Homework #3: Selection and order statistics, heaps/heapsort
Due Date: Tuesday, 30 April 2002

Reading: Chapters 7, 10 in CLR, 6, 9 in CLRS.

Recall that *exercises* are for you to work out on your own; *problems* are to be handed in.

Exercise 3-1. Do Exercise 10.1–2 on page 187 of CLR, 9.1–2 on page 185 of CLRS.

Exercise 3-2. Do Exercise 10.3–8 on page 192 of CLR, 9.3–8 on page 198 of CLRS.

Exercise 3-3. Do Exercise 7.2–5 on page 144 of CLR, 6.2–6 on page 132 of CLRS.

Exercise 3-4. Do Exercise 7.3–3 on page 147 of CLR, 6.3–3 on page 135 of CLRS.

Exercise 3-5. Do Exercise 7.4–2 on page 149 of CLR, 6.4–3 on page 136 of CLRS.

Problem 1. Weighted statistics [50 points].

For n distinct elements $x_1, x_2, \ldots, x_n$ with positive weights $w_1, w_2, \ldots, w_n$ such that $\sum_{i=1}^{n} w_i = W$, the **weighted 3-median** is the element x_k satisfying

$$\sum_{x_i<x_k} w_i \leq \frac{W}{3}$$

and

$$\sum_{x_i>x_k} w_i \leq \frac{2W}{3} \ .$$

(a) Show how to compute the weighted 3-median of n elements in $O(n \lg n)$ worst-case time using sorting.

(b) Show how to compute the weighted 3-median in $\Theta(n)$ worst-case time using a linear-time median algorithm such as SELECT from the text.

Problem 2. Do Exercise 7.5–5 on page 151 of CLR, 6.5–7 on page 142 of CLRS [10 points].

Problem 3. Do Problem 7–1 on page 152 of CLR, 6–1 on page 142 of CLRS [20 points].