Linear Algebra Review

CS 205A:
Mathematical Methods for Robotics, Vision, and Graphics

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**Midterm**

10/28, in class

Includes this week’s material.
What Have We Done?

\[ \mathbf{A} \mathbf{x} = \mathbf{b} \]
Gaussian Elimination

- Exactly the same as what you did on paper
- Phases: Forward substitution, back substitution (pivoting)
- Elimination matrices: convenience for understanding theory
LU Factorization

- Allows for solving linear systems via forward/backward substitution
- Might not exist – need pivots (e.g. LUP)
Cholesky Factorization: $LL^\top$

For symmetric, positive definite matrices
QR Factorization

- $R$ is upper triangular
- $Q$ has orthonormal columns; usually basis for column space of $A$
- Two strategies: Gram-Schmidt and Householder
- Least-squares without squaring the condition number
Diagonalizability: \[ A = X^{-1}AX \]

- Only if there is a full eigenspace
- Spectral theorem: If \( A \) is symmetric/Hermitian, full orthogonal eigenspace
- Eigenvalue algorithms: Mostly variations of power method
Singular Value Decomposition

\[ A = U \Sigma V^T \]
Variational Approach

Define energy measuring something desirable and minimize it.
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$$E = \| Ax - \vec{b} \|_2^2$$
Variational Approach

Define energy measuring something desirable and minimize it.

\[ E = \| A\vec{x} - \vec{b} \|_2^2 \]

Lagrange multipliers!
Look for Special Structure

- Symmetric
- Positive definite
- Sparse
- Normal equations
- Square
- Full rank
- Block
- Triangular
Reduce to Known Algorithm

Show that a specific problem is equivalent to:

- Least squares (kernel trick)
- Eigenvectors (ODEs, embedding)
- Factorization (metric learning)
- SVD (principal components analysis)
Stability and Conditioning

Complement algorithmic analysis with understanding quality of output
Advice

Experiment.
Advice

Experiment.

Ask for help.