Homework #3: Selection and order statistics, heaps/heapsort
Due Date: Wednesday, 3 February 2016

Reading: Chapters 6, 9 in CLRS.

Problem 1. Weighted order statistics [50 points, 20 points for part (a) and 30 points for part (b)].

For n distinct elements $x_1, x_2, \ldots, x_n$ with positive weights $w_1, w_2, \ldots, w_n$ such that $\sum_{i=1}^{n} w_i = W$, the **weighted 3-median** is the element x_k satisfying

$$\sum_{x_i < x_k} w_i \leq \frac{W}{3}$$

and

$$\sum_{x_i > x_k} w_i \leq \frac{2W}{3} .$$

(a) Show how to compute the weighted 3-median of n elements in $O(n \lg n)$ worst-case time using sorting.

(b) Show how to compute the weighted 3-median in $\Theta(n)$ worst-case time using a linear-time median algorithm such as SELECT from the text (SELECT must be treated as a black box, *not* modified).

Problem 2. Do Exercise 6.5–8 on page 166 of CLRS [15 points].

Problem 3. Do Problem 6–2 on page 167 of CLRS [15 points, 3 points per part].