Homework 1: Numerics & Intro to LU

CS 205A: Mathematical Methods for Robotics, Vision, and Graphics (Spring 2016)
Stanford University
Due Tuesday, April 12, 11:00am (via gradescope)

This homework is one of the more difficult assignments in CS 205A. Please make ample use of office hours, Piazza, and other resources, and get started early.

Note: As mentioned in lecture, although it is not worth course credit you may wish as an exercise on your own to implement algorithms we discuss in class. We are happy to help you debug and experiment with your implementations in office hours and on Piazza. From last week's lectures, the obvious choices for implementation are Gaussian elimination (with or without pivoting) and LU factorization.

Textbook problems: 2.5 (20 points), 3.12(a-e) (35 points)

Sum Bad Pi: (45 points) One of the lousier ways to compute π is by using the Gregory series

$$G_n = 4\sum_{k=1}^{n} \frac{(-1)^{k+1}}{2k-1} \xrightarrow{n\to\infty} \pi = 3.14159265358979323846264338327950288419716939937510582...$$

Its convergence is slow and, well, unusual. In this question, you will explore how many correct digits can you compute of π using this formula–the matching digits may not all be contiguous!

1. Write a simple program, using double-precision floating point arithmetic, that computes G_n. Evaluate G_n for $n = 10000, 100000, 1000000$ and 5000000. Compare your results to π. How many digits match?

2. Next, implement the more careful Kahan Sum algorithm from the textbook (§2.3.2), and compare your results for the n values used previously. How many digits match?

3. One might be bothered by the alternating nature of this series. Consider the modified non-alternating series where each term is a positive number,

$$G_n = 4\sum_{\substack{k=1\\k\text{ odd}}}^{n} \left(\frac{1}{2k-1} - \frac{1}{2(k+1)-1}\right) = 2\sum_{\substack{k=1\\k\text{ odd}}}^{n} \frac{1}{(k-\frac{1}{2})(k+\frac{1}{2})} \qquad (n\text{ even}).$$

   Compare your results for this summation (with/without Kahan Sum) to the previous values. Is this approach better or worse?

4. (Extra credit; 5 points) Taking n as large as you want, which of these Gregory-based approaches (or a slightly modified one) is the best way to compute π in double precision?

Extra credit: 2.9 (5 points)