Sampling

Sampling process
Aliasing
Nyquist frequency
Reconstruction process
The sampling theorem
Antialiasing
Convolution of a “Spike”

<table>
<thead>
<tr>
<th>Signal/Image</th>
<th>Filter</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 0 1 0 0 0 0 ...</td>
<td>1 2 1</td>
</tr>
</tbody>
</table>

Result: copy of the filter centered at the spike

| 0 1 2 1 0 0 0 ... |
Convolution

**Convolution Theorem:** Multiplication in the frequency domain is equivalent to convolution in the space domain.

\[ f \otimes g \leftrightarrow F \times G \]

**Symmetric Theorem:** Multiplication in the space domain is equivalent to convolution in the frequency domain.

\[ f \times g \leftrightarrow F \otimes G \]
Imagers = Signal Sampling

Imagers convert a continuous image to a discrete image. Each sensor integrates light over its active area.

\[ R(i, j) = \int_{A_{i,j}} E(x, y) P_{i,j}(x, y) \, dx \, dy \]

Examples:
- Retina: photoreceptors
- Digital camera: CCD or CMOS array
- Vidicon: phosphors

How should this be done in computer graphics?
Aliasing
“Aliases”

Aliases \([\theta \text{ and } (2\pi - \theta)]\) are indistinguishable after sampling!
**Sampling a “Zone Plate”**

Zone plate: \( \sin x^2 + y^2 \)

Sampled at 128x128

Reconstructed to 512x512

Using a 30-wide Kaiser windowed sinc

Left rings: part of signal

Right rings: prealiasing

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**Nyquist Frequency**

Definition: The Nyquist Frequency is 1/2 the sampling frequency

A periodic signal with a frequency above the Nyquist frequency cannot be differentiated from a periodic signal below the Nyquist frequency

The various indistinguishable signals are called aliases
Sampling: Spatial Domain

\[ f(x) \]

\[ \sum_{n=-\infty}^{\infty} \delta(x - nT) f(nT) \]

\[ \Pi(x) = \sum_{n=-\infty}^{\infty} \delta(x - nT) \]
Sampling: Frequency Domain

\[ F(\omega) \]

\[ \mathcal{H}_{1/T}(\omega) = \sum_{n=-\infty}^{\infty} \delta(\omega - n/T) \]

Undersampling: Aliasing

\[ \mathcal{H}_{1/T}(\omega) \]
Displays = Signal Reconstruction

All physical displays recreate a continuous image from a discrete sampled image by using a finite sized source of light for each pixel.

Examples:
- LCDs: Finite aperture
- DACs: sample and hold
- Cathode ray tube: phosphor spot and grid
Jaggies

Retort image by Don Mitchell

Staircase pattern or jaggies

Perfect Reconstruction: Freq. Domain

Rect function of width $T$:

$$\Pi_T(x) = \begin{cases} 1 & \frac{T}{2} \leq |x| \leq \frac{T}{2} \\ 0 & \text{otherwise} \end{cases}$$
Perfect Reconstruction: Spatial Domain

Sinc function:

\[ \text{sinc} \left( \frac{x}{T} \right) \]

\[ \text{sinc}(x) = \frac{\sin \pi x}{\pi x} \]
Reconstruction: Freq. Domain

Ideally, use a perfect low-pass filter - the sinc function - to bandlimit the sampled signal and thus remove all copies of the spectra introduced by sampling.

Unfortunately,

- The sinc has infinite extent and we must use simpler filters with finite extents. Physical processes in particular do not reconstruct with sincs.
- The sinc may introduce ringing which are perceptually objectionable.
Sampling a “Zone Plate”

Zone plate: $\sin x^2 + y^2$

Sampled at 128x128
Reconstructed to 512x512
Using optimal cubic

Left rings: part of signal
Right rings: prealiasing
Middle rings: postaliasing

Antialiasing

Retort image by Don Mitchell
The Sampling Theorem

Sampling and Reconstruction
Sampling Theorem

This result is known as the Sampling Theorem and is due to Claude Shannon who first discovered it in 1949.

A signal can be reconstructed from its samples without loss of information, if the original signal has no frequencies above 1/2 the Sampling frequency.

For a given bandlimited function, the rate at which it must be sampled is called the Nyquist Frequency.

Sampling in Computer Graphics

Artifacts due to sampling - Aliasing
- Jaggies
- Moire
- Flickering small objects
- Sparkling highlights
- Temporal strobing

Preventing these artifacts - Antialiasing
Antialiasing by Prefiltering

Frequency Space
Prefiltering by Computing Coverage

Pixel area = Box filter
Equivalent to area of pixel covered by the polygon

25% 50% 75% 100%

Point vs. Area Sampled

Point

Exact Area

Checkerboard sequence by Tom Duff