Homework 6: Optimization
Stanford University
Due Friday, May 26, 11:59pm

Textbook problems: 9.1 (10 points), 9.3 (20 points), 9.5 (30 points)

Julia Programming: This question uses the following two functions,
\[ f(x) = 1 + x + x^2 + x^3 + x^4 \] (easy)
\[ h(x) = \exp \left( -\frac{1}{x^2} \right) \] (hard)
(which you should plot for your own information).

Do the following:

- Implement a function to perform **Newton’s method for minimization** on a scalar function of one variable, that takes a function as input, and an initial guess \( x_0 \). Use ForwardDiff to compute any derivatives you need (see https://github.com/JuliaDiff/ForwardDiff.jl). (8 points)
  - Apply it to \( f \), starting at \( x = 1 \), and report the first 20 values at each step. (3 points)
  - Apply it to \( h \), starting at \( x = 1 \), and report the first 20 values at each step. (3 points)
  - How close to the true minima can you get in each case? (4 points)
  - Why is the \( h \) function so hard to optimize? (2 points)

- Implement a function to perform **golden section search** that takes a function as input, and an initial search interval. (8 points)
  - Apply it to \( f \), starting on the interval \([-1,1]\], and report the first 20 values at each step. (3 points)
  - Apply it to \( h \), starting on the interval \([-1,1]\], and report the first 20 values at each step. (3 points)
  - How close to the true minima can you get in each case? (4 points)
  - Is golden section search more reliable than Newton’s method for \( h \)? Why? (2 points)