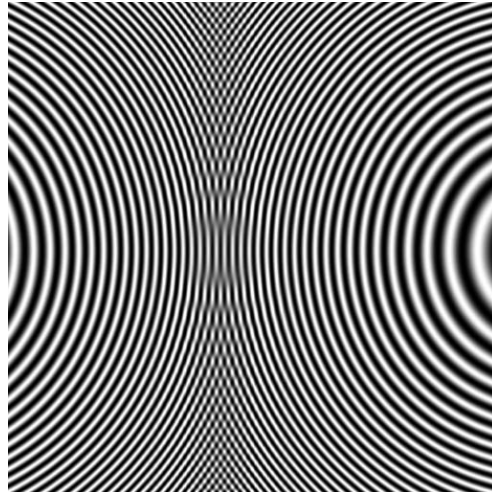


CS148: Introduction to Computer Graphics and Imaging

# Sampling and Aliasing



## Key Concepts

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**Sampling**

**Aliasing**

**Nyquist frequency**

**Filters and convolution**

**Antialiasing**

# Sampling

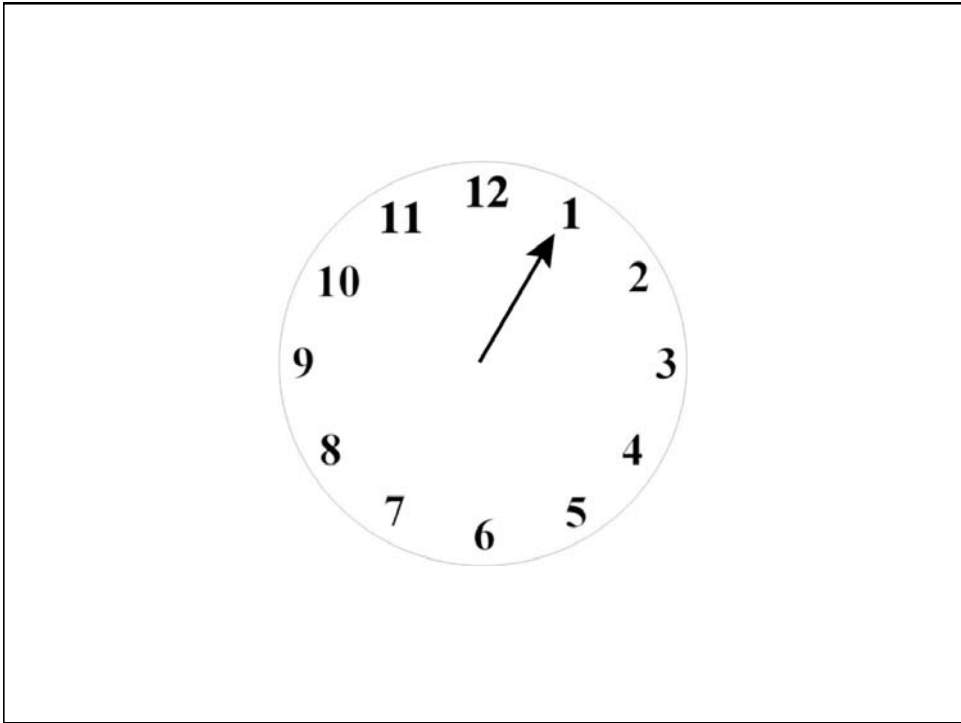
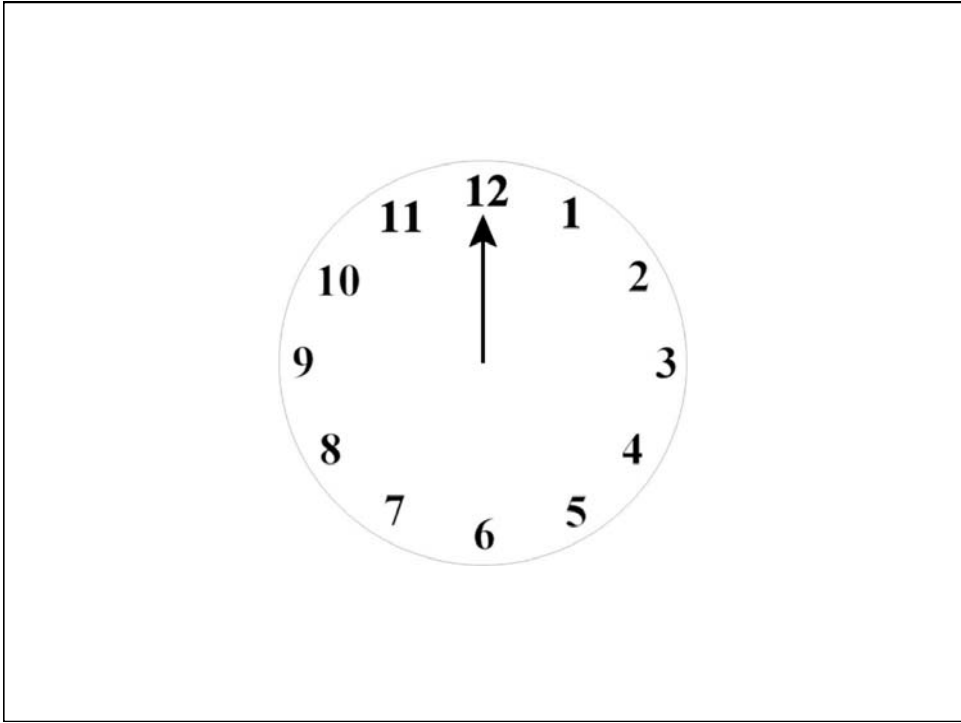
## Image Generation = Sampling

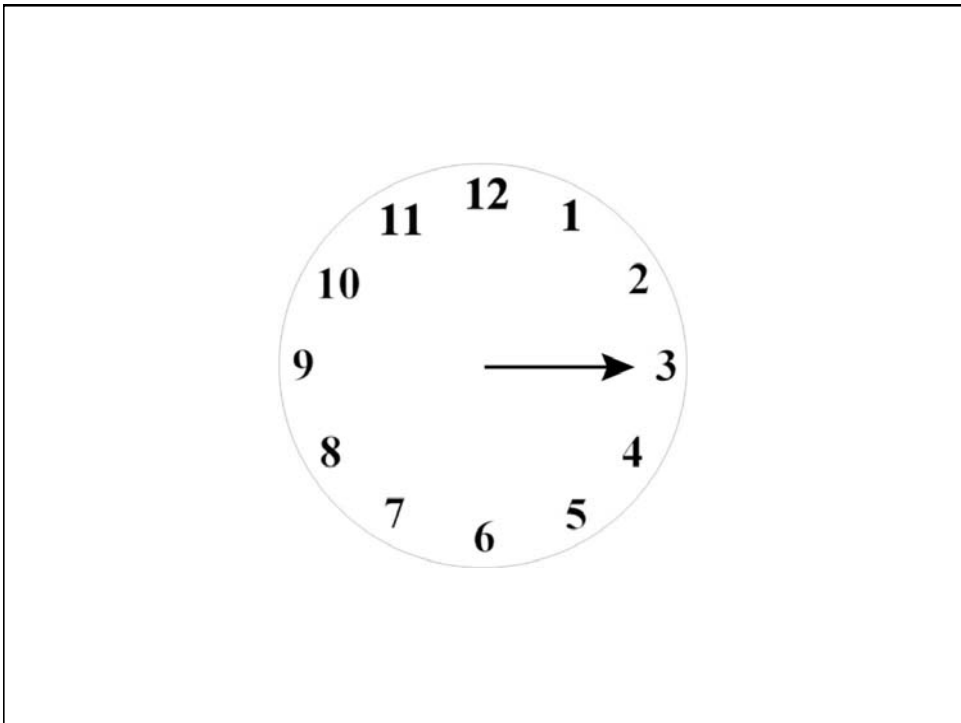
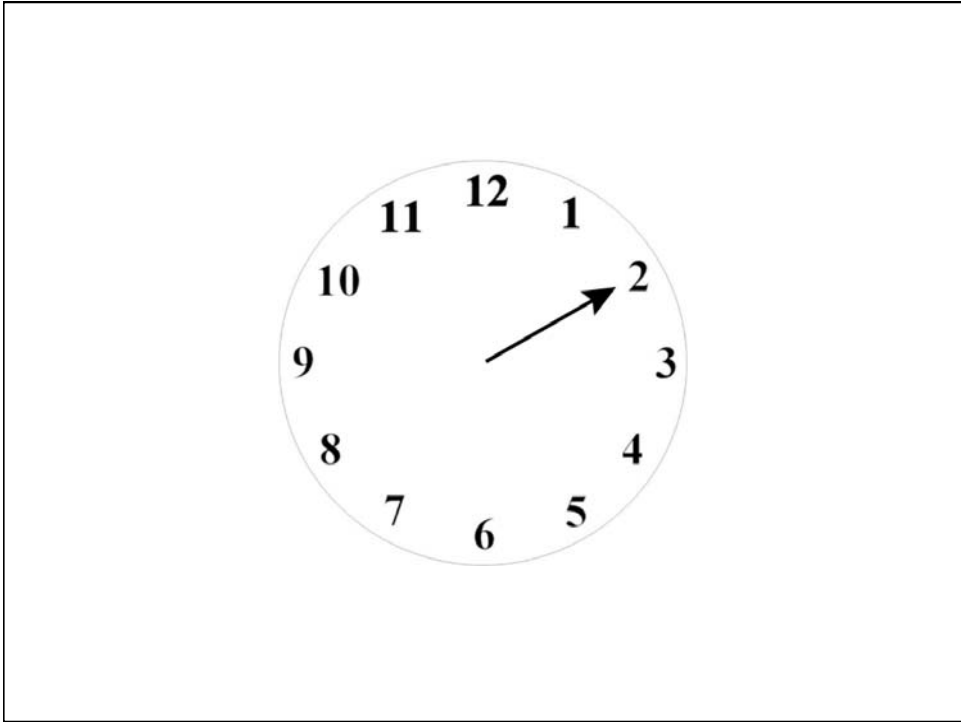
Rasterization is performed by testing whether a point is inside a triangle

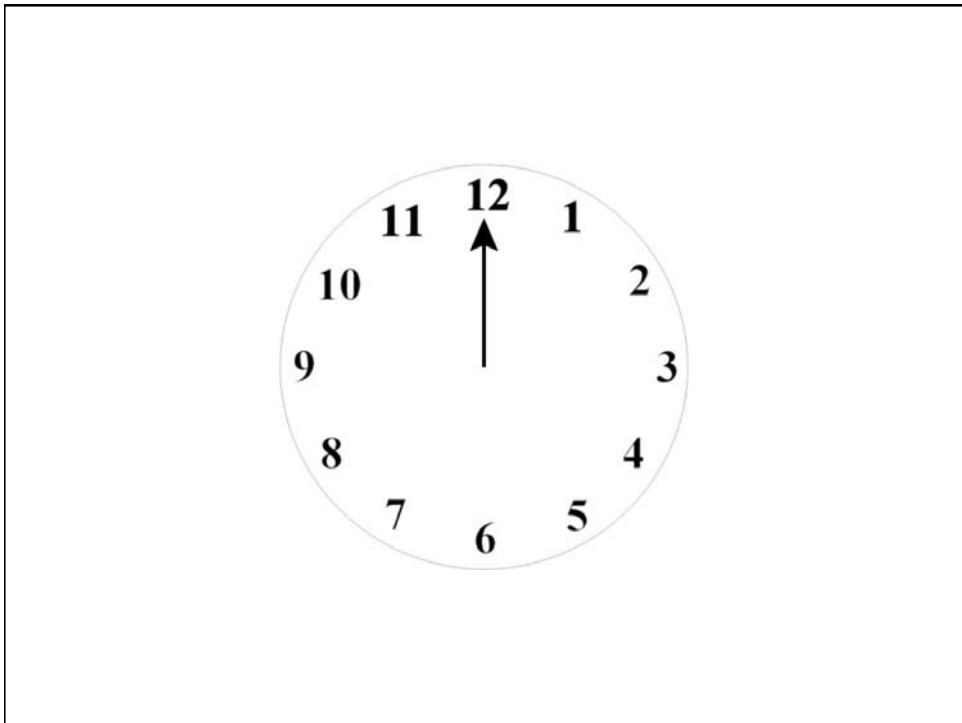
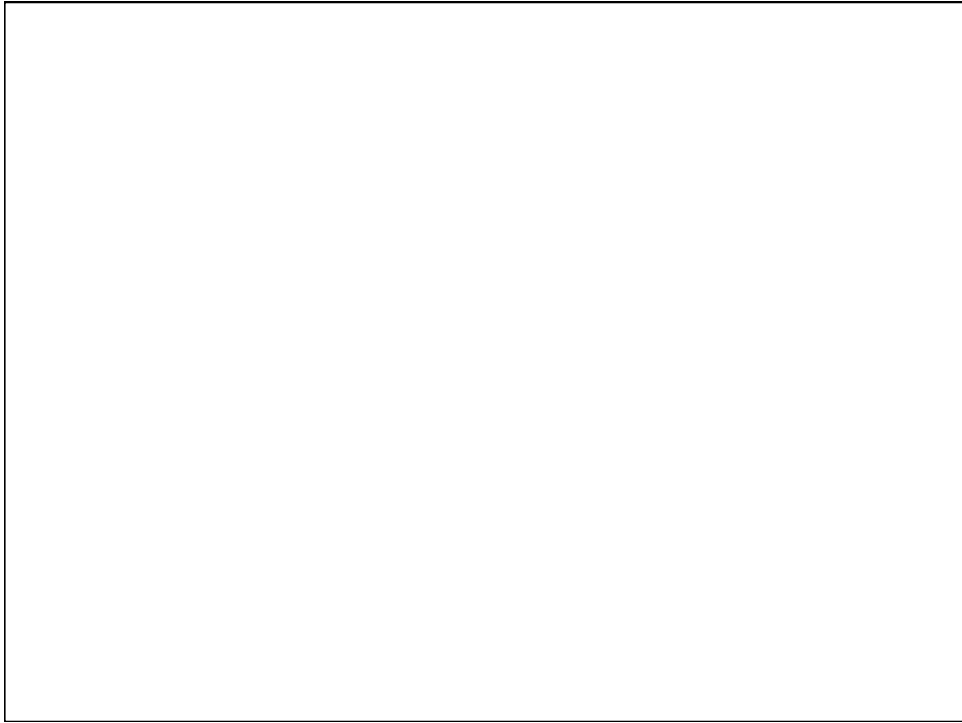
More generally, evaluate a function at a point

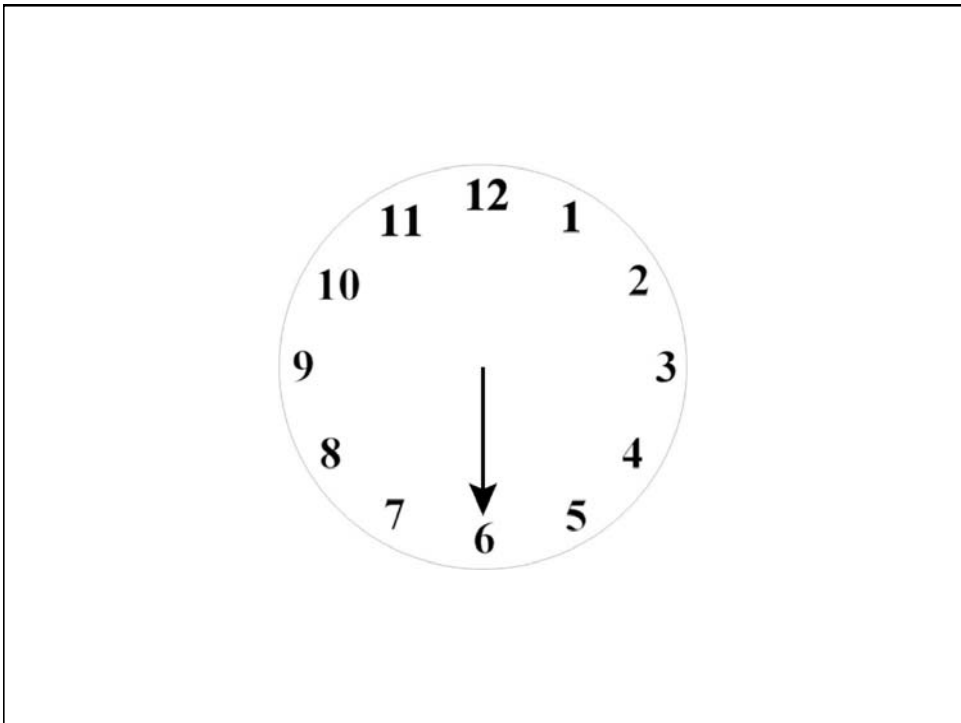
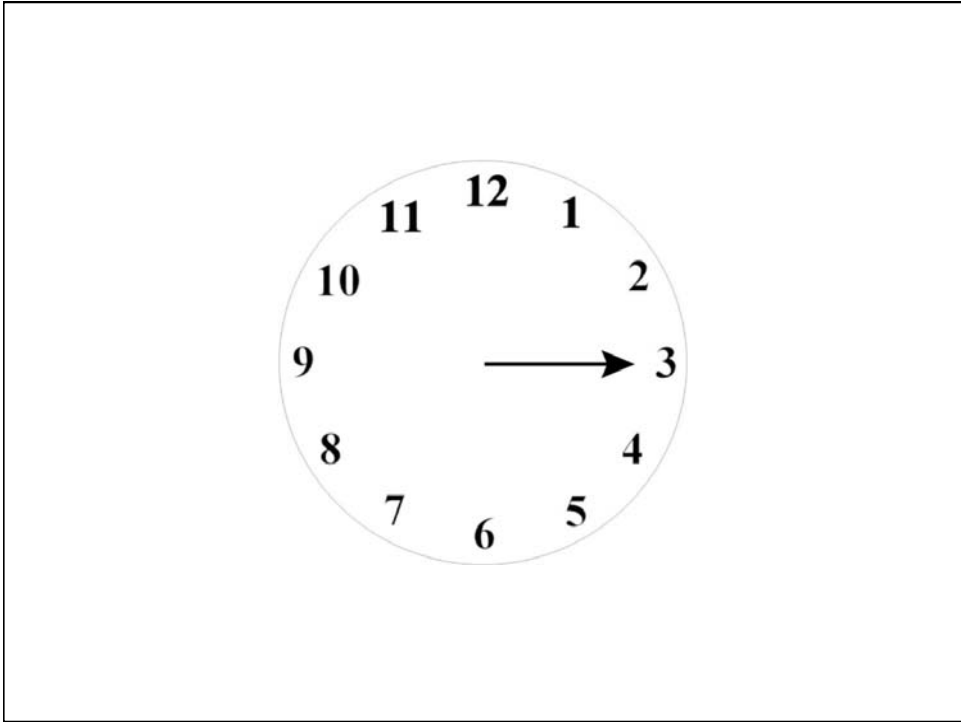
```
for( x = 0; x < xmax; x++ )  
    for( y = 0; y < ymax; y++ )  
        image[x][y] = f(x,y);
```

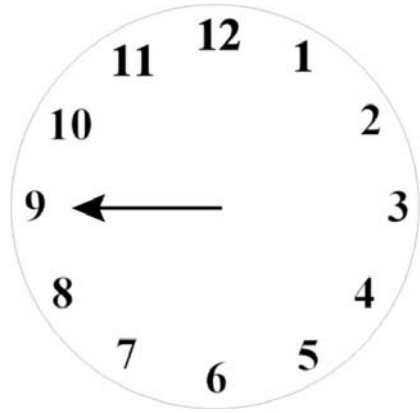
Take a continuous function and convert to set of samples, a discrete representation of the func.

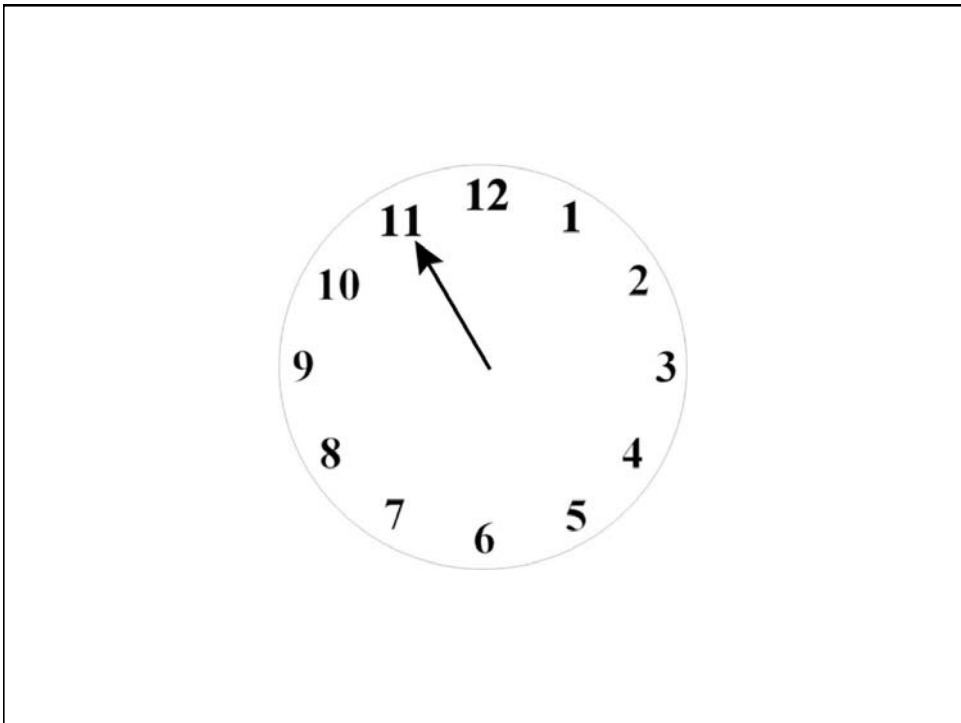
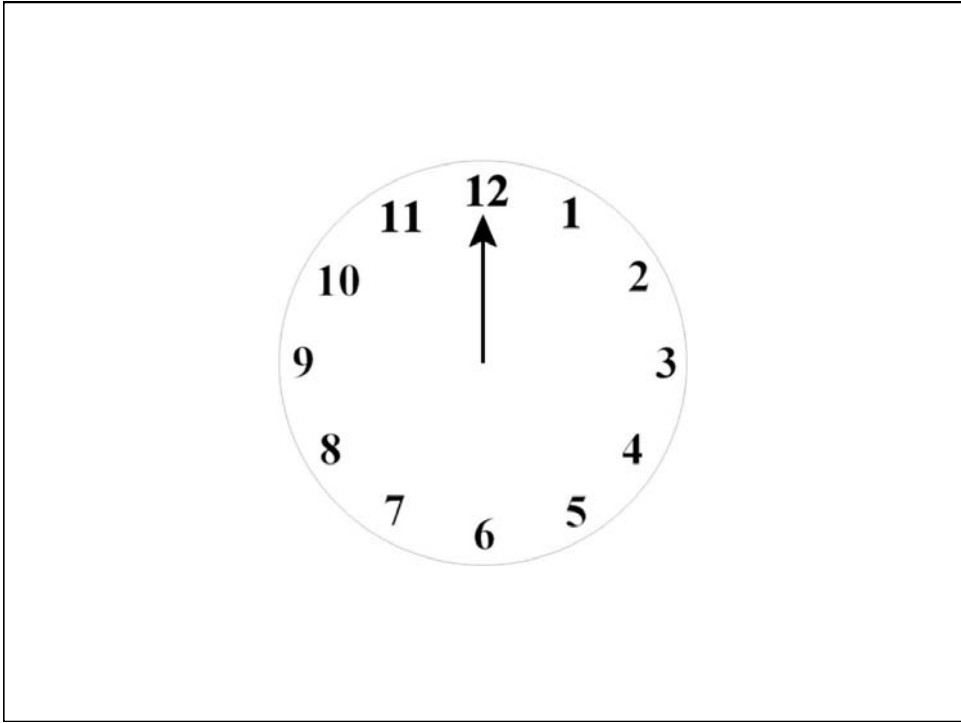


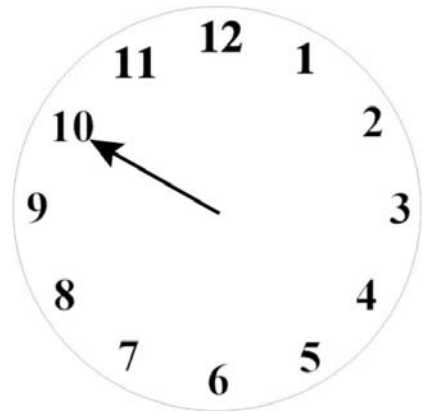










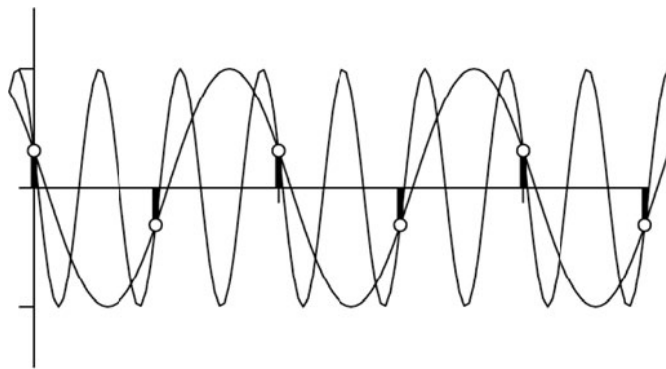


## **Wagon Wheel Effect**

[http://www.michaelbach.de/ot/mot\\_wagonWheel/](http://www.michaelbach.de/ot/mot_wagonWheel/)

# Aliasing

## “Aliases”



**Two frequencies are indistinguishable**

## Nyquist Frequency

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**Definition:** The Nyquist frequency is  $\frac{1}{2}$  the sampling frequency =  $1/(2 T_s)$

**A periodic signal above the Nyquist frequency cannot be distinguished from a periodic signal below the Nyquist frequency**

**Indistinguishable frequencies look alike  
Hence, they are called aliases**

## Sampling in Computer Graphics

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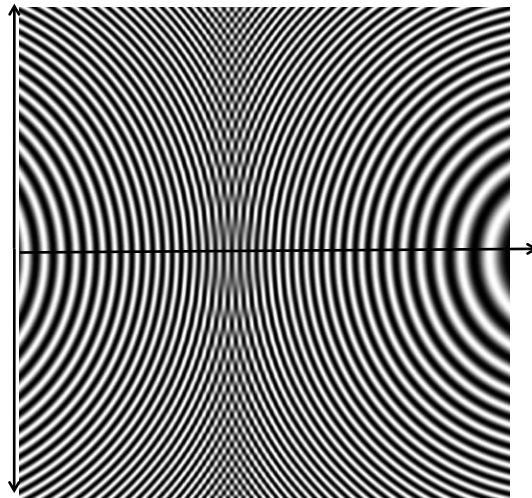
**Artifacts due to sampling - Aliasing**

- Jaggies
- Moire
- Flickering small objects
- Sparkling highlights
- Temporal strobing

**Preventing these artifacts - Antialiasing**

## Zone Plate

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$$\sin(x^2 + y^2)$$

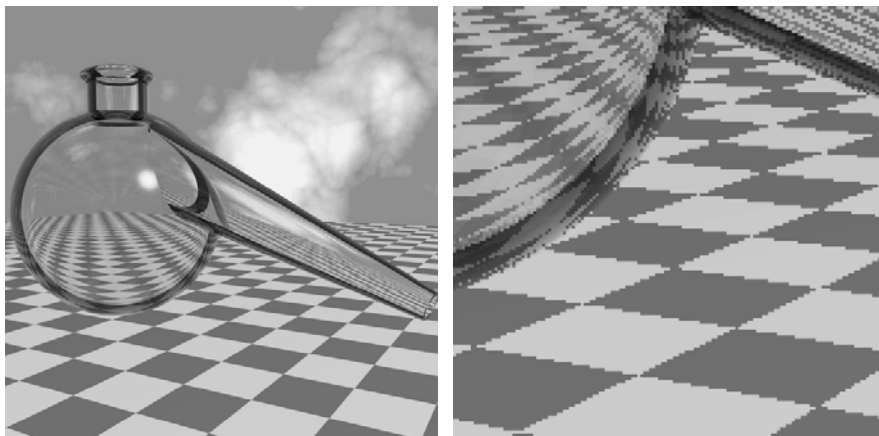
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## Jaggies

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Retort sequence by Don Mitchell



Staircase pattern or jaggies

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## **Antialiasing**

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**Simple idea:**

**Remove frequencies above the Nyquist frequency  
before sampling**

**How? Filtering**

**Filters = Convolution**

# Convolution

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Signal / Image

1	3	0	4	2	1
---	---	---	---	---	---

Filter

1	2
---	---

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# Convolution

---

1	3	0	4	2	1
---	---	---	---	---	---

1	2
---	---

$$1 * 1 + 3 * 2 = 7$$

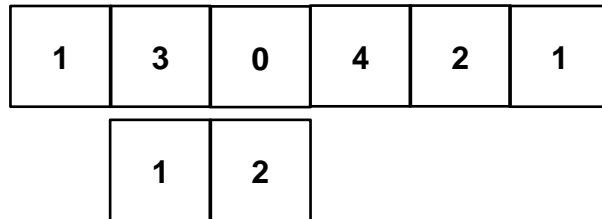
7					
---	--	--	--	--	--

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## Convolution

---



$$3 * 1 + 0 * 2 = 3$$

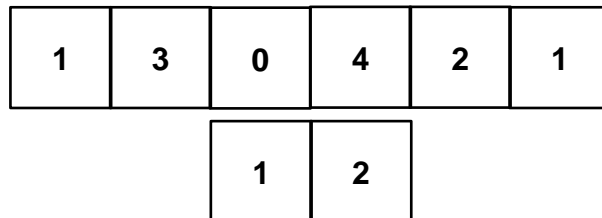


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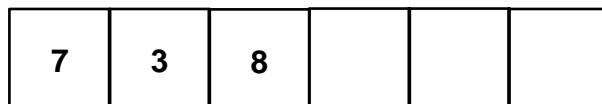
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## Convolution

---



$$0 * 1 + 4 * 2 = 8$$

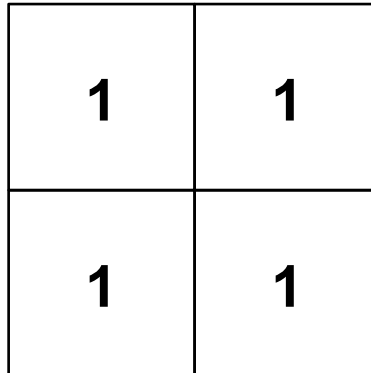


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## Box Filter

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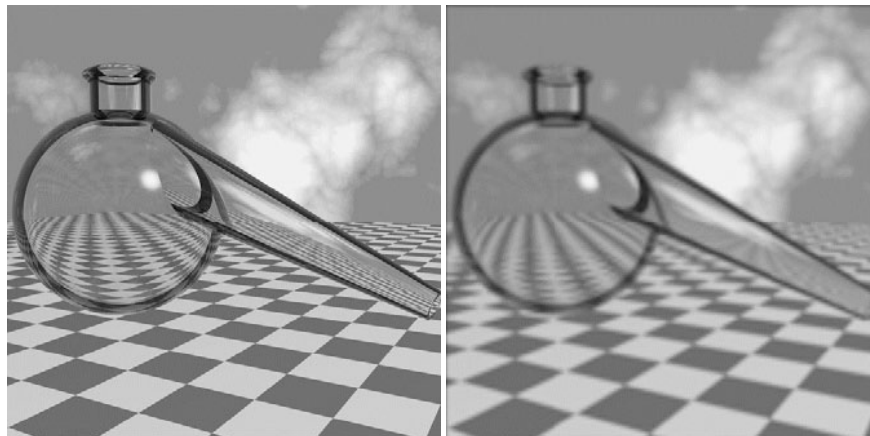
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## Box Filter = Low-Pass Filter

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Low-Pass means low frequencies are “passed”  
High frequencies are removed by the filter



Original

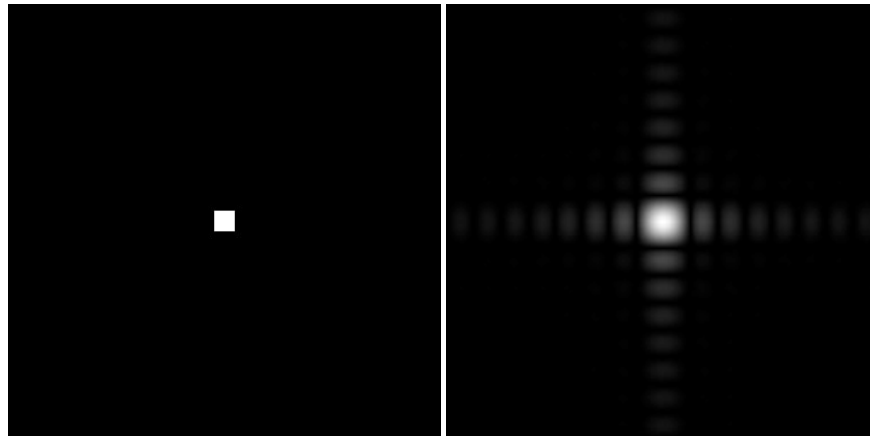
Box-Filter

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## Box Filter = Low-Pass Filter

---



Spatial Domain

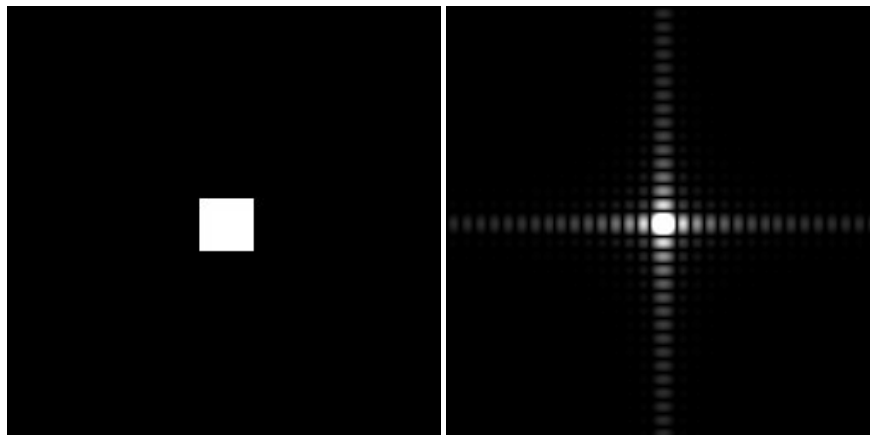
Frequency Domain

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## Wider Filters, Lower Frequencies

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Spatial Domain

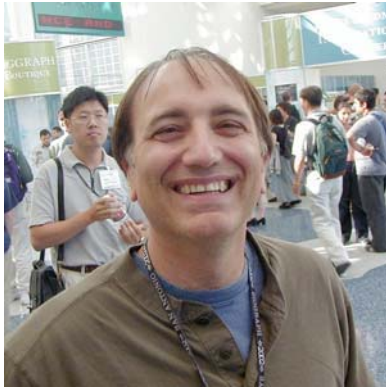
Frequency Domain

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## My Frequencies

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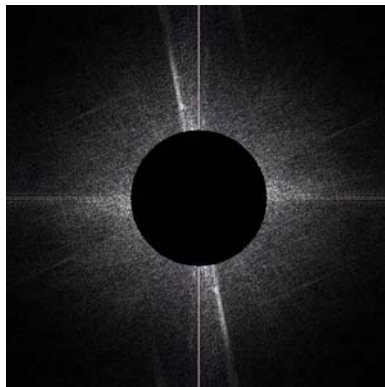


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## High-Pass Filtering Me

---



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# Antialiasing

## Antialiasing

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**Simple idea:**

**Remove frequencies above the Nyquist frequency  
before sampling**

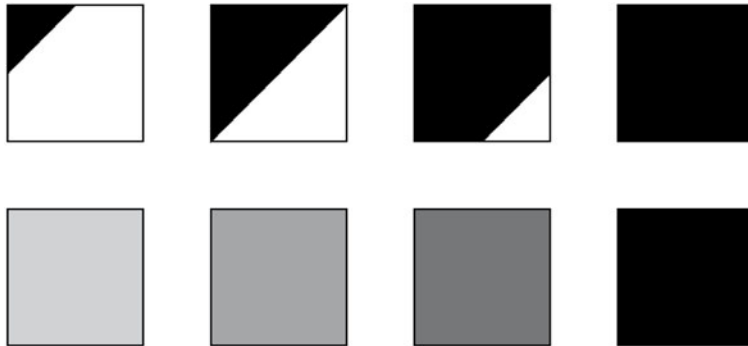
**How? Filtering**

**Filter during rasterization**

## Prefiltering by Computing Coverage

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Pixel Area = Box Filter

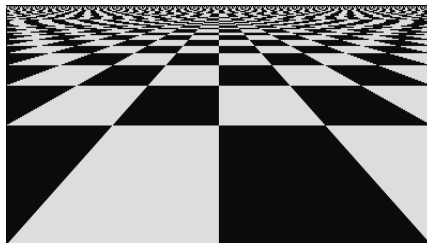
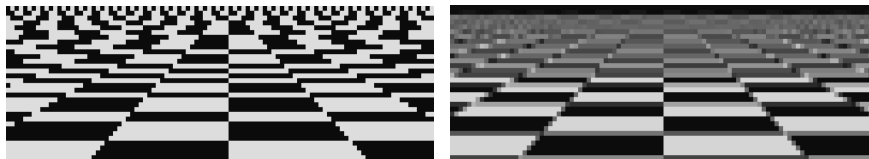


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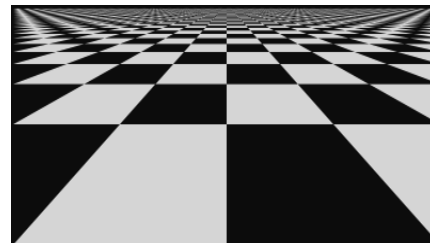
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## Point- vs. Area-Sampled

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Point



Area

Checkerboard sequence by Tom Duff

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## Point-sampling vs. Super-sampling

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Point

4x4 Super-sampled

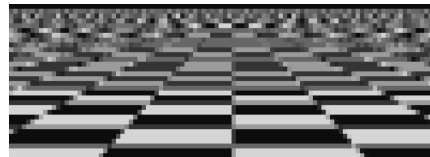
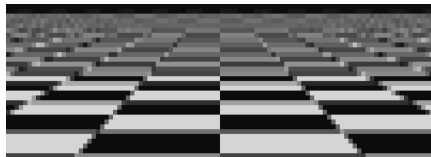
Checkerboard sequence by Tom Duff

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## Area-Sampling vs. Super-sampling

---



Exact Area

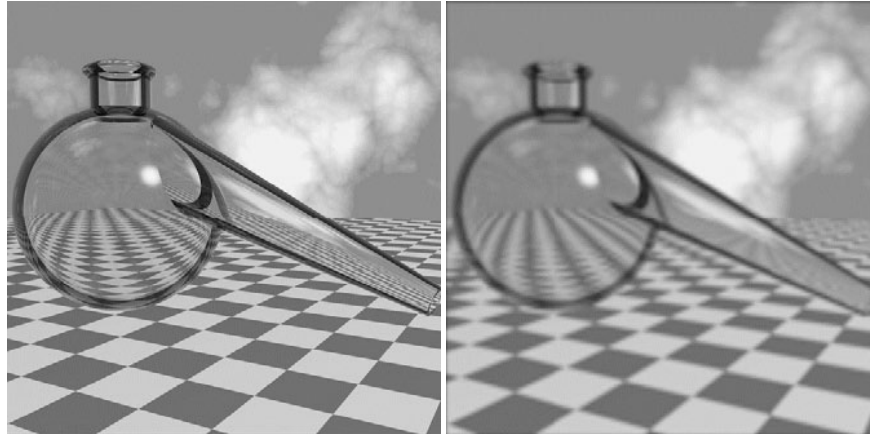
4x4 Super-sampled

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## Box Filter = Low-Pass Filter

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**Original**

**Box-Filter**

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## Things to Remember

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**Sampling converts continuous to discrete**

**Image generation involves sampling**

**May also sample geometry, motion, ...**

**Nyquist frequency is  $\frac{1}{2}$  the sampling rate**

**Frequencies above the Nyquist frequency can alias**

**That is, appear as other frequencies**

**Antialiasing – Filter before sampling**

**Output is coverage; removes jaggies**

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