Key Concepts

- Lossless vs. lossy compression
- Kolmogorov complexity
- Entropy and Huffman coding
- JPEG / Discrete cosine transform (DCT)
- JPEG2000 / Haar Wavelets
Image and Video Data Rates

One full screen Image
■ 1024 x 768 x 24b = ~2.5MB

DVD
■ 720 x 480 x 24b x 30f/s = ~30 MB/s

High Definition DVD
■ 1920 x 1080 x 24b x 30f/s = ~178MB/s

Film
■ 4000 x 3000 x 36b x 30f/s = ~1.5GB/s
■ 8 TB for a 90 minute movie!

Lossless vs. Lossy Compression

Lossless
■ All information stored
■ Original can be reconstructed exactly

Lossy
■ Some information discarded
■ Much higher compression ratios possible
■ Strategies
■ Discard information that rarely occurs
■ Discard information humans won’t notice
Kolmogorov Complexity

What is the shortest program that can generate this fractal image?

64 Byte C program

\[ z = z0 = \text{Complex}(x,y) \]
for ( i=0; i<1000; i++) {
  \[ z = z^2 + c \]
}
setpixel(\(x,y, |z-z0|\))

24 Bytes of parameters

Re(c), Im(c),
\(x_{min}, x_{max}, y_{min}, y_{max}\)

17 KB JPEG

Lossless Compression
Image Entropy

An image has low entropy
if its pixel values are predictable
An image has high entropy
if its pixel values are random (unpredictable)

0 entropy  Some entropy  High entropy

Example

Alphabet: A, B, C, D
Frequencies: ¼, ¼, ¼, ¼
Code: 00, 01, 10, 11

ABCDBDAC = 8 characters
00 01 10 11 01 11 00 11 = 16 bits

¼ * 2 + ¼ * 2 + ¼ * 2 + ¼ * 2 = 2 bits/ch
Example

Alphabet: A, B, C, D
Frequencies: \( \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8} \)
Code: 0, 10, 110, 111

ABACADAB = 8 characters
0 10 0 110 0 111 0 10 = 14 bits

\( \frac{1}{2} \times 1 + \frac{1}{4} \times 2 + \frac{1}{8} \times 3 + \frac{1}{8} \times 3 = 1 \frac{3}{4} \) bits/ch

Definition of Entropy

Given a random variable \( X \) with possible values \( x_1, x_2, \ldots, x_n \), each with probability \( p(x_1), p(x_2), \ldots, p(x_n) \), the entropy is given as:

\[
H(X) = \sum_{i=1}^{n} p(x_i) \log_2 \left( \frac{1}{p(x_i)} \right)
\]

\[= - \sum_{i=1}^{n} p(x_i) \log_2 p(x_i)\]

\( \log 1/p(x_i) \) is the number of bits to code \( x_i \)
\( H \) is the average number of bits
Huffman Coding

Input
- Symbols and their probabilities

Output
- Binary code for each symbol

Less frequent values have longer codes than more frequent values
Huffman Coding

Algorithm

- Create a node for each value
- Repeat until there is only one node left:
  - Merge the two nodes with lowest probabilities into a new node whose probability is the sum of its children
  - The path from the root node to the leaf node is the code for each value
Transform Coding

If you transform the input data, it may be much more predictable

For example, taking differences

\[ X = 2 \ 2 \ 2 \ 2 \ 3 \ 4 \ 5 \ 6 \ 6 \ 6 \ 6 \quad H(X) = 2.0049 \]

Difference Operator

\[ D(X) = 2 \ 0 \ 0 \ 0 \ 1 \ 1 \ 1 \ 1 \ 0 \ 0 \ 0 \quad H(D(X)) = 1.3222 \]
Lossy Compression

Two Observations on Images

In natural images, low frequencies are more common than high frequencies
Therefore, more bits should be allocated to low frequencies

The human visual system is less sensitive to high frequencies
Therefore, distortion or errors in high frequencies are harder to see (to an extent)
**JPEG - DCT**

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**Discrete Cosine Basis**

\[ X_k = \sum_{n=0}^{N-1} x_n \cos \left[ \frac{\pi}{N} \left( n + \frac{1}{2} \right) \right] \]

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Discrete Cosine Transform (DCT)

Image space

Frequency space

262,144 terms

Lossy Compression
Lossy Compression

262,144 terms

43384/262144 = 16%

Largest terms

Lossy Compression

262,144 terms

8353/262144 = 3.2%

Largest terms
Quantization

Def: Quantization is the process of mapping a high precision value to a low precision value. Compresses, but introduces error.

256 levels
8 bits/pixel

4 levels
2 bits/pixel

Peak Signal-to-Noise Ratio

Original Image (I) and new image (I’)

\[ MSE = \frac{1}{N} \sum_{i=0}^{N} \| I(i) - I'(i) \|^2 \]

\[ RMSE = \sqrt{MSE} \]

\[ PSNR = 10 \log_{10} \left( \frac{I_{max}^2}{MSE} \right) \]

\[ = 20 \log_{10} \left( \frac{I_{max}}{RMSE} \right) \]
Quantize Frequencies Differently

\[
\begin{bmatrix}
-415 & -30 & -61 & 27 & 56 & -20 & -2 & 0 \\
4 & -22 & -61 & 10 & 13 & -7 & -9 & 5 \\
-47 & 7 & 76 & -25 & -29 & 10 & 5 & -6 \\
-49 & 12 & 34 & -15 & -10 & 6 & 2 & 2 \\
12 & -7 & -13 & -4 & -2 & 2 & 3 & 3 \\
-8 & 3 & 2 & -6 & -2 & 1 & 4 & 2 \\
-1 & 0 & 0 & -2 & -1 & -3 & 4 & -1 \\
0 & 0 & -1 & -4 & -1 & 0 & 1 & 2 \\
\end{bmatrix}
\div
\begin{bmatrix}
16 & 11 & 10 & 16 & 24 & 40 & 51 & 61 \\
12 & 12 & 14 & 19 & 26 & 58 & 60 & 55 \\
14 & 13 & 16 & 24 & 40 & 57 & 69 & 56 \\
14 & 17 & 22 & 29 & 51 & 87 & 80 & 62 \\
18 & 22 & 37 & 56 & 68 & 109 & 103 & 77 \\
24 & 35 & 55 & 64 & 81 & 104 & 113 & 92 \\
49 & 64 & 78 & 87 & 103 & 121 & 120 & 101 \\
72 & 92 & 95 & 98 & 112 & 100 & 103 & 99 \\
\end{bmatrix}
\]

\[
= \begin{bmatrix}
-26 & -3 & -6 & 2 & 2 & -1 & 0 & 0 \\
0 & -2 & -4 & 1 & 1 & 0 & 0 & 0 \\
-3 & 1 & 5 & -1 & -1 & 0 & 0 & 0 \\
-4 & 1 & 2 & -1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

Quantized DCT

Error

\[
\text{error} = \text{abs}(\text{original} - \text{compressed}) \times 8
\]
JPEG2000 - Wavelets

Fourier vs. Haar Basis

Cosines

Haar Wavelets
Haar Wavelets

Scaling function = average

Wavelet = difference

Transform:
S = (A + B)/2
D = (A - B)/2

Inverse transform
A = S + D
B = S - D
Haar Wavelets

\[ 6 \ 8 \ 5 \ 9 \ 5 \ 5 \ 6 \ 6 \]

\[ 7 \ 7 \ 5 \ 6 \ \boxed{-1} \ \boxed{-2} \ 0 \ 0 \]

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Averages
Smoothed version of signal
Lower resolution image

Differences
Local, high frequencies
Details missing from low resolution part
Haar Wavelets

6 8 5 9 5 5 6 6

7 7 5 6 -1 -2 0 0

7 5.5 -1 -2 0 0

Haar Wavelets

6 8 5 9 5 5 6 6

7 7 5 6 -1 -2 0 0

7 5.5 -1 -2 0 0
Haar Wavelets

6 8 5 9 5 5 6 6

7 7 5 6 -1 -2 0 0

7 5.5 0 -0.5 -1 -2 0 0

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Haar Wavelets

6 8 5 9 5 5 5 6 6

7 7 5 6 -1 -2 0 0

7 5.5 0 -0.5 -1 -2 0 0

6.25 0 -0.5 -1 -2 0 0

Haar Wavelets

6 8 5 9 5 5 5 6 6

7 7 5 6 -1 -2 0 0

7 5.5 0 -0.5 -1 -2 0 0

6.25 0.75 -0.5 -1 -2 0 0
Haar Wavelets

6 8 5 9 5 5 6 6

Full Transform

6.25 7.5 0 -0.5 -1 -2 0 0

High Resolution Details
Medium Res. Details
Low Resolution Details
Average Resolution Value

2D: Standard Decomposition

Stollnitz, Derose, Salesin, Wavelets for computer graphics

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2D: Non-Standard Decomposition

transform rows

transform columns

Stollnitz, Derose, Salesin, Wavelets for computer graphics

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What else needs compression?

Video
Textures
Geometry (including animated geometry)

...
Things to Remember

Lossless
- Compression depends on the entropy

Lossy
- Visual system less sensitive to high frequencies
- Distribute error to higher frequencies
- Measure error vs. number of bits (PSNR)

Different basis functions
- JPEG / Discrete cosine transform (DCT) - good
- JPEG2000 / Wavelets - better