Basic Signal Processing: Sampling and Aliasing

Key Concepts

Frequency space
Filters and convolution
Sampling
Aliasing and the Nyquist frequency
Antialiasing
Frequency Space

Constant

Spatial Domain          Frequency Domain
\[ \sin (2 \pi 16 x) \]

Spatial Domain
16 cycles / width (= frequency)

Frequency Domain

Spatial Domain

\[ \sin (2 \pi 32 x) \]

Spatial Domain

Frequency Domain
\[ \sin (2 \pi 32 y) \]

**Spatial Domain**

**Frequency Domain**

\[ \sin (2 \pi 16 x) \sin (2 \pi 32 y) \]

**Spatial Domain**

**Frequency Domain**
Gaussian (r/16)

Spatial Domain  Frequency Domain

Gaussian (r/32)

Spatial Domain  Frequency Domain
My Humble Frequencies

Spatial Domain  Frequency Domain

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Filtering
Remove High Frequencies (Blur)

Spatial Domain  Frequency Domain

Remove Low Frequencies (Edges)

Spatial Domain  Frequency Domain
Remove Low and High Frequencies

Spatial Domain

Frequency Domain

Remove Low and High Frequencies

Spatial Domain

Frequency Domain
Filters = Convolution

Convolution

\[
\begin{array}{cccccc}
1 & 3 & 0 & 4 & 2 & 1 \\
\end{array}
\]

\[
\begin{array}{cc}
1 & 2 \\
\end{array}
\]
Convolution

\[
\begin{array}{cccccc}
1 & 3 & 0 & 4 & 2 & 1 \\
1 & 2 \\
7 & & & & & \\
\end{array}
\]

\[3 \times 1 + 0 \times 2 = 3\]

\[
\begin{array}{cccccc}
7 & 3 & & & & \\
\end{array}
\]
Convolution

\[
\begin{array}{cccccc}
1 & 3 & 0 & 4 & 2 & 1 \\
& & 1 & 2 & & \\
\end{array}
\]

\[0 \times 1 + 4 \times 2 = 8\]

\[
\begin{array}{cccc}
7 & 3 & 8 & \text{ } \\
\text{ } & & & \text{ } \\
\end{array}
\]

Convolution Theorem

A filter can be implemented in the spatial domain using convolution.

A filter can also be implemented in the frequency domain:
- Convert image to frequency domain
- Convert filter to frequency domain
- Multiply filter times image in frequency domain
- Convert result to the spatial domain

Filter properties are easier to understand in the frequency domain.
Box Filter

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Box Filter = Low-Pass Filter

Spatial Domain  Frequency Domain
Wider Filters, Lower Frequencies

Spatial Domain  Frequency Domain

Size of Filter

As a filter is localized in space, it spreads out in frequency. Conversely, as a filter is localized in frequency, it spreads out in space.

A box filter is very localized in space; it has infinite extent in frequency space.
Efficiency?

When would it be faster to apply the filter in the spatial domain?

When would it be faster to apply the filter in the frequency domain?

Sampling
Image Generation = Sampling

Evaluating a function at a point is sampling

for( int x = 0; x < xmax; x++ )
for( int y = 0; y < ymax; y++ )
l[x][y] = f( float(x), float(y));

Rasterization is equivalent to evaluating the function inside(triangle,x,y)

Aliasing
Wagon Wheel Effect

http://www.michaelbach.de/ot/mot_wagonWheel/

"Aliases"

These two frequencies are indistinguishable. Indistinguishable frequencies are called aliases.
Nyquist Frequency

Definition: The Nyquist frequency is \( \frac{1}{2} \) the sampling frequency (1 / Ts)

Frequencies above the Nyquist frequency appear as aliases

No aliases appear if the function being sampled has no frequencies above the Nyquist frequency

Sampling in Computer Graphics

All of computer graphics involve sampling

Artifacts due to sampling - Aliasing
- Wagon wheel effect – sampling in time
- Jaggies – sampling in space
- Temporal strobing – sampling in space-time
- Moire – sampling texture coordinates
- Sparkling highlights – sampling normals

Preventing these artifacts - Antialiasing
Zone Plate $\sin x^2$

Zone Plate

$\sin(x^2 + y^2)$

Aliases

(0,0)
Jaggies

Retort sequence by Don Mitchell

Staircase pattern or jaggies

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Antialiasing
Antialiasing

Simple idea:

Remove frequencies above the Nyquist frequency before sampling

How? Filtering before sampling

Filter during rasterization

Prefiltering by Computing Coverage

1 pixel box filter removes frequencies greater than the pixel sampling rate

Box filter and sample = computing coverage/alpha
**Point- vs. Area-Sampled**

Point vs. Area

Checkerboard sequence by Tom Duff

**Supersampling**

Approximate a box filter by taking more samples and averaging them together

4 x 4 supersampling
Point-sampling vs. Super-sampling

Point  4x4 Super-sampled

Checkerboard sequence by Tom Duff

Area-Sampling vs. Super-sampling

Exact Area  4x4 Super-sampled
Antialiasing

Jaggies

Prefilter

Antialiasing vs. Blurred Aliases

Blurred Jaggies

Prefilter
Things to Remember

Signal processing
- Frequency domain vs. spatial domain
- Filters in the frequency domain
- Filters in the spatial domain = convolution

Sampling and aliasing
- Image generation involves sampling
- May also sample geometry, motion, ...
- Nyquist frequency is ½ the sampling rate
- Frequencies above the Nyquist frequency appear as other frequencies – aliases
- Antialiasing – Filter before sampling