Homework 0: Math Review

CS 205A: Mathematical Methods for Robotics, Vision, and Graphics (Fall 2013), Stanford University

Due Monday, September 30, in class

This homework is designed to help students in CS 205A review the basic mathematics that we will be using for the rest of the course. Make sure you are 100% confident in your solution to each problem: We will use these techniques repeatedly for the rest of the course! Depending on your background, you may not have seen some of these topics; make ample use of office hours and section to help fill any gaps in your knowledge or understanding.

Problem 1 (20 points). *Take $C^1(\mathbb{R})$ to be the set of continuously differentiable functions $f : \mathbb{R} \to \mathbb{R}$. Why is $C^1(\mathbb{R})$ a vector space? Show that $C^1(\mathbb{R})$ has dimension ∞.*

Problem 2 (20 points). *Suppose the rows of $A \in \mathbb{R}^{m \times n}$ are given by the transposes of $\vec{r}_1, \ldots, \vec{r}_m \in \mathbb{R}^n$ and the columns of $A \in \mathbb{R}^{m \times n}$ are given by $\vec{c}_1, \ldots, \vec{c}_n \in \mathbb{R}^m$. That is,*

$$A = \begin{pmatrix} - & \vec{r}_1^\top & - \\ - & \vec{r}_2^\top & - \\ & \vdots & \\ - & \vec{r}_m^\top & - \end{pmatrix} = \begin{pmatrix} | & | & & | \\ \vec{c}_1 & \vec{c}_2 & \cdots & \vec{c}_n \\ | & | & & | \end{pmatrix}.$$

Give expressions for the elements of $A^\top A$ and $AA^\top$ in terms of these vectors.

Problem 3 (20 points). *Give a linear system of equations satisfied by minima of the energy $f(\vec{x}) = \|A\vec{x} - \vec{b}\|_2$ with respect to $\vec{x}$, for $\vec{x} \in \mathbb{R}^n$, $A \in \mathbb{R}^{m \times n}$, and $\vec{b} \in \mathbb{R}^m$.*

Problem 4 (20 points). *Suppose $A, B \in \mathbb{R}^{n \times n}$. Formulate a condition for vectors $\vec{x} \in \mathbb{R}^n$ to be critical points of $\|A\vec{x}\|_2$ subject to $\|B\vec{x}\|_2 = 1$. Also, give an alternative expression for the optimal values of $\|A\vec{x}\|_2$.*

Problem 5 (20 points). *Fix some vector $\vec{a} \in \mathbb{R}^n \backslash \{\vec{0}\}$ and define $f(\vec{x}) = \vec{a} \cdot \vec{x}$. Give an expression for the maximum of $f(\vec{x})$ subject to $\|\vec{x}\| = 1$.*