Outline

✦ examples of camera sensor noise
  • don’t confuse it with JPEG compression artifacts
✦ probability, mean, variance, signal-to-noise ratio (SNR)
✦ laundry list of noise sources
  • photon shot noise, dark current, hot pixels, fixed pattern noise, read noise
✦ SNR (again), dynamic range (DR), bits per pixel
✦ ISO
✦ denoising
  • by aligning and averaging multiple shots
  • by image processing will be covered in a later lecture
Nokia N95 cell phone at dusk

- 8×8 blocks are JPEG compression
- unwanted sinusoidal patterns within each block are JPEG’s attempt to compress noisy pixels
Canon 5D II at dusk

- ISO 6400
- f/4.0
- 1/13 sec
- RAW w/o denoising
Canon 5D II at dusk

• ISO 6400
• f/4.0
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• RAW w/o denoising
Canon 5D II at dusk

- ISO 6400
- f/4.0
- 1/13 sec
Photon shot noise

- the number of photons arriving during an exposure varies from exposure to exposure and from pixel to pixel, even if the scene is completely uniform
- this number is governed by the Poisson distribution
Poisson distribution

- expresses the probability that a certain number of events will occur during an interval of time
- applicable to events that occur
  - with a known average rate, and
  - independently of the time since the last event
- if on average $\lambda$ events occur in an interval of time, the probability $p$ that $k$ events occur instead is

$$p(k; \lambda) = \frac{\lambda^k e^{-\lambda}}{k!}$$
Mean and variance

✦ the mean of a probability density function $p(x)$ is
\[ \mu = \int x p(x) \, dx \]
✦ the variance of a probability density function $p(x)$ is
\[ \sigma^2 = \int (x - \mu)^2 p(x) \, dx \]
✦ the mean and variance of the Poisson distribution are
\[ \mu = \lambda \]
\[ \sigma^2 = \lambda \]
✦ the standard deviation is
\[ \sigma = \sqrt{\lambda} \]

Deviation grows slower than the average.
Signal-to-noise ratio (SNR)

\[ SNR = \frac{\text{mean pixel value}}{\text{standard deviation of pixel value}} = \frac{\mu}{\sigma} \]

\[ SNR \text{ (dB)} = 20 \log_{10} \left( \frac{\mu}{\sigma} \right) \]

✧ example

- if SNR improves from 100:1 to 200:1, then it improves by
  \[ 20 \log_{10}(200) - 20 \log_{10}(100) = +6 \text{ dB} \]
Photon shot noise (again)

- photons arrive in a Poisson distribution

\[ \mu = \lambda \]
\[ \sigma = \sqrt{\lambda} \]

- so

\[ SNR = \frac{\mu}{\sigma} = \sqrt{\lambda} \]

- shot noise scales as square root of number of photons

- examples
  - doubling the width and height of a pixel increases its area by 4x, hence # of photons by 4x, hence SNR by 2x or +6 dB
  - opening the aperture by 1 f/stop increases the # of photons by 2x, hence SNR by \( \sqrt{2} \) or +3 dB
Dark current

- electrons dislodged by random thermal activity
- increases linearly with exposure time
- increases exponentially with temperature
- varies across sensor, and includes its own shot noise

Canon 20D, 612 sec exposure

(https://theory.uchicago.edu/~ejm/pix/20d/tests/noise/)
Hot pixels

- electrons leaking into well due to manufacturing defects
- increases linearly with exposure time
- increases with temperature, but hard to model
- changes over time, and every camera has them
Fixing dark current and hot pixels

✦ example
  • Aptina MT9P031 (in Nokia N95 cell phone)
  • full well capacity = ~8500 electrons/pix
  • dark current = 25 electrons/pix/sec at 55°C

✦ solution #1: chill the sensor
  • Retiga 4000R bioimaging camera
  • Peltier cooled 25°C below ambient
  • full well capacity = 40,000 electrons/pix
  • dark current = 1.64 electrons/pix/sec

✦ solution #2: dark frame subtraction
  • available on high-end SLRs
  • compensates for average dark current
  • also compensates for hot pixels and FPN
Fixed pattern noise (FPN)

- manufacturing variations across pixels, columns, blocks
- mainly in CMOS sensors
- doesn’t change over time, so read once and subtract

Canon 20D, ISO 800, cropped
Read noise

- thermal noise in readout circuitry
- again, mainly in CMOS sensors
- not fixed pattern, so only solution is cooling

Canon 1Ds Mark III, cropped

this image tainted by JPEG artifacts?
Recap

- **photon shot noise**
  - unavoidable randomness in number of photons arriving
  - grows as the square root of the number of photons,
    so brighter lighting and longer exposures will be less noisy

- **dark current noise**
  - grows with exposure time and sensor temperature
  - minimal for most exposure times used in photography
  - correct by subtraction, but only corrects for average dark current

- **hot pixels, fixed pattern noise**
  - caused by manufacturing defects, correct by subtraction

- **read noise**
  - electronic noise when reading pixels, unavoidable

Questions?
Signal-to-noise ratio
(with more detailed noise model)

\[ SNR = \frac{\text{mean pixel value}}{\text{standard deviation of pixel value}} = \frac{\mu}{\sigma} \]

\[ = \frac{P Q_e t}{\sqrt{P Q_e t + D t + N_r^2}} \]

where

- \( P \) = incident photon flux (photons/pixel/sec)
- \( Q_e \) = quantum efficiency
- \( t \) = exposure time (sec)
- \( D \) = dark current (electrons/pixel/sec), including hot pixels
- \( N_r \) = read noise (rms electrons/pixel), including fixed pattern noise

SNR changes with scene brightness, aperture, and exposure time

(formula from http://learn.hamamatsu.com/articles/ccdsnr.html)
Signal-to-noise ratio
(with more detailed noise model)

\[
SNR = \frac{\text{mean pixel value}}{\text{standard deviation of pixel value}} = \frac{\mu}{\sigma}
\]

\[
= \frac{P Q_e t}{\sqrt{P Q_e t + D t + N_r^2}}
\]

✦ examples

• Retiga 4000R = \((1000 \times 55\%) / \sqrt{(1000 \times 55\% + 1.64 + 12^2)}\)
  = 20.8:1 assuming 1000 photons/pixel/sec for 1 second

• Aptina MT9P031 = \((1000\div11\times69\%) / \sqrt{(1000\div11\times69\% + 25 + 2.6^2)}\)
  = 6.5:1 assuming pixels are 1/11 as large as Retiga’s

✦ for 10 photons/pixel/sec for 100 seconds

• Retiga = 18.7:1
• Aptina = 1.2:1

Don’t use your cell phone for astrophotography!
**Dynamic range**

\[ DR = \frac{\text{max output swing}}{\text{noise in the dark}} = \frac{\text{saturation level} - D_t}{\sqrt{D_t + N_r^2}} \]

- **examples**
  - Retiga 4000R = \((40,000 - 1.64) / \sqrt{1.64 + 12^2}\)  
    = 3,313:1 (11.7 bits) for a 1 second exposure
  - Aptina MT9P031 = \((8500 - 25) / \sqrt{25 + 2.6^2}\)  
    = 1500:1 (10.5 bits) for a 1 second exposure

- determines precision required in ADC, and useful # of bits in RAW image
- any less than ~10 bits would be < 8 bits after gamma correction for JPEG encoding, and you would see quantization artifacts

To reiterate the difference between SNR and DR, signal-to-noise ratio (SNR) tells you how noisy an image will be at a particular light level, and a sensor will have a different SNR for each possible light level, while dynamic range (DR) is a single number giving the maximum possible range between saturation (for bright scenes) and the noise floor (for dark scenes). DR tells you nothing about how noisy a low-light image will be; it just says that it will be (barely) distinguishable from pure noise. So a cell phone might have as large a dynamic range as an SLR, but if its low-light images are very noisy (as they typically are), you wouldn’t want to use it for low-light photography.
Low-light cameras

\[ DR = \frac{\text{max output swing}}{\text{noise in the dark}} = \frac{\text{saturation level} - D_t}{\sqrt{D_t + N_r^2}} \]

- Andor iXon+888 back-illuminated CCD
  - $40,000
  - performance
    - \( DR = (80,000 - 0.001) / \sqrt{(0.001 + 6^2)} \)
      - \( = 13,333:1 \) (13.7 bits) for a 1 second exposure
  - “electron multiplication” mode
    - \( DR = (80,000 - 0.001) / \sqrt{(0.001 + <1^2)} \)
      - \( \approx 80,000:1 \) (16.2 bits)
    - “can see a black cat in a coal mine”

- compare to 10.5 bits for Aptina
- don’t use your cell phone for fluorescence microscopy!

if cooled to -75º C

can reliably detect a single photon
ISO - signal gain

- doubling ISO doubles the signal
  - linear with light, so same as 2× exposure time, or −1 f/stop
  - implemented as analog amplification on Canon 5D II up to ISO 6400; higher ISOs are implemented using digital multiplication after ADC?

- you want to amplify as early as possible during readout
  - if you amplify before read noise is added, and RN is independent of signal amplitude, then the amplified signal will have better SNR

- you especially want to amplify before quantization by ADC
  - if you quantize a low signal, then brighten it in Photoshop, you will see quantization artifacts (contouring)
  - if you quantize a very low signal, you may get zero (black)

- raising exposure typically improves SNR faster than raising ISO
  - thus, you should maximize exposure time until stopped by object motion blur, camera shake blur, or saturation; if stopped by blur, then raise ISO until stopped by saturation (i.e. don’t clip whites)
The signal amplification pipeline

- raising the ISO is usually implemented as analog amplification (of voltages) before analog-to-digital conversion (ADC), but for high ISOs, some cameras may also perform digital multiplication (of numbers) after ADC

- analog amplification is better than digital multiplication, for the reasons given on the previous slide

To reiterate the “recipe” I gave in class, here’s how to take a picture that minimizes noise:

1. Make your aperture as wide as you want it for depth of field.
2. Make your exposure as long as you dare make it, given handshake or object motion blur.
3. Raise the ISO to ensure an image that fills the range of numbers representable in the RAW or JPEG file, i.e. until the brightest object in the scene that you don’t want to appear saturated just reaches white on the histogram.

All of these are done in the camera during shooting. Don’t use Photoshop to brighten an image (except minor adjustments), because it will enhance noise more than raising the ISO will, and it may introduce quantization artifacts (contouring).
SNR and ISO over the years

- SNR has been improving with better sensor designs
- but total # of megapixels has risen to offset these improvements, making pixels smaller, so SNR in a pixel has remained static
- display resolutions have not risen as fast as megapixels, so we’re increasingly downsizing our images for display
- if you average 4 camera pixels to produce 1 for display, SNR doubles, so for the same display area, SNR has been improving

After lecture, Jesse pointed out to me that as displays match and begin to exceed human retinal acuity, it no longer matters how many pixels they have, only how many pixels we can see. This in turn depends on screen size and viewing distance. He’s right, but except for a few high-end smartphones that hasn’t happened yet, so my metric is still meaningful.
Implicit in this example is the notion that averaging down 4 pixels to make one pixel has a similar effect on SNR as having a pixel 4x as large. The effect isn't identical, because the contributions by read noise are different, but read noise is less important to SNR than photon shot noise.
SNR and ISO over the years

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- display resolutions have not risen as fast as megapixels, so we’re increasingly downsizing our images for display
- if you average 4 camera pixels to produce 1 for display, SNR doubles, so for the same display area, SNR has been improving
- this allows higher ISOs to be used in everyday photography

(http://www.dxomark.com/index.php/eng/Insights/SNR-evolution-over-time)
Nikon D3S, ISO 25,600, denoised in Lightroom 3, photograph by Fredo Durand
Nikon D3S, ISO 25,600, denoised in Lightroom 3, photograph by Fredo Durand
RAW image from camera, before denoising in Lightroom
Fredo said it was too dark to read the menu...
tone mapped to show the scene as Fredo might have experienced it
single frame in dark room using iPhone 4
average of ~30 frames using SynthCam

SNR increases as $\sqrt{\text{# of frames}}$
Recap

- **signal-to-noise ratio** (SNR) is mean/stddev of pixel value
  - rises with $\sqrt{\text{brightness and/or exposure time}}$
  - depends also on dark current and read noise
  - poor for short exposures and very long exposures

- **dynamic range** (DR) is max swing / noise in the dark
  - fixed for a particular sensor and exposure time
  - determines # of useful bits in RAW image

- **ISO** is amplification of signal before conversion to digital
  - maximize exposure time until camera or object blurs, then maximize ISO, making sure not to saturate
  - can combine multiple short-exposure high-ISO pictures

Questions?
Slide credits

✦ Eddy Talvala

✦ Filippov, A., *How many bits are really needed in the image pixels?* (sic), http://www.linuxdevices.com/articles/AT9913651997.html