Sampling and Aliasing

Key Concepts

Sampling
Aliasing
Nyquist frequency
Filters and convolution
Antialiasing
Image Generation = Sampling

Rasterization is performed by testing whether a point is inside a triangle

More generally, evaluate a function at a point

\[
\text{for ( int x = 0; x < xmax; x++ )}
\]
\[
\text{for ( int y = 0; y < ymax; y++ )}
\]
\[
I[x][y] = f( \text{float(x)}, \text{float(y)});
\]

Take a continuous function f and convert to set of samples I, a discrete representation of the func.
Zone Plate \( \sin x^2 \)
Aliasing
Wagon Wheel Effect

http://www.michaelbach.de/ot/mot_wagonWheel/

“Aliases”

Two frequencies are indistinguishable
Nyquist Frequency

Definition: The Nyquist frequency is $\frac{1}{2}$ the sampling frequency $= \frac{1}{2T_s}$

A periodic signal above the Nyquist frequency cannot be distinguished from a periodic signal below the Nyquist frequency

Indistinguishable frequencies look alike
Hence, they are called aliases

Sampling in Computer Graphics

Artifacts due to sampling - Aliasing

- Jaggies
- Moire
- Flickering small objects
- Sparkling highlights
- Temporal strobing

Preventing these artifacts - Antialiasing
Jaggies

Retort sequence by Don Mitchell

Staircase pattern or jaggies

Antialiasing
Antialiasing

Simple idea:

Remove frequencies above the Nyquist frequency before sampling

How? Filtering

Filters
My Frequencies

High-Pass Filtering Me
Box Filter = Low-Pass Filter

Low-Pass means low frequencies are “passed”
High frequencies are removed by the filter

Original
Box-Filter

Filters = Convolution
Convolution

Signal / Image

\[
\begin{bmatrix}
1 & 3 & 0 & 4 & 2 & 1
\end{bmatrix}
\]

Filter

\[
\begin{bmatrix}
1 & 2
\end{bmatrix}
\]

\[1 \times 1 + 3 \times 2 = 7\]
Convolution

\[
\begin{array}{cccccc}
1 & 3 & 0 & 4 & 2 & 1 \\
\end{array}
\]

\[
\begin{array}{cc}
1 & 2
\end{array}
\]

\[3 \times 1 + 0 \times 2 = 3\]

\[
\begin{array}{cc}
7 & 3
\end{array}
\]

Convolution

\[
\begin{array}{cccccc}
1 & 3 & 0 & 4 & 2 & 1 \\
\end{array}
\]

\[
\begin{array}{cc}
1 & 2
\end{array}
\]

\[0 \times 1 + 4 \times 2 = 8\]

\[
\begin{array}{ccc}
7 & 3 & 8
\end{array}
\]
Box Filter

1 1
1 1

Box Filter = Low-Pass Filter

Spatial Domain
Frequency Domain
Wider Filters, Lower Frequencies

Spatial Domain

Frequency Domain

Antialiasing
Antialiasing

Simple idea:

Remove frequencies above the Nyquist frequency before sampling

How? Filtering

Filter during rasterization

Prefiltering by Computing Coverage

Pixel Area = Box Filter
Point- vs. Area-Sampled

Checkerboard sequence by Tom Duff

Point-sampling vs. Super-sampling

Checkerboard sequence by Tom Duff
Area-Sampling vs. Super-sampling

Exact Area  

4x4 Super-sampled

Antialiasing

Jaggies  

Prefilter
Antialiasing vs. Blurred Aliases

Blurred Jaggies  Prefilter

Things to Remember

Sampling converts continuous to discrete
Image generation involves sampling
  May also sample geometry, motion, ...
Nyquist frequency is ½ the sampling rate
Frequencies above the Nyquist frequency can aliases
  That is, appear as other frequencies
Antialiasing – Filter before sampling
Output is coverage; removes jaggies