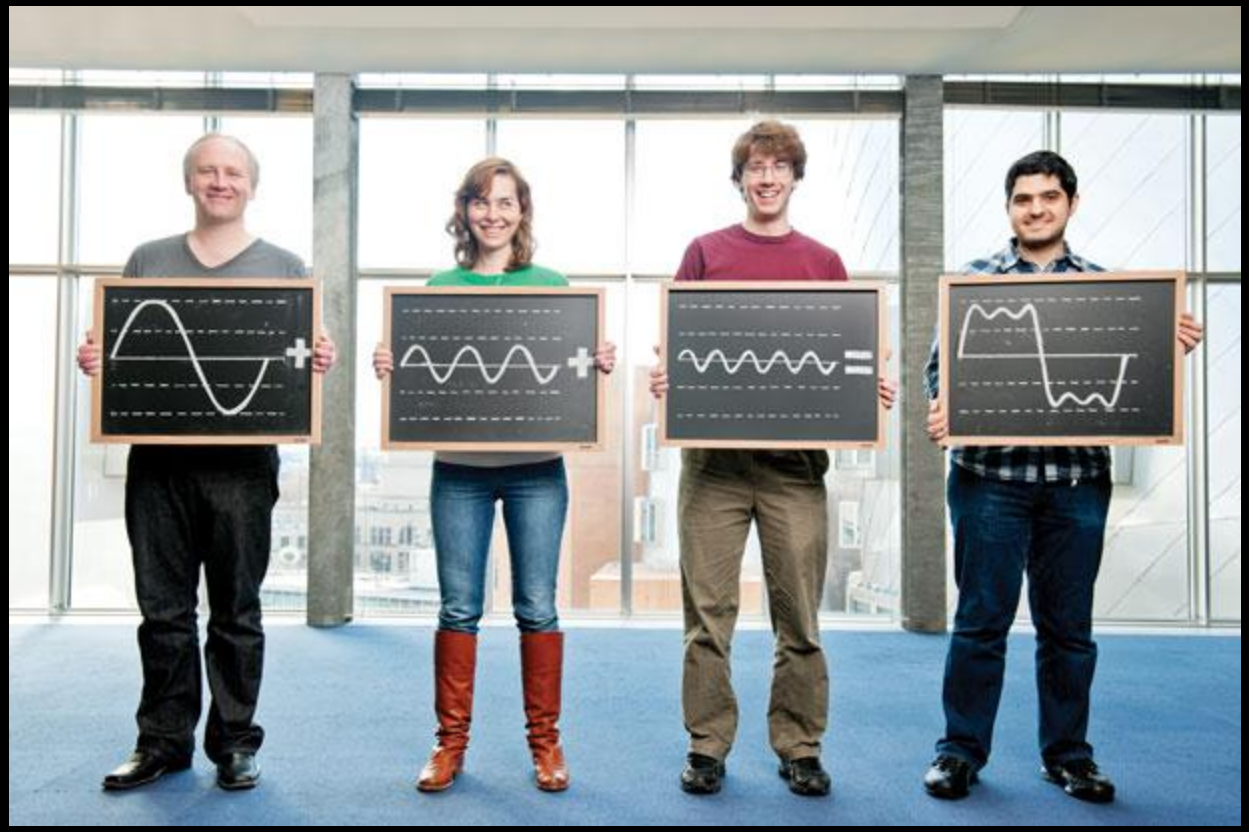
http://www.technologyreview.com/article/427676/a-faster-fourier-transform/

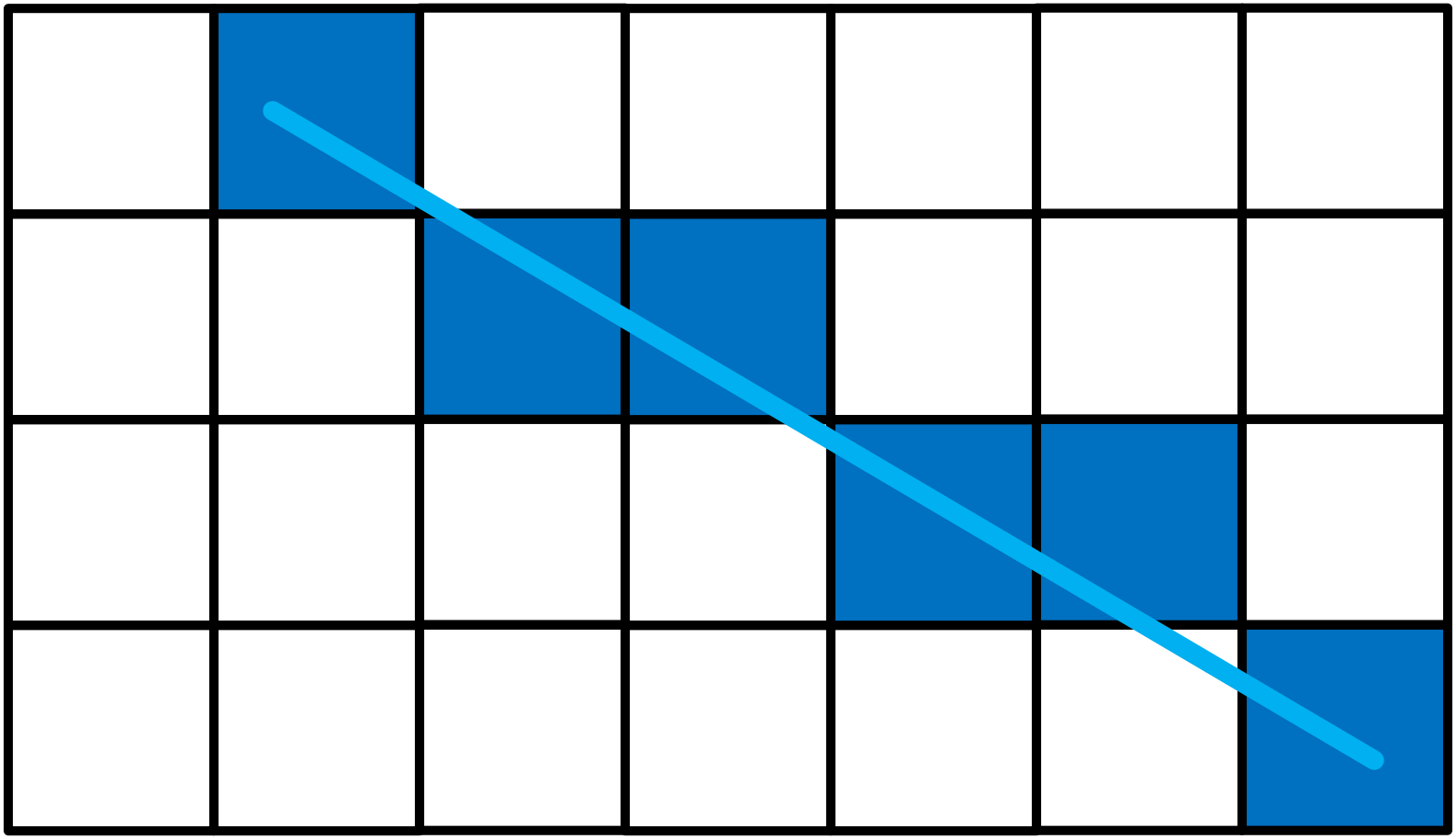


Sampling and Fourier Theory

CS 148, Summer 2012
Introduction to Computer Graphics and Imaging

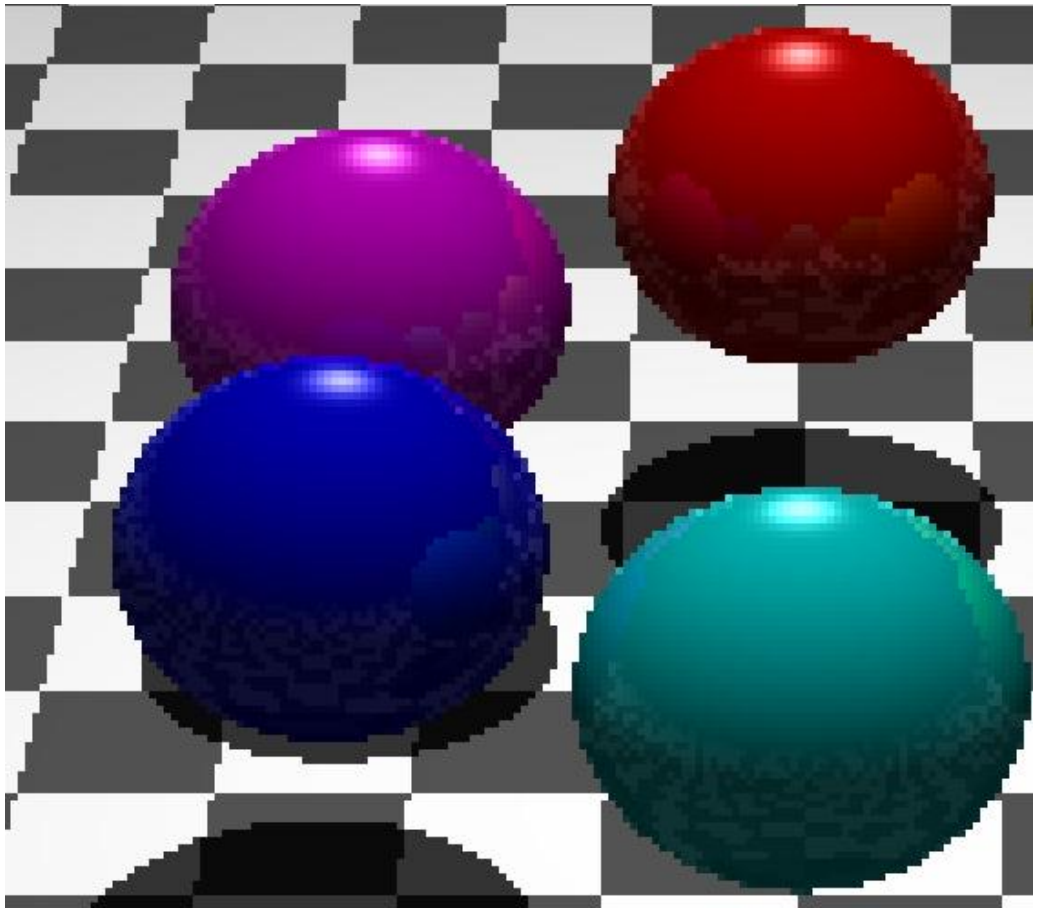
Justin Solomon

Important Issues We've Ignored



Jaggies

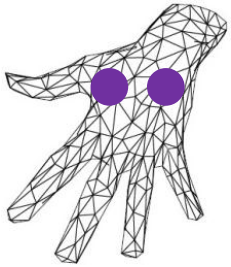
Important Issues We've Ignored



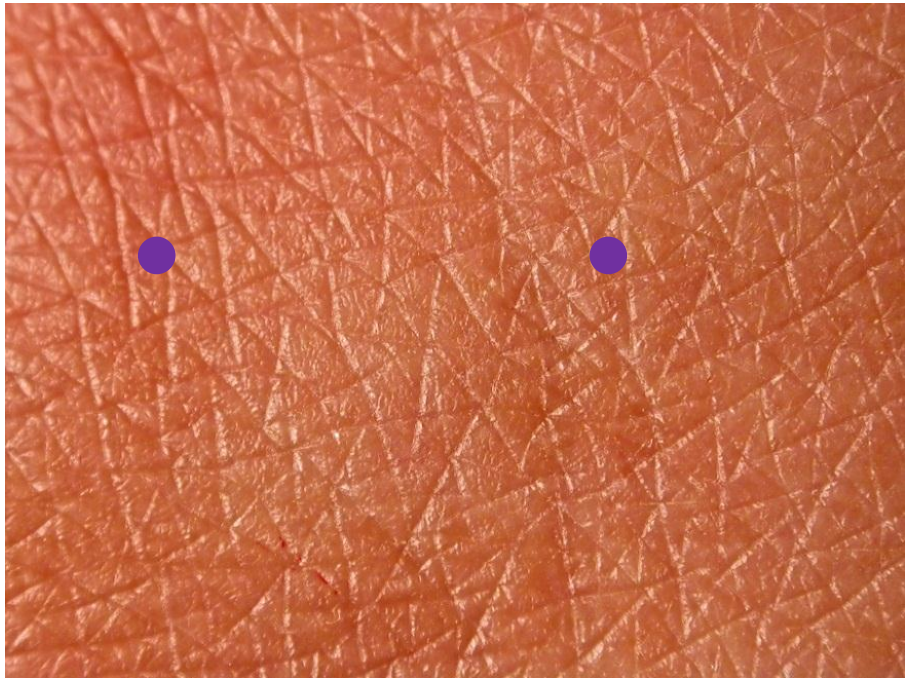
Sampling issues

Important Issues We've Ignored

Small image



Large texture



Texture lookup spacing

Important Issues We've Ignored

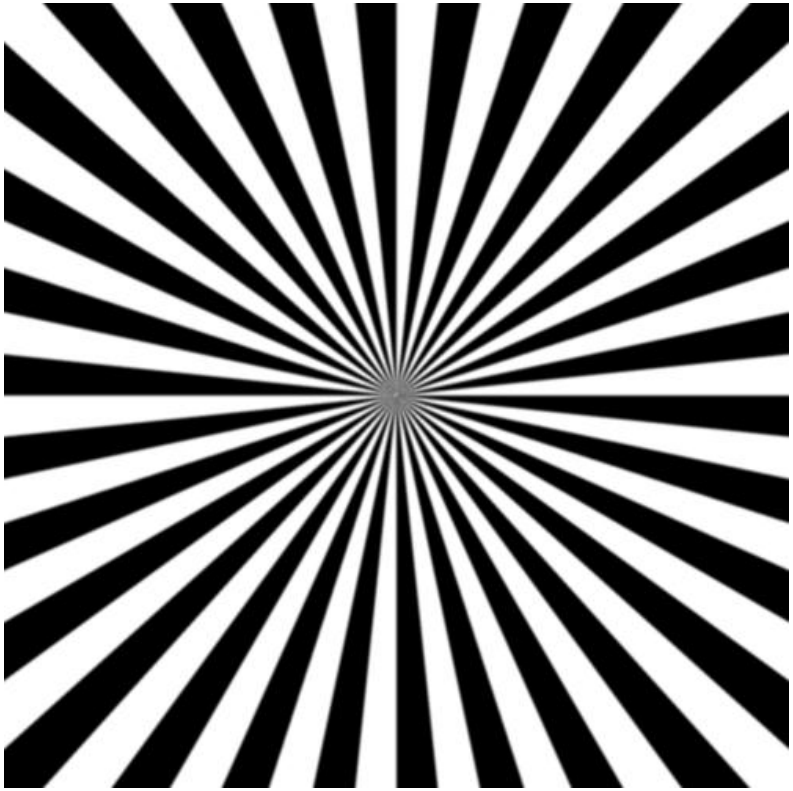
Moiré patterns

More Curiosities

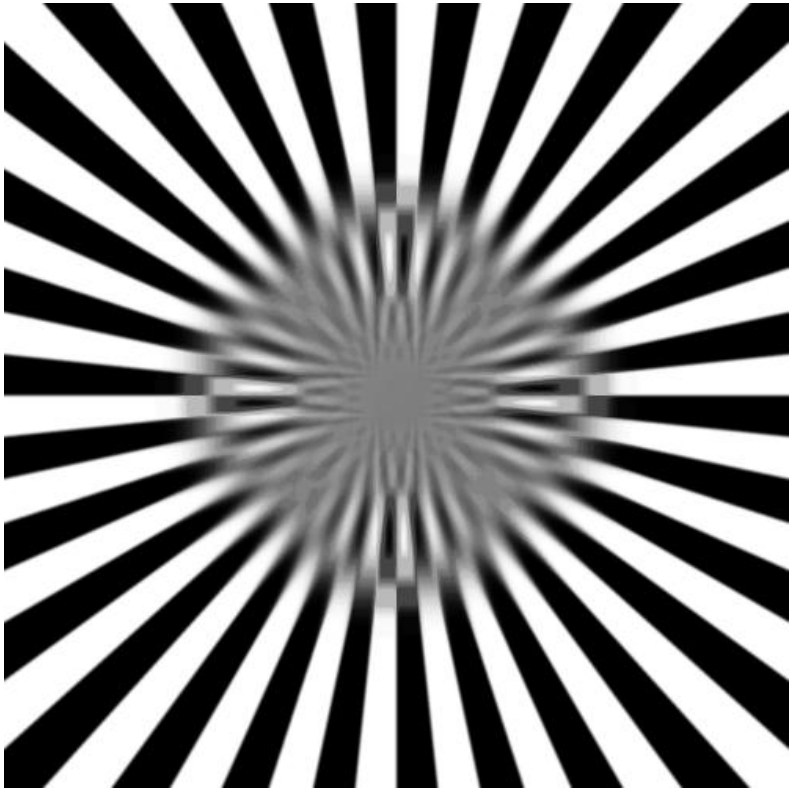
http://www.youtube.com/watch?v=jHS9JGkEOmA

Wagon wheel effect

More Curiosities



Original



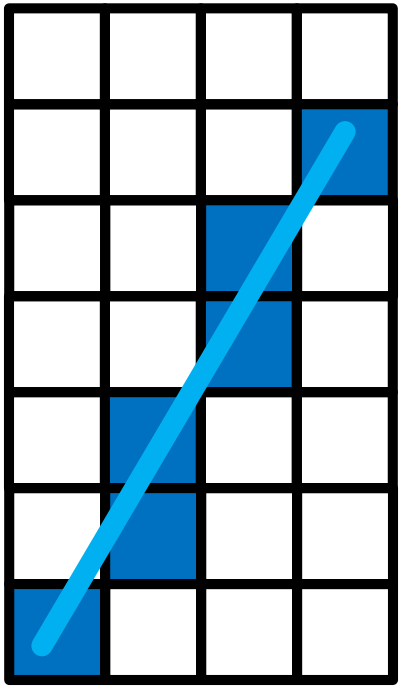
Photoshop "Median"

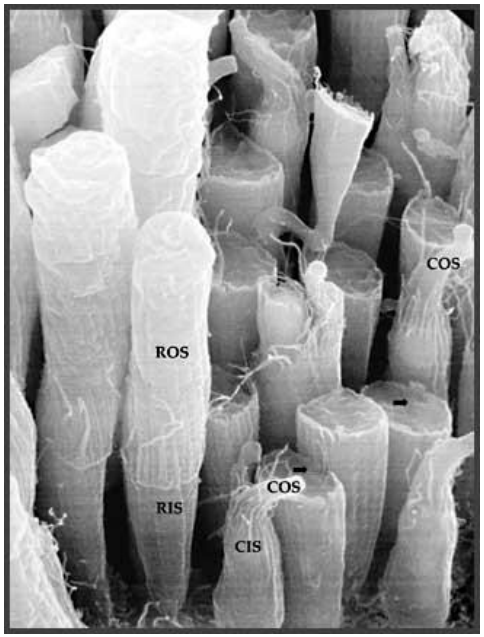
Kass and Solomon, "Smoothed Local Histogram Filters," SIGGRAPH 2010

Broken image filtering

What's Broken?

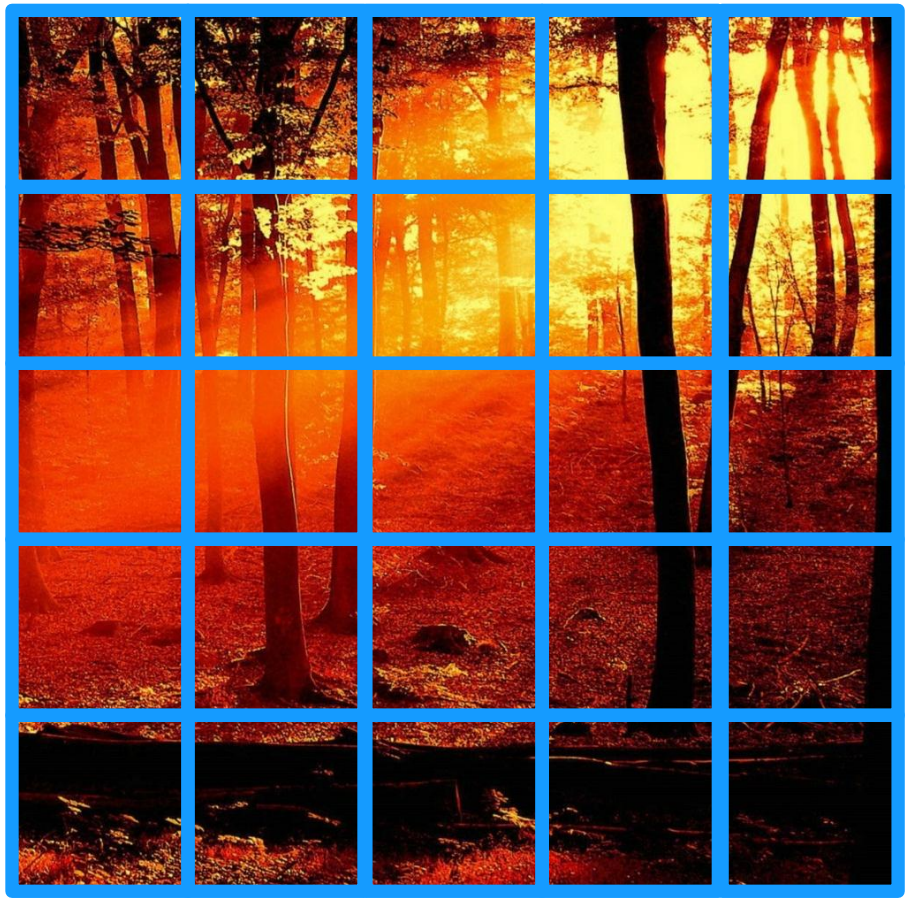
Continuous world





Discrete sensors and displays

What's Broken?



What goes in each pixel?

Our Strategy

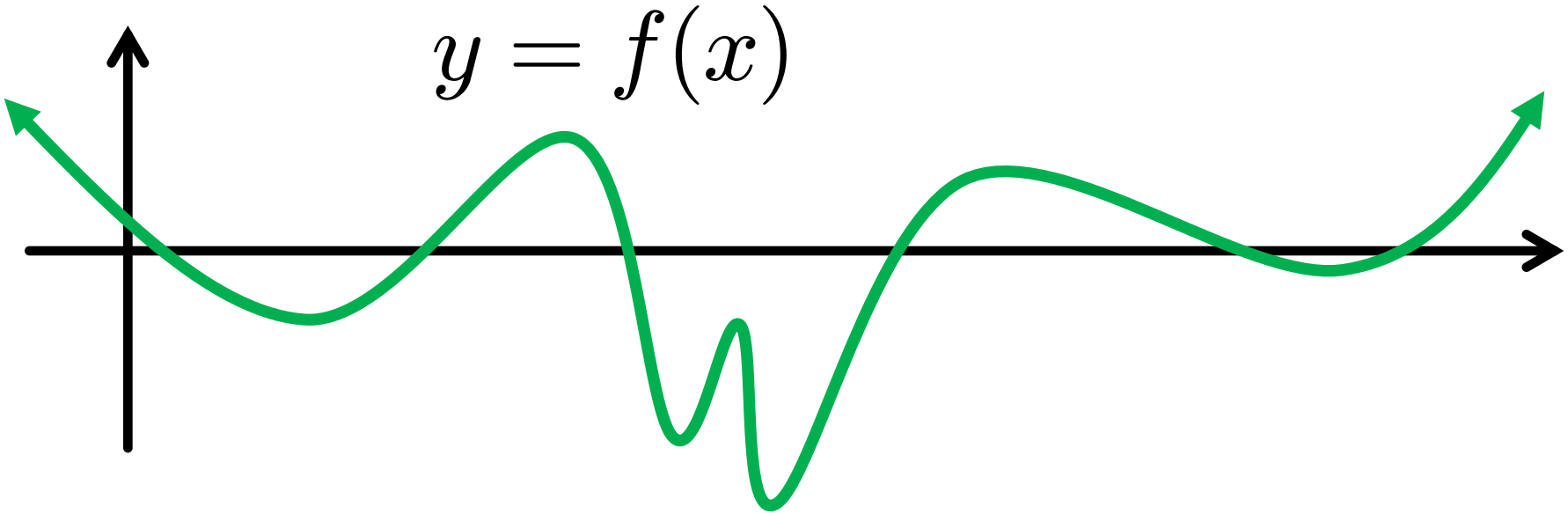
Employ mathematical and perceptual tricks to compensate for our inexact sampling.

Today

Employ mathematical and perceptual tricks to compensate for our inexact sampling.

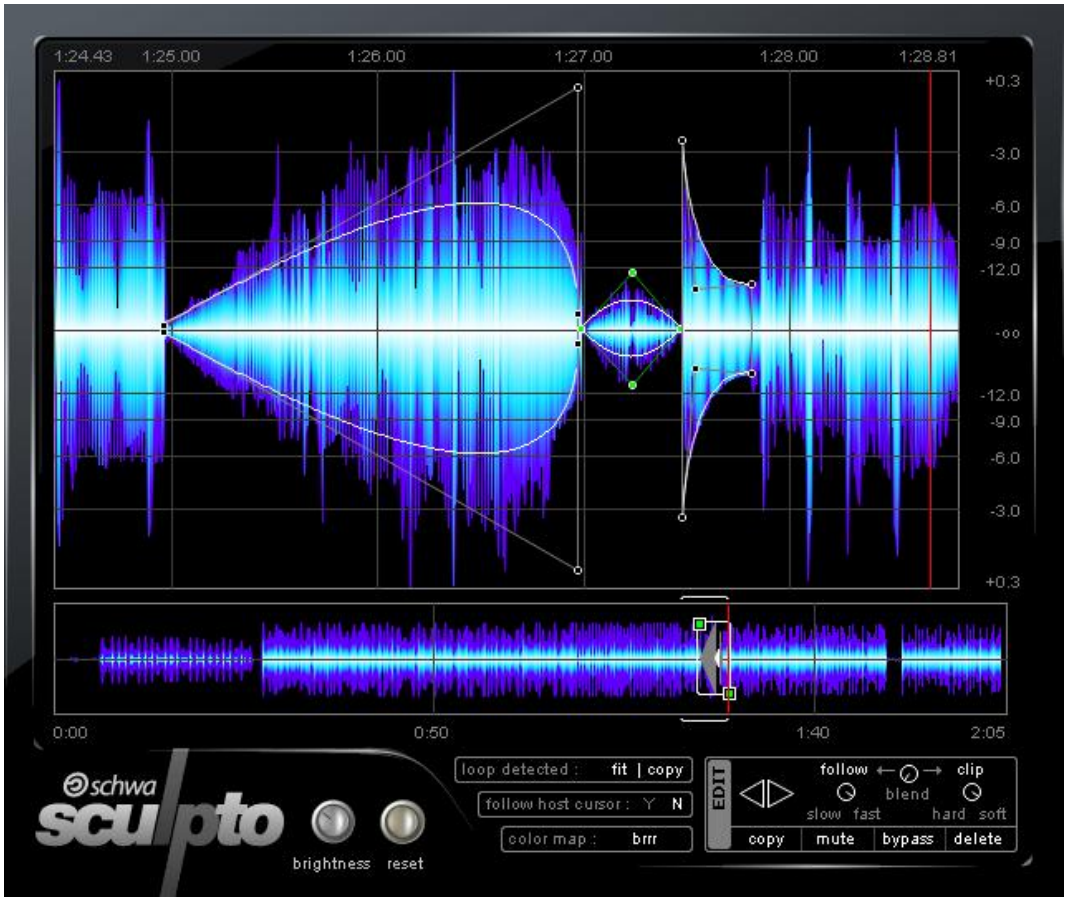

WARNING
THE OPPOSITE
OF RIGOROUS

Our Model



1D signals

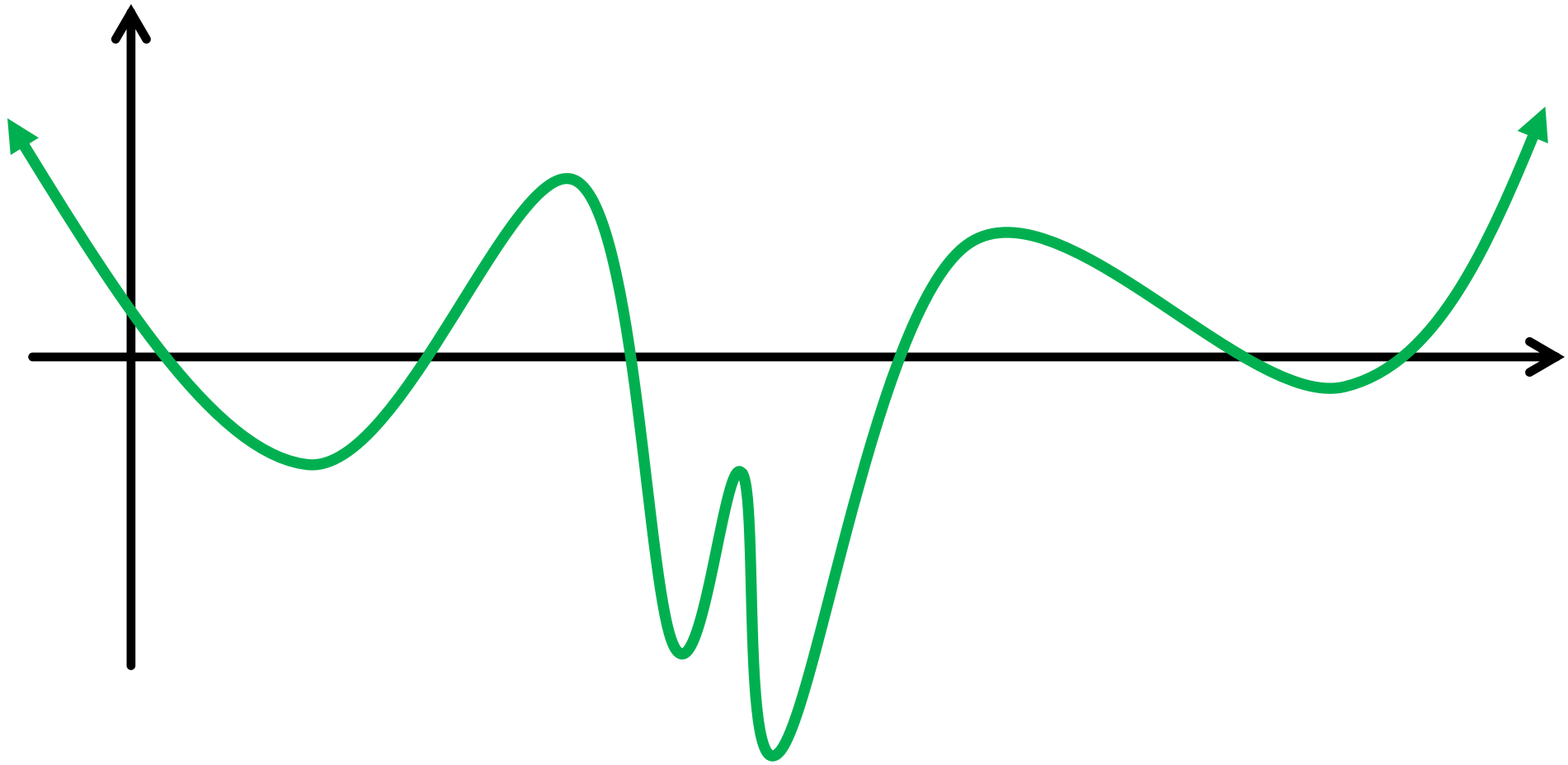
Our Model



http://cdn05.stillwellaudio.com/wp-content/uploads/2007/10/sculpto-1.png

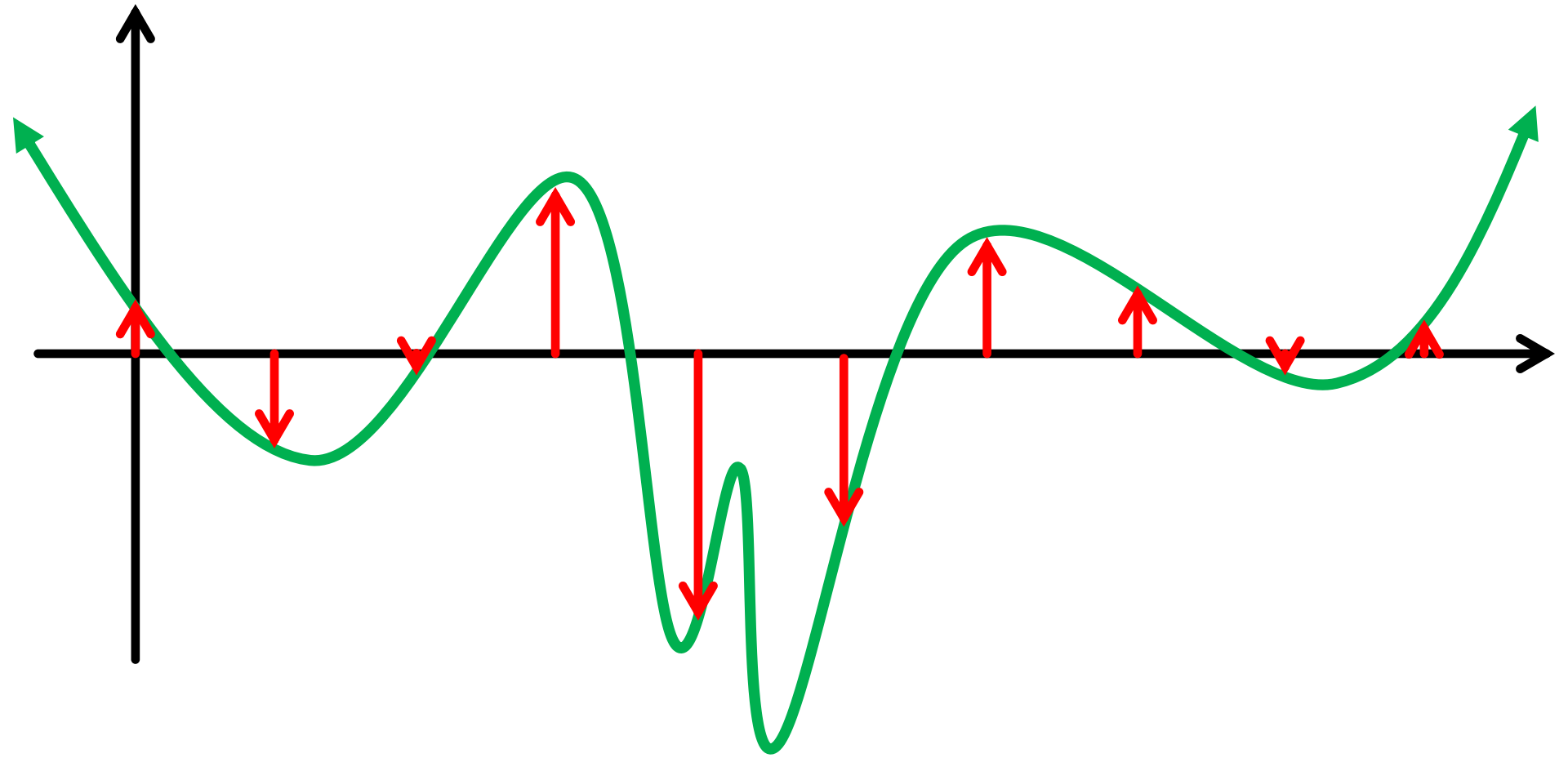
1D signals

Sampling



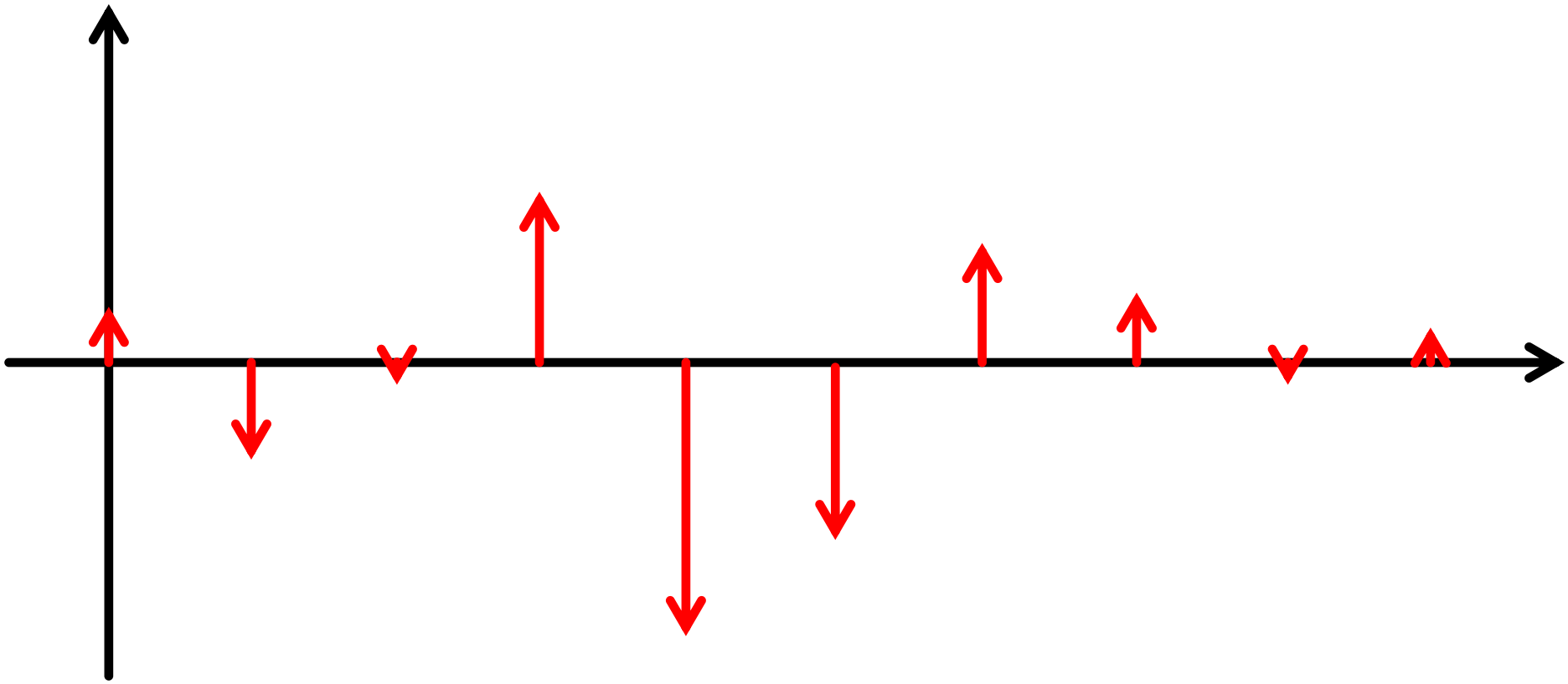
Multiply by "impulse train"

Sampling



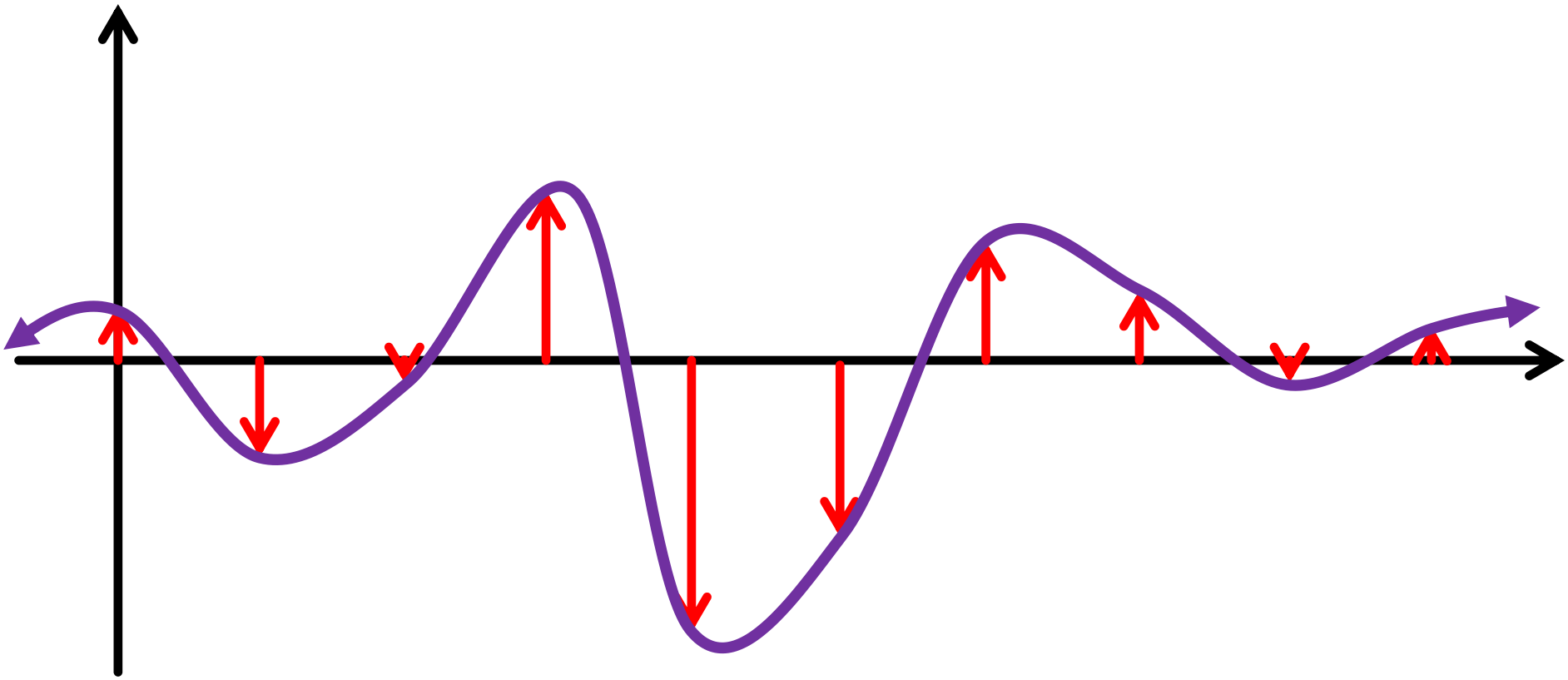
Multiply by "impulse train"

Sampling



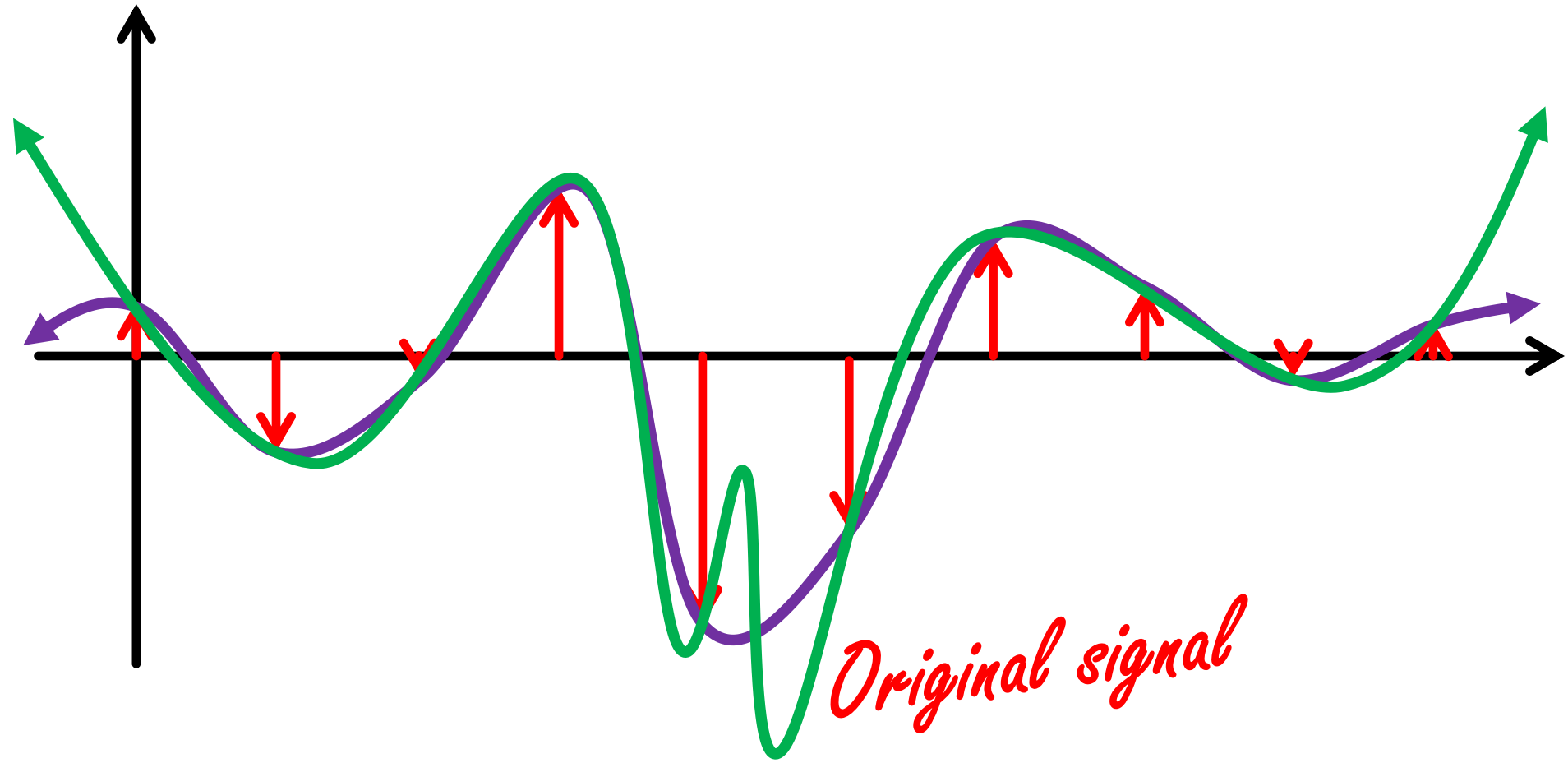
Multiply by "impulse train"

Sampling



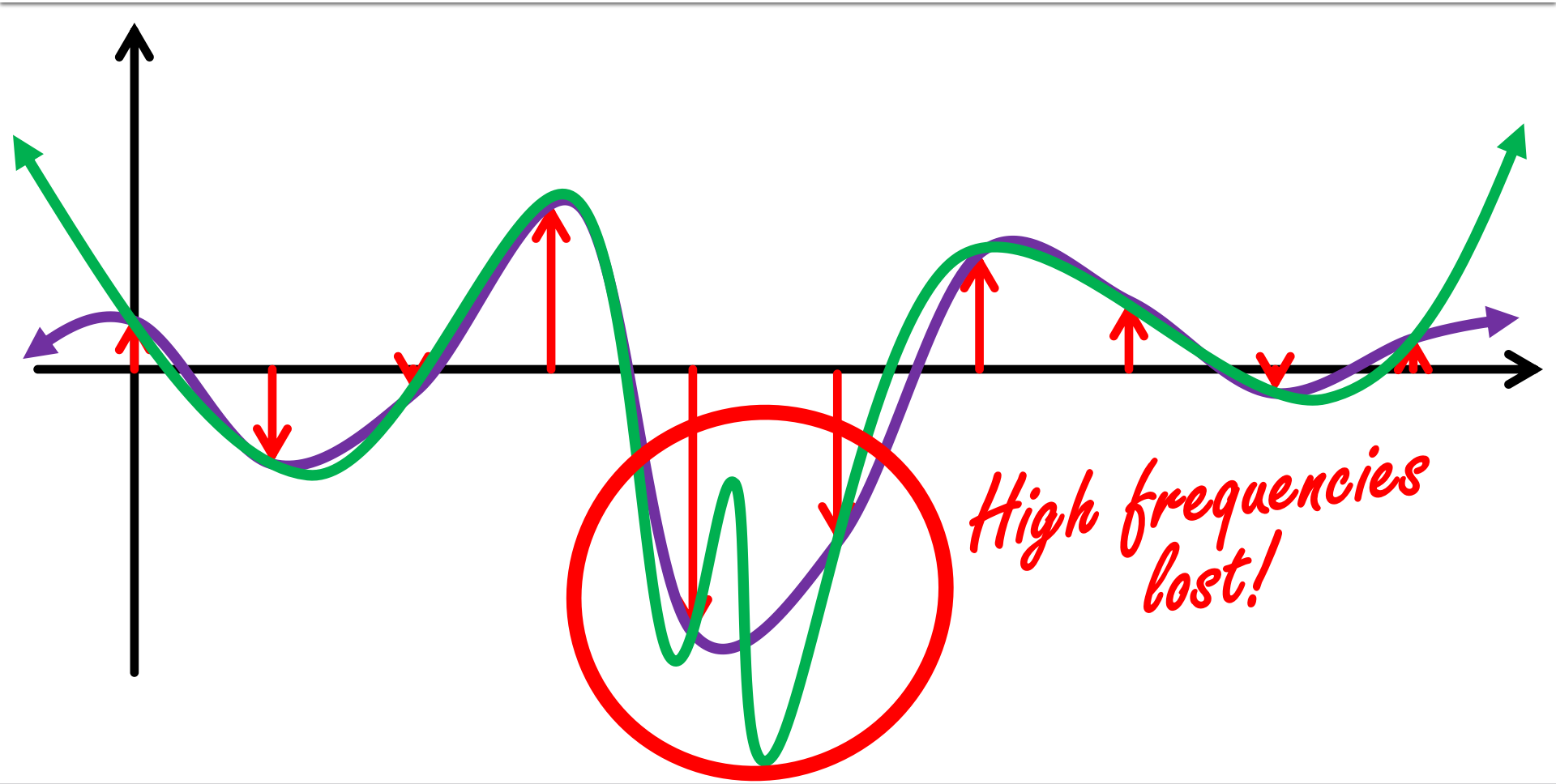
Reconstruct

Sampling



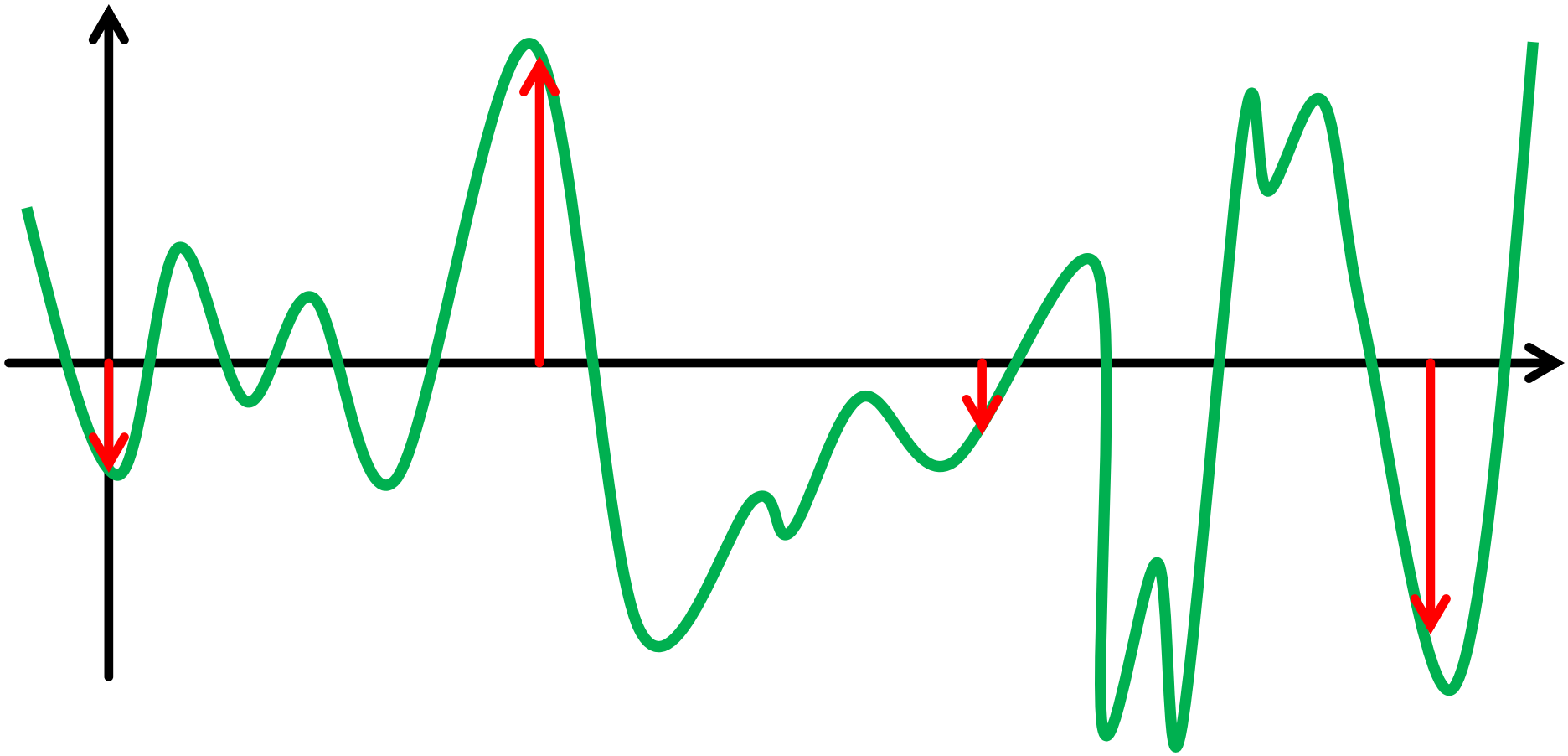
Reconstruct

Sampling



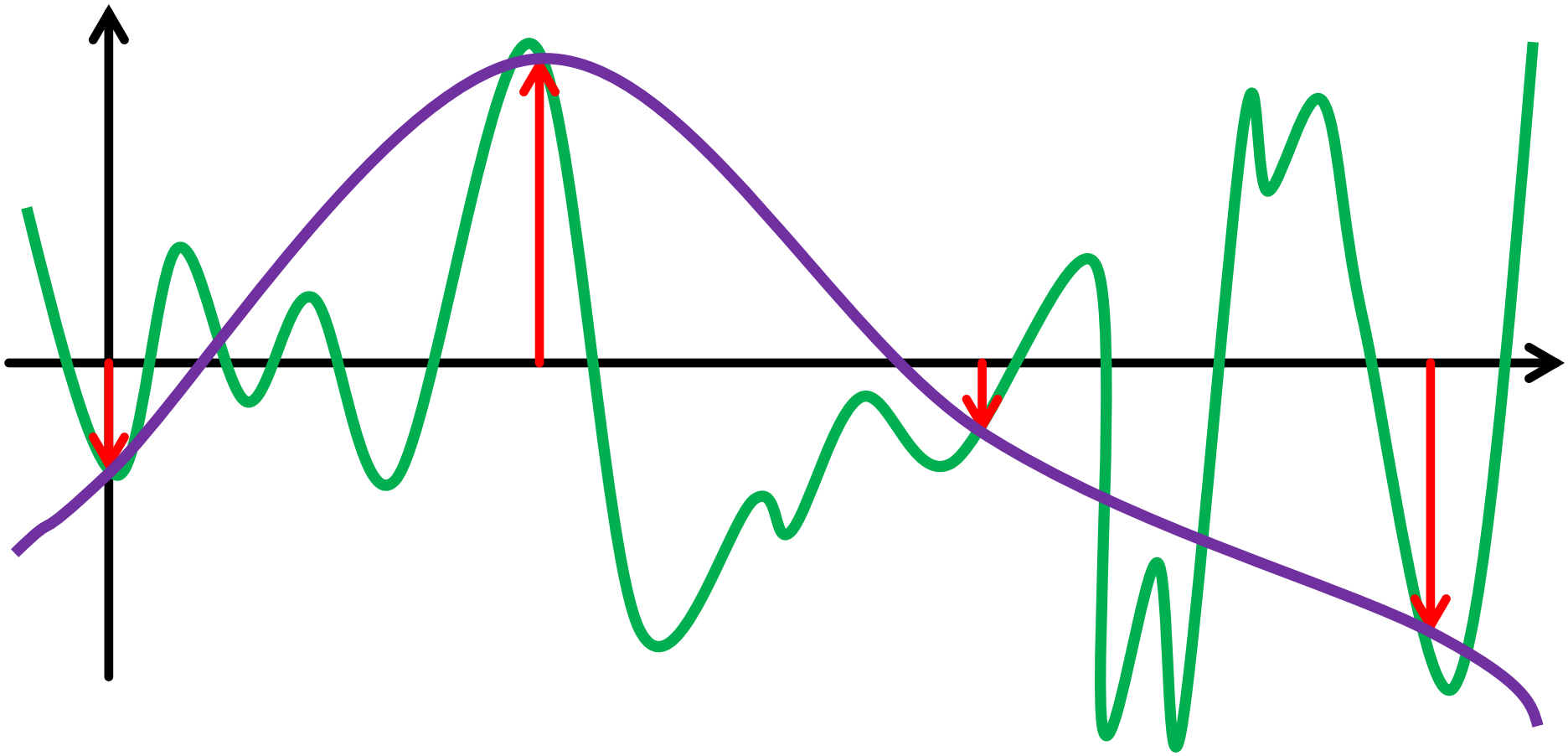
Reconstruct

More Obvious Example



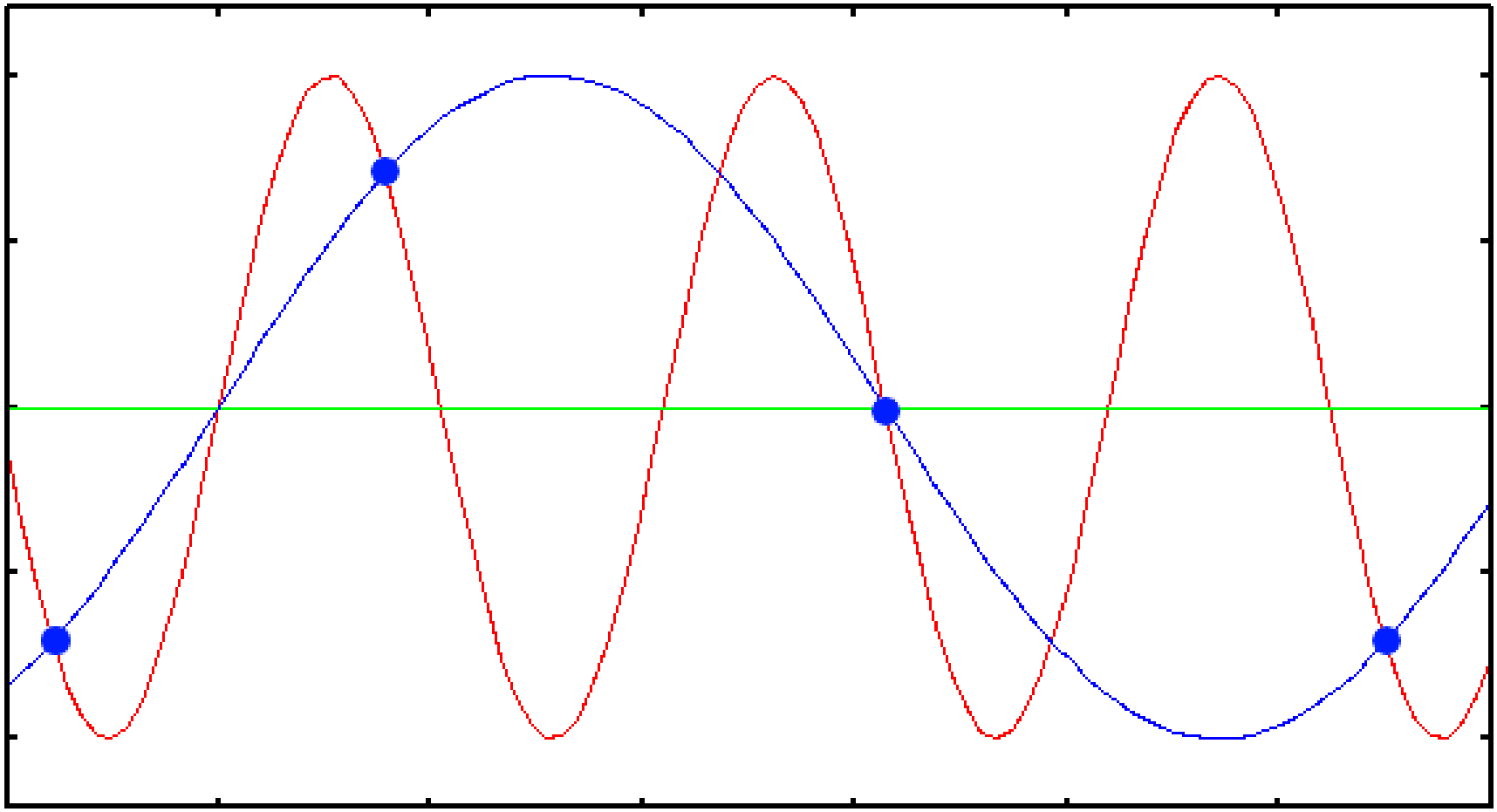
Four samples won't suffice!

More Obvious Example



Four samples won't suffice!

Worst Possible Example



http://www.svi.nl/wikiimg/Aliasing-plot.png

The Issue at Hand

Too many wiggles between consecutive samples.

The Issue at Hand

Too many wiggles between consecutive samples.

Need a way to formalize

Fourier Transform

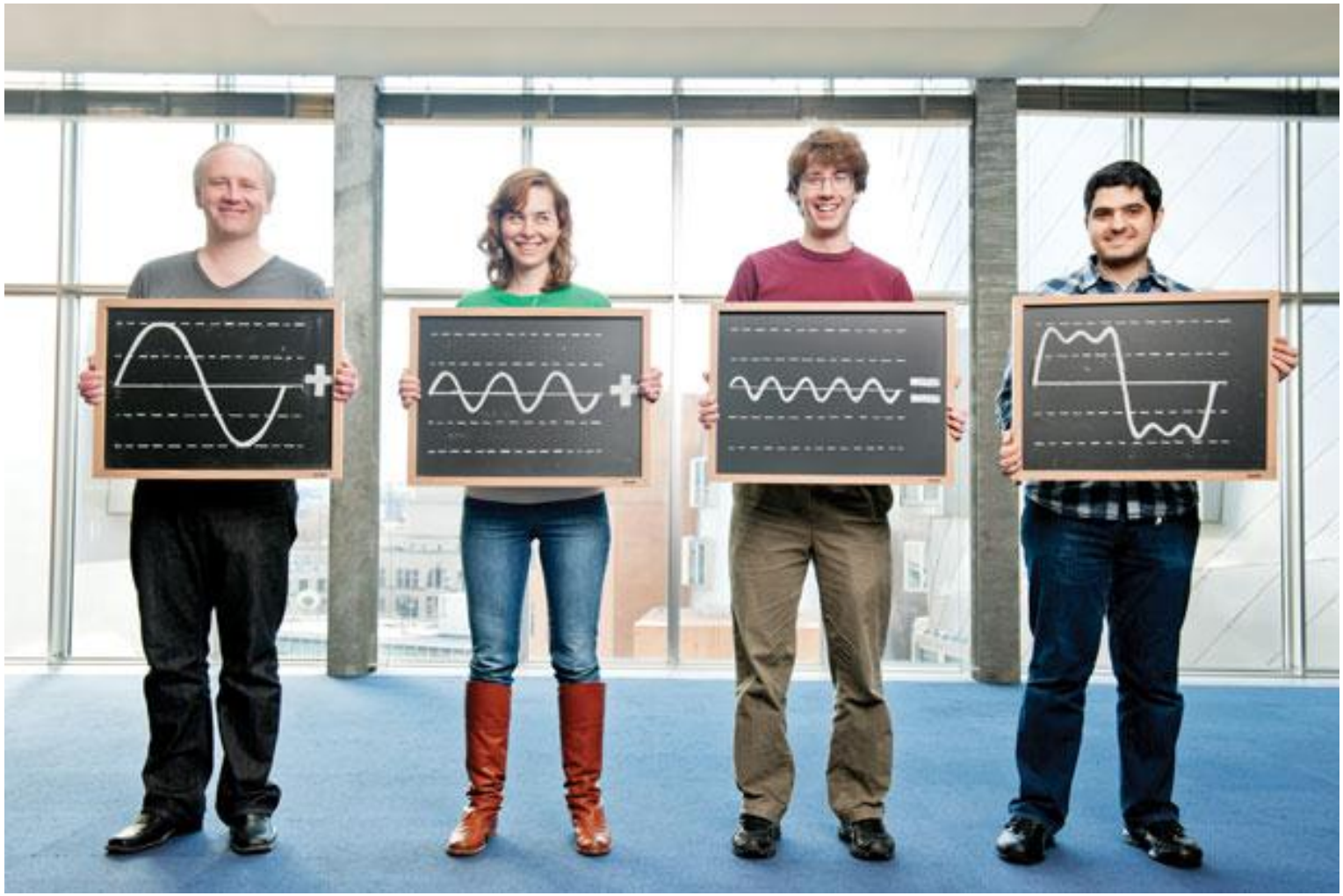
Joseph Fourier

1768-1830

THEOREM(-ISH).

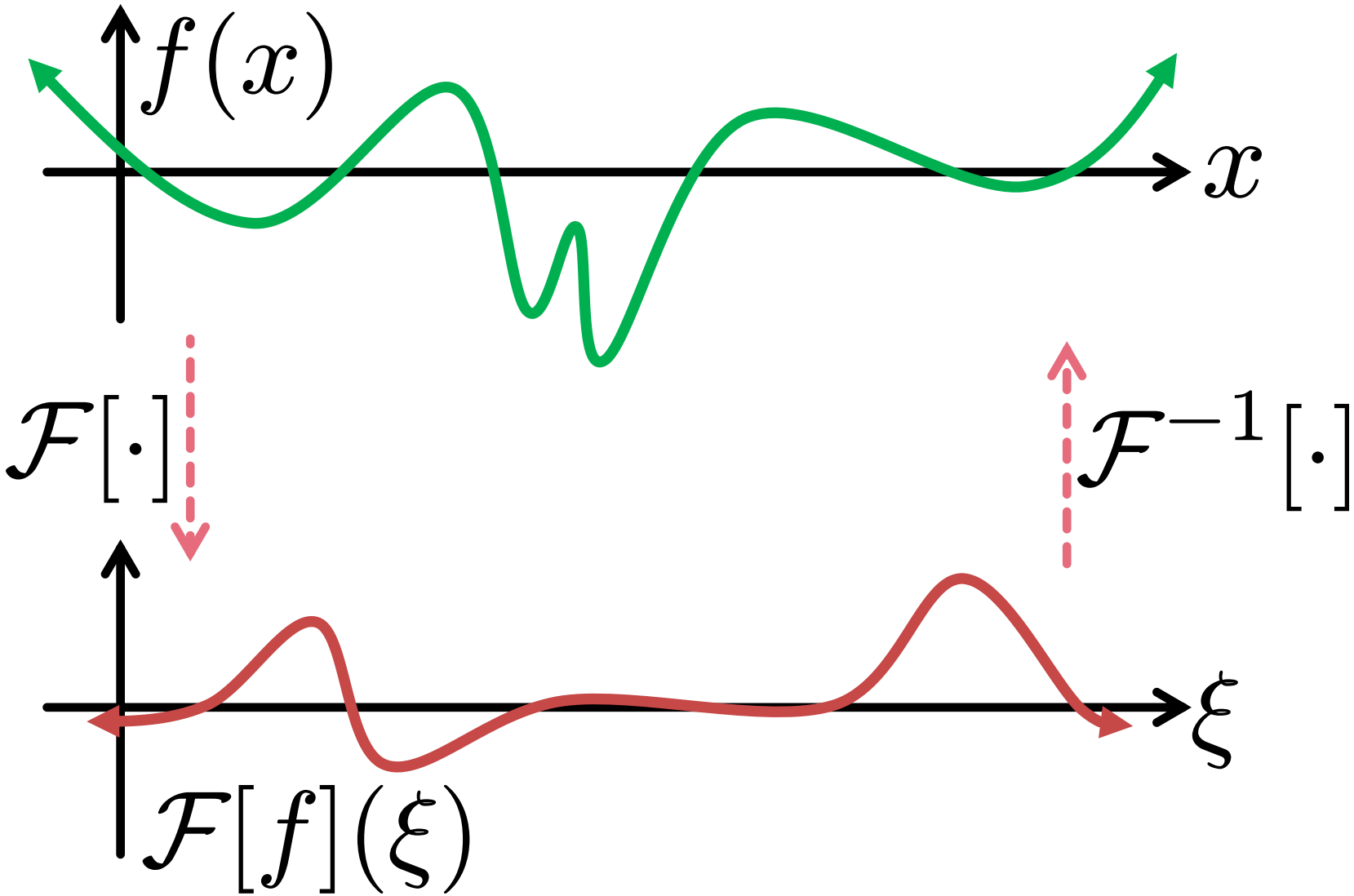
Any function can be written as a combination of simple wiggles.

Fourier Transform

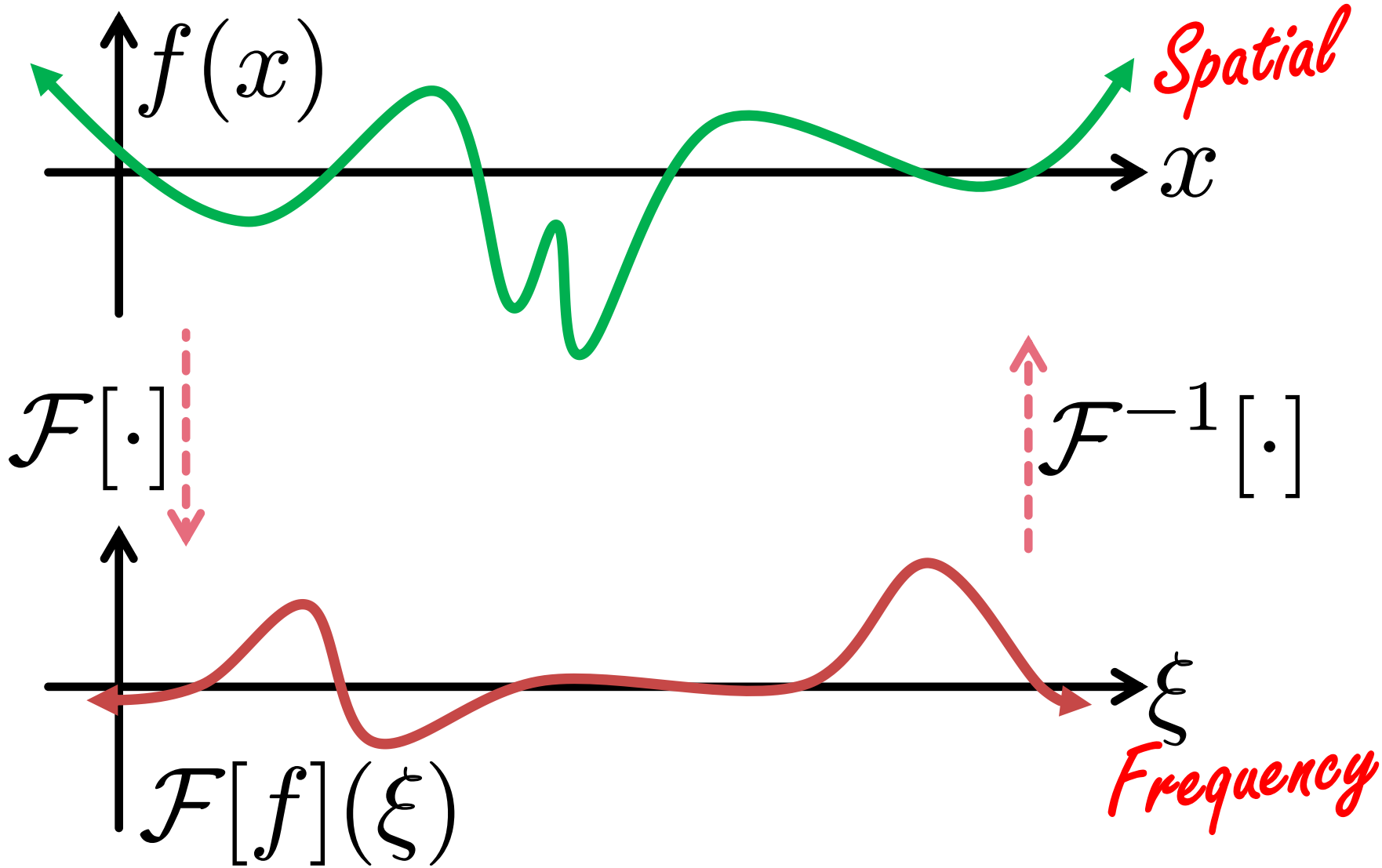


http://www.technologyreview.com/article/427676/a-faster-fourier-transform/

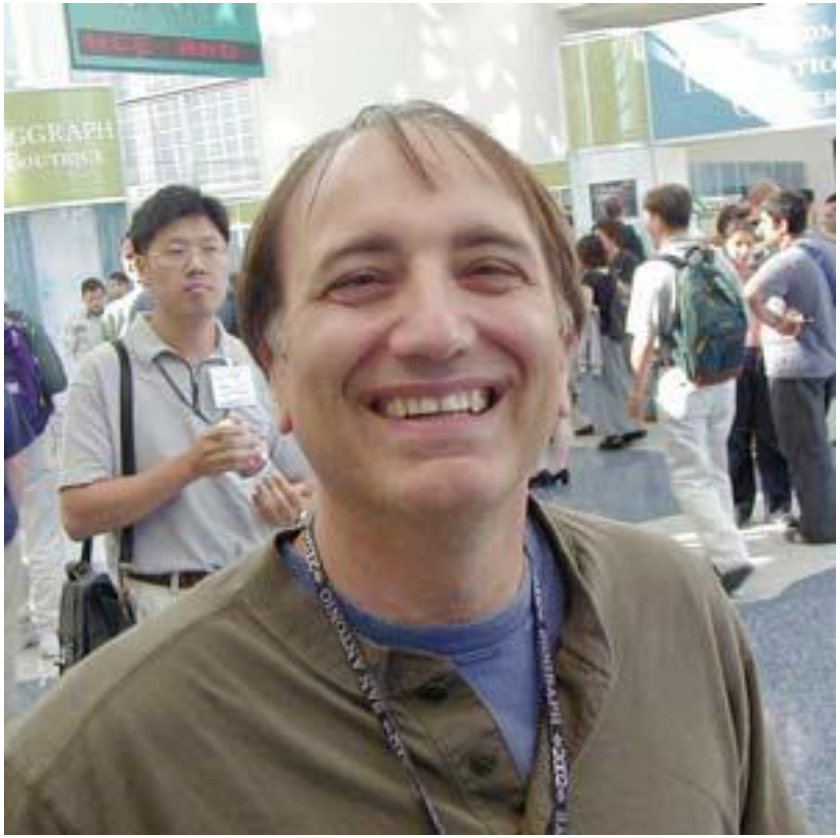
Fourier Transform

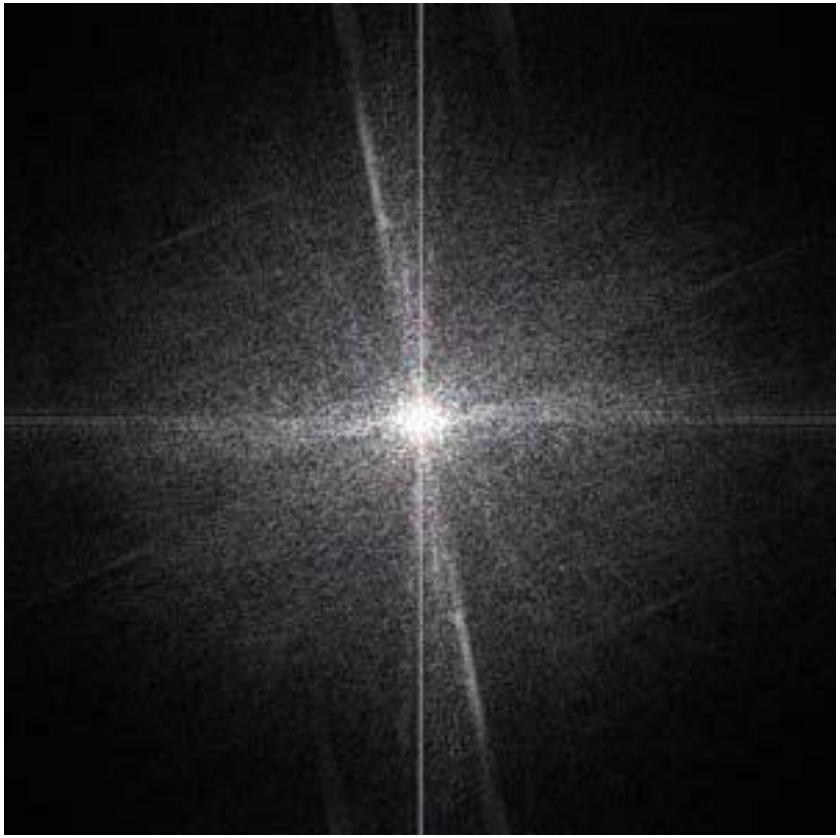


Fourier Transform



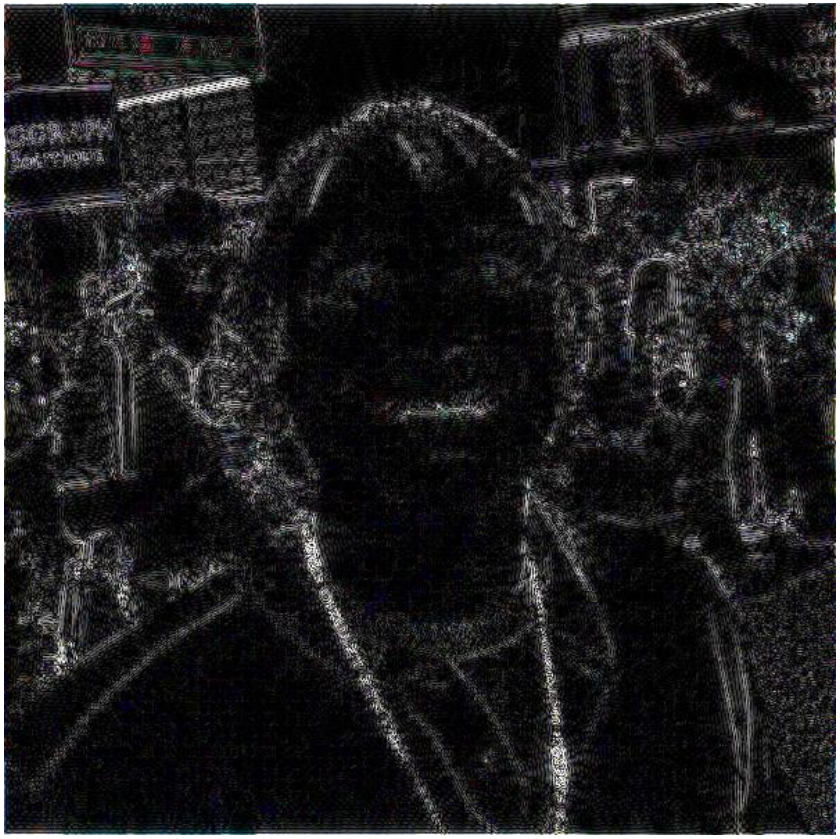
Frequency Domain Filtering

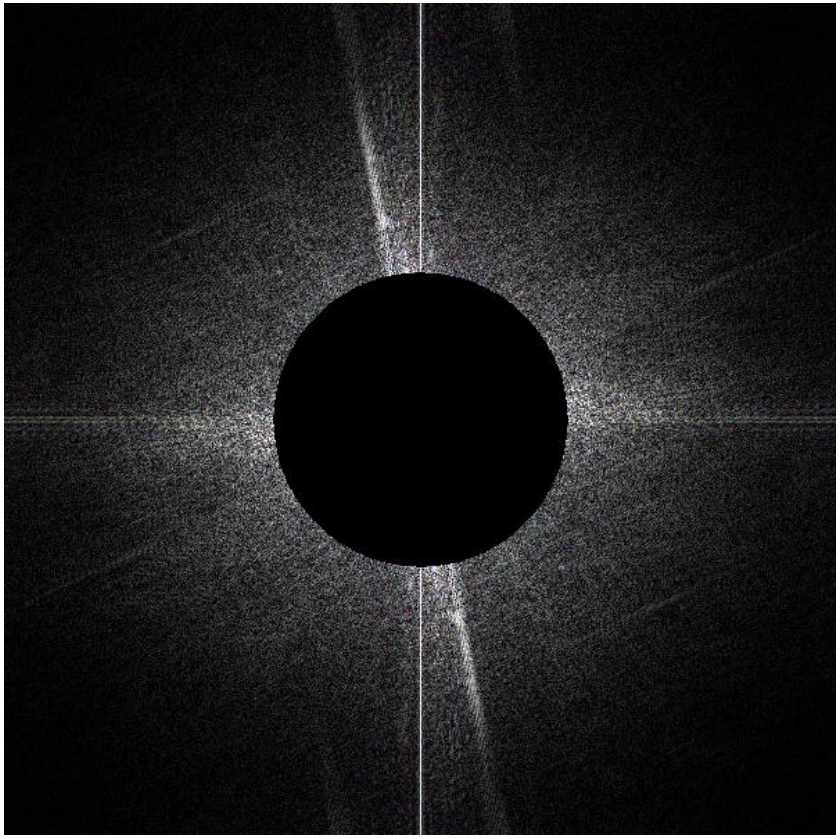




2D Fourier transform

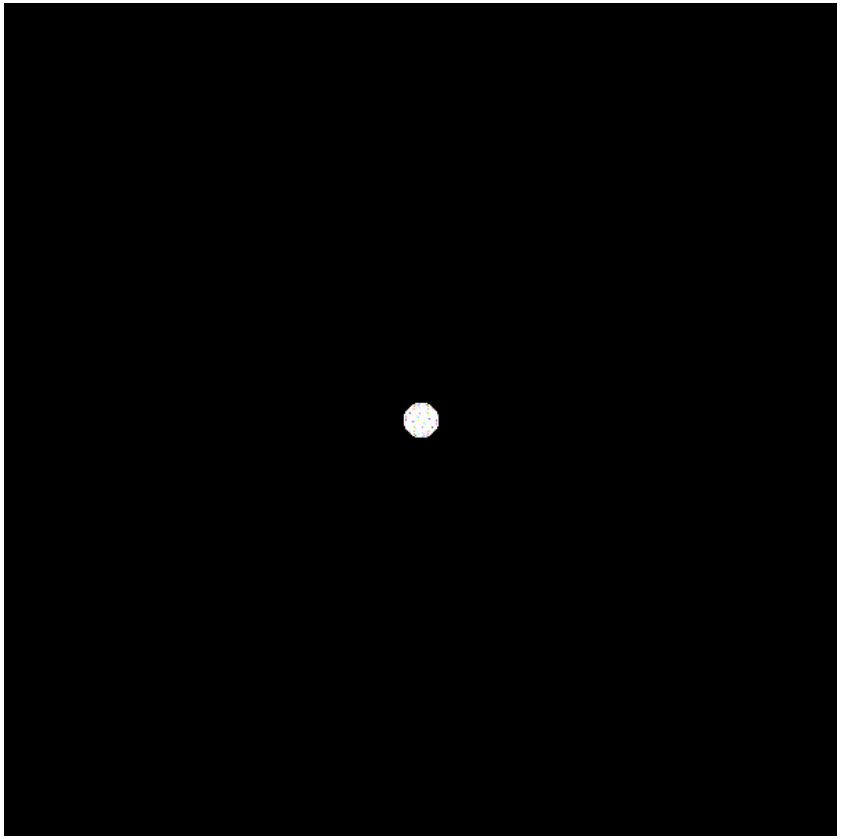
Frequency Domain Filtering





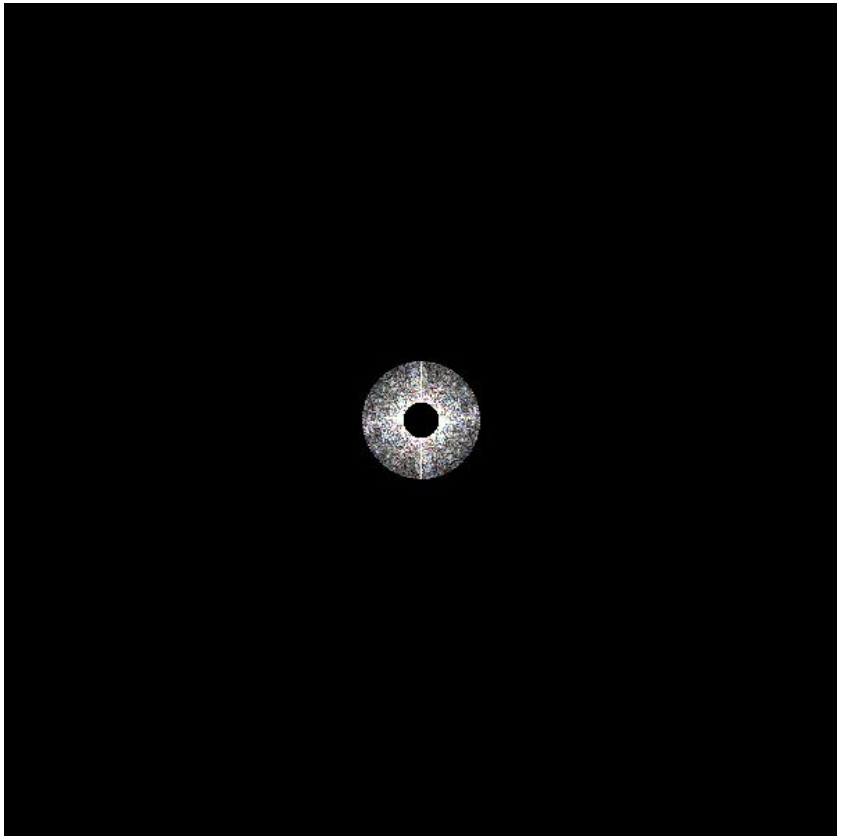
High pass filter

Frequency Domain Filtering



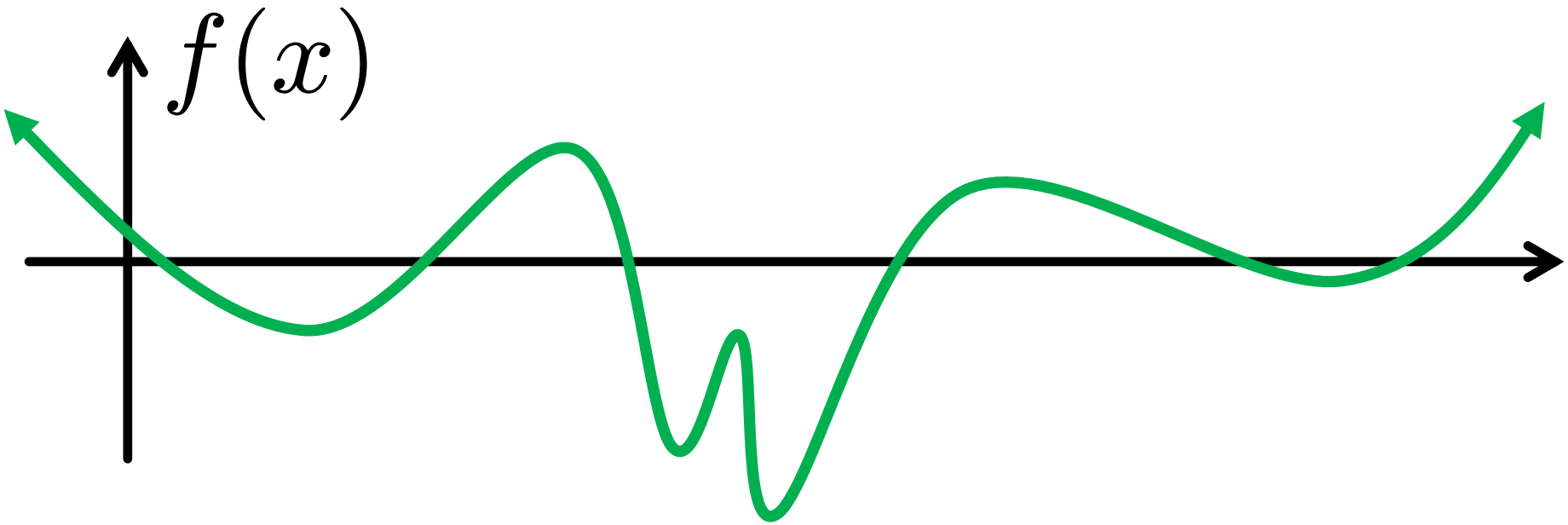
Low pass filter

Frequency Domain Filtering



Band pass filter

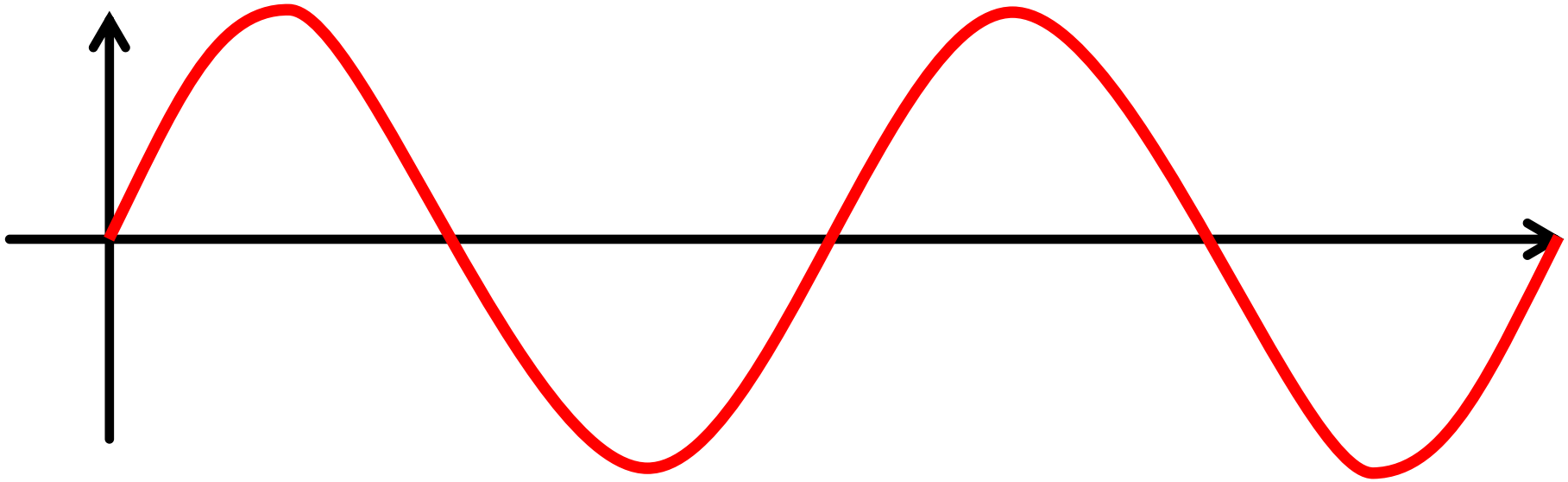
Evaluating Fourier Transform



Simplest possible thing

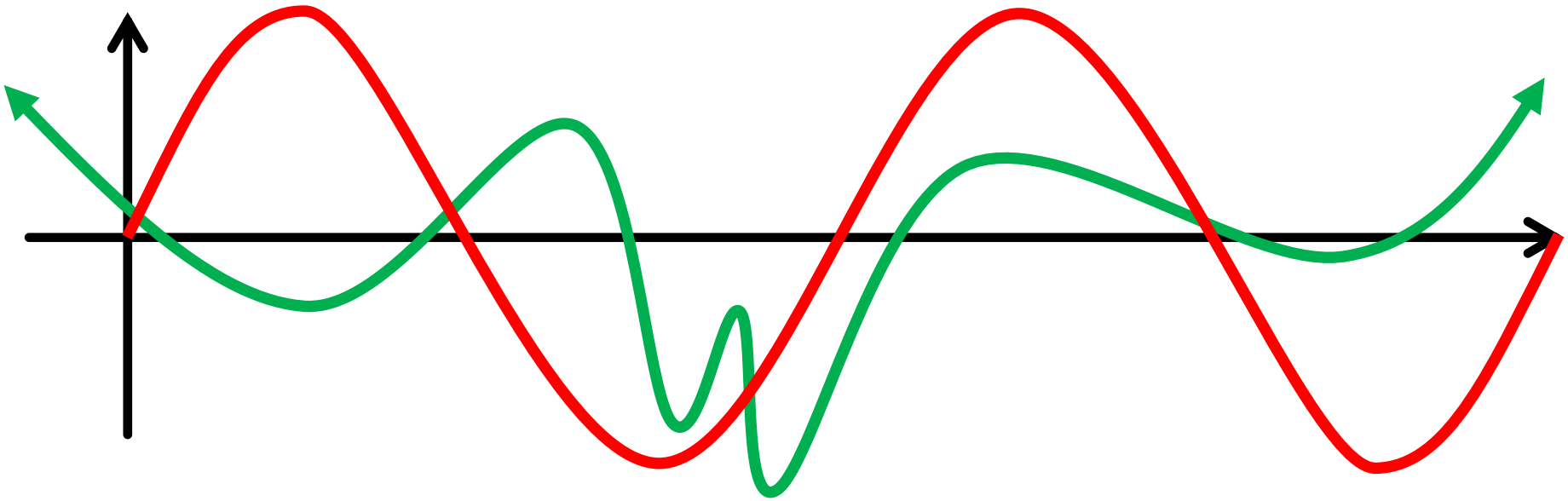
Evaluating Fourier Transform

$$\sin(2\pi\xi x)$$



Simplest possible thing

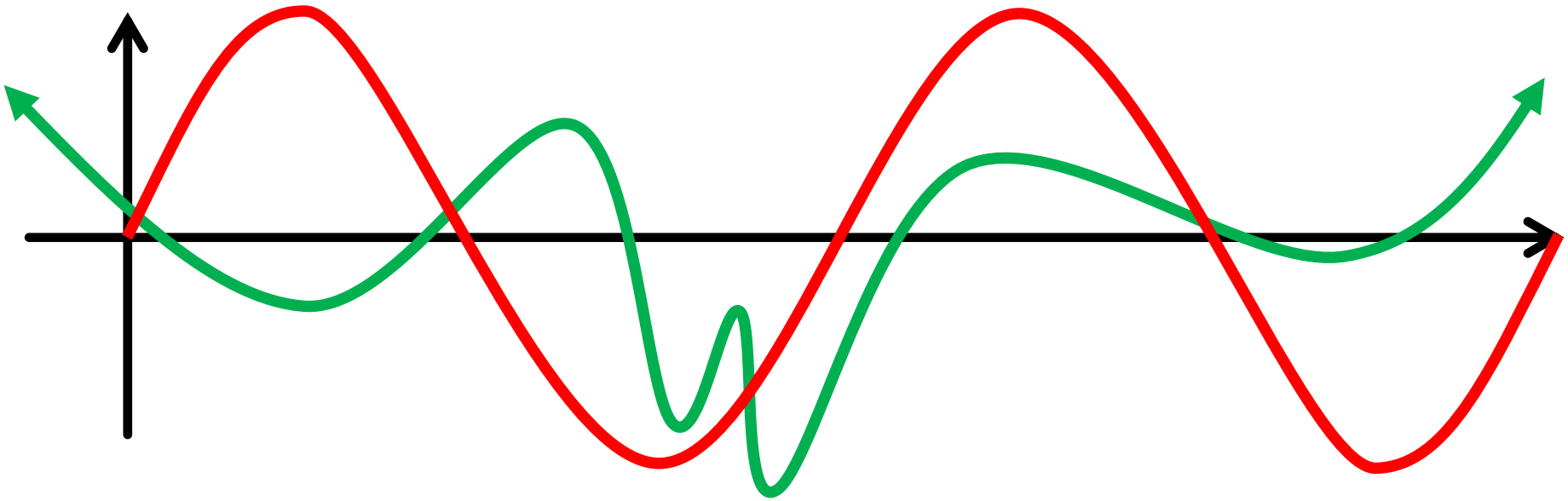
Evaluating Fourier Transform



Multiply and integrate.

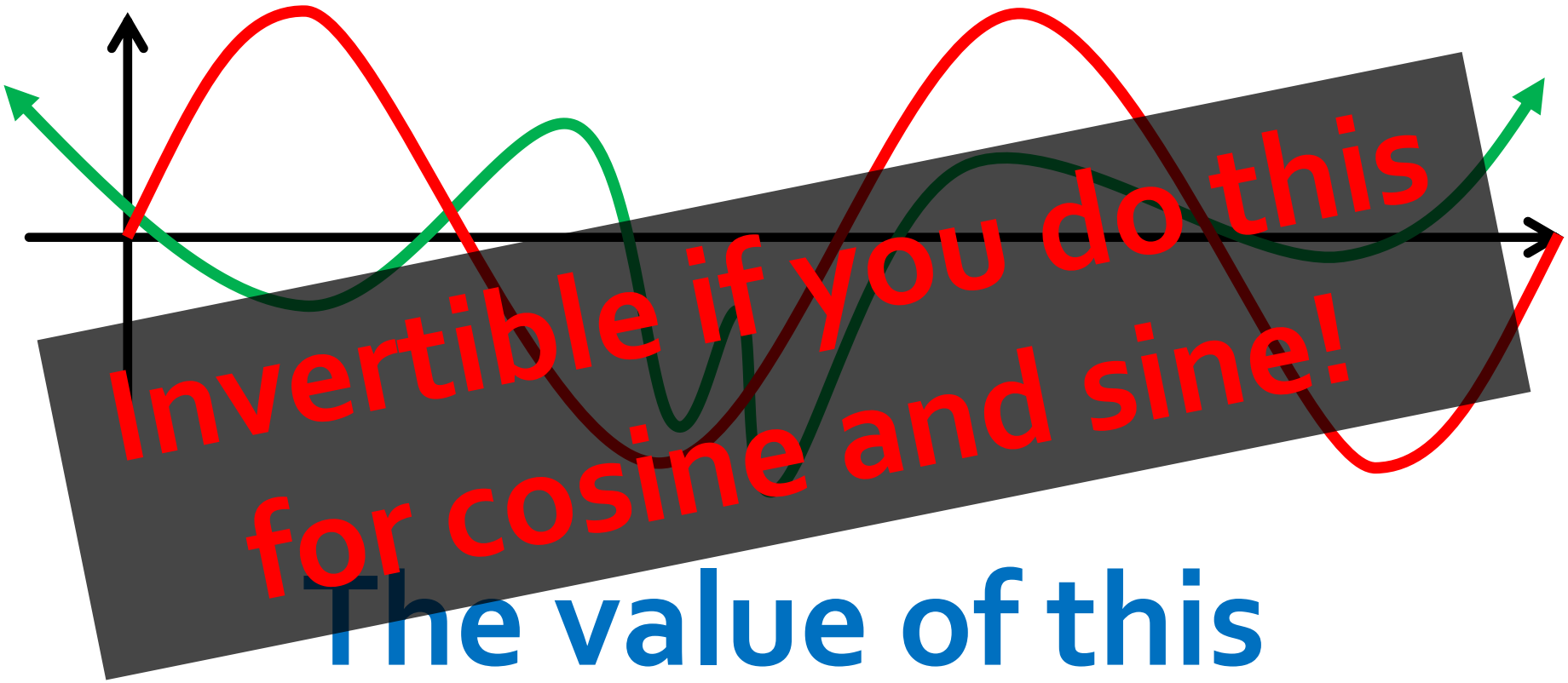
Simplest possible thing

Definition of Fourier Transform



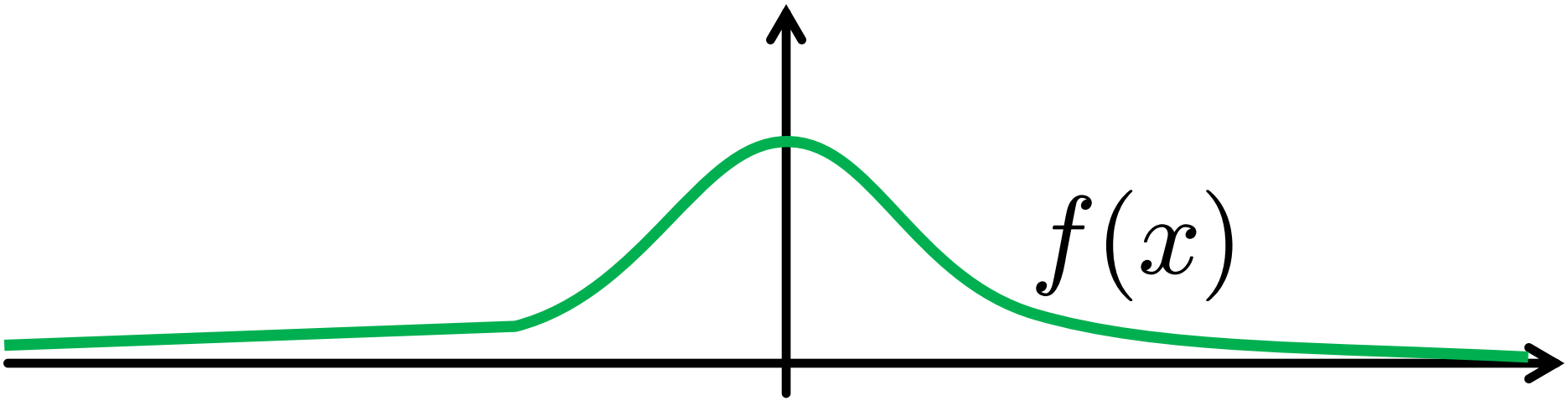
The value of this integral for all ξ.

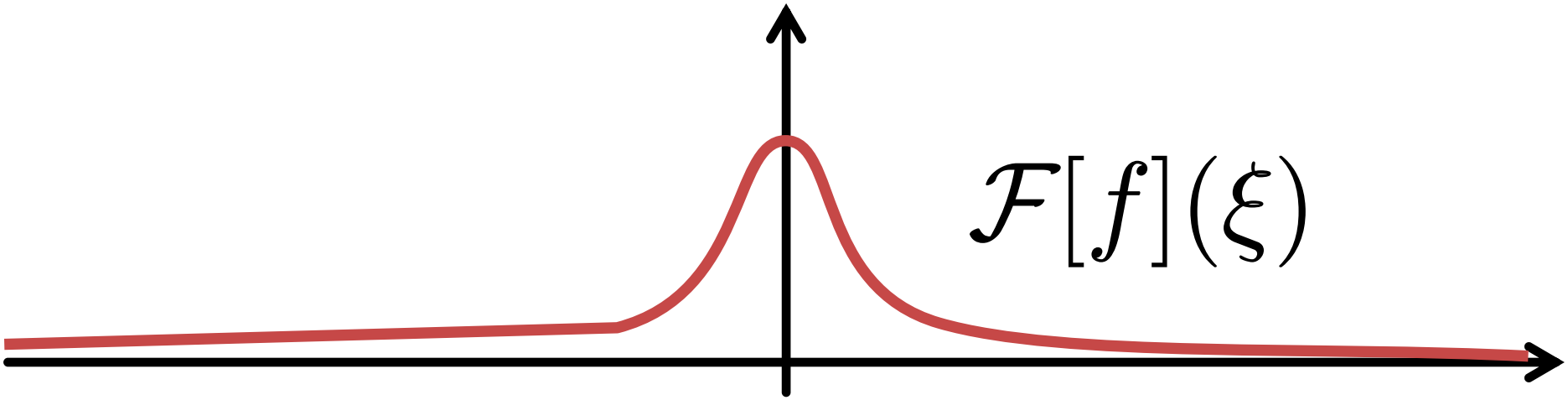
Definition of Fourier Transform



The value of this integral for all ξ.

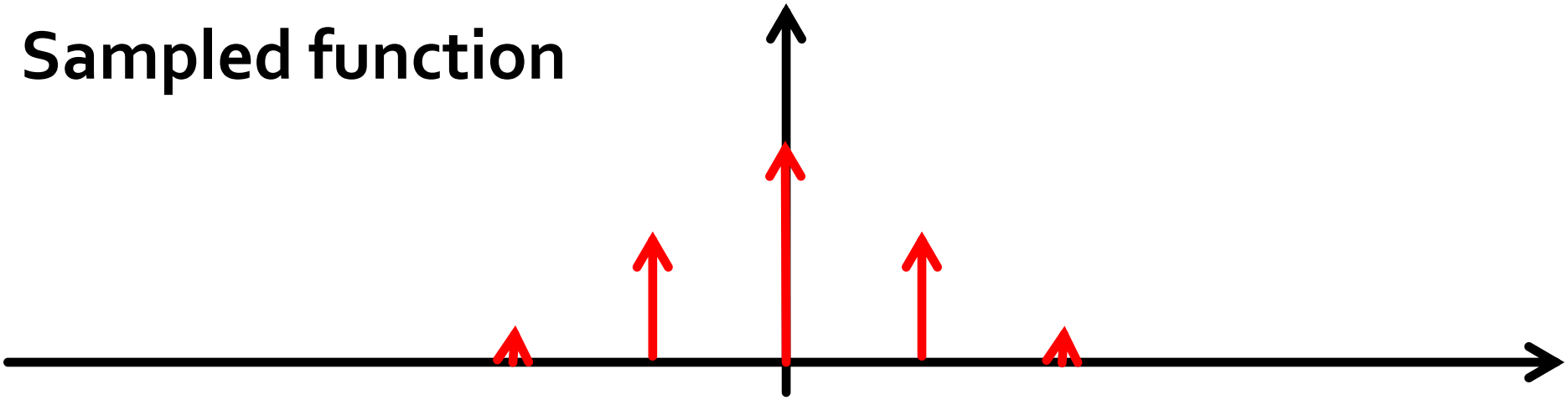
An Unfortunate Phenomenon



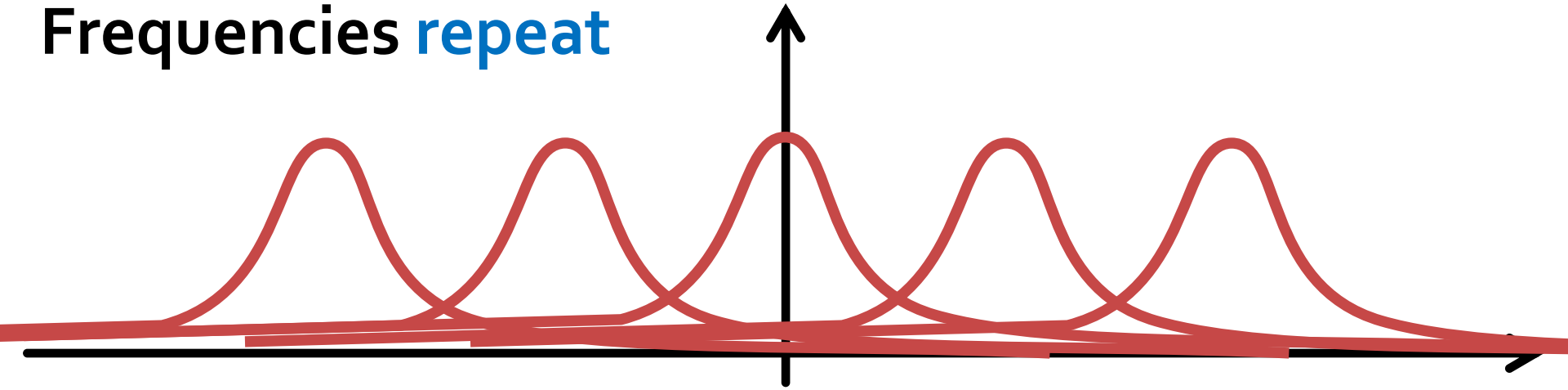


An Unfortunate Phenomenon

Sampled function

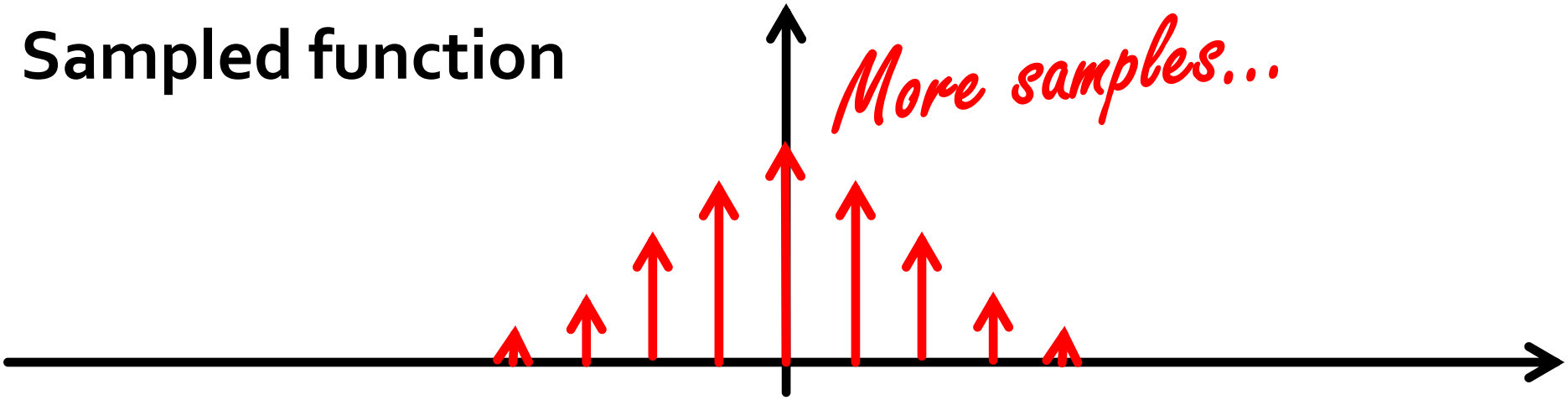


Frequencies repeat

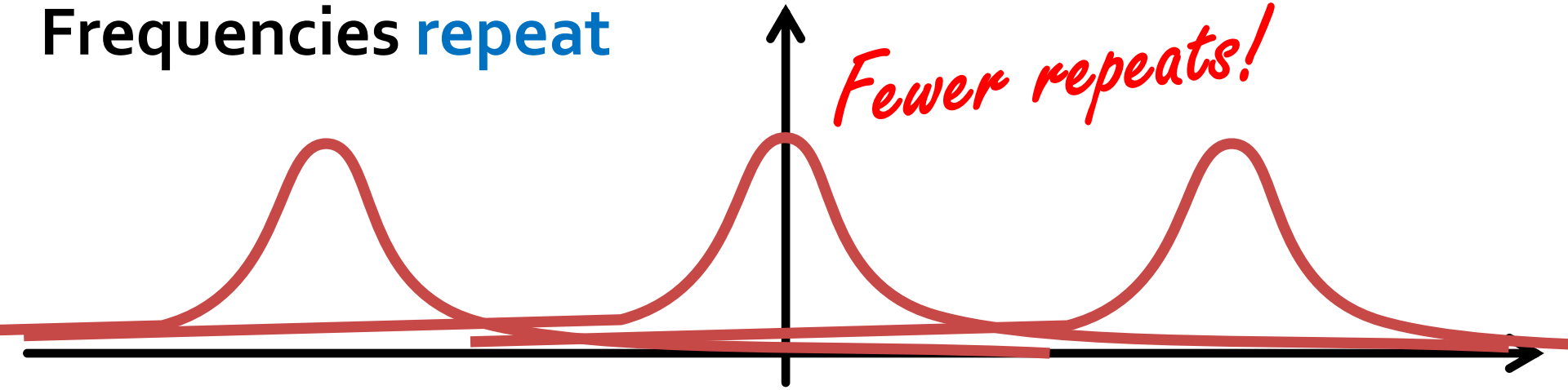


An Unfortunate Phenomenon

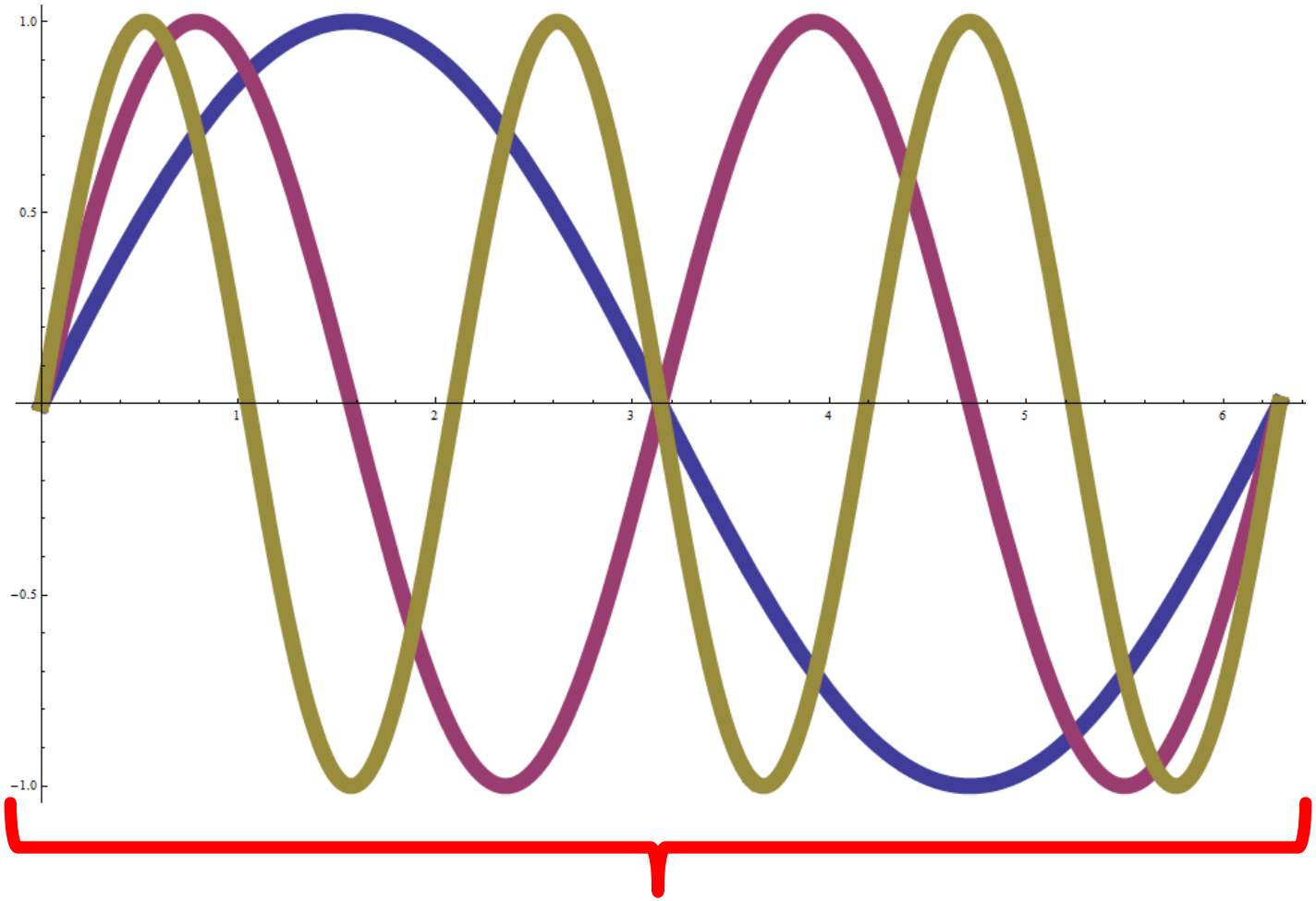
Sampled function



Frequencies repeat

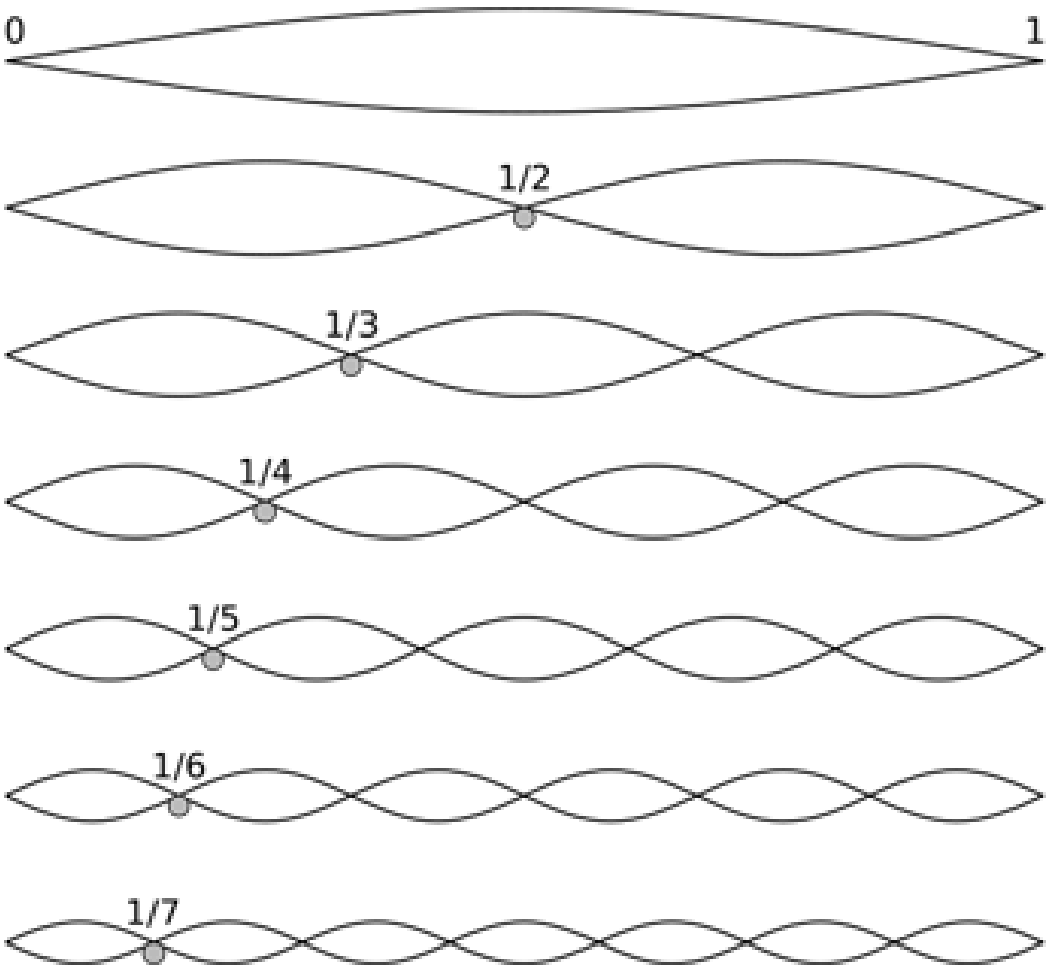


What's Going On?



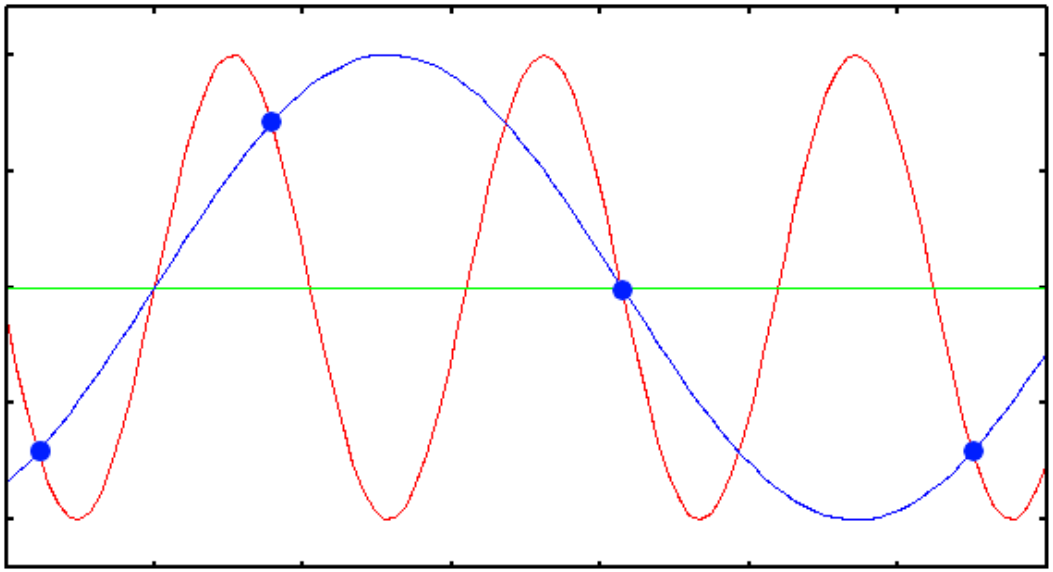
One interval fits multiple frequencies!

String Harmonics



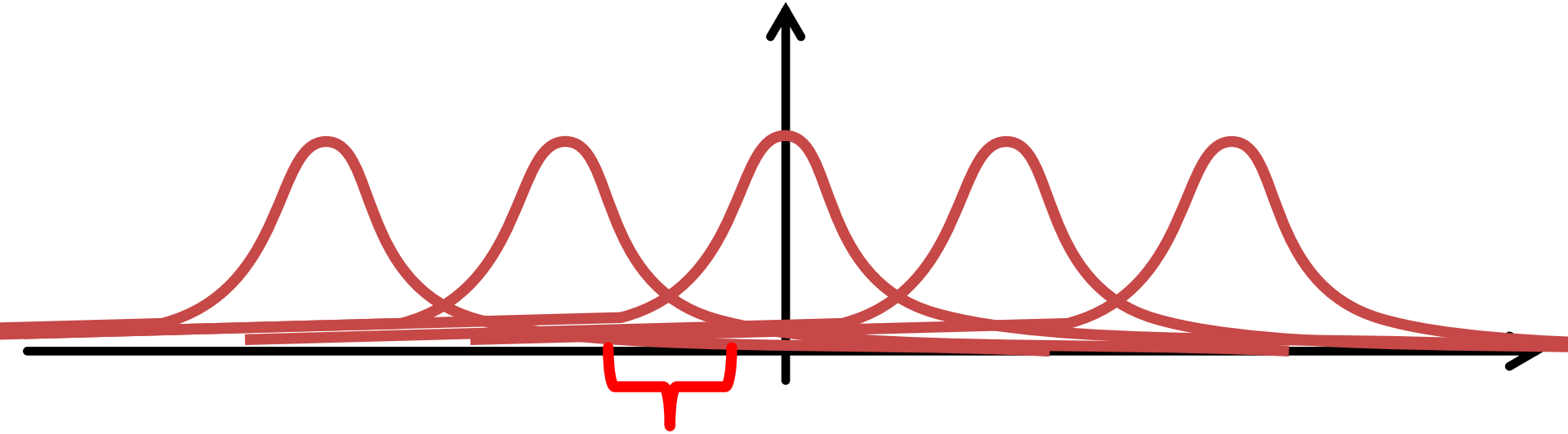
http://www.physicscentral.com/explore/action/images/string.gif

Why is this a problem?



Can "confuse" frequencies

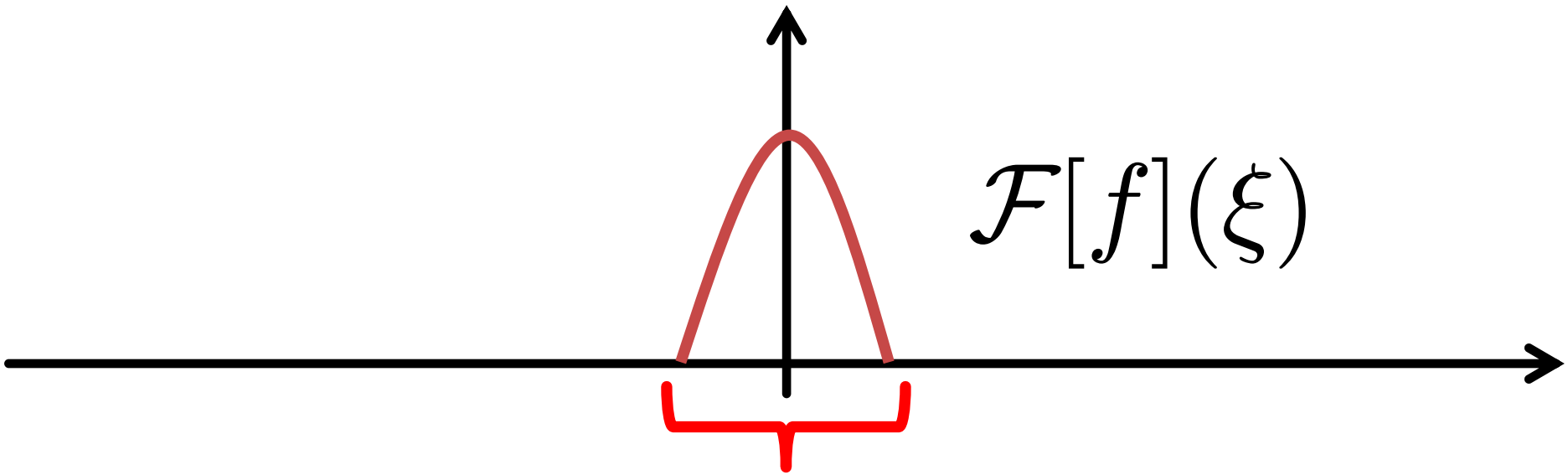
Why is this a problem?



Irrecoverable mixing!

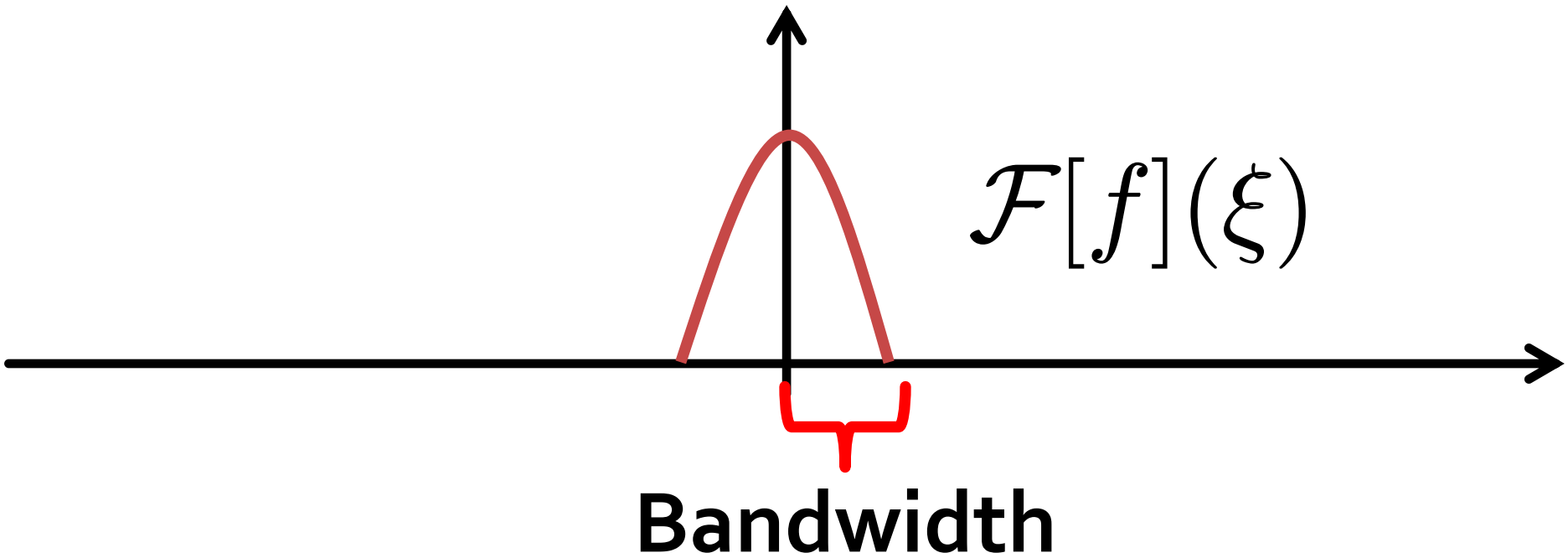
Can "confuse" frequencies

When Isn't This a Problem?

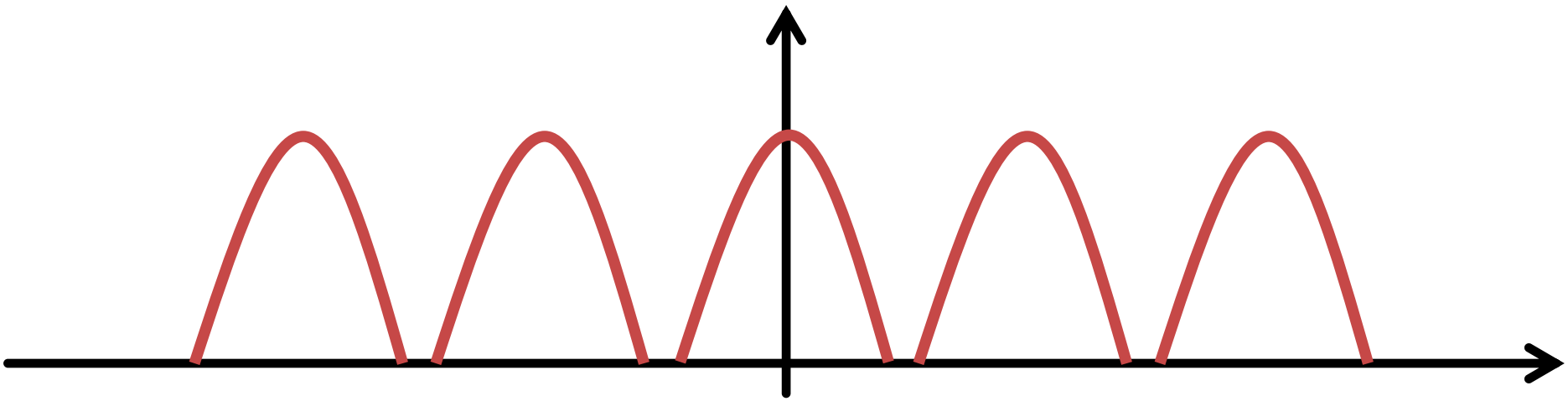


Everywhere else zero

When Isn't This a Problem?



When Isn't This a Problem?



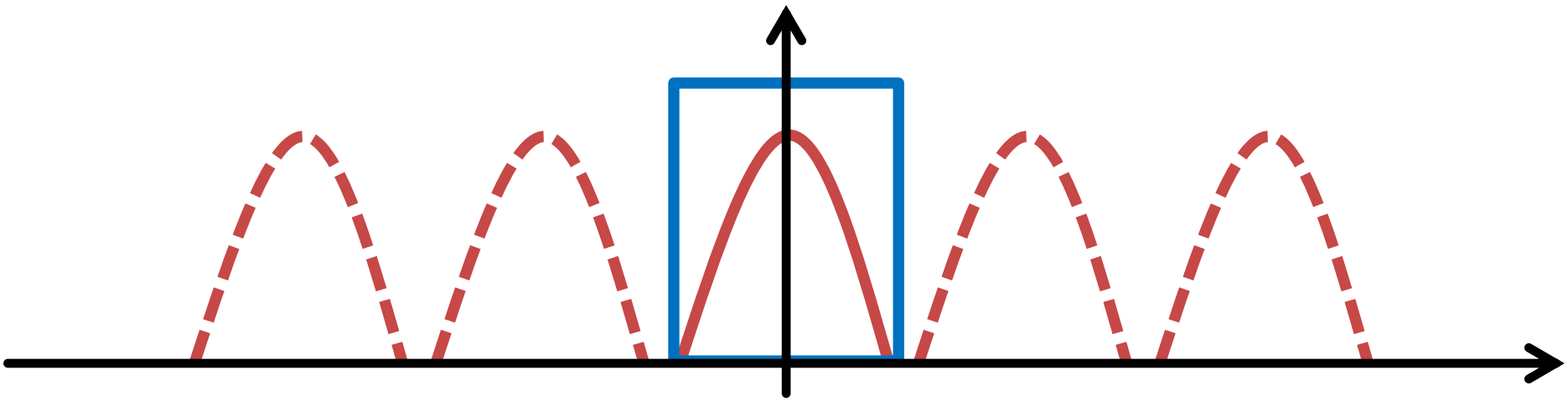
No interference

Nyquist rate

[**nahy**-kwist reyt]:

The lowest alias-free sample rate; two times the bandwidth of a band-limited signal.

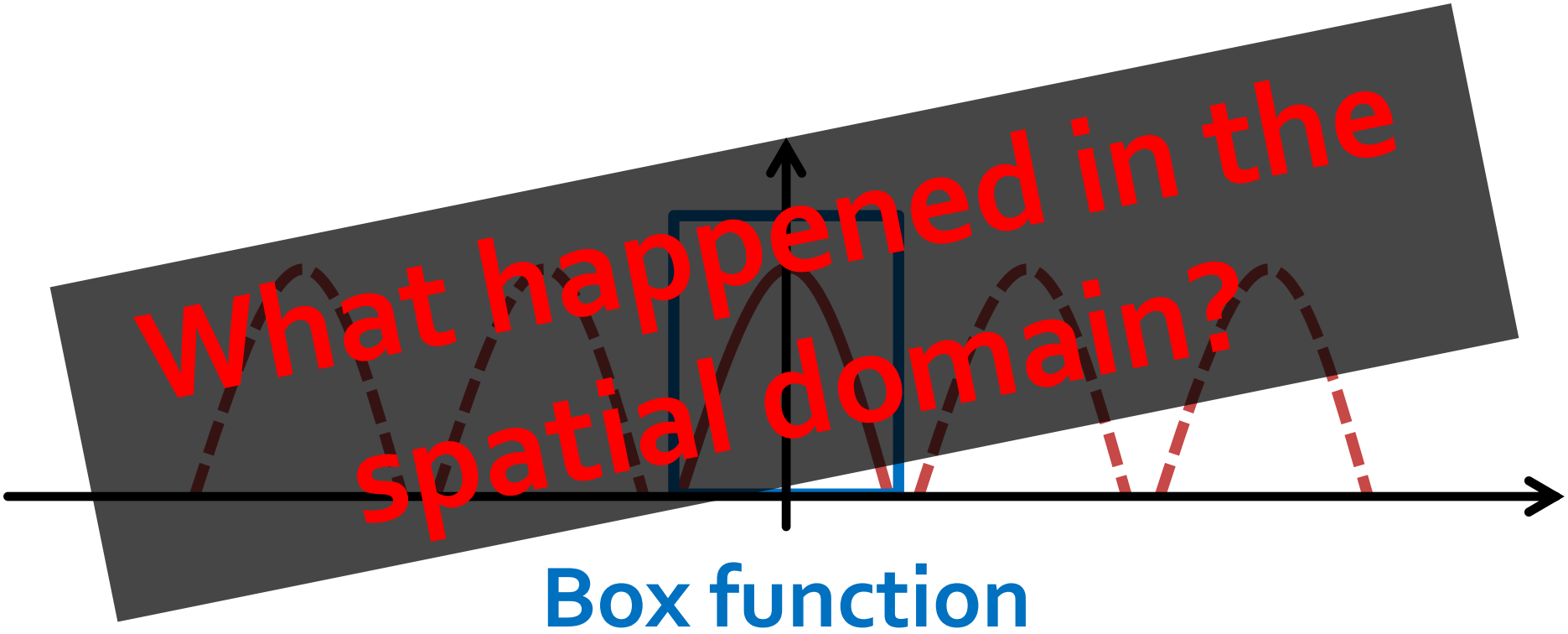
Perfect Reconstruction



Box function

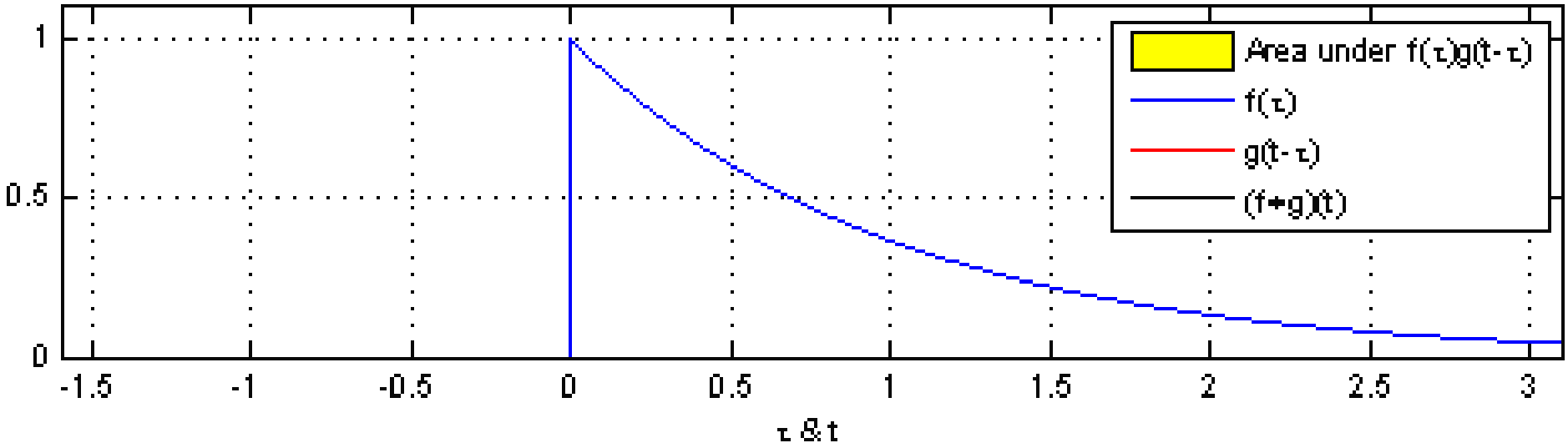
Multiply in frequency space

Perfect Reconstruction



Multiply in frequency space

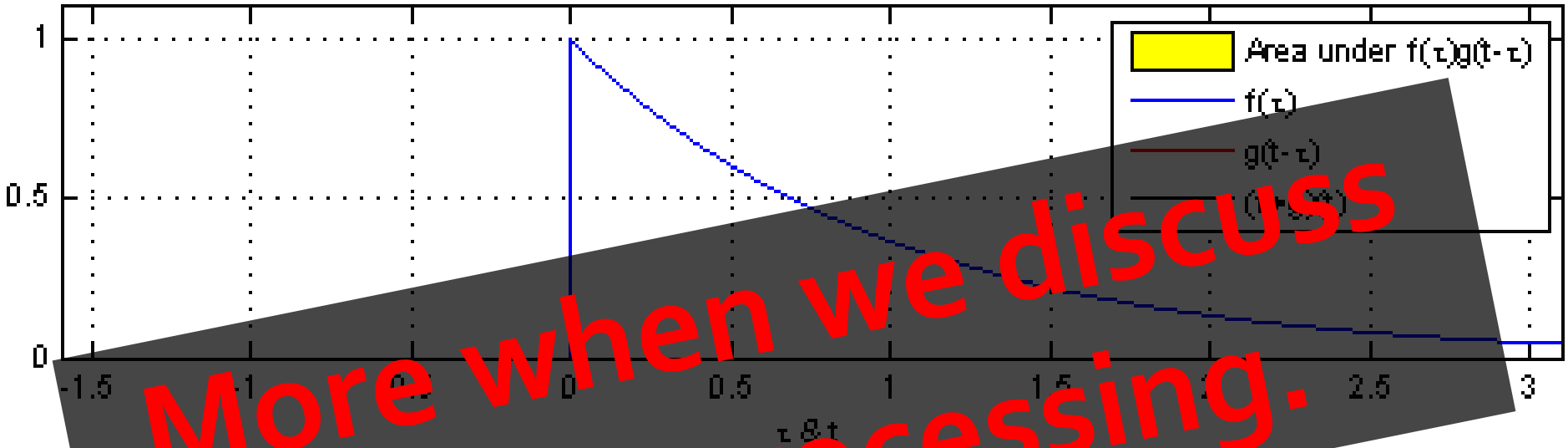
Convolution



http://upload.wikimedia.org/wikipedia/commons/b/b9/Convolution_of_spiky_function_with_box2.gif

Replace function with weighted average

Convolution



http://upload.wikimedia.org/wikipedia/commons/b/b9/Convolution_of_spiky_function_with_box2.gif

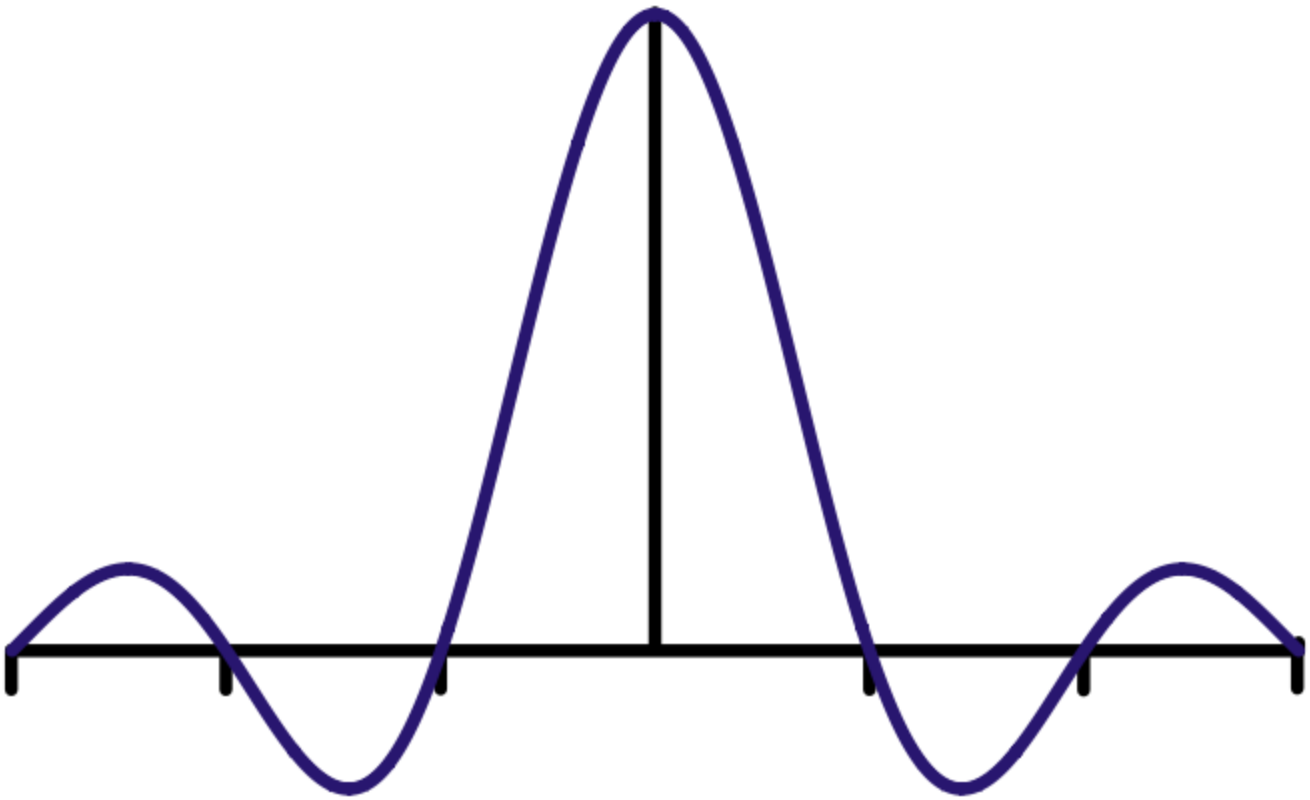
More when we discuss image processing.

Replace function with weighted average

Convolution Theorem

Multiplication in frequency domain is convolution in spatial domain.

Sinc

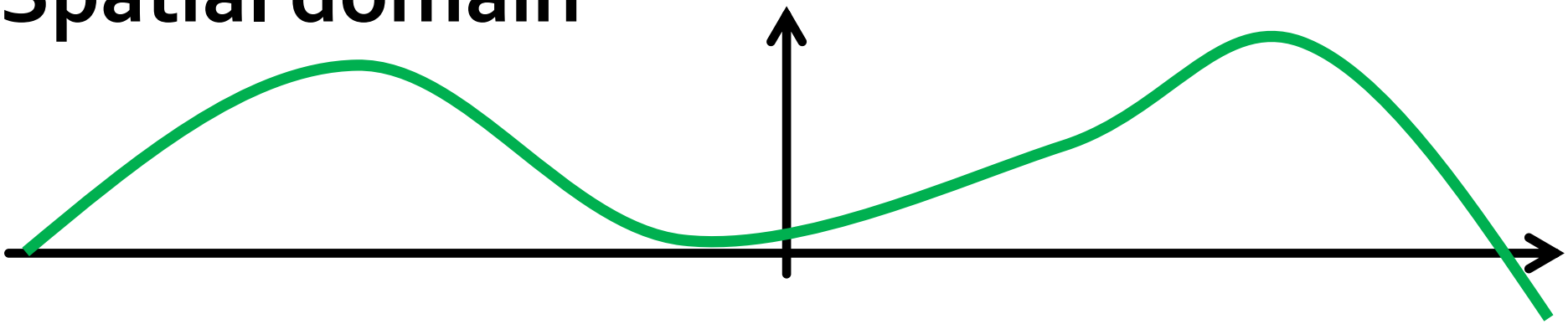


https://upload.wikimedia.org/wikipedia/commons/thumb/2/2b/Sinc_Filter.svg/700px-Sinc_Filter.svg.png

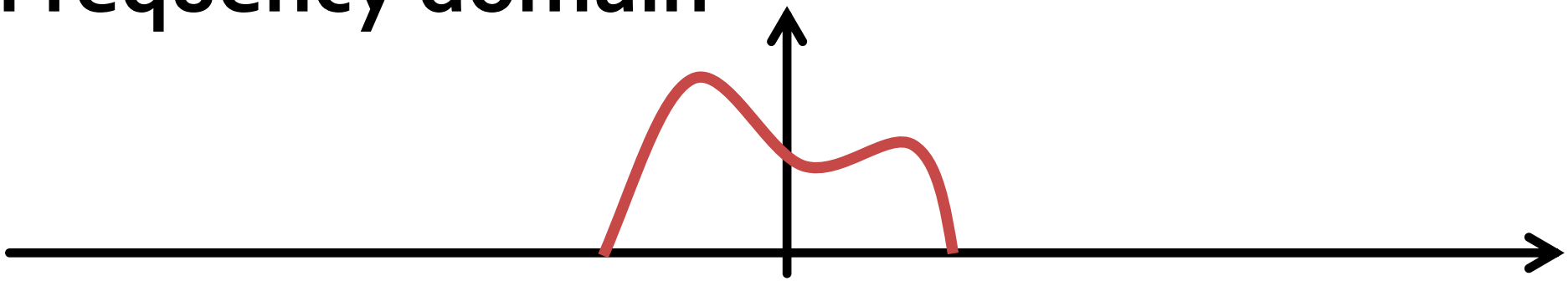
Fourier transform of box

A Perfect Story

Spatial domain



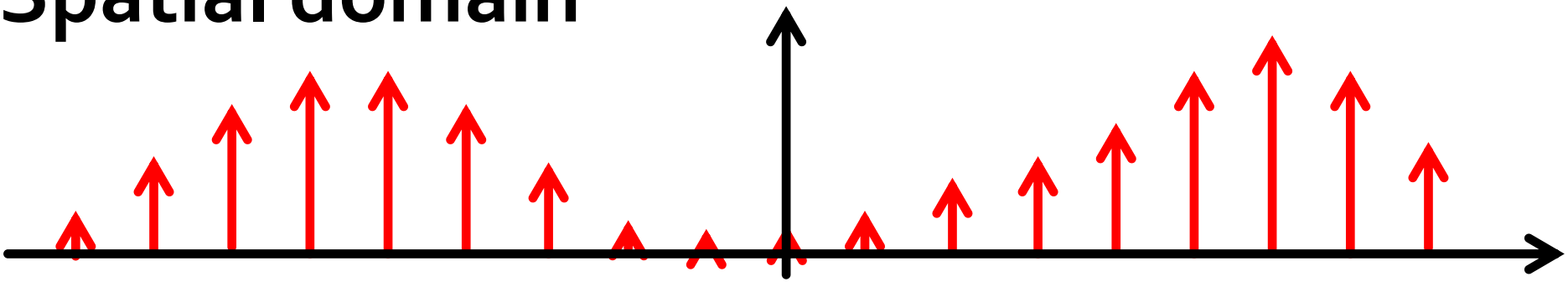
Frequency domain



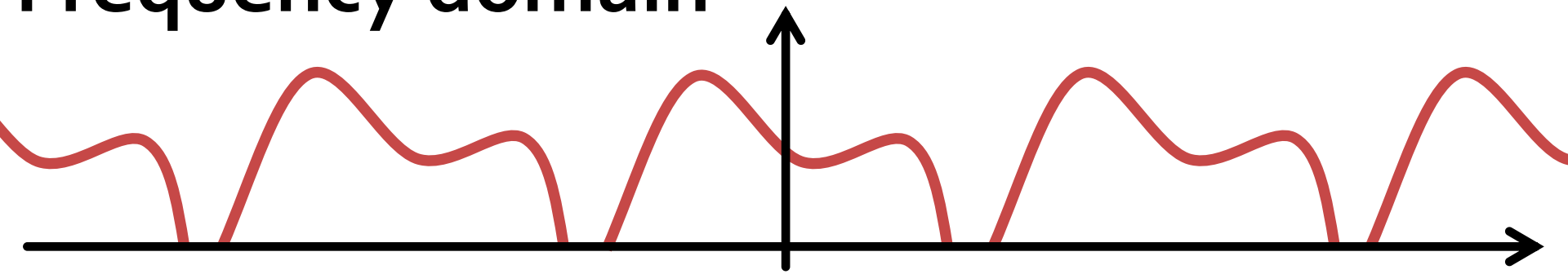
Input function

A Perfect Story

Spatial domain



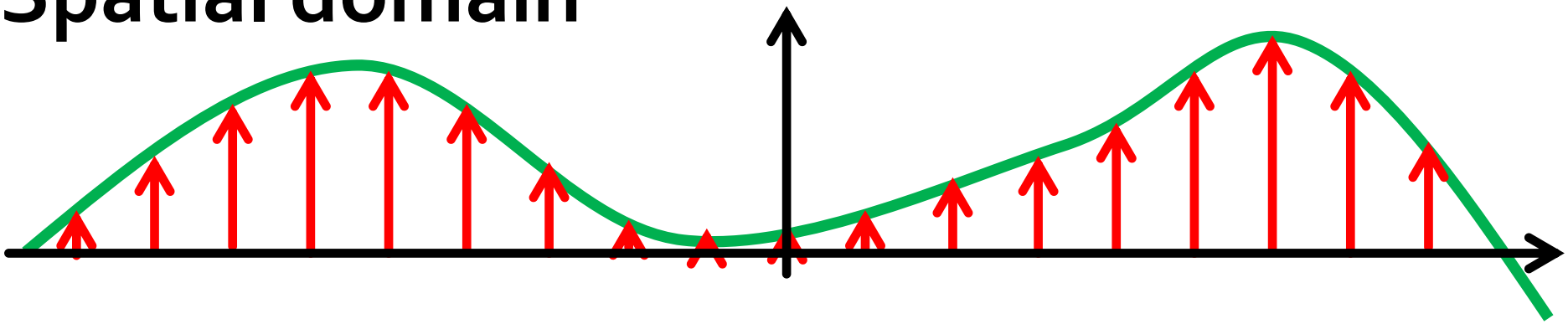
Frequency domain



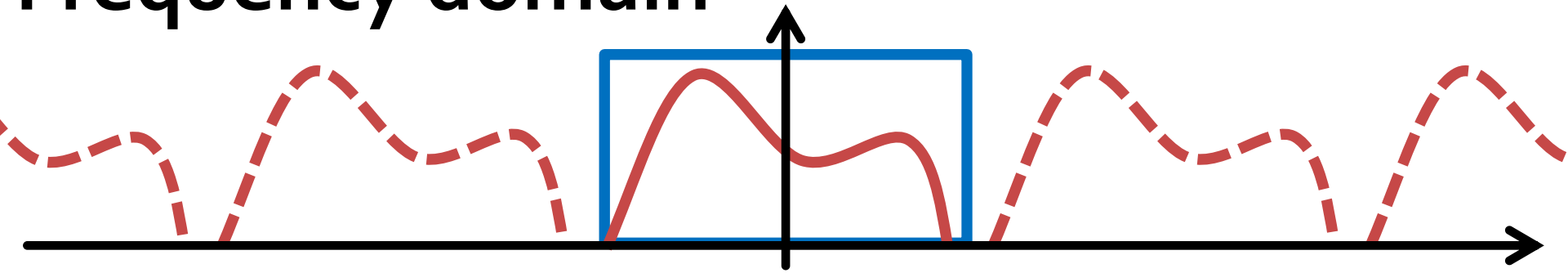
1. Sample the function

A Perfect Story

Spatial domain

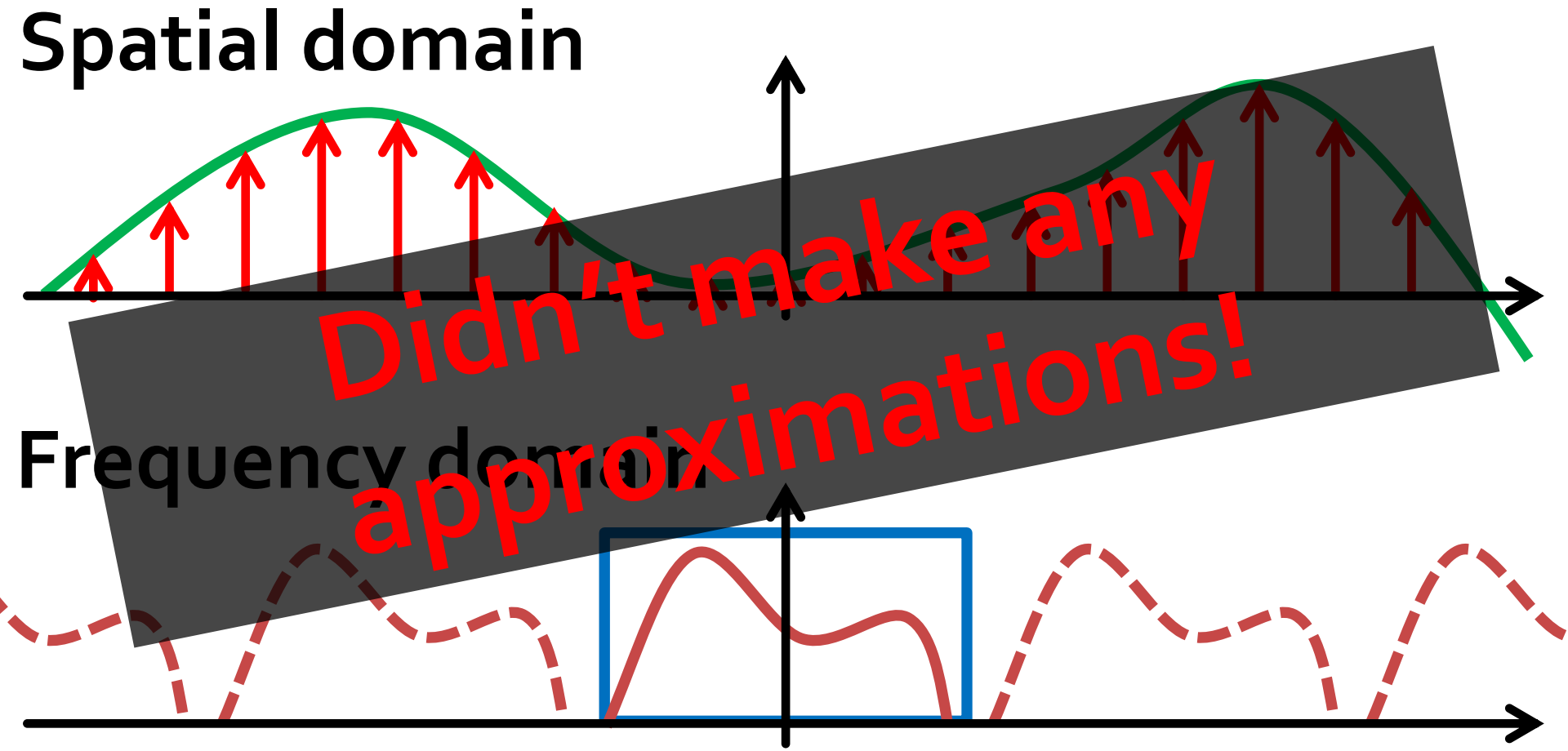


Frequency domain



2. Reconstruct using sinc

A Perfect Story



2. Reconstruct using sinc

Back to Reality

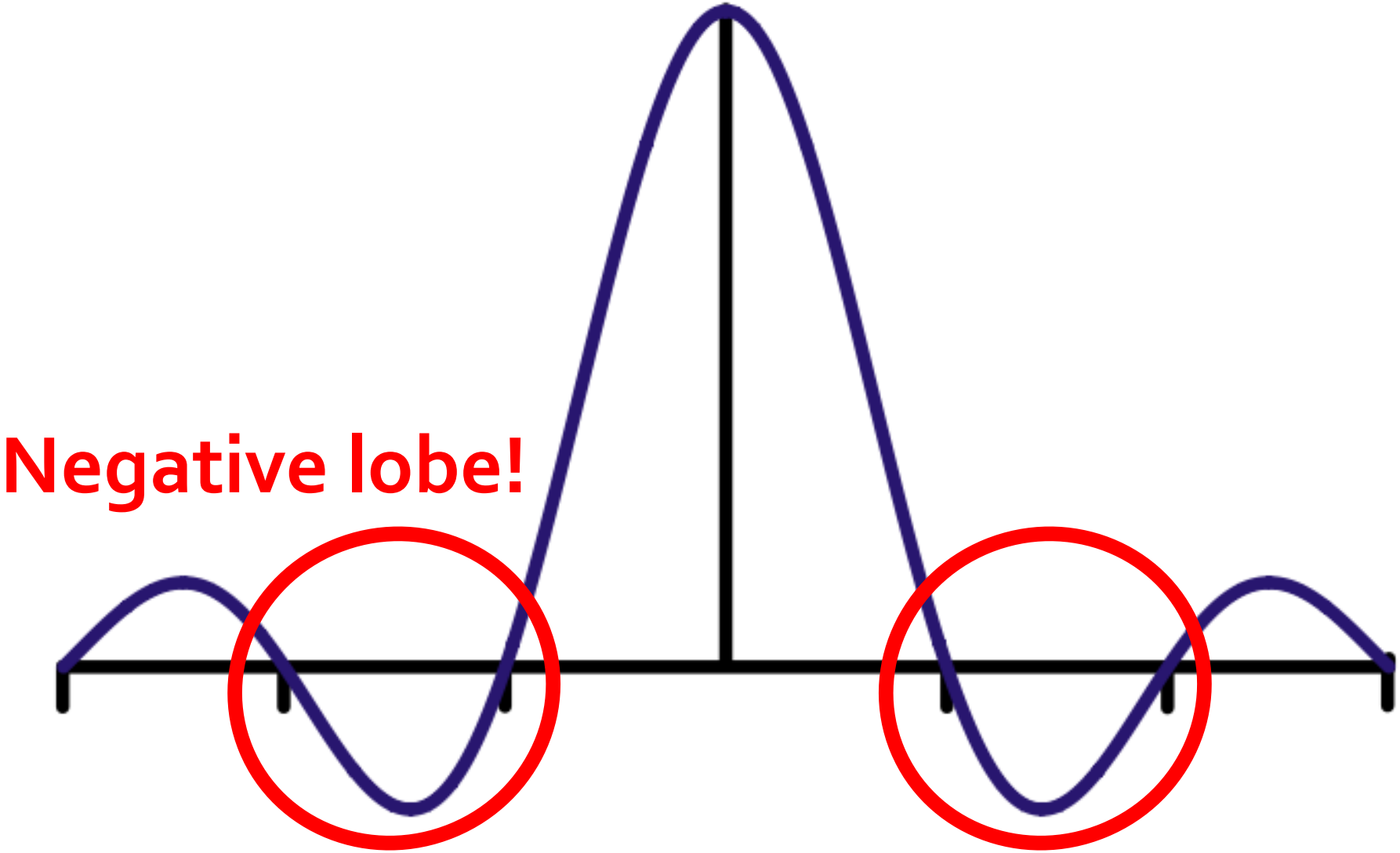
Practical signals cannot have finite bandwidth.

Back to Reality

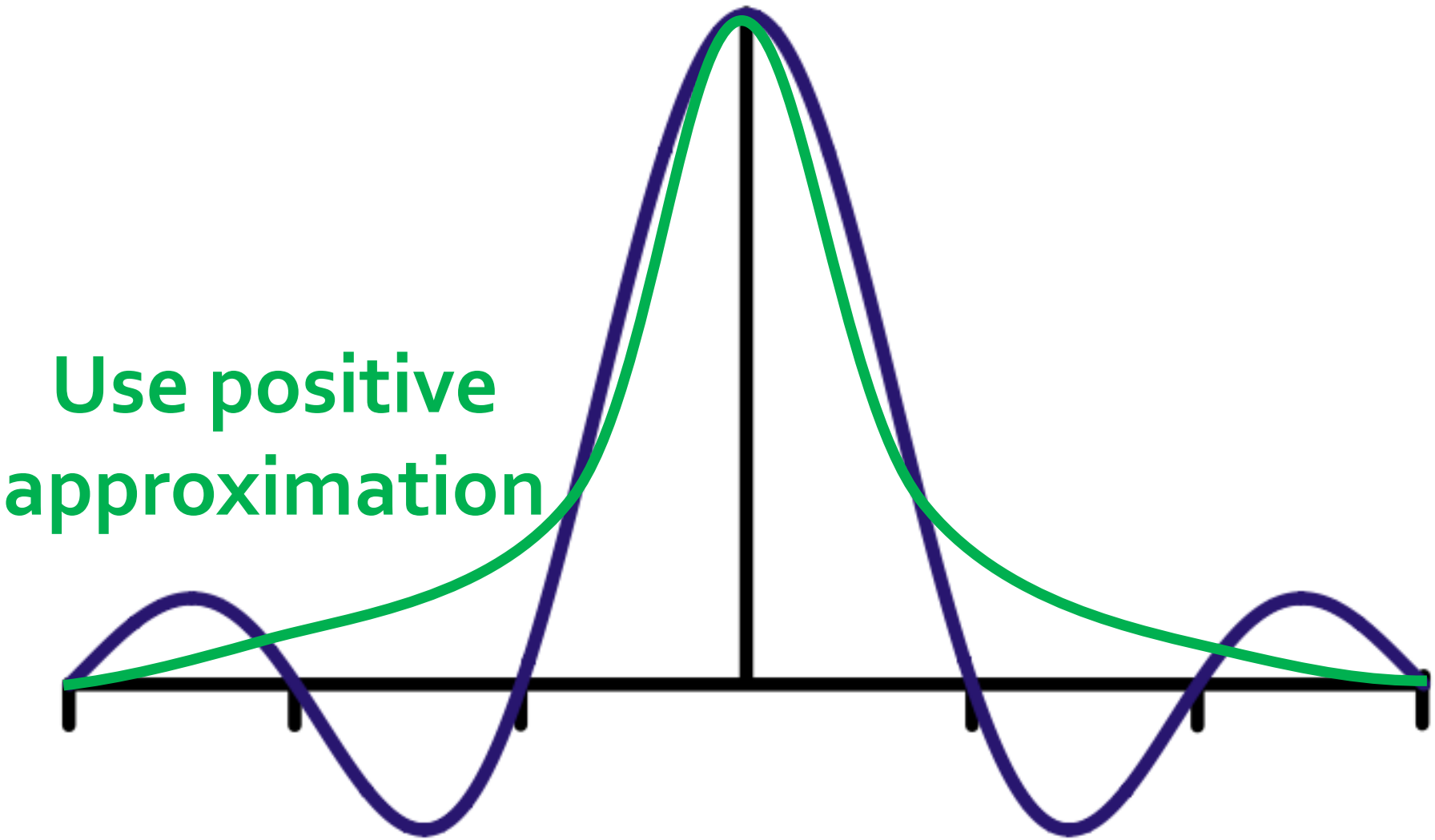
Practical signals cannot have finite bandwidth.

"Heisenberg Uncertainty Principle"

Back to Reality



Back to Reality



Back to Reality

Sharp edges need special treatment!

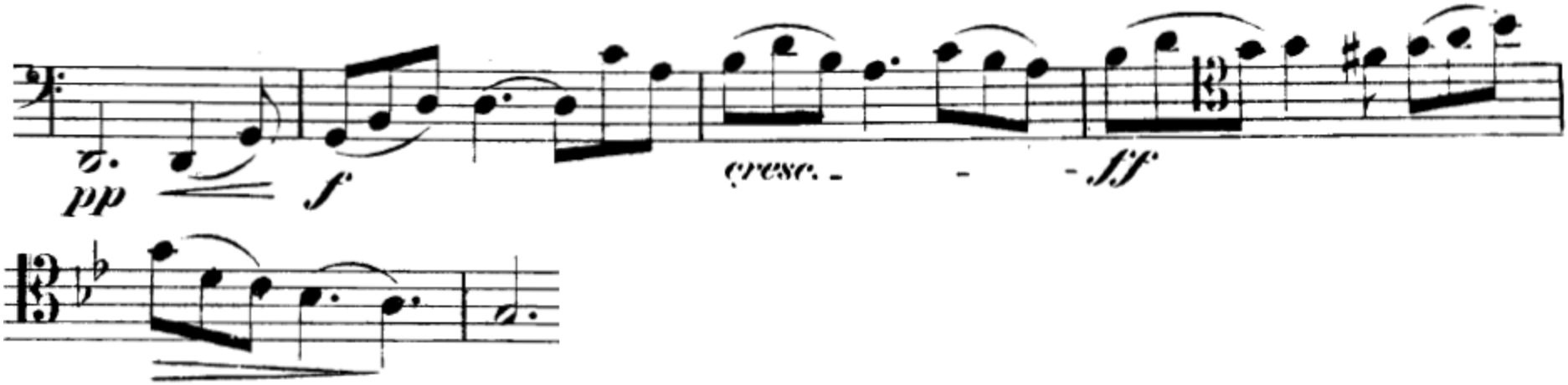
http://vcg.isti.cnr.it/~corsini/publications/thumbs/tn_fast-bilateral-filter.png

Frequency paradigm isn't perfect

Practical Conclusions

Sample above the Nyquist rate.

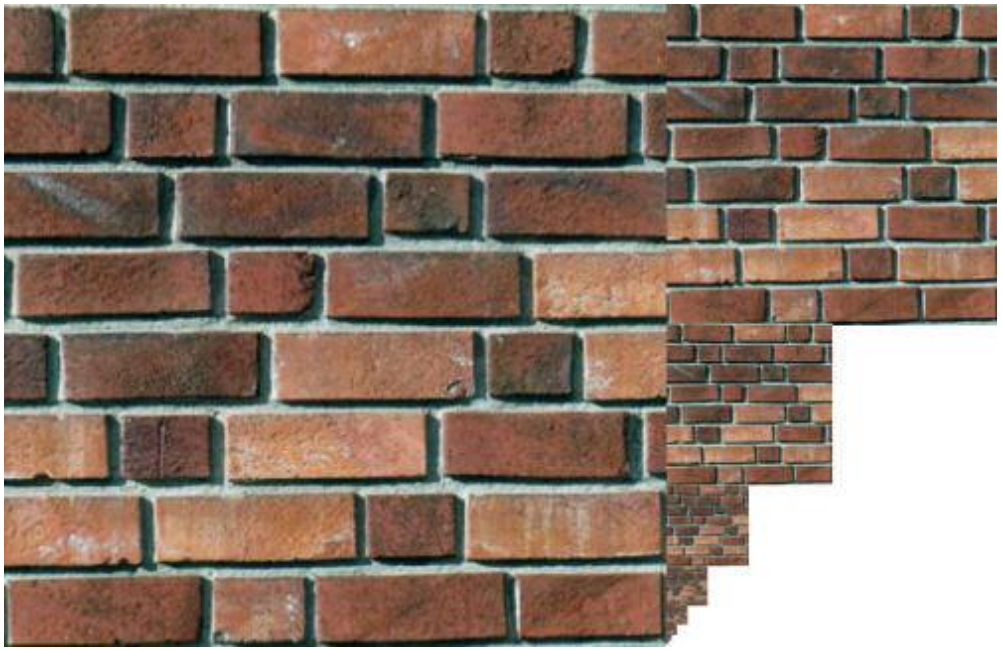
Sampling Below the Nyquist Rate




```
y=wavrecord(750*20,750);
wavplay(y*300,750)
```

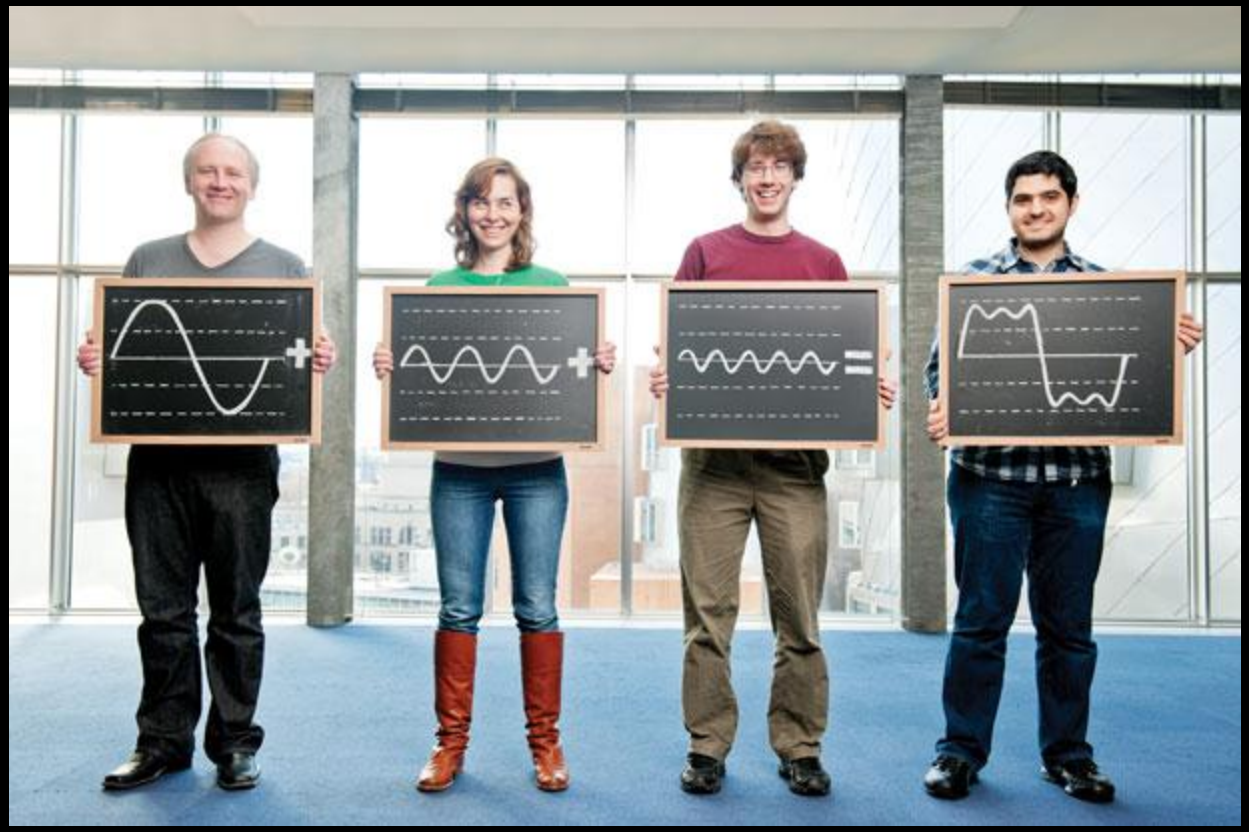
Cello Concerto in D Minor, Edouard Lalo

Practical Conclusions



Reconstruct using filters that look like sinc.

http://www.technologyreview.com/article/427676/a-faster-fourier-transform/



Sampling and Fourier Theory

CS 148, Summer 2012
Introduction to Computer Graphics and Imaging

Justin Solomon