

Linear Algebra Review

CS 205A:
Mathematical Methods for Robotics, Vision, and Graphics

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Midterm Exam

Tuesday 5/9, in class

- ▶ Does not include today's material—only linalg.
- ▶ Closed book exam.
- ▶ Can use 1 page of notes (two-sided, human readable)
- ▶ No calculators or computers allowed (or needed)
- ▶ SCPD: Arrange proctor for exam at similar time.
- ▶ Previous midterm exams on website (similar material)
- ▶ Simple HW this week – short programming question.

What Have We Done?

$$A\vec{x} = \vec{b}$$

Gaussian Elimination

- ▶ Codifies the typical approach taken on paper
- ▶ **Phases:** Forward substitution, back substitution (pivoting)
- ▶ **Elimination matrices:** Notational convenience, algorithmically *slow*!

LU Factorization

- ▶ $O(n^3)$ time to compute
- ▶ Allows for solving linear systems via forward/backward substitution ($O(n^2)$ time)
- ▶ Might not exist – need pivots (e.g. LUP)

Cholesky Factorization: LL^T

For symmetric, positive
definite matrices

QR Factorization

- ▶ R is **upper triangular**
- ▶ Q has **orthonormal columns**
- ▶ **Many algorithms:**
Gram-Schmidt, Householder, Givens
- ▶ **Least-squares** w/o squaring condition #

Diagonalizability: $D = X^{-1}AX$

- ▶ Diagonalizable iff there is a full eigenspace
- ▶ **Spectral theorem:** symmetric/Hermitian
 \implies full, orthogonal eigenbasis
- ▶ **Computation:** Variations of power method
- ▶ **Note:** $AX = XD$ (usually $AX \neq DX$!!)

Singular Value Decomposition

$$A = U \Sigma V^T$$

Variational Approach

Define energy measuring something desirable and minimize it.

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Lagrange multipliers!

Regularization

Improves conditioning of ill-posed problems

E.g., Tikhonov regularization.

$$E(\vec{x}) = \|A\vec{x} - \vec{b}\|_2^2 + \alpha \|\vec{x}\|_2^2$$

Multiple formulations.

Connection to truncated SVD.

Look for Special Structure

- ▶ Symmetric
- ▶ Positive definite
 - ▶ Sparse
- ▶ Normal equations
 - ▶ Square
 - ▶ Full rank
 - ▶ Block
- ▶ Triangular

Reduce to Known Algorithm

Show that a specific problem is equivalent to:

- ▶ Least squares (curve fitting)
- ▶ Eigenvectors (ODEs, embedding)
- ▶ Factorization (metric learning)
- ▶ SVD (principal components analysis)

Stability and Conditioning

Complement algorithmic
analysis with understanding
quality of output

Julia

Not on written midterm,
but useful for study.

Advice

Draw matrix pictures.

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Draw matrix pictures.

Experiment.

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Ask for help.