Elementary Differential Geometry

cs164

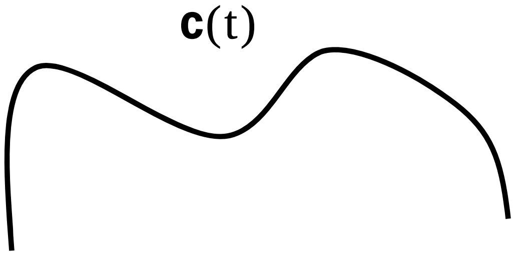
Outline

- Curvature formula for 2d curves
- Applications
- Curvature of space curves and surfaces
- Digital geometry processing

Arbitrary Parameterization

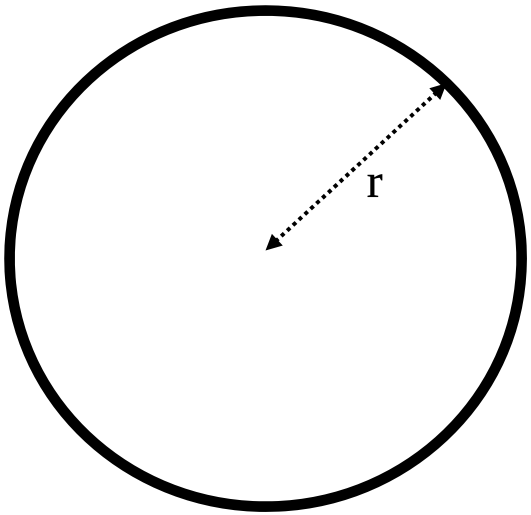
- Given a 2d parametric curve **c**(t)=(x(t),y(t))’, its signed curvature is given by

$$k(t) = \frac{x''(t)y'(t) - x'(t)y''(t)}{(x'(t)^2 + y'(t)^2)^{\frac{3}{2}}}$$



Example1

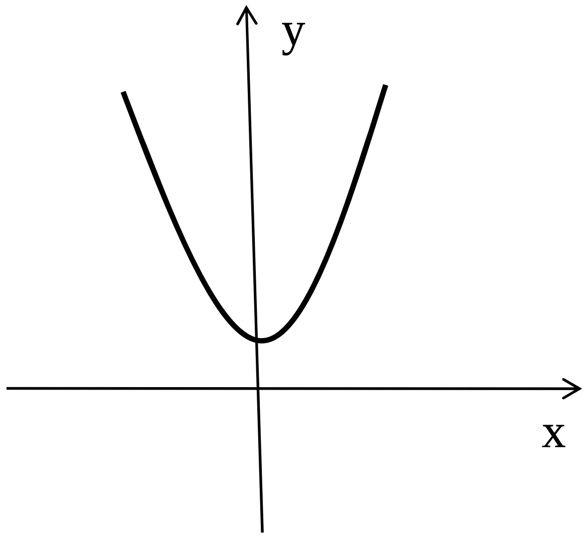
- The curvature of 2d circle with radius r is



$$k = \frac{1}{r}$$

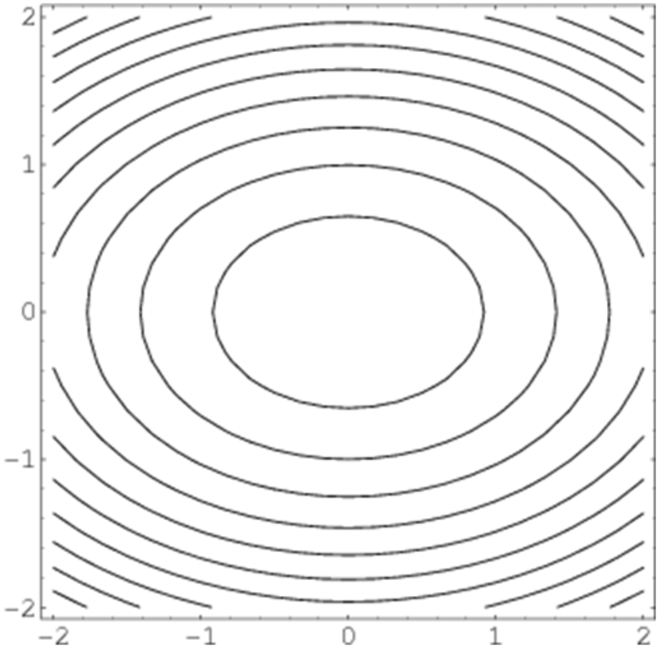
Example 2

- The curvature of a parabola $y = 1/2kx^2 + c$ is given by



$$k(x) = \frac{k}{\left(1+(kx)^2\right)^{\frac{3}{2}}}$$

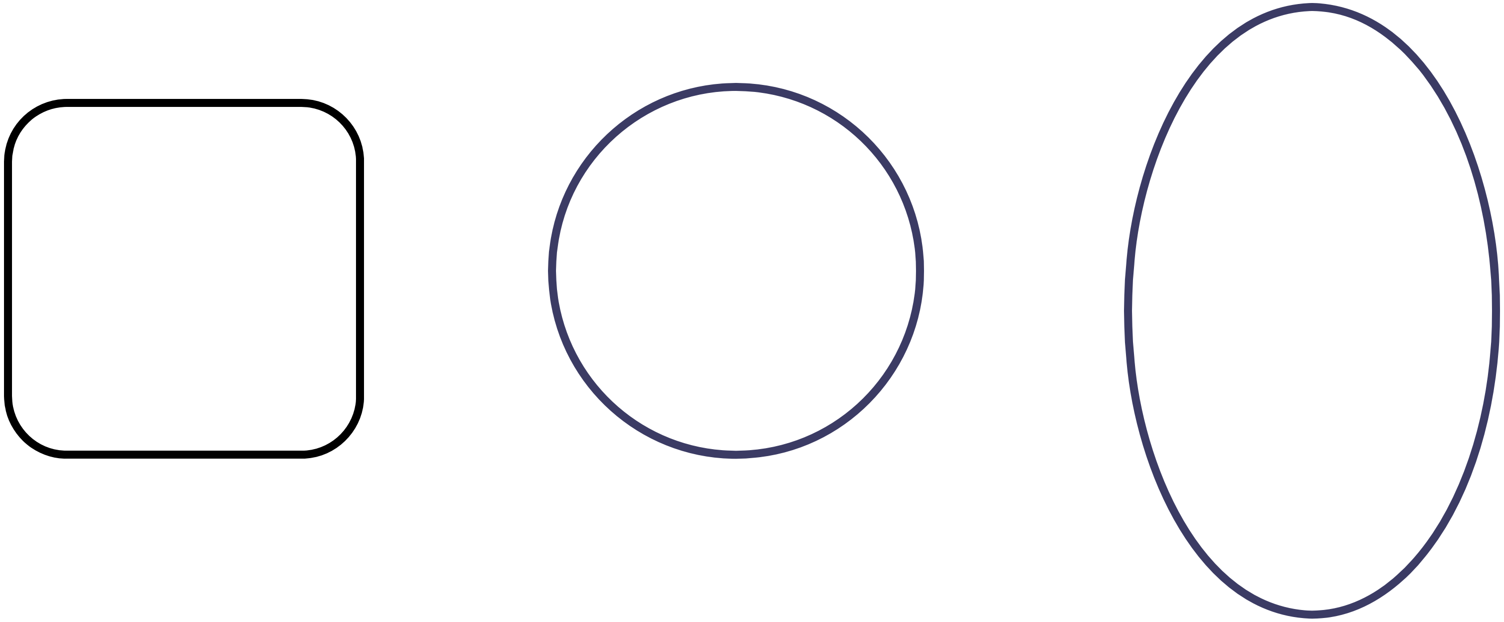
Implicit Curve-Exercise



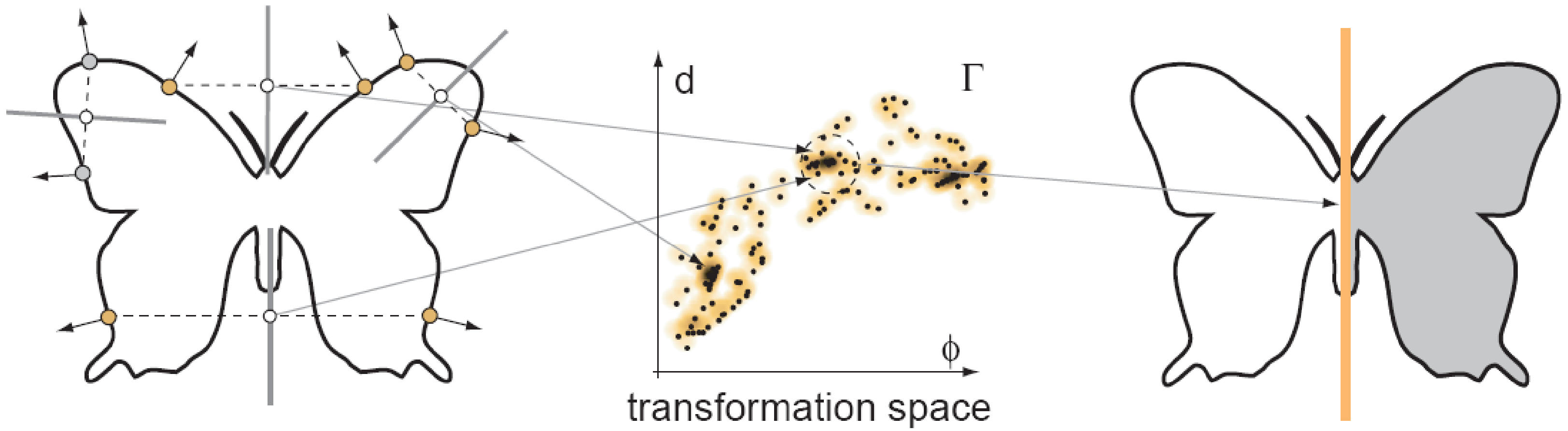
$$f(x,y) = c$$

Object Retrieval

- Compare curvature distributions

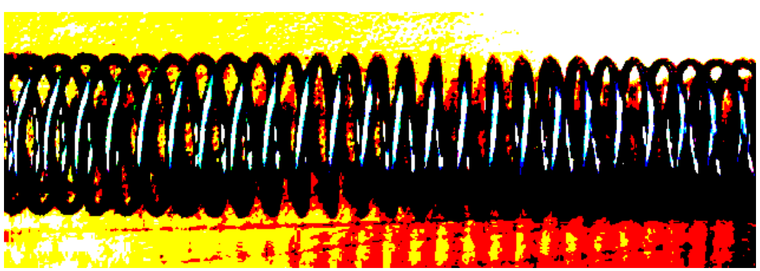


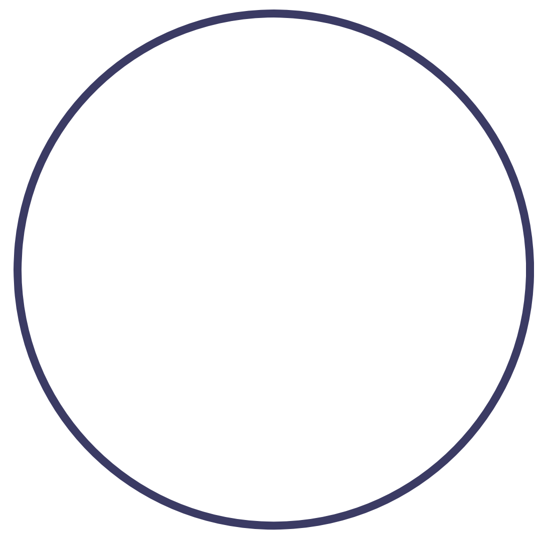
Symmetry Detection



Curvature of space curves

- Space curve does not lie on a plane.
- What is the difference between a spring and a circle





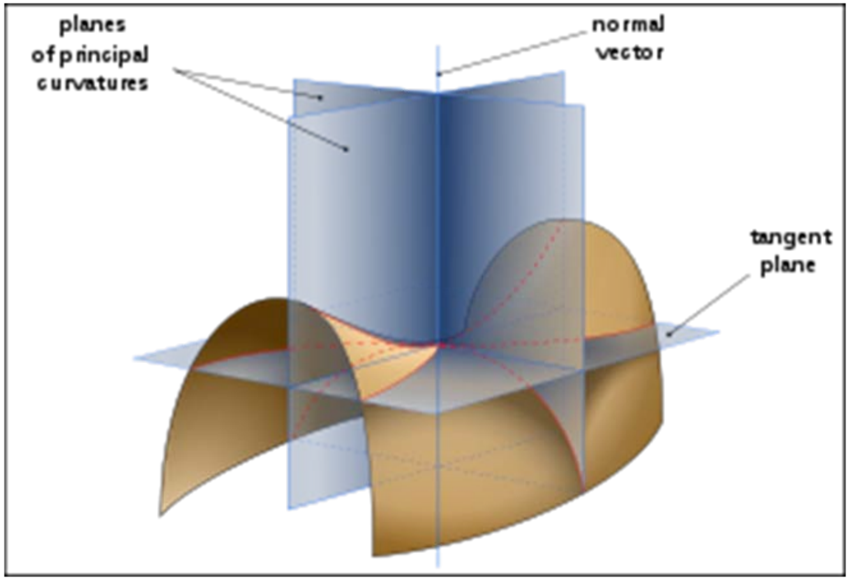
Curvature and Torsion

$$k(t) = \frac{r'(t) \times r''(t)}{\left\| r'(t) \right\|^3} \qquad \tau(t) = \frac{\det(r'(t), r''(t), r'''(t))}{\left\| r'(t) \times r''(t) \right\|^2}$$

- Property: A plane curve with non-vanishing curvature has zero torsion at all points. Conversely, if the torsion of a regular curve is identically zero then this curve belongs to a fixed plane.

Curvature of surfaces

- Two principal curvatures and two principal directions.
- Mean curvature
- Gaussian curvature

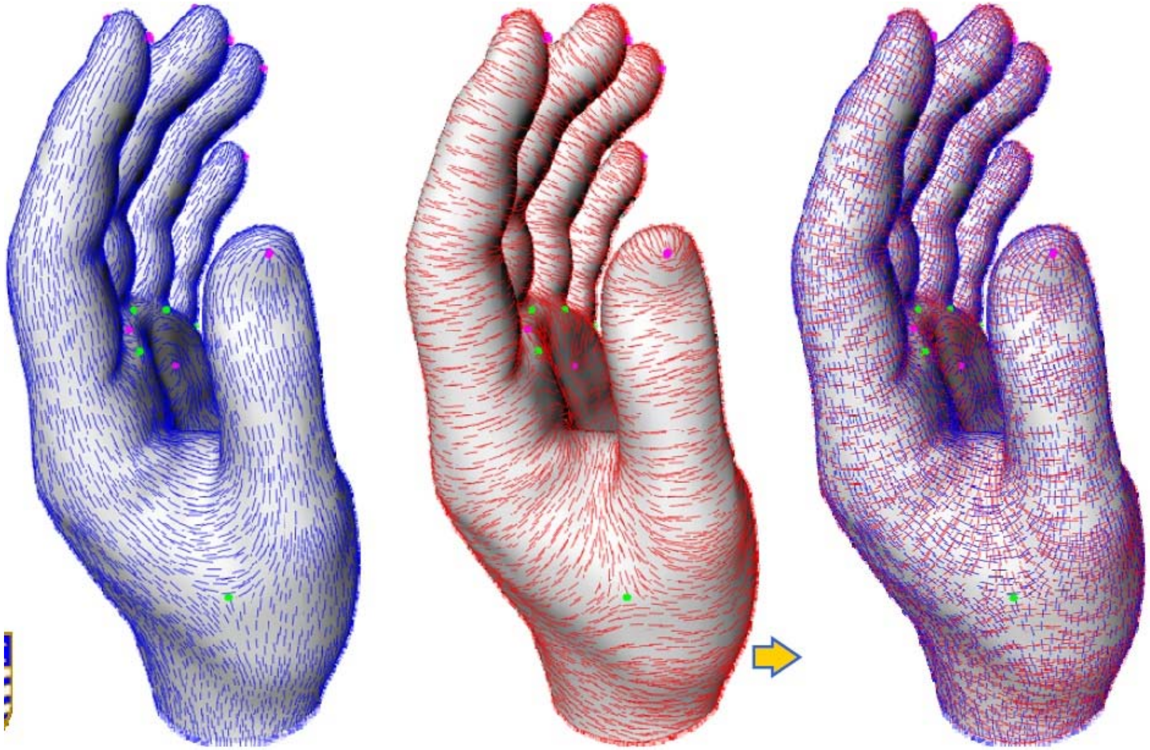


Example

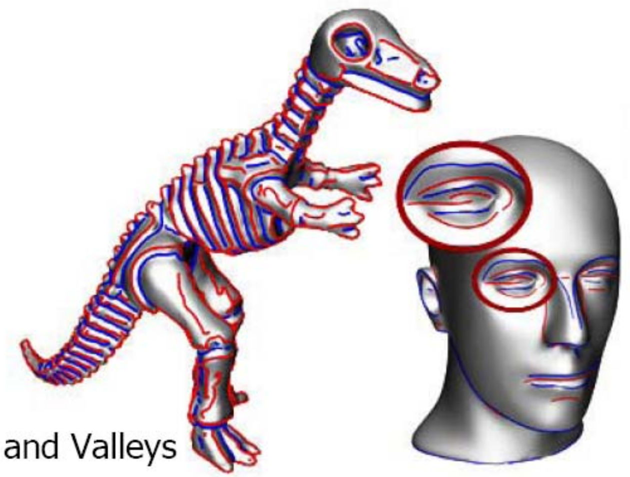
- The principal curvatures of a parabola $z = 1/2ax^2+1/2by^2$ are a and b.
- The principal directions are x and y axis.

Digital Geometry Processing

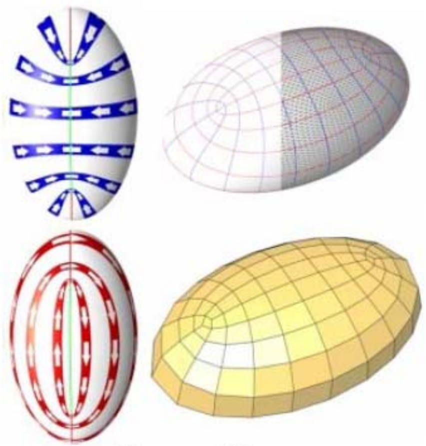
- Principal directions



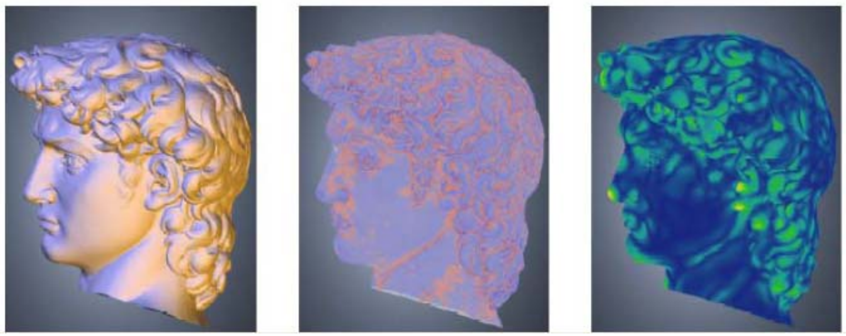
More



Ridges and Valleys



Remeshing



Viewpoint
selection

Design