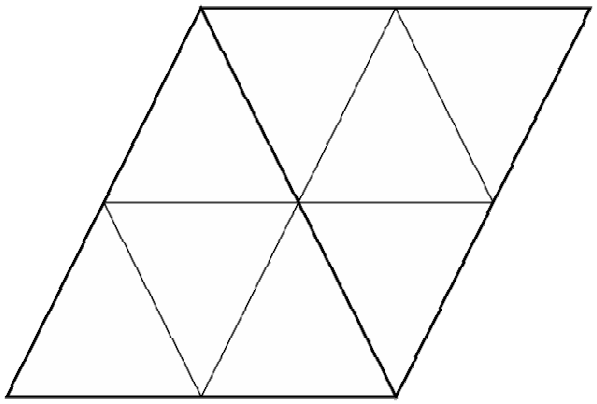
Subdivision Surfaces



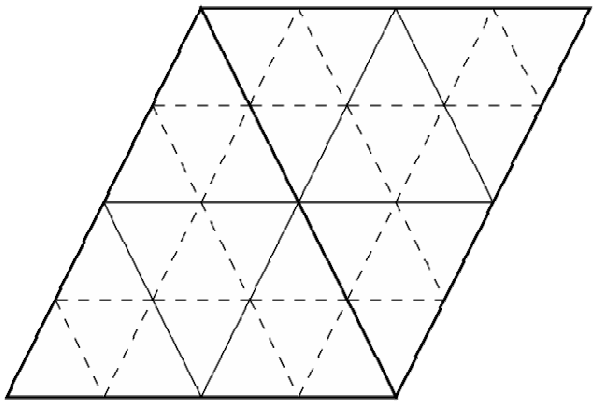
Triangle Mesh



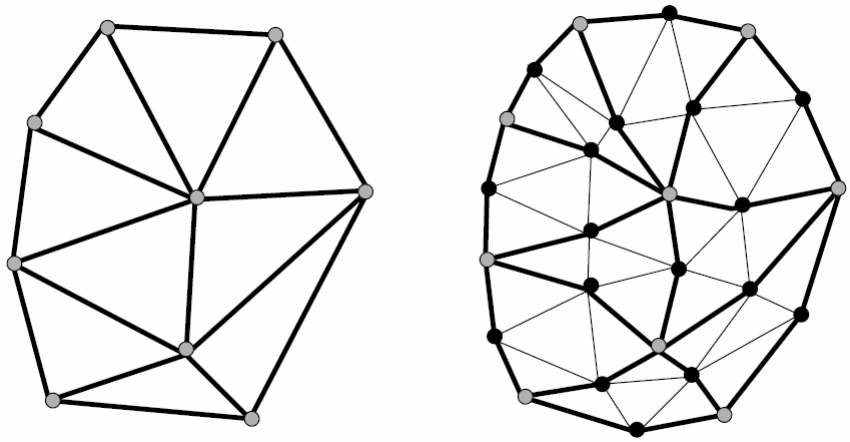
Triangle Mesh – Topological Subdivide



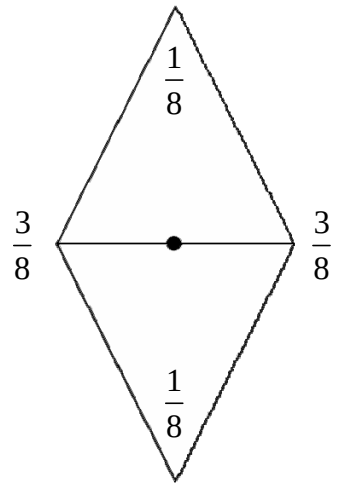
Triangle Mesh – Topological Subdivide



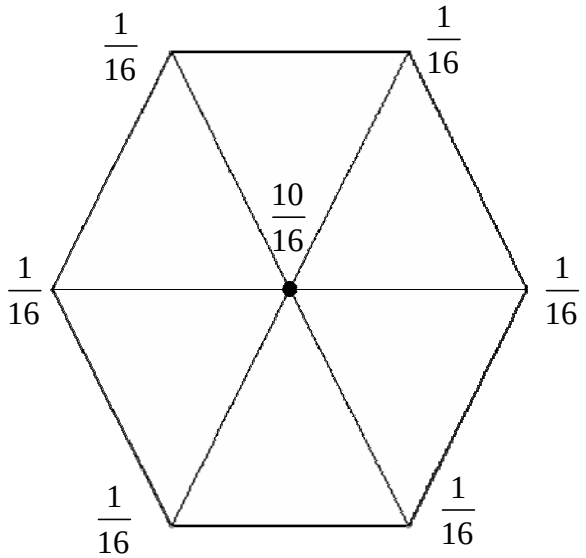
Triangle Mesh – Subdivide



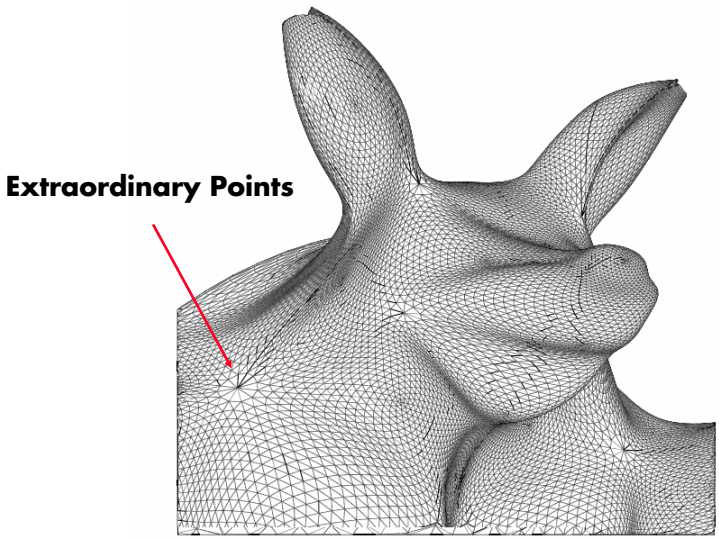
Loop Algorithm - Edge



Loop Algorithm - Vert

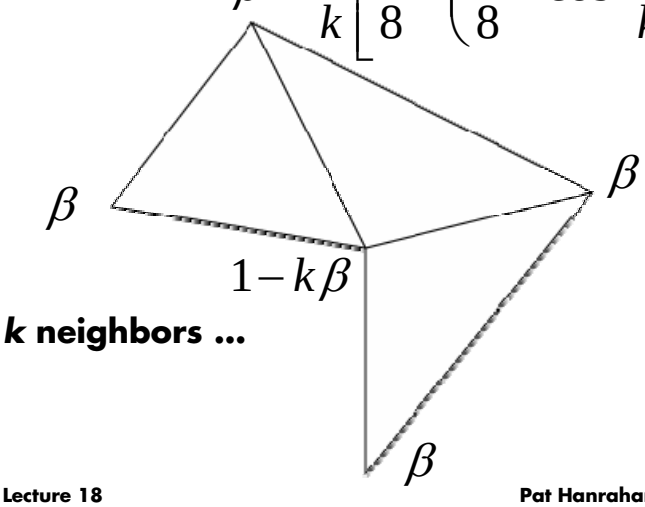


Semi-Regular Meshes



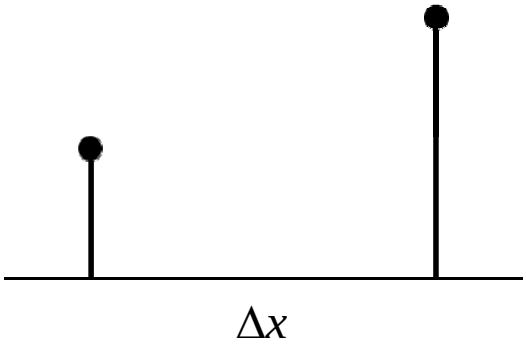
Loop Subdivision – Extraordinary Vertex

$$\beta = \frac{1}{k}\left[\frac{5}{8} - \left(\frac{3}{8} + \cos\frac{2\pi}{k}\right)^2\right]$$

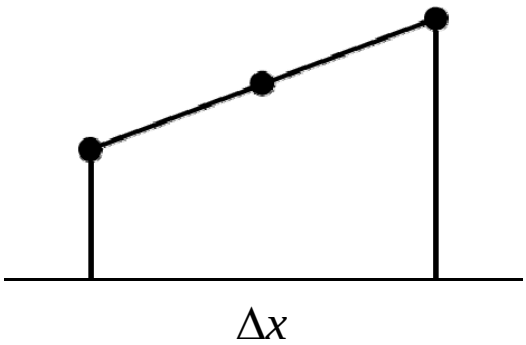


***k* neighbors ...**

Fractal Subdivision

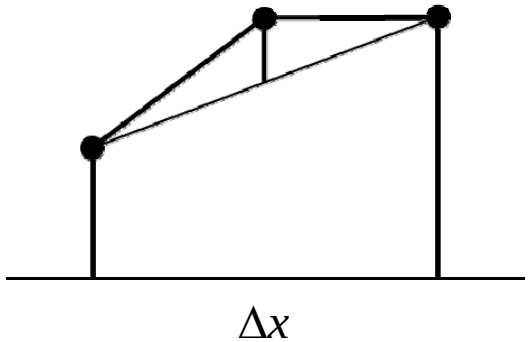


Fractal Subdivision

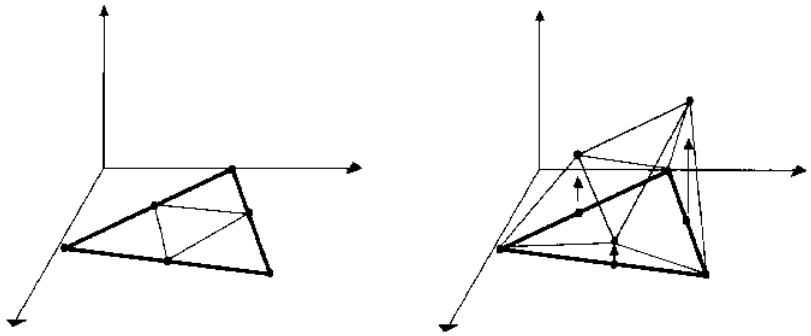


Fractal Subdivision

$$\Delta y = \text{random}() \bullet \Delta x$$



Fractal Subdivision: Height Field



Summary

Digital geometry processing ala signal processing

Three common representations

- **Dense polygon meshes**
- **Bicubic surfaces**
- **Subdivision surfaces**

Common operations

- **Instancing**
- **Transformation: linear and non-linear (bend)**
- **Compressing and simplifying**
- **...**