Homework #1: Algorithm analysis, asymptotic notation
Due Date: Wednesday, 20 January 2016

Homework policies

CS161 is technical course involving algorithms design as well as proofs and analysis about algorithms, so doing the homework is the only way to acquire a working knowledge of the material presented. We encourage you strongly to start working on the homework problems right away—the problems below, as well as those to follow, have from modest to considerable technical depth and you are unlikely to be able to solve them if you wait until the evening before the due date.

Collaboration in solving the problems is encouraged in this class—you have a lot to learn from your fellow students. However, in order to make grading the homeworks a meaningful way to measure your effort and your understanding of the material, we must put some restrictions:

- *On theoretical (mathematical) problems, you may work together in groups of up to three students on finding solutions, but each of you must then write up your favorite solutions independently. Please list the names of your collaborators on your homework.*
- *On the programming project, groups of up to three students can work together as a team, handing in a single body of code and documentation for their joint effort.*

It is important that the homeworks be turned in on time. Paper-and-pencil homeworks must be submitted by the day they are due, in electronic form, by 5:00 pm PDT. The programming project and write up must submitted by running the provided submission script, again by 5:00 pm PDT the day they are due. Each student is allowed two 24-hour grace periods during the quarter. That means a single homework can be handed in late by two days, or two homeworks may be handed in late by one day each. Other than these grace periods, late homeworks will not be accepted.

For the complete set of homework policies in CS161 please consult the appropriate section of the class web page — especially regarding the Honor Code.

Unless otherwise stated, proofs will be required for all claims made on your homework assignments.

Note that references to CLRS Exercises or Problems are always to the Third Edition of the book.

Problem 1-1. Suppose we are comparing implementations of insertion sort and merge sort on the same machine. For inputs of size n, insertion sort runs in $8n^2$ steps, while merge sort runs in $64n\lg n$ steps (recall that lg is the log base 2 function). For which values of n does insertion sort beat merge sort? [10 points]

Problem 1-2. Do Problem 2–1 on page 39 of CLRS [insertion sort on small arrays in merge sort]. [20 points — each of the parts (a), (b), (c), (d) is worth 5 points]

Problem 1-3. Let $f(n)$ and $g(n)$ be asymptotically nonnegative functions. Using the basic definition of the Θ notation, prove that $\max(f(n), g(n)) = \Theta(f(n) + g(n))$. [10 points]

Problem 1-4. Asymptotic relations among common functions [40 points]

Rank the following 25 functions by order of growth, *i.e.*, find an ordering $g_1, g_2, \ldots, g_{25}$ of the functions such that each consecutive pair satisfies either $g_i = \Omega(g_{i+1})$ or $g_i = \Theta(g_{i+1})$ for $i = 1, 2, \ldots, 24$. Note that multiple functions may form equivalence groups under the Θ relation – so that within an equivalence group any ordering is fine. Justify the relationship between each pair. *Hint:* Read Section 3.2 from the text.

$(3/2)^n$	$(\sqrt{2})^{\lg n}$	$\lg^* n$	n^2	$(\lg n)!$
n^3	$\lg^2 n$	$\lg(n!)$	2^{2^n}	$n^{1/\lg n}$
$\lg\lg n$	$n \cdot 2^n$	$n^{\lg\lg n}$	$\ln n$	2^n
$2^{\lg n}$	$(\lg n)^{\lg n}$	$4^{\lg n}$	$(n+1)!$	$\sqrt{\lg n}$
$n!$	$2^{\sqrt{2\lg n}}$	n	$n\lg n$	1

The following identities can be helpful (please verify them):

$$\begin{aligned}
(\lg n)^{\lg n} &= n^{\lg\lg n} \\
4^{\lg n} &= n^2 \\
2^{\lg n} &= n \\
2^{\sqrt{2\lg n}} &= n^{\sqrt{2/\lg n}} \\
n^{1/\lg n} &= 2.
\end{aligned}$$

Asymptotic bounds for Stirling's formula, as in Lecture 1, are also helpful in ranking the expressions with factorials:

$$\begin{aligned}
n! &= \Theta(n^{n+1/2}e^{-n}) \\
\lg(n!) &= \Theta(n\lg n) \\
(\lg n)! &= \Theta((\lg n)^{\lg n+1/2}e^{-\lg n}).
\end{aligned}$$