

Linear Algebra Review

CS 205A:
Mathematical Methods for Robotics, Vision, and Graphics

Doug James (and Justin Solomon)



Warning

Midterm

Tuesday 5/3, in class

Includes today's material. See Piazza for FAQ.

What Have We Done?

$$A\vec{x} = \vec{b}$$

Gaussian Elimination

- ▶ Codifies the typical approach taken on paper
- ▶ **Phases:** Forward substitution, back substitution (pivoting)
- ▶ **Elimination matrices:** Notational convenience, algorithmically *slow*!

LU Factorization

- ▶ $O(n^3)$ time to compute
- ▶ Allows for solving linear systems via forward/backward substitution ($O(n^2)$ time)
- ▶ Might not exist – need pivots (e.g. LUP)

Cholesky Factorization: LL^T

For symmetric, positive
definite matrices

QR Factorization

- ▶ R is **upper triangular**
- ▶ Q has **orthonormal columns**
- ▶ **Many algorithms:**
Gram-Schmidt, Householder, Givens
- ▶ **Least-squares** w/o squaring condition #

Diagonalizability: $D = X^{-1}AX$

- ▶ Diagonalizable iff there is a full eigenspace
- ▶ **Spectral theorem:** symmetric/Hermitian
 \implies full, orthogonal eigenbasis
- ▶ **Computation:** Variations of power method
- ▶ **Note:** $AX = XD$ (usually $AX \neq DX$!!)

Singular Value Decomposition

$$A = U \Sigma V^T$$

Variational Approach

Define energy measuring something desirable and minimize it.

Variational Approach

Define energy measuring something desirable and minimize it.

$$E(\vec{x}) = \|A\vec{x} - \vec{b}\|_2^2$$

Variational Approach

Define energy measuring something desirable and minimize it.

$$E(\vec{x}) = \|A\vec{x} - \vec{b}\|_2^2$$

Lagrange multipliers!

Look for Special Structure

- ▶ Symmetric
- ▶ Positive definite
 - ▶ Sparse
- ▶ Normal equations
 - ▶ Square
 - ▶ Full rank
 - ▶ Block
- ▶ Triangular

Reduce to Known Algorithm

Show that a specific problem is equivalent to:

- ▶ Least squares (kernel trick)
- ▶ Eigenvectors (ODEs, embedding)
- ▶ Factorization (metric learning)
- ▶ SVD (principal components analysis)

Stability and Conditioning

Complement algorithmic
analysis with understanding
quality of output

Advice

Draw matrix pictures.

Advice

Draw matrix pictures.

Experiment.

Advice

Draw matrix pictures.

Experiment.

Ask for help.