Homework 5: Root-Finding

CS 205A: Mathematical Methods for Robotics, Vision, and Graphics (Spring 2017)
Stanford University
Due Friday, May 19, 11:59pm

Textbook problems: 8.1 (5 points), 8.3 (10 points), 8.4 (10 points), 8.5 (5 points), 8.9 (20 points), 8.10 (10 points)

Julia Programming Question: (40 points) Implement the method from problem 8.10, which can automatically find the separated roots of a polynomial. Use it to estimate *all* roots of

$$p(x) = x^6 - \frac{223x^5}{140} + \frac{319x^4}{315} - \frac{37x^3}{112} + \frac{59x^2}{1008} - \frac{3x}{560} + \frac{1}{5040}$$

on the interval $x \in [0, 1]$. You may use any automated sub-method for individual root finding (bisection, secant, Newton, etc.) provided that you implement it yourself. Correctness is important, but speed is not a concern. Determine the following:

1. How many unique roots are there?
2. What are the root x-values to at least 10 digits of accuracy?
3. Are all of them "simple roots?" Why?

Note that you may plot/visualize the results to understand the function and confirm your results, but that may not be the basis of your automated root-finding method.

(Hint: You can find derivatives of all orders of a polynomial easily by exploiting a simple coefficient rule:

$$p(x) = \sum_{k=0}^{n} c_k x^k \Longrightarrow p'(x) = \sum_{k=0}^{n-1} c'_k x^k$$

where $c'_k = (k+1)c_{k+1}$.)

Bonus: (5 points) What happens when you try your Julia method on the following polynomial?

$$p(x) = x^6 - \frac{33x^5}{20} + \frac{493x^4}{450} - \frac{271x^3}{720} + \frac{17x^2}{240} - \frac{x}{144} + \frac{1}{3600}$$

What is the problem here?