Linear Algebra Review

CS 205A:
Mathematical Methods for Robotics, Vision, and Graphics

Justin Solomon

Warning

Midterm

10/28, in class

Includes this week's material.

What Have We Done?

$$A\vec{x} = \vec{b}$$

Gaussian Elimination

- Exactly the same as what you did on paper
- Phases: Forward substitution, back substitution (pivoting)
- Elimination matrices: convenience for understanding theory

LU Factorization

- Allows for solving linear systems via forward/backward substitution
- Might not exist – need pivots (e.g. LUP)

Cholesky Factorization: $LL^\top$

For symmetric, positive definite matrices

QR Factorization

- R is upper triangular
- Q has orthonormal columns; usually basis for column space of A
- Two strategies: Gram-Schmidt and Householder
- Least-squares without squaring the condition number

Diagonalizability: $A = X^{-1}AX$

- Only if there is a full eigenspace
- Spectral theorem: If A is symmetric/Hermitian, full orthogonal eigenspace
- Eigenvalue algorithms: Mostly variations of power method

Singular Value Decomposition

$$A = U\Sigma V^\top$$

Variational Approach

Define energy measuring something desirable and minimize it.

Variational Approach

Define energy measuring something desirable and minimize it.

$$E = \|A\vec{x} - \vec{b}\|_2^2$$

Variational Approach

Define energy measuring something desirable and minimize it.

$$E = \|A\vec{x} - \vec{b}\|_2^2$$

Lagrange multipliers!

Look for Special Structure

- Symmetric
- Positive definite
- Sparse
- Normal equations
- Square
- Full rank
- Block
- Triangular

Reduce to Known Algorithm

Show that a specific problem is equivalent to:

- Least squares (kernel trick)
- Eigenvectors (ODEs, embedding)
- Factorization (metric learning)
- SVD (principal components analysis)

Stability and Conditioning

Complement algorithmic analysis with understanding quality of output

Advice

Experiment.

Advice

Experiment.

Ask for help.

Next