Homework #6: Graph Algorithms
Due Date: Wednesday, 2 March 2016

Problem 1. Amortized analysis for two stacks [30 points]

Suppose there are two stacks called A and B, manipulated by the following operations:

push-A(d): Pushes a datum d onto stack A. Real Cost $= 1$.

push-B(d): Pushes a datum d onto stack B. Real Cost $= 1$.

multi-pop-A(k): Removes $min(k, size(A))$ elements from stack A. Real Cost $= min(k, size(A))$.

multi-pop-B(k): Removes $min(k, size(B))$ elements from stack B. Real Cost $= min(k, size(B))$.

transfer(k): Repeatedly pops elements from stack A and pushes them onto stack B, until either k elements have been moved, or A is empty. Real Cost=number of elements moved. (Note that you can transfer *only* from A to B.)

(a) (10 points) Give amortized costs to each operation using the accounting method. Using your amortized costs show an $O(n)$ worst case bound on the cost of n operations.

(b) (10 points) Give a potential function such that the amortized cost of each of the operations is constant, and evaluate the constant for each of the operations.

(c) (10 points) Using your potential function show an $O(n)$ worst case bound on the cost of n operations.

Problem 2. Bipartite graphs [50 points (25 per part)]

An undirected graph $G = (V, E)$ is called *bipartite* if the nodes can be partitioned into two subsets A and B in such a way that all edges go between A and B.

(a) Prove that a graph is bipartite iff it can be 2-colored, that is iff all nodes can be colored with two colors so that no two adjacent nodes have the same color.

(b) Give an efficient algorithm to decide whether a graph is bipartite. A by-product of your algorithm should be the partition of V into A and B.