Homework 0: Math Review

CS 205A: Mathematical Methods for Robotics, Vision, and Graphics (Spring 2017)
Stanford University
Due Thursday, April 13, 11:59pm (via GradeScope)

The written part of this homework is designed to help students in CS 205A review the basic mathematics that we will be using for the rest of the course. Make sure you are 100% confident in your solution to each problem: We will use these techniques repeatedly for the rest of the course! Depending on your background, you may not have seen some of these topics; make ample use of office hours and section to help fill any gaps in your knowledge or understanding.
The programming question will help you explore defining matrices, functions, plot(), and using Markdown comments to explain your results.

Textbook problems: 1.5, 1.7, 1.9, 1.10, 1.15 (15 points each)

Julia Programming Assignment (25 points): Question 1.13 considers a time-dependent matrix $\mathbf{A}(t)$, and the rate of change of its inverse $\mathbf{A}^{-1}(t)$. Use Julia to numerically verify the stated identity,

$$\frac{d(\mathbf{A}^{-1})}{dt} = -\mathbf{A}^{-1}\frac{d\mathbf{A}}{dt}\mathbf{A}^{-1}$$

for the special case of the N-by-N time-dependent matrix $\mathbf{A}(t)$ whose (m, n) entry is

$$A_{m,n} = \sin(mnt)$$

where $m, n = 1 \dots N$. You may use the fact that $(\frac{d\mathbf{A}}{dt})_{m,n} = mn\cos(mnt)$, but $\frac{d(\mathbf{A}^{-1})}{dt}$ should be implemented using centered finite differences,

$$\frac{d(\mathbf{A}^{-1})}{dt}(t) \approx \frac{(\mathbf{A}(t+h))^{-1} - (\mathbf{A}(t-h))^{-1}}{2h} \qquad \text{(an } O(h^2) \text{ approximation)}$$

where h is a small parameter of your choosing. Verify the approximation at $t = 1$ for a matrices of different sizes of N, specifically consider $N = 10$ and then $N = 1000$.

(a) How small can you make the relative error of your finite-difference approximation vs the formula in each case? Note: use the default Float64 precision, and compute matrix norms using the Frobenius norm (`vecnorm(A)` in Julia).

(b) Plot the relative error as a function of N for a sampling of values. What do you observe?

To simplify submission to GradeScope with your other written homework, export a **PDF** of your Julia 0.5.1 Notebook.