

CS233, CME251: Geometric and Topological Data Analysis

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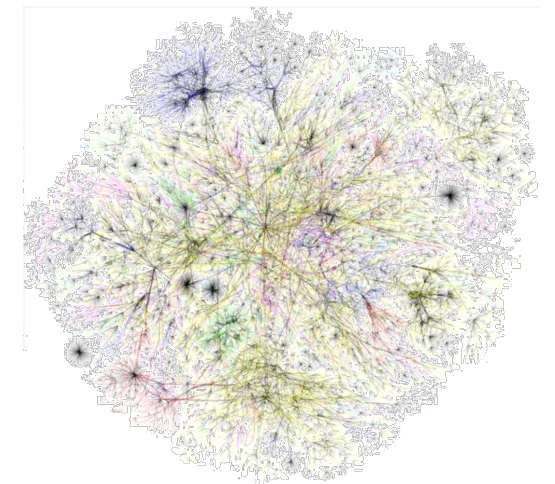
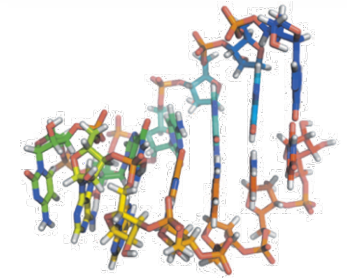
Lecture 1
29 March 2021



Introduction

Big Data Era

- Data from many kinds of sensors
- Data from simulations
- Data from the activities of individuals on the internet



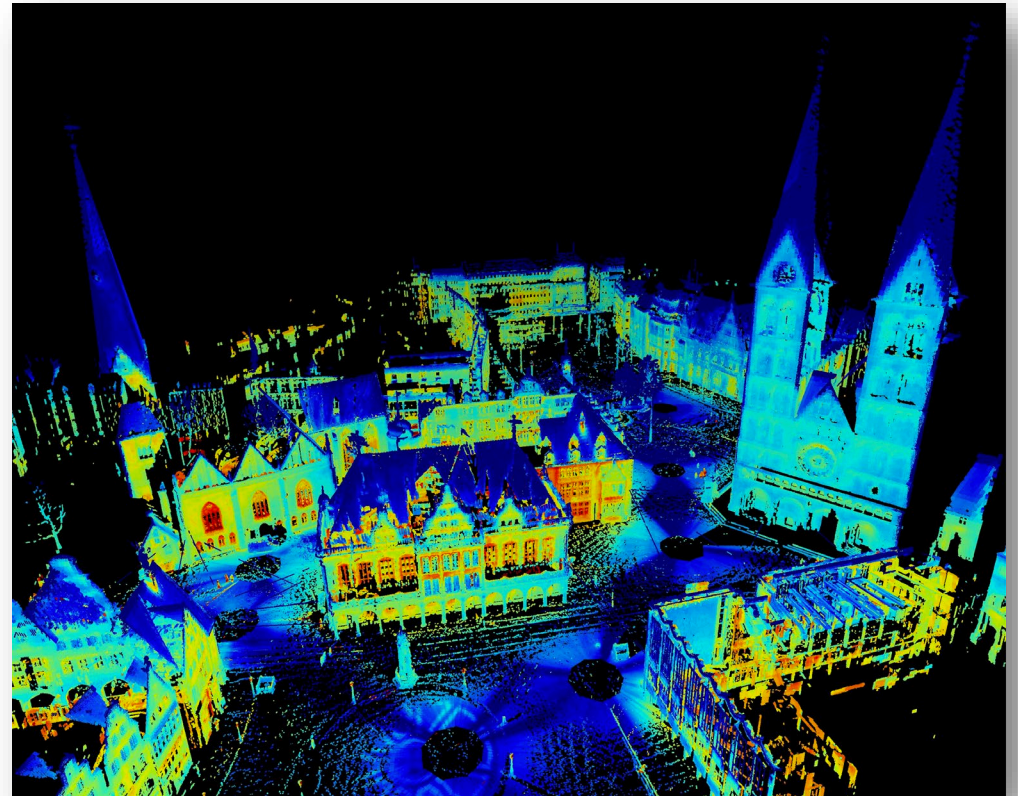
“Data Science”

Data Sets of Geometric Character

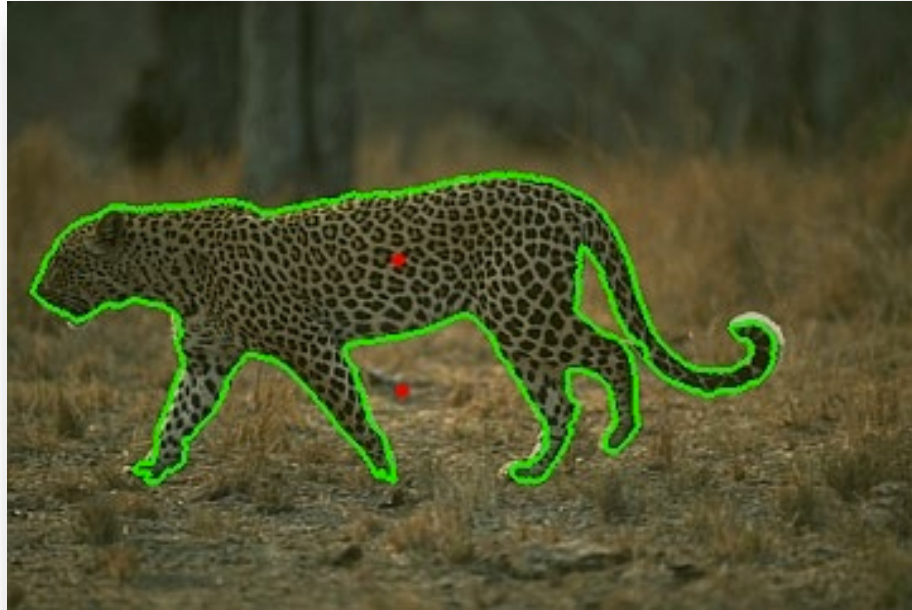


Vehicle GPS traces (Beijing)

3D city scans (Bremen)



Geometry in Ordinary Images and Videos

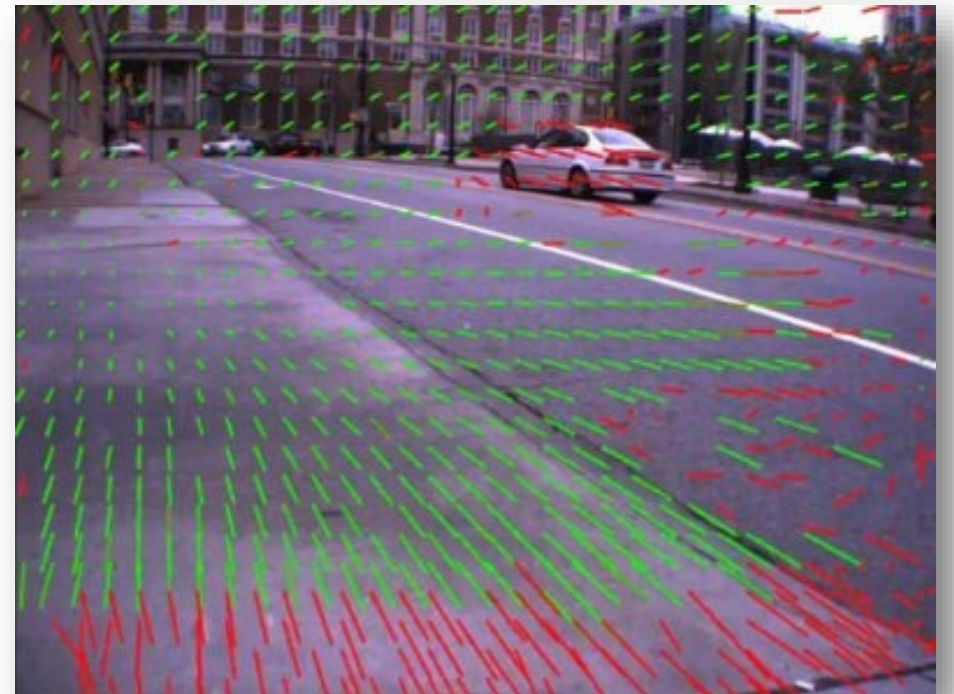


Object recognition

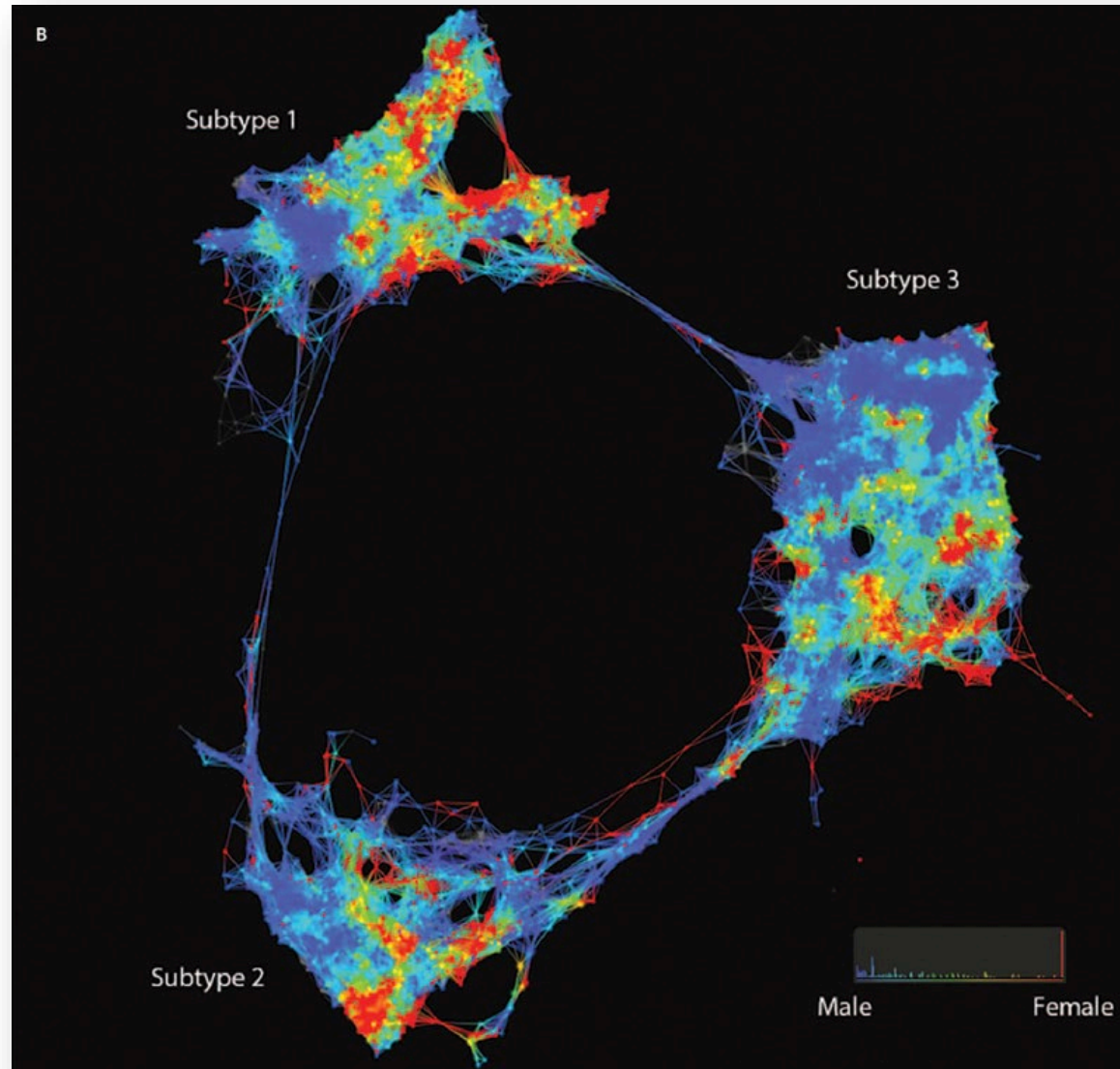
Segmentation involves geometry

Motion involves geometry

Optical flow

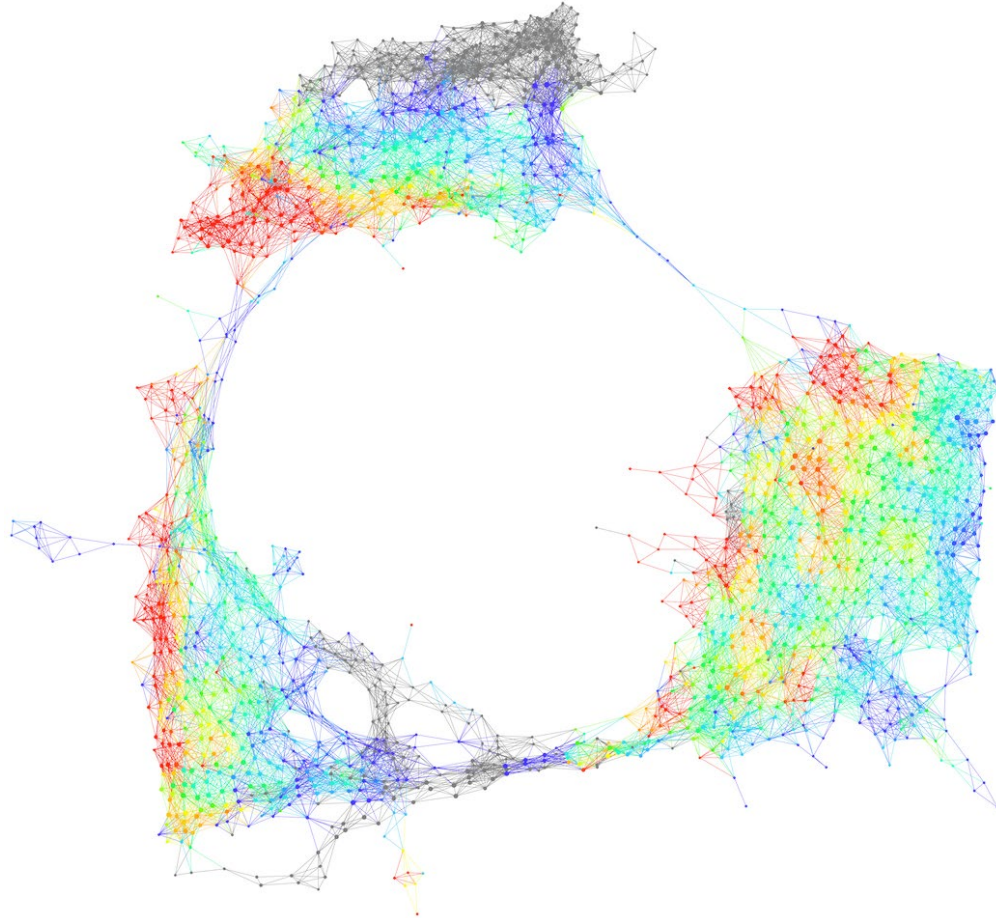


Non-Geometric Data



Diabetes II
subtypes

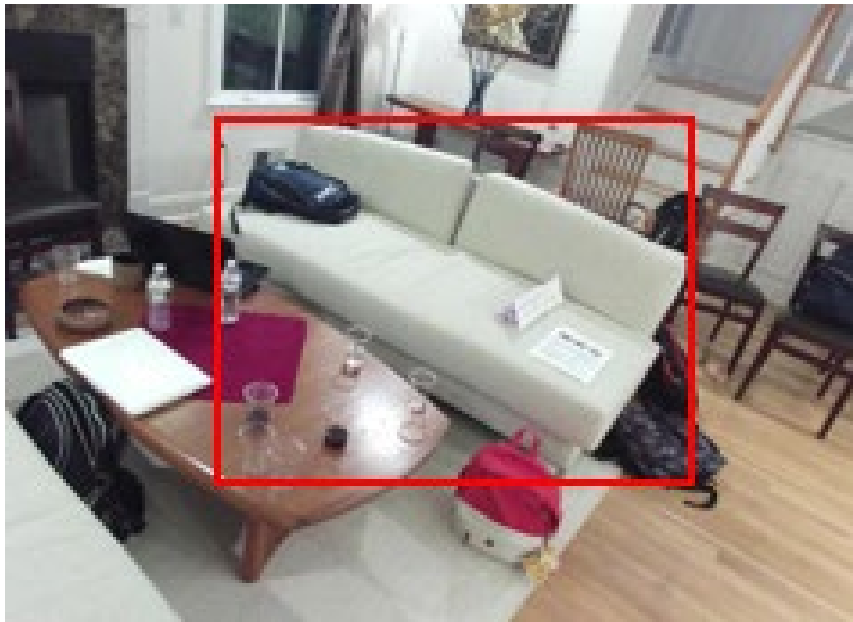
Data Has Shape



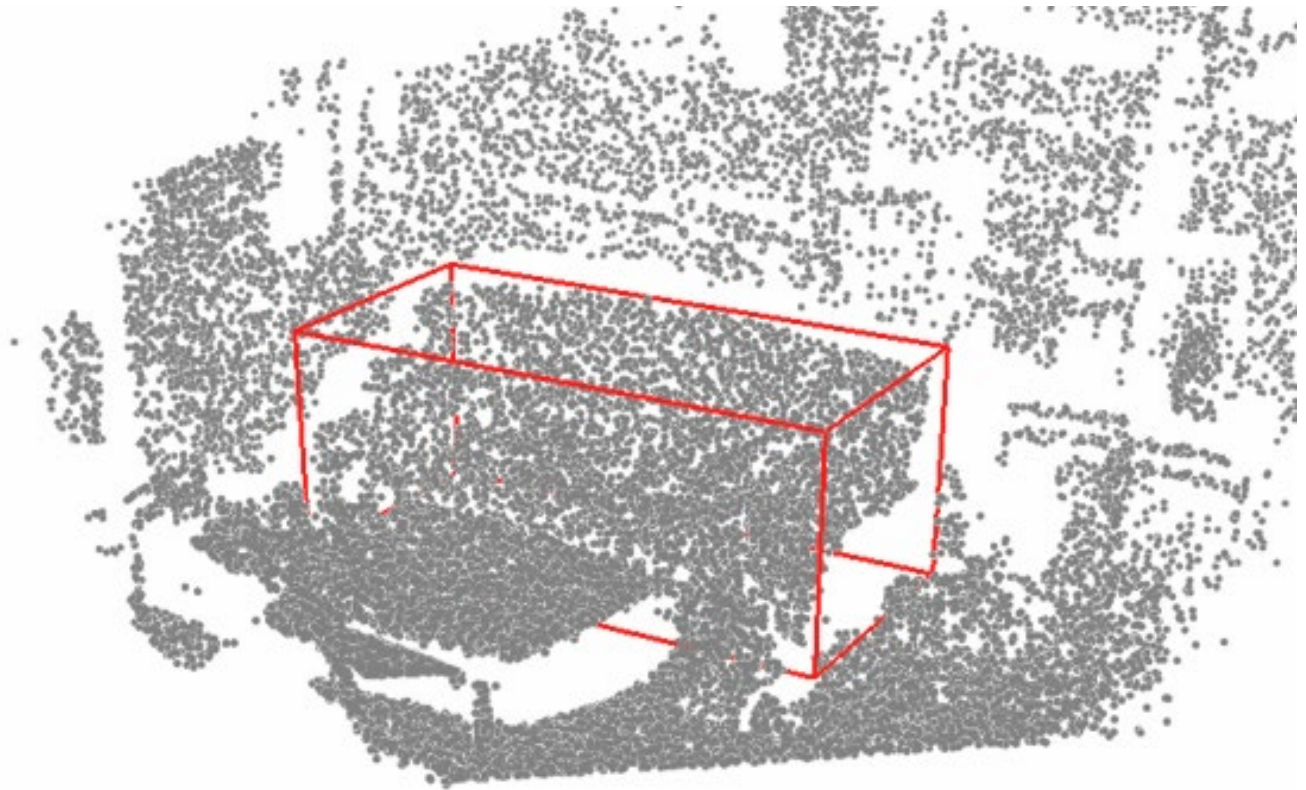
CS233 Key Course Goals

- Cover basic tools for **geometric and topological data analysis**, both supervised and unsupervised
- Discuss mathematical ways, based on geometry and topology, to **encode and transfer knowledge** about the data
- Introduce methods for **joint data analysis and joint machine learning**
– benefiting from the “wisdom of the collection”

Integrate Data Across Multiple Modalities



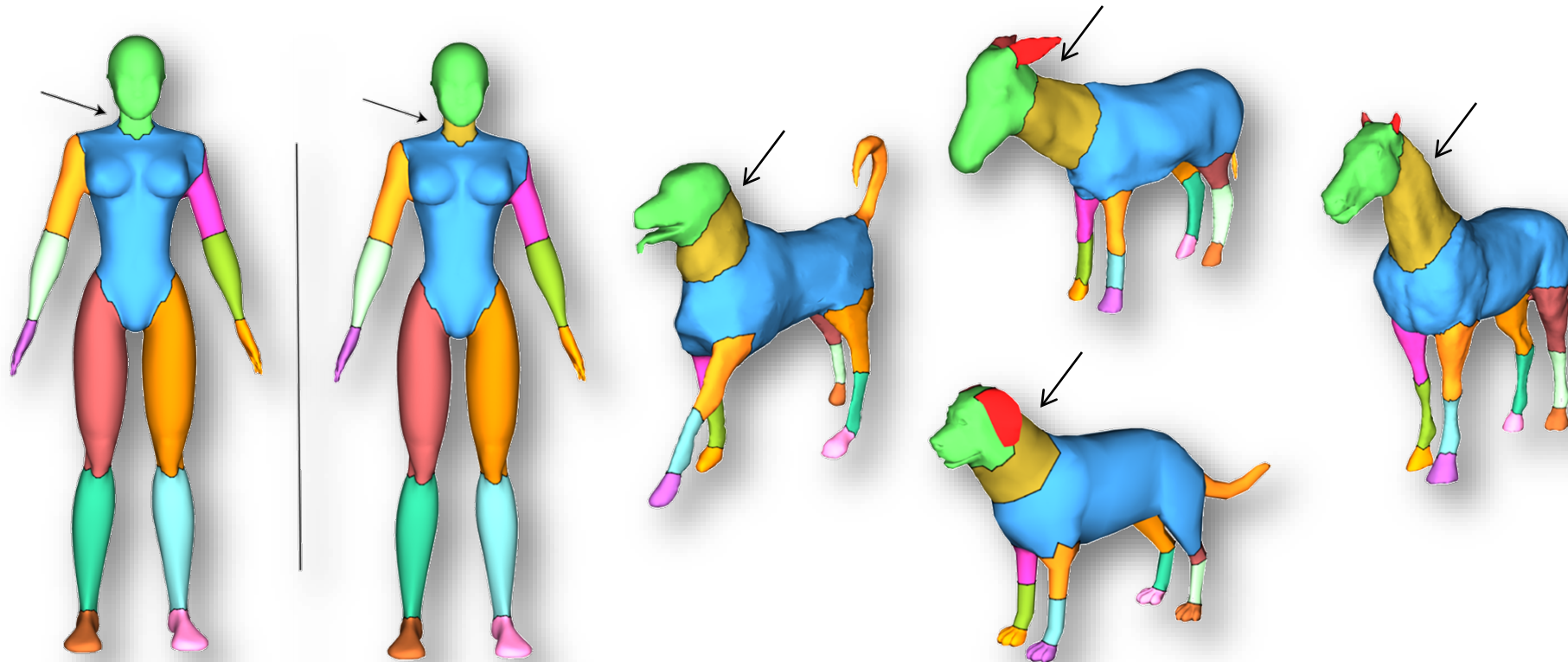
- + High resolution
- + Dense coverage
- - Subject to many imaging artifacts



- + Absolute depth and scale
- - Sparse, low rez

Each Data Set Is Not Alone

- The interpretation of a particular piece of geometric data is deeply influenced by our interpretation of other related data



3D Segmentation

Look at Data from Many View Points

- Algebraic
- Graph theory based
- Geometric
- Topological



Getting to the semantics of data

Caveats

- Relatively new course in a fast evolving area – still some rough edges, especially in the current all virtual format
- Will cover a very wide variety of techniques: LA, ML, Stat, optimization, geometry processing, computer vision, algebraic topology ...

geometric data analysis

- Data sets will be primarily visual/geometric (images, 2D/3D point clouds, meshes, or CAD models)

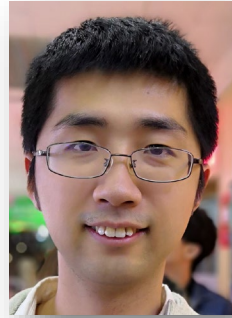
Prerequisites / Overlaps

- Presumes some knowledge of linear algebra, optimization, algorithmic thinking, basic geometry ...
- As compared to CS229, STATS216, much greater emphasis on diverse, irregular data types (e.g., point clouds, graphs) and unsupervised methods, in addition to the geometry / topology angle

The Team

The Principals

- Leonidas (Leo) Guibas (CS & EE)
 - Instructor
- Davis Rempe (CS)
 - Course Assistant
- Yueqi Duan (CS)
 - Course Assistant
- Kaichun Mo (CS)
 - Guest lecturer
- Samir Chowdhury (Neuropsychiatry)
 - Guest lecturer
- Carrie Petersen (CS)
 - Admin



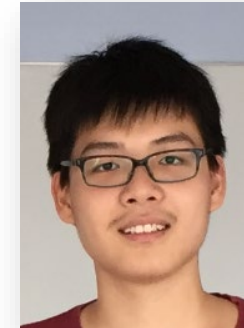
Yueqi



Davis



Leo



Kaichun



Samir



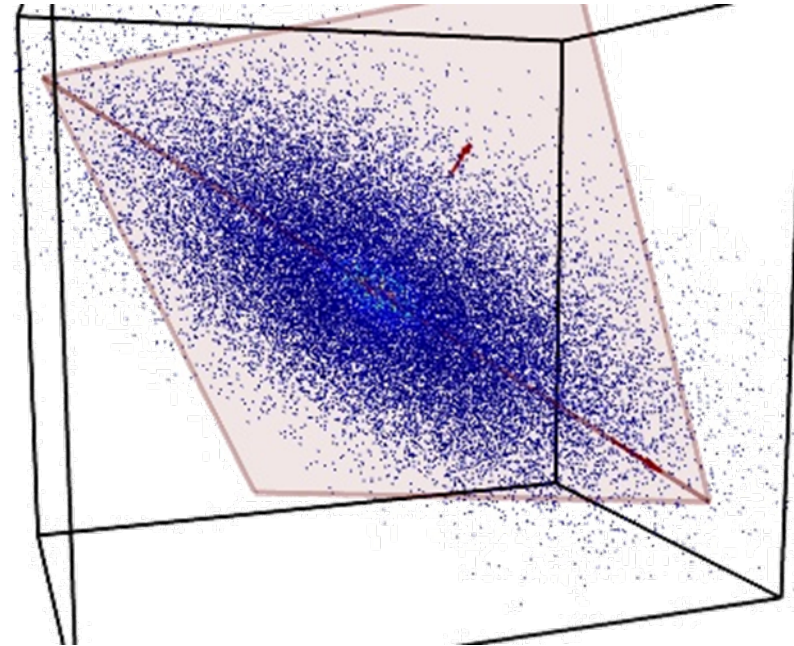
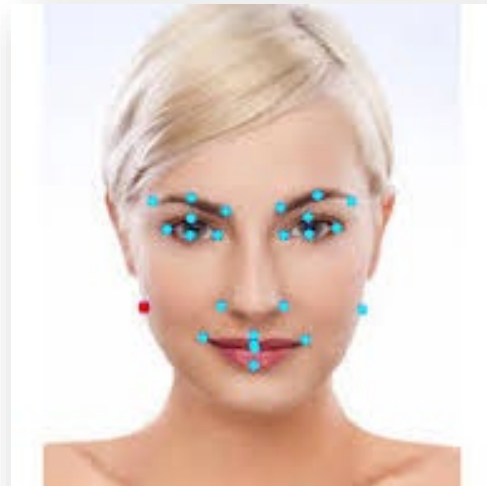
Carrie

Unsupervised Methods

The Linear Space View of Data

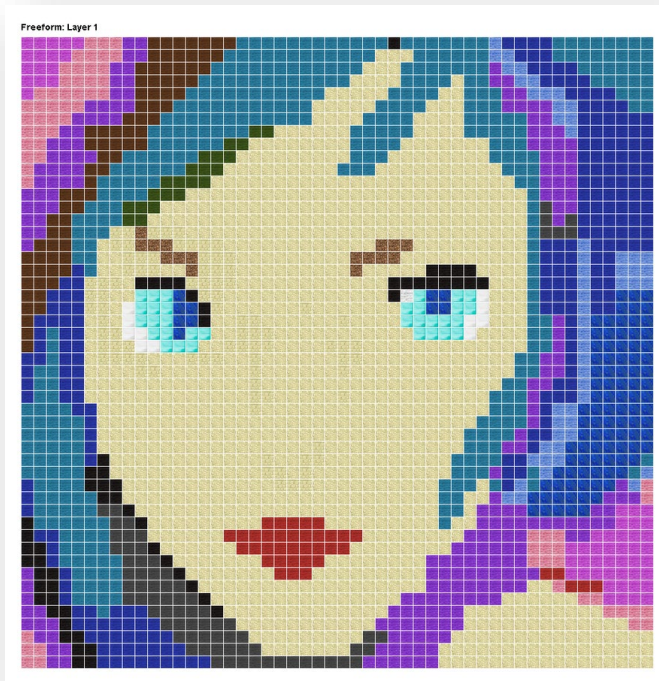
Embedding Data into a Euclidean Space

- Attributes or features can be used to map data to a Euclidean space

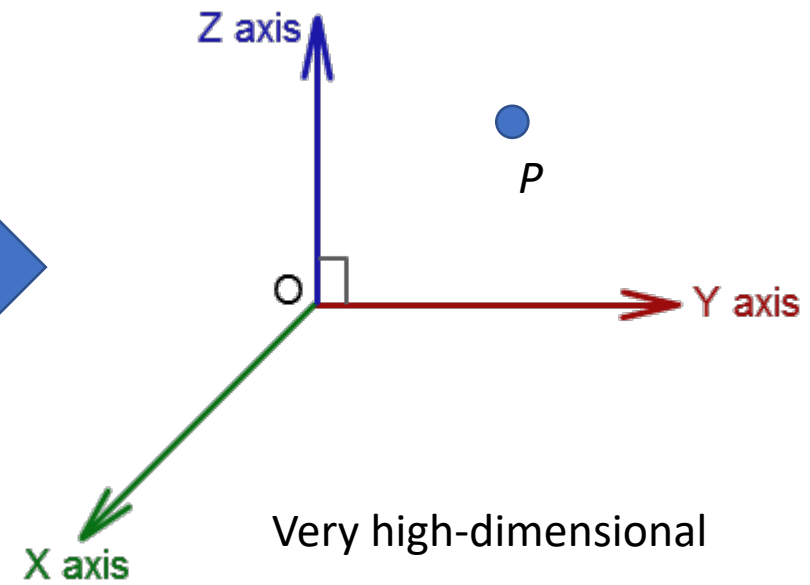


Direct Embeddings

◆ Input



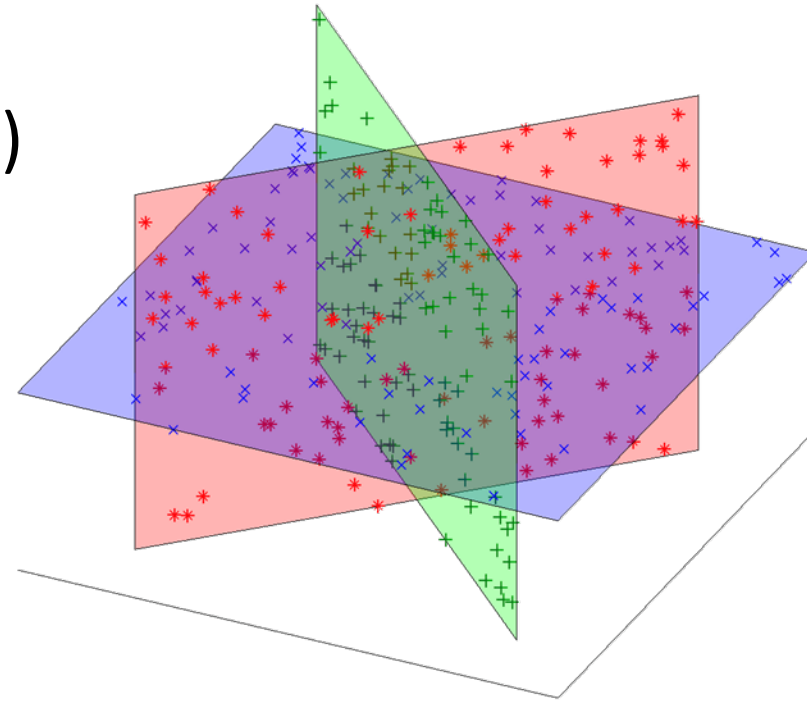
◆ Latent code



Linear Space Methods

- Principal components analysis (PCA)
- Canonical correlation analysis (CCA)

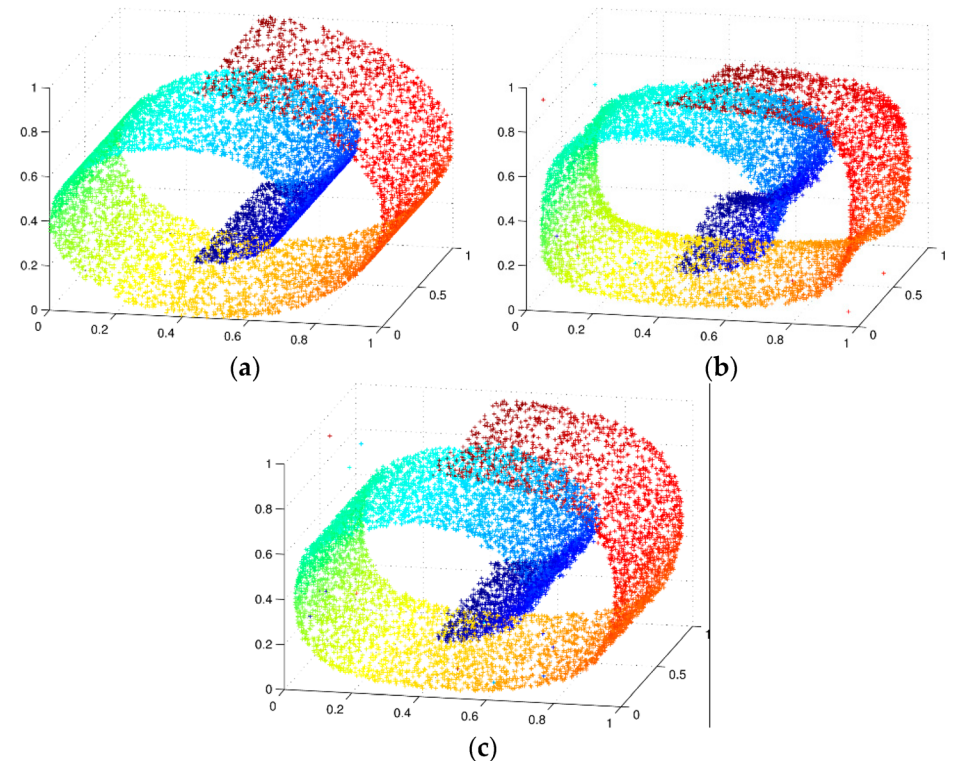
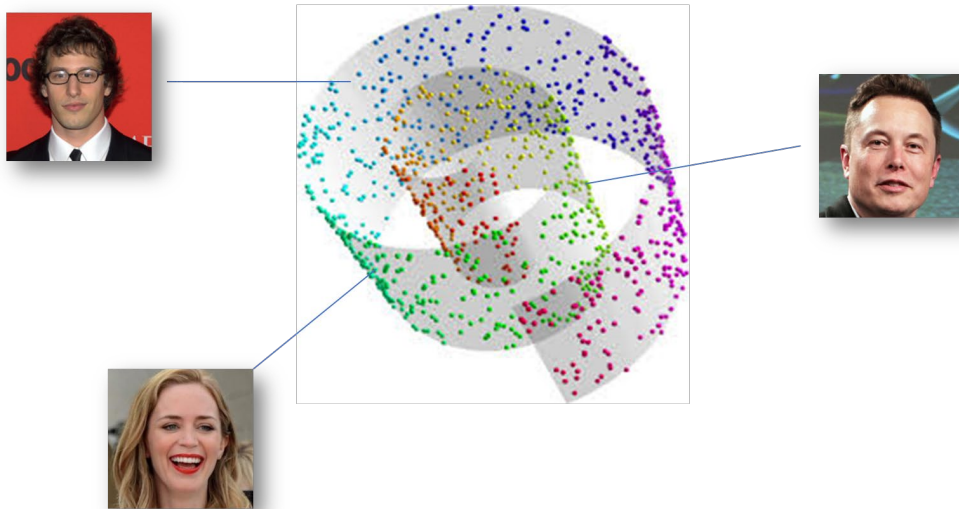
Dimensionality reduction



Low-d data (intrinsic dimension) living on a linear subspace,
inside a high-d space (extrinsic dimension)

Data as Points on a Manifold

- Non-linear dimensionality reduction
- Low-d data inside high-d space may lie on a non-flat manifold

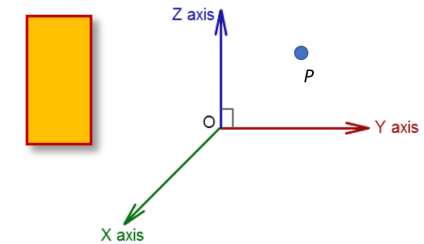
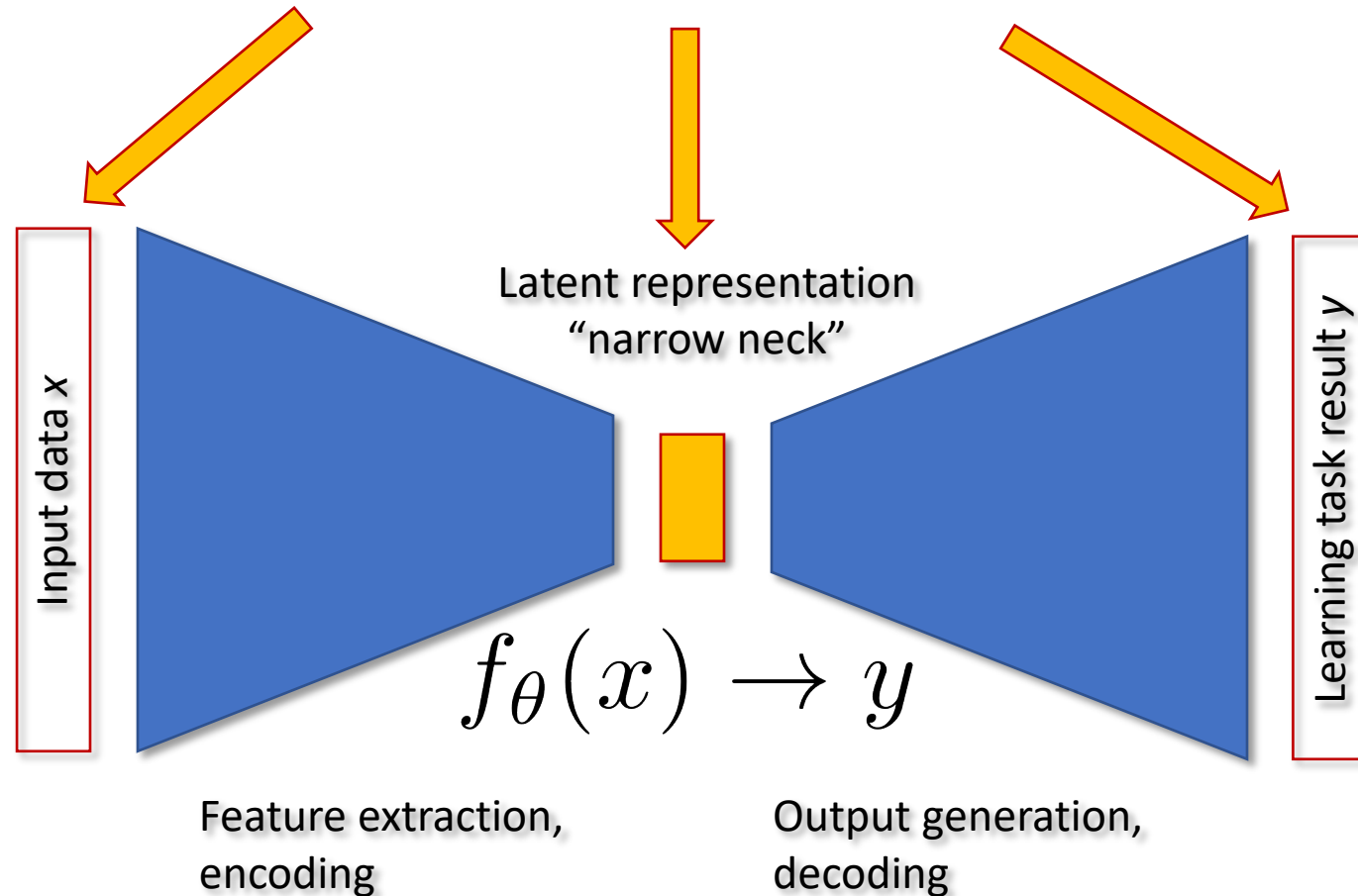


Isomap, locally linear embeddings, Laplacian eigenmaps, t-SNE

Supervised Methods

Deep Learning

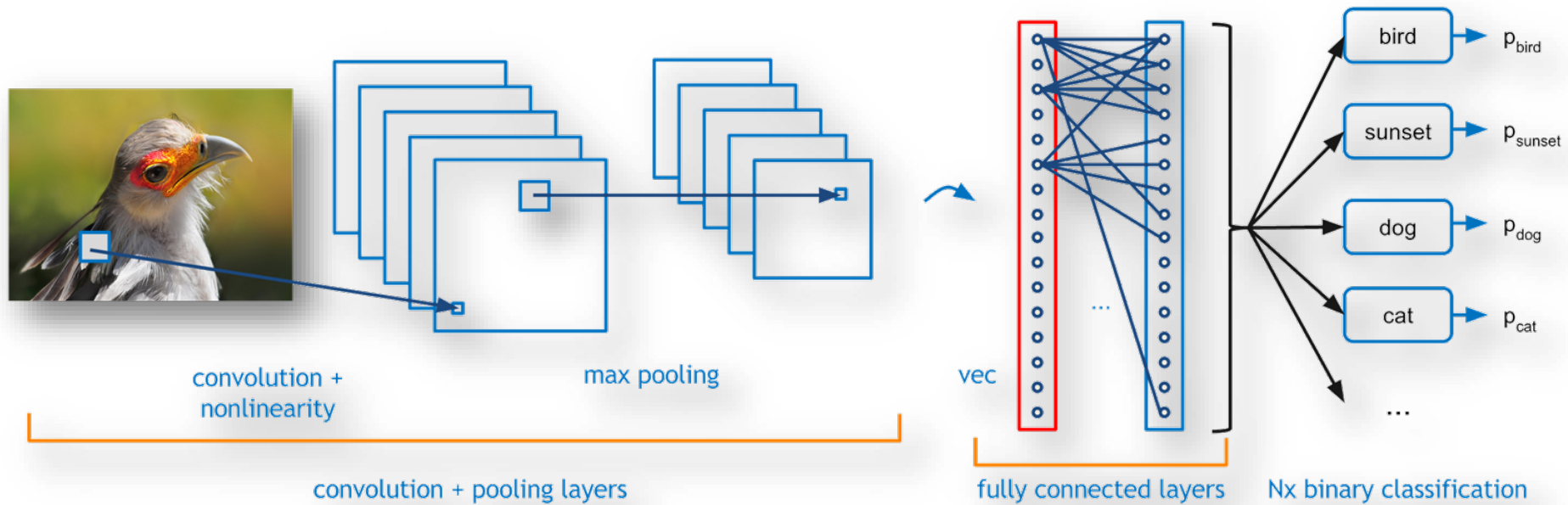
Trainable Feed-Forward Learning Pipelines



A latent code (= a point in a Euclidean feature space) acts as a low-d proxy for high-d input data, w.r.t. a learning task.

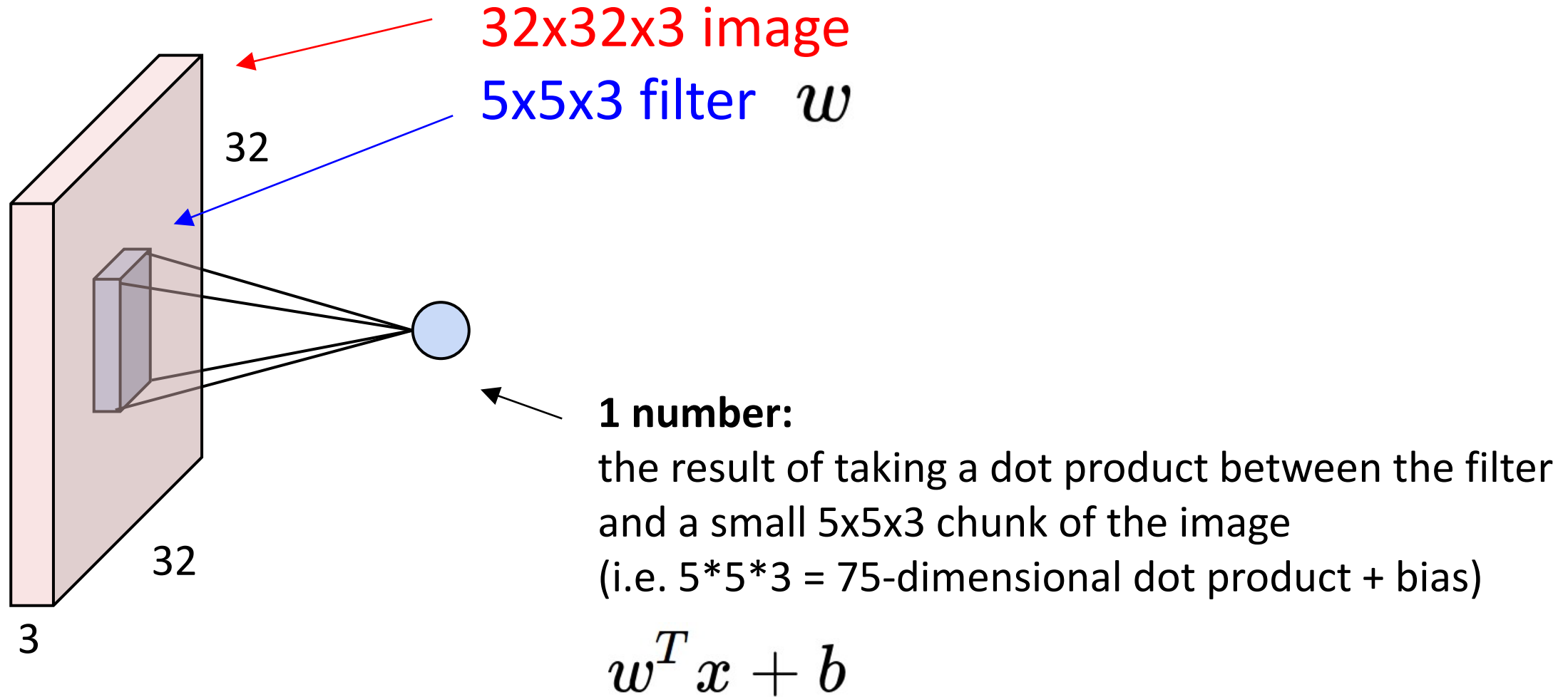
Image Deep Learning Networks

- Feed forward networks

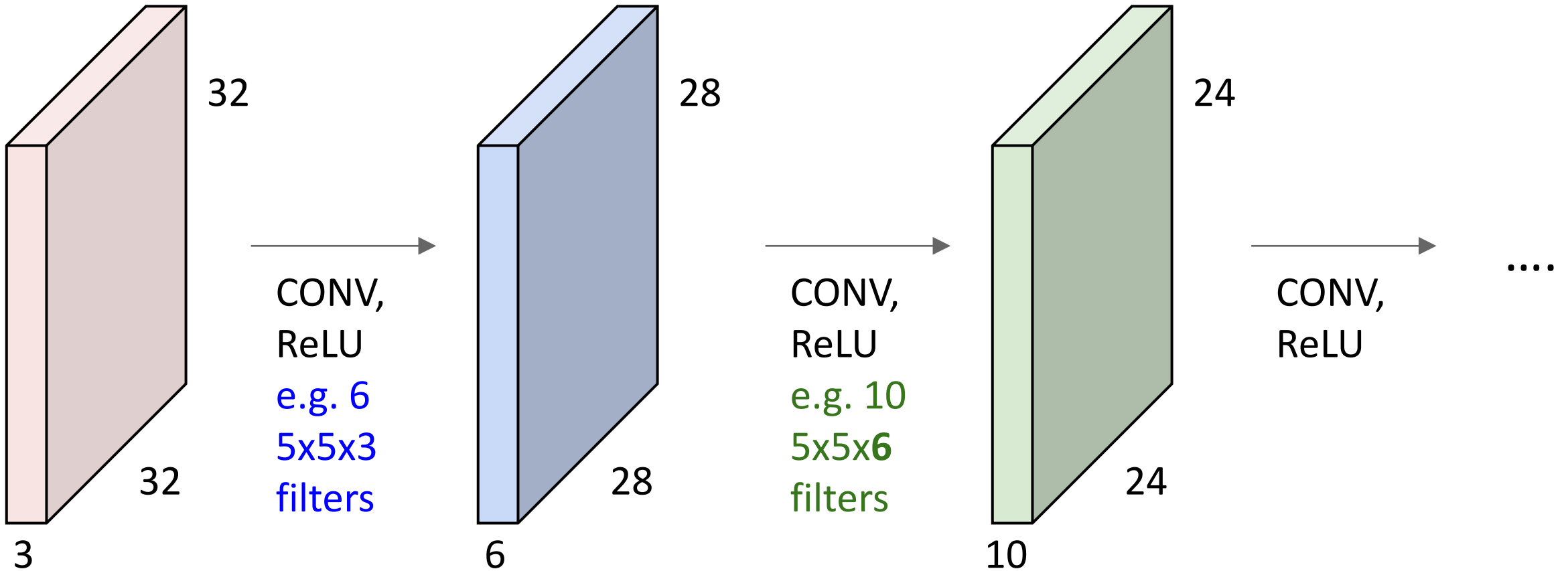


Classification

Convolutional Layers



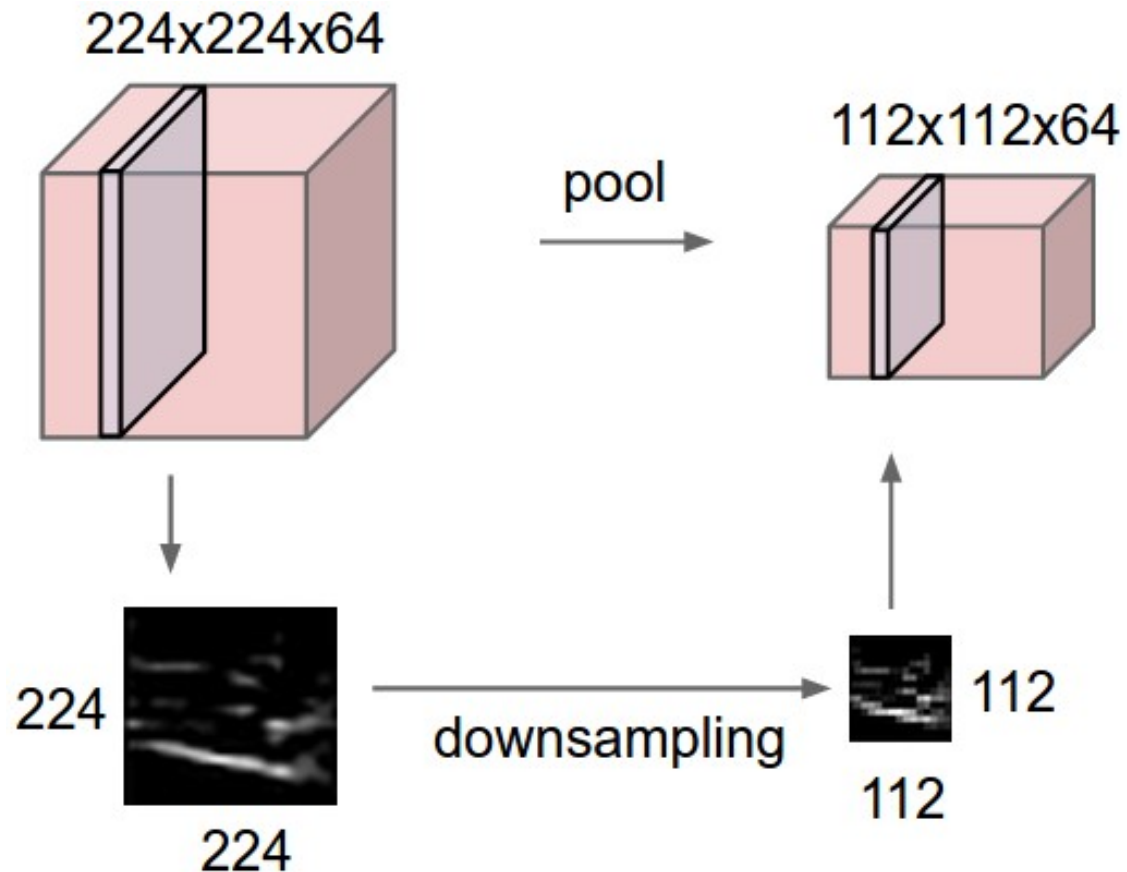
Convolutional Architectures



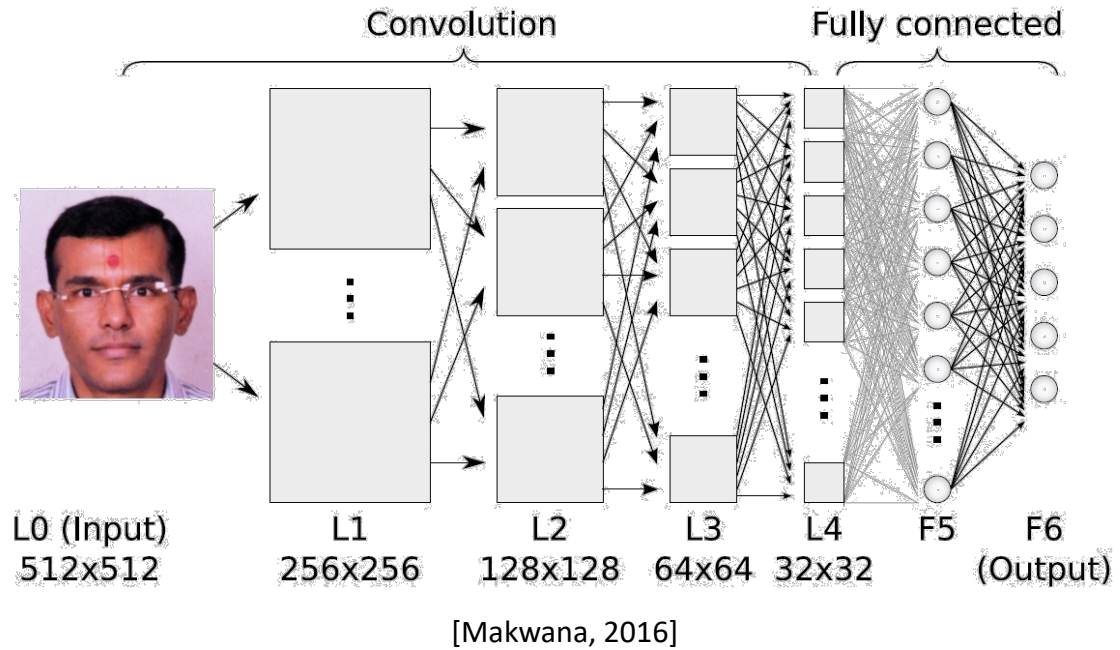
ConvNets are a sequence of convolutional layers, interspersed with activation functions

Pooling Layer

- makes the representations smaller and more manageable
- operates over each activation map independently:



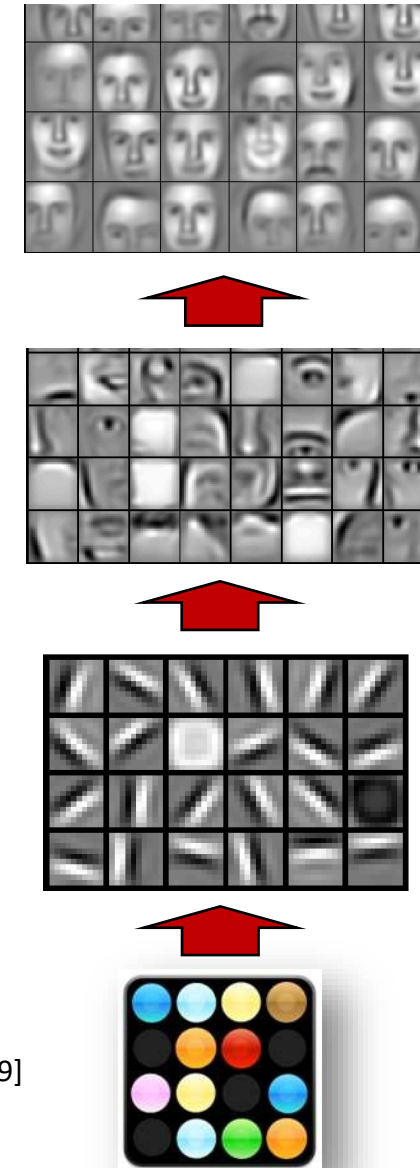
Data Abstraction Across Layers



Data-driven feature learning at ascending abstraction layers

“Vertical” networks

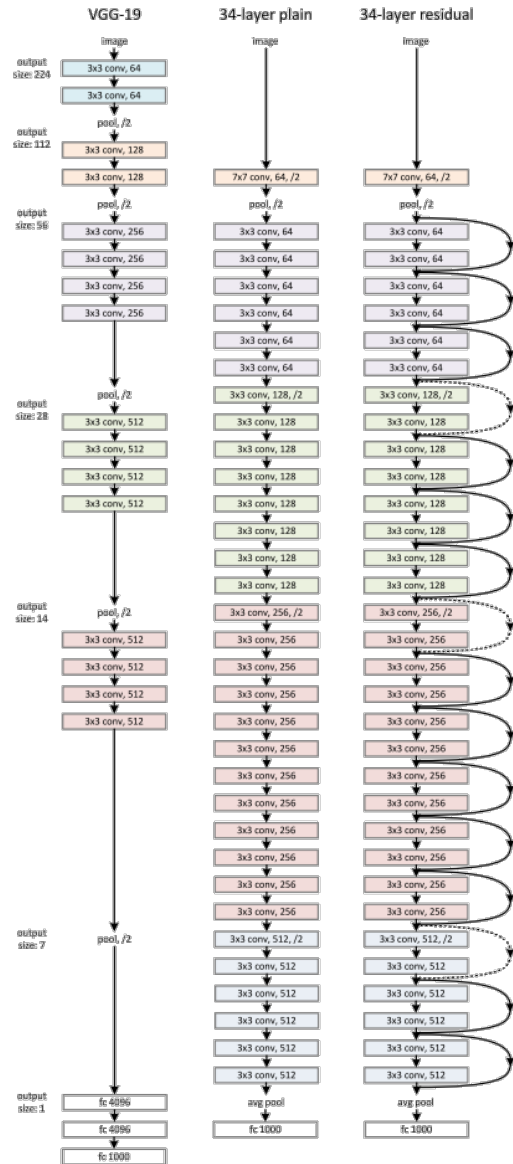
[Lee et al., 2009]



Success Made Possible By

[He et al., 2015]

Novel deep architectures



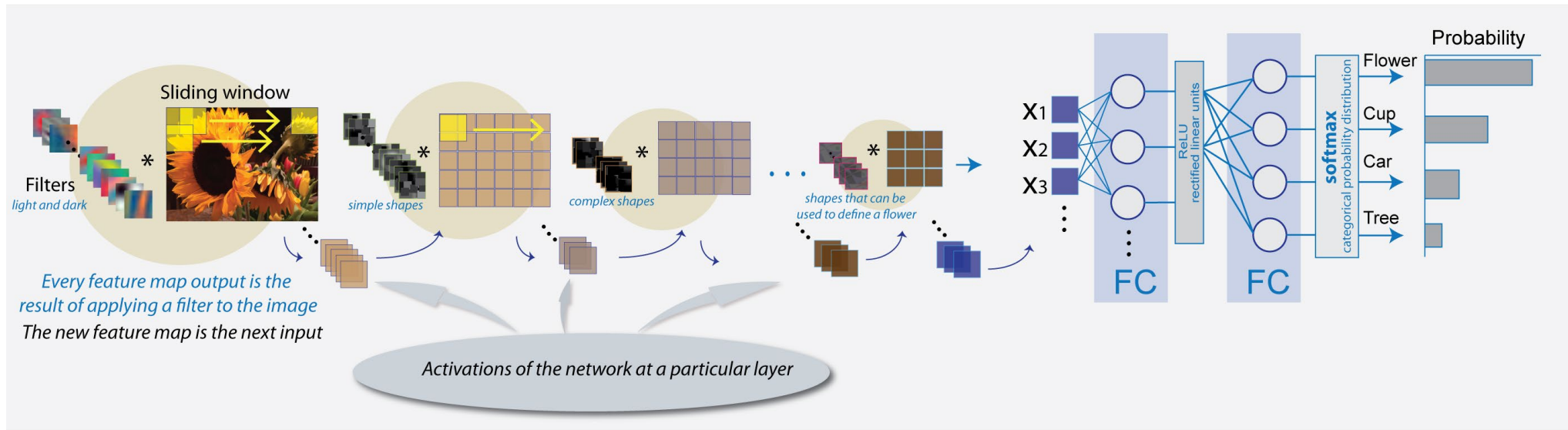
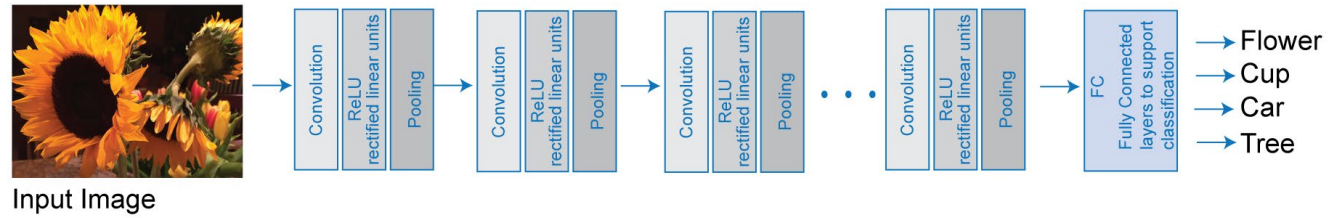
Plenty of annotated data

Lots of computing power



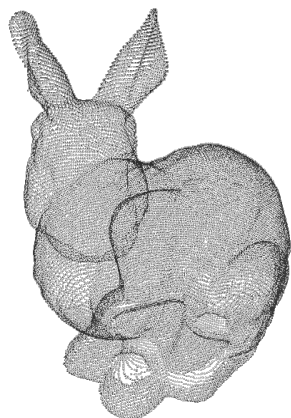
Learning on Irregular Data

Convolutional Networks

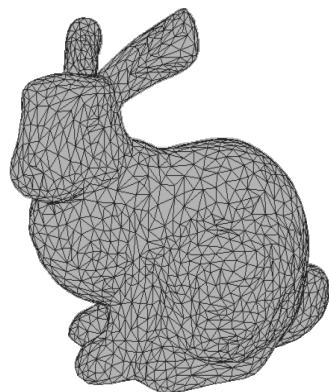


Convolutions require regular array type inputs for weight sharing and other optimizations

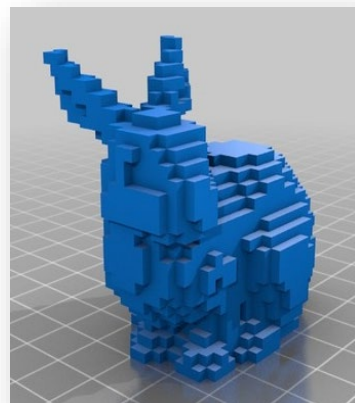
Multiple Representations for 3D Geometric Data



Point Cloud



Mesh



Volumetric



Projected View

RGB(D)

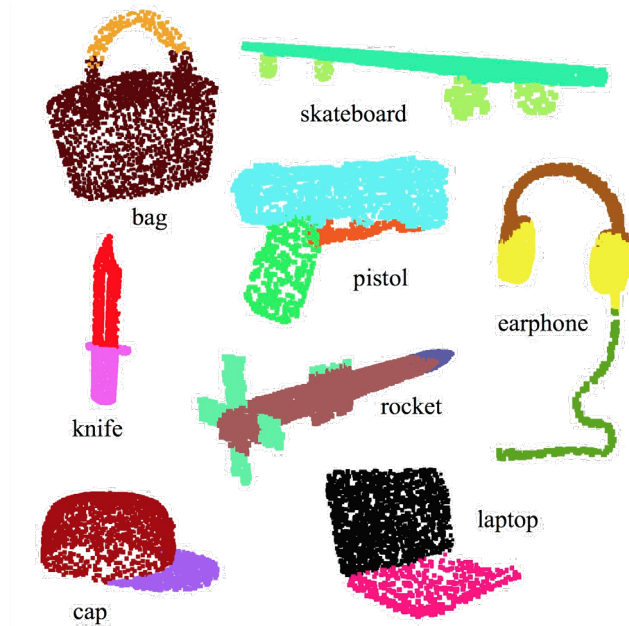
...

These are irregular
representations

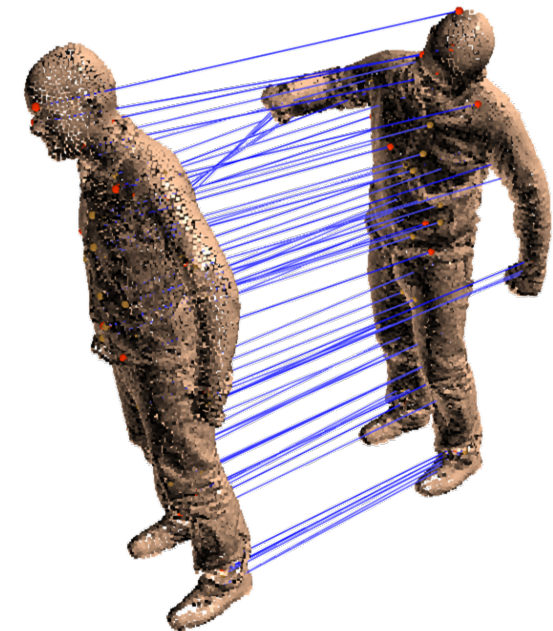
Deep Learning for 3D Geometry Analysis



Classification



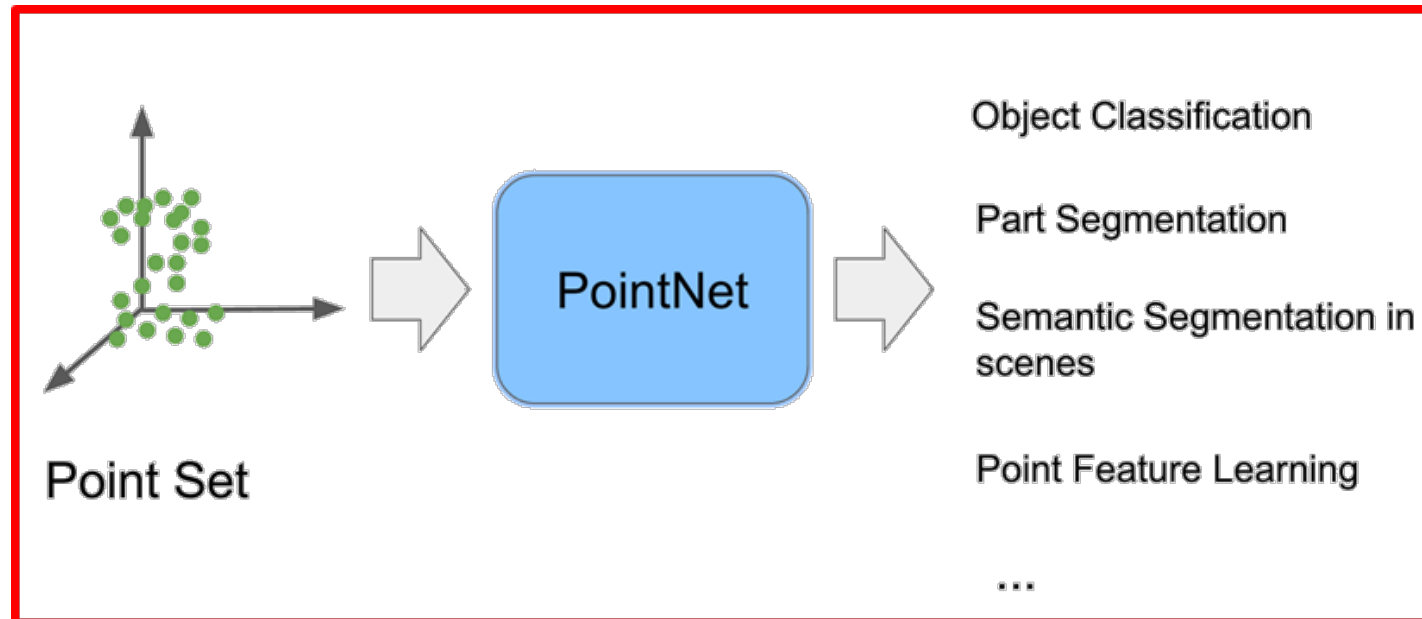
Segmentation/Parsing
(object/scene)



Correspondences

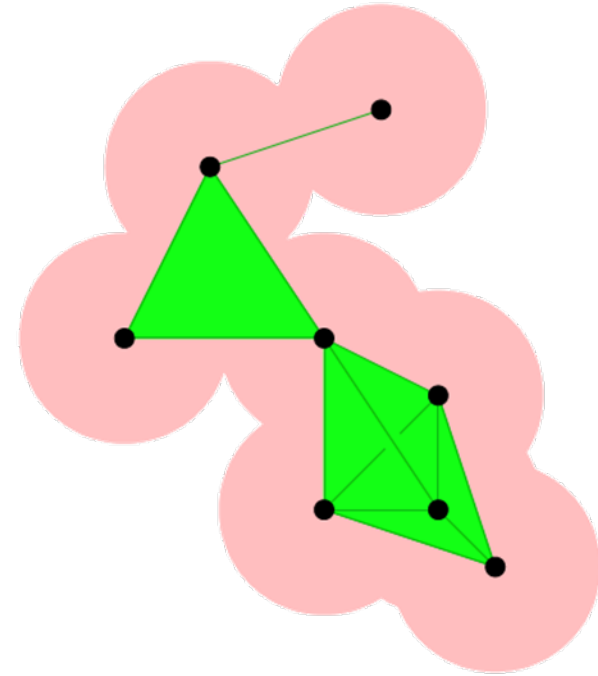
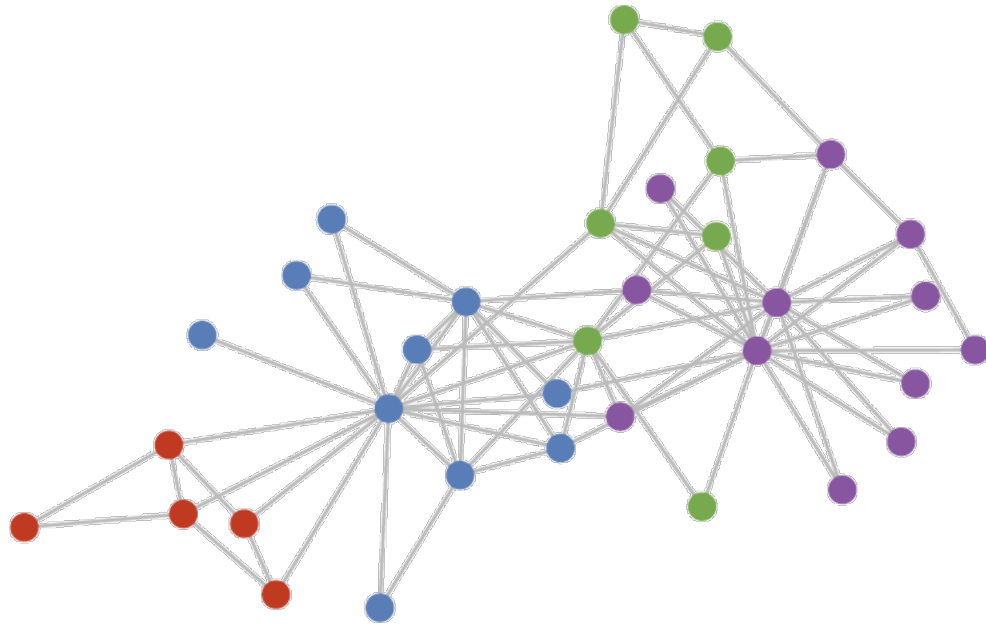
PointNet: Working Directly with Point Cloud Data

- Goal: design a NN architecture that can work directly with point clouds
- Must deal with unstructured, unordered data

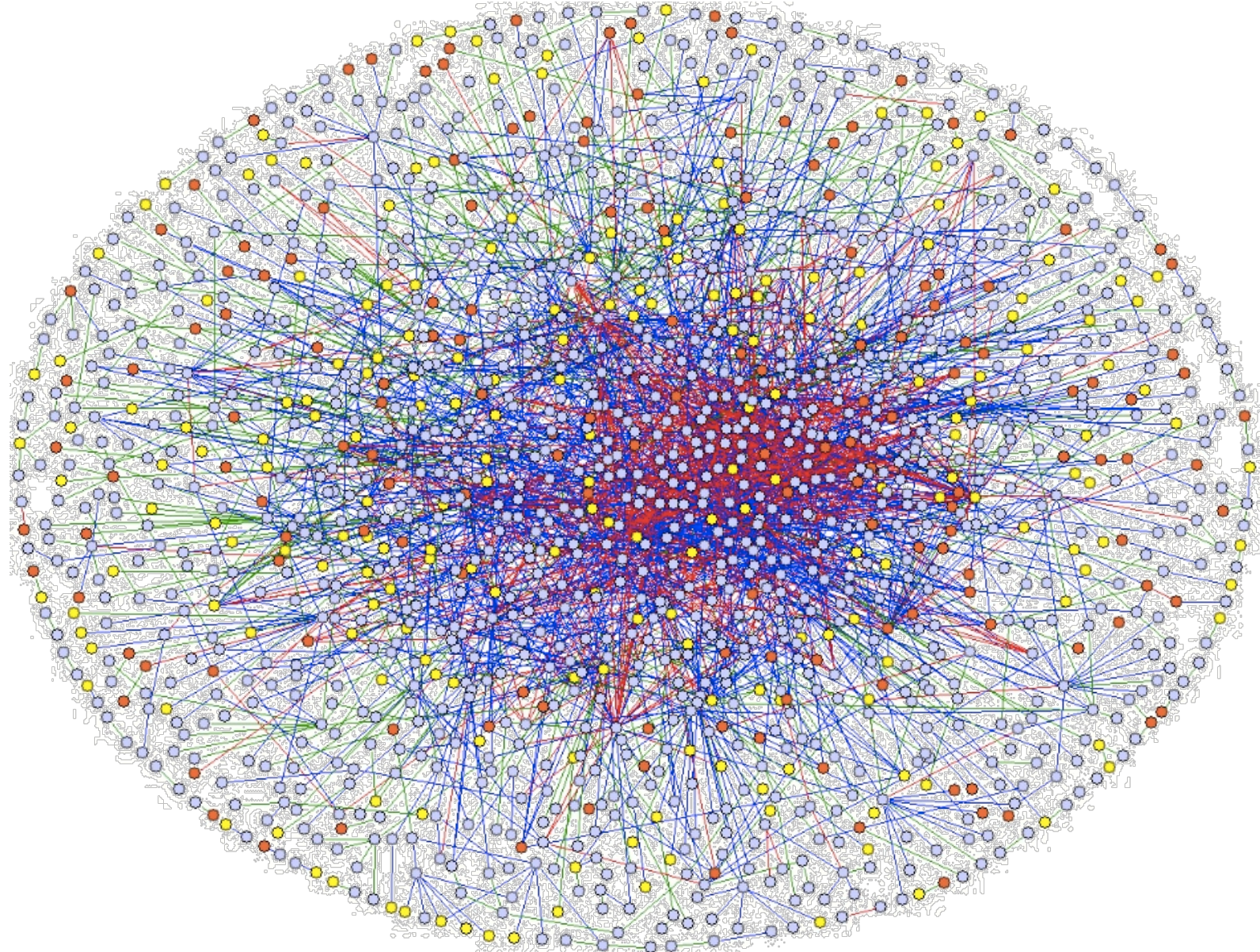


The Graph View of Data

Graphs and Simplicial Complexes

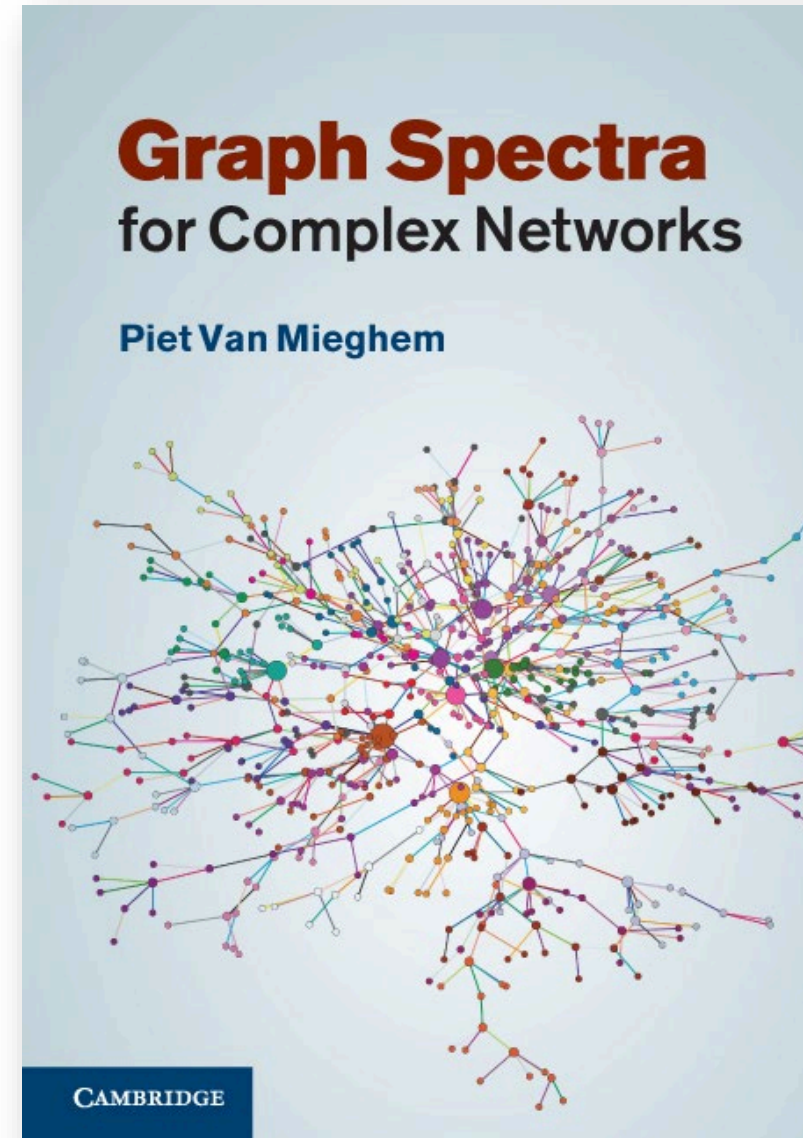


A Graph View of Data



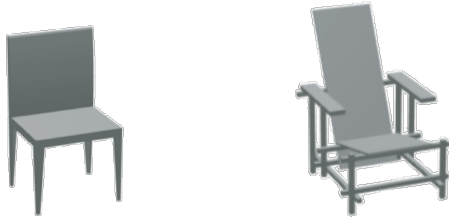
Spectral Methods in Graph Theory

- Linking the graph-theoretic and linear algebraic view of data.

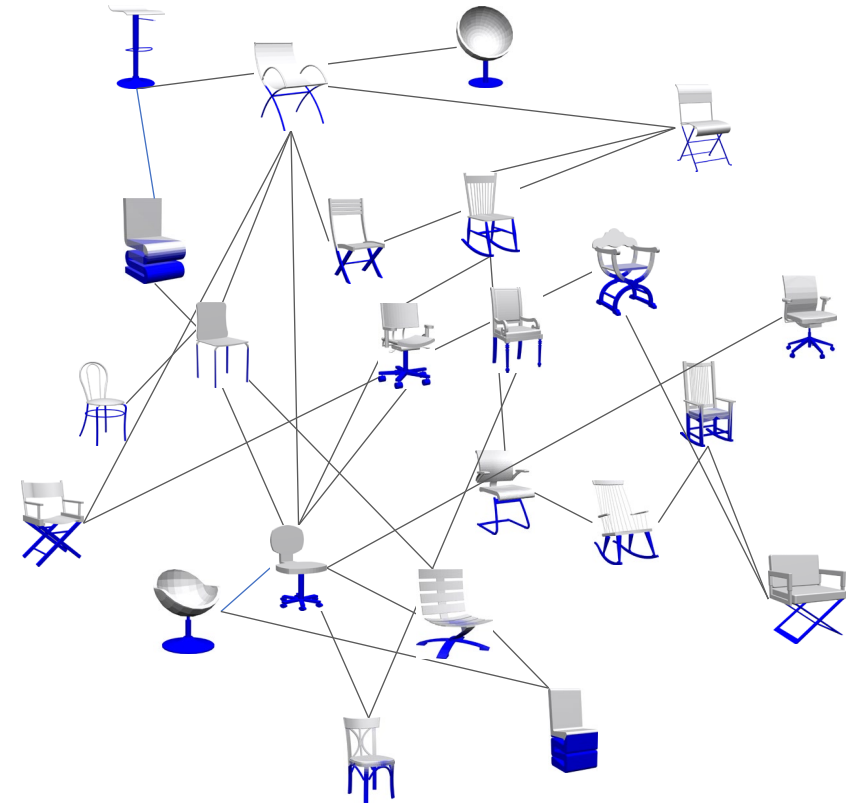


From Feature Vectors to Feature Sets

- What to do if the number of features per data set is variable?
 - “simple” vs “complex” objects



- Distance or similarity metrics
- Multidimensional scaling

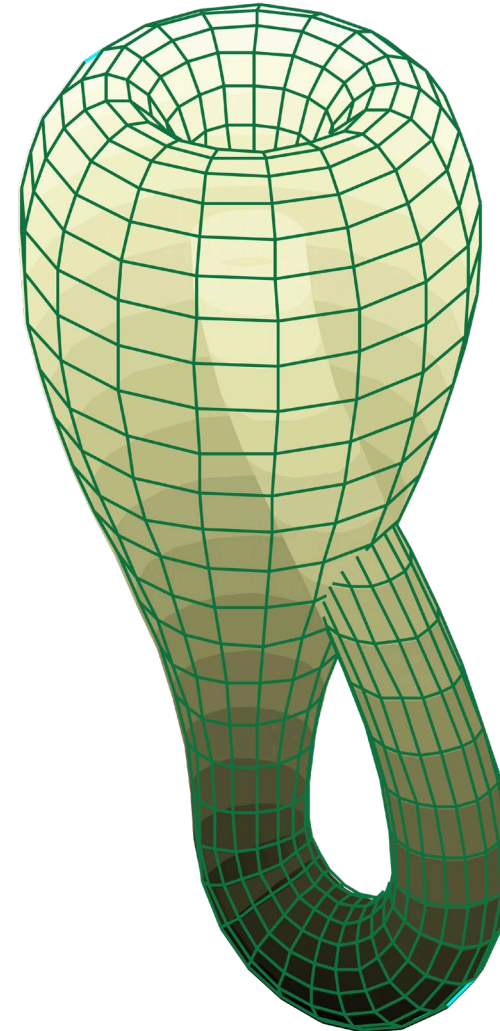


A “Procrustean” approach

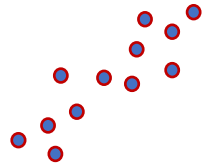
Topological Data Analysis (TDA)

Computational Topology

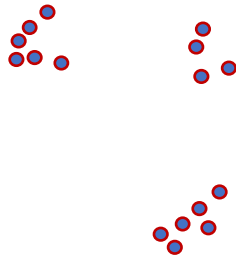
- Topology is the branch of mathematics that does not take distances too seriously. G. Carlsson
- Large distances (aka “similarity metrics” are often suspect ...



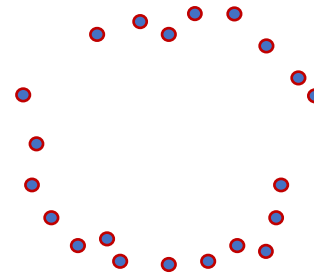
In TDA, Sampled Spaces: “The Shape of Data”



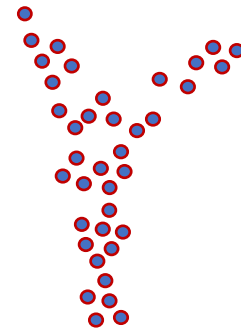
Regression



Cluster



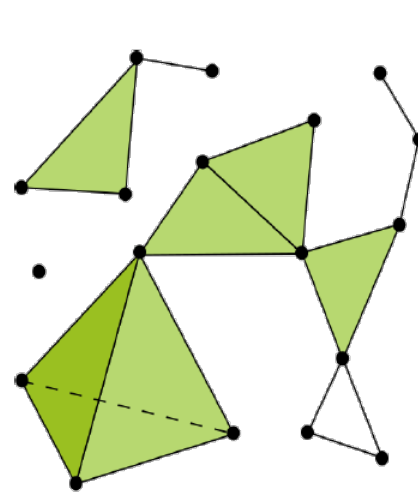
Loop



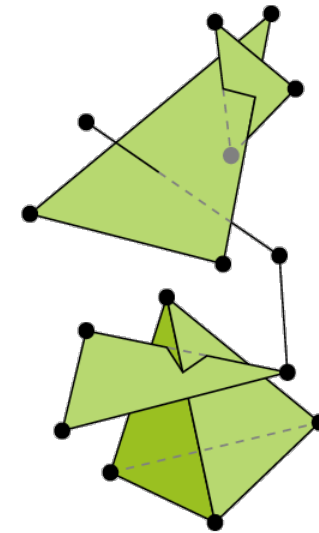
Flared

Simplicial Complexes

- A **simplicial complex** K is a finite set of simplices such that
 1. $\sigma \in K, \tau \leq \sigma \Rightarrow \tau \in K$,
 2. $\sigma, \sigma' \in K \Rightarrow \sigma \cap \sigma' \leq \sigma, \sigma'$ or $\sigma \cap \sigma' = \emptyset$.
- The **dimension** of K is $\dim K = \max\{\dim \sigma \mid \sigma \in K\}$.
- The **vertices** of K are the zero-simplices in K .
- A simplex is **principal** if it has no proper coface in K .

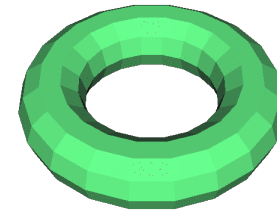
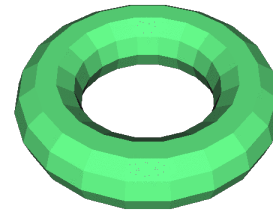
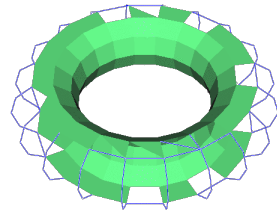
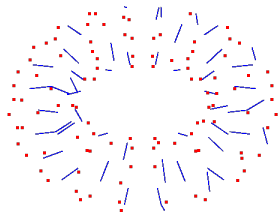
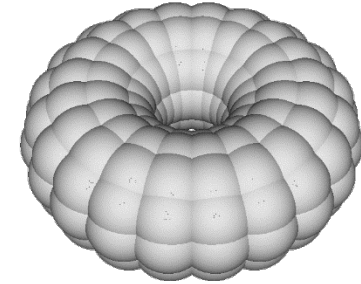
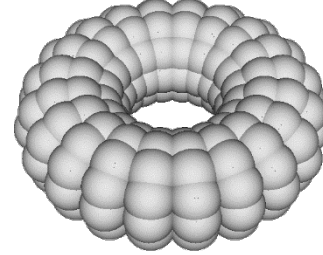
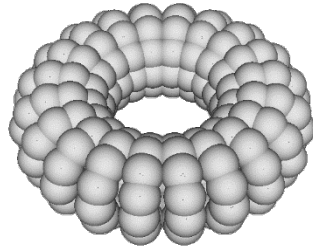
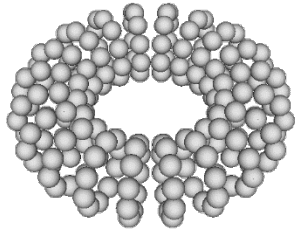


(left) an example



(right) a non example

A Multiscale View of Data

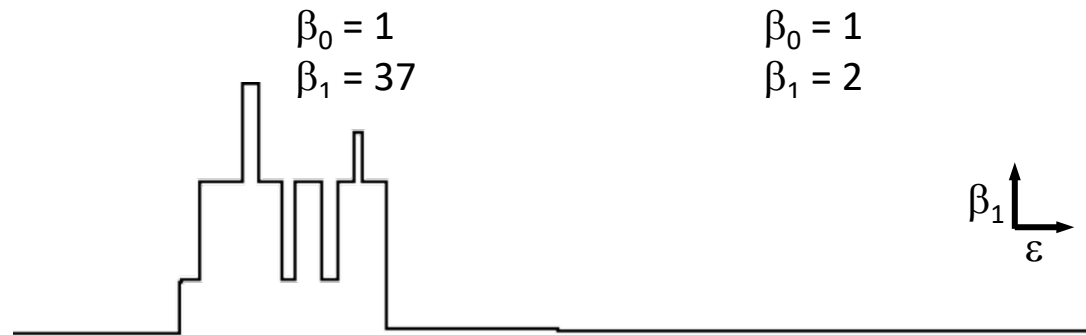


$\beta_0 = 150$
 $\beta_1 = 0$

$\beta_0 = 1$
 $\beta_1 = 37$

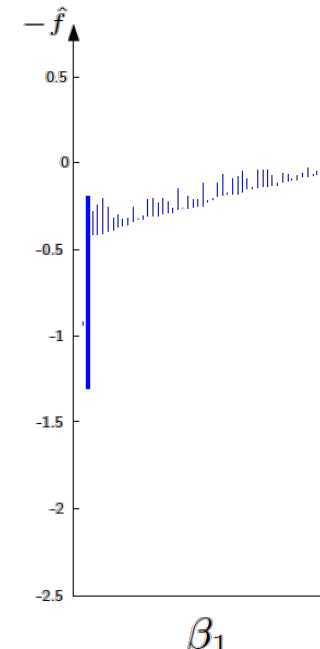
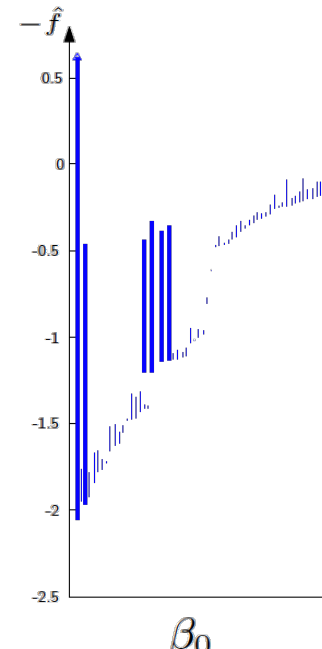
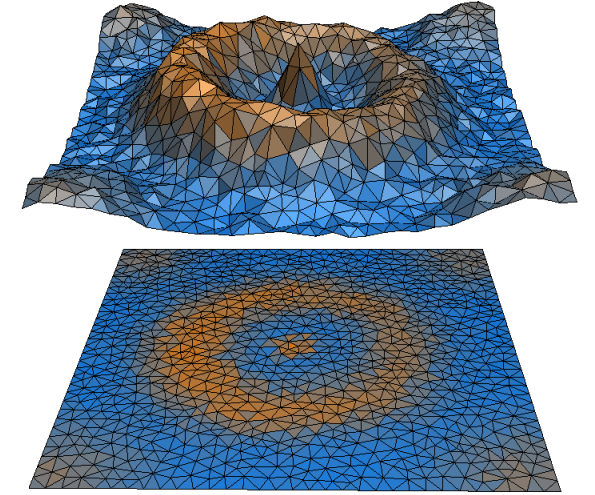
$\beta_0 = 1$
 $\beta_1 = 2$

$\beta_0 = 1$
 $\beta_1 = 1$

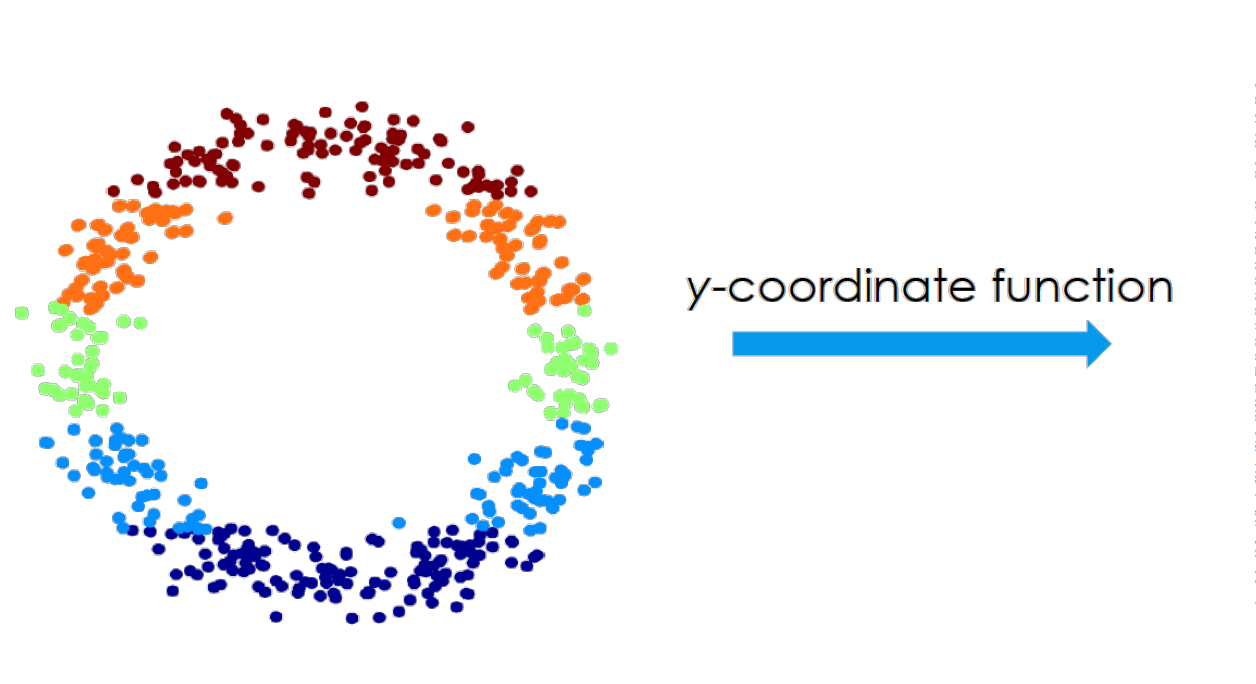


Persistent Homology

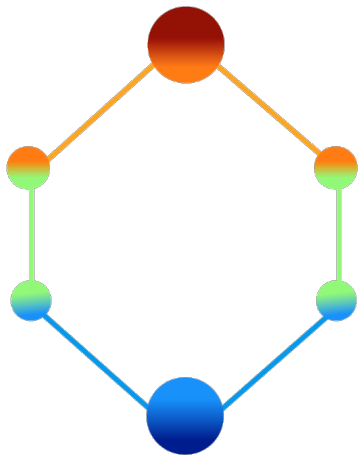
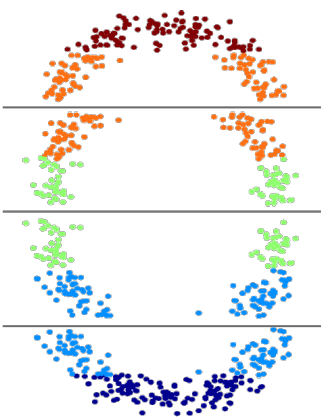
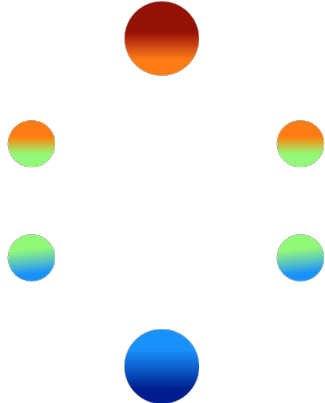
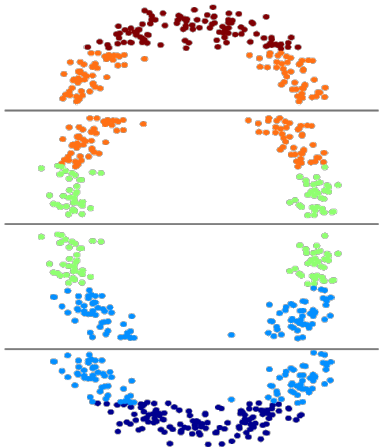
- Persistence of topological features
- Barcodes and persistence diagrams
- Data embeddings into non_Euclidean spaces



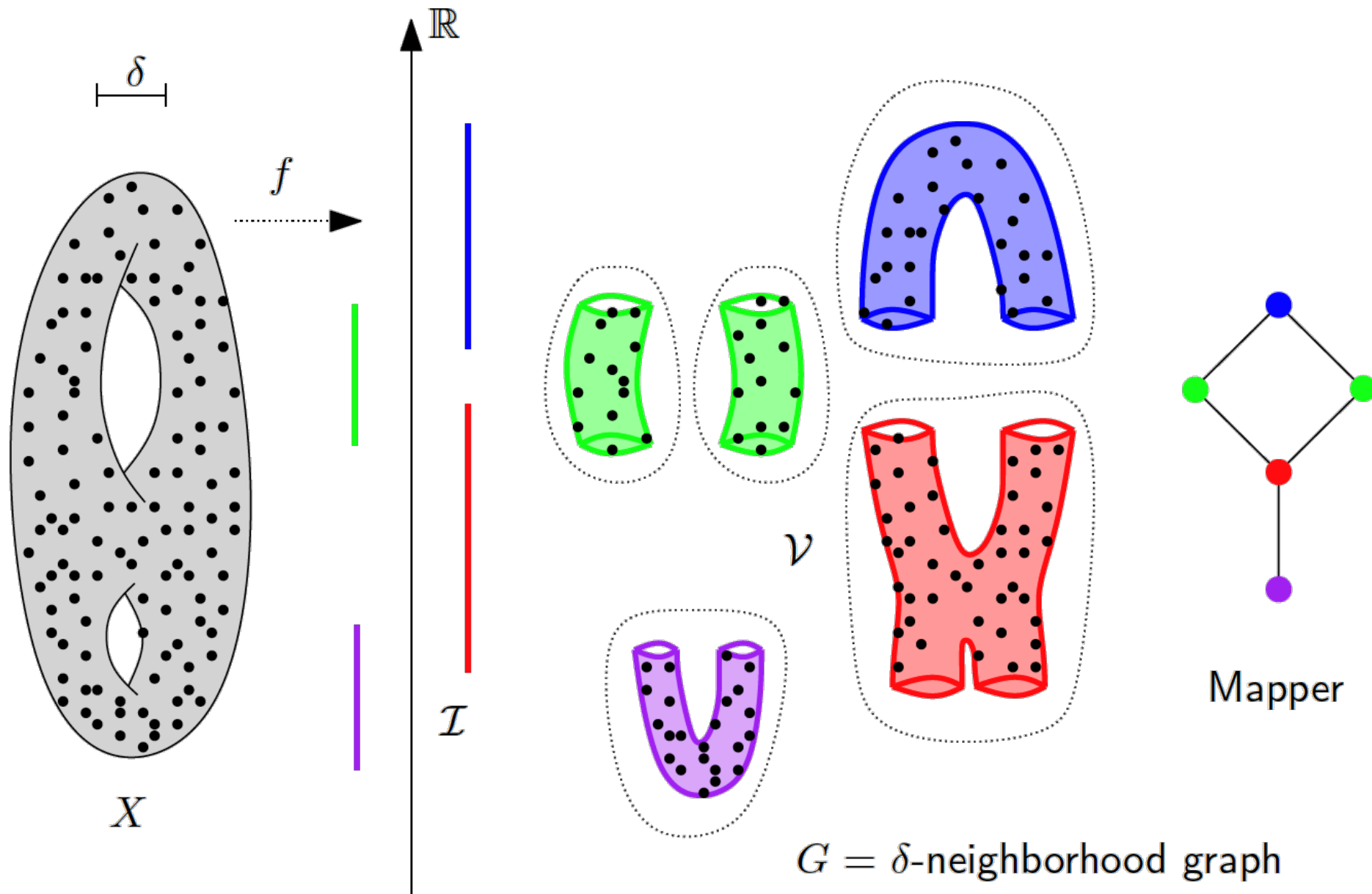
Functions on the Data as Filters



A Simplicial Complex

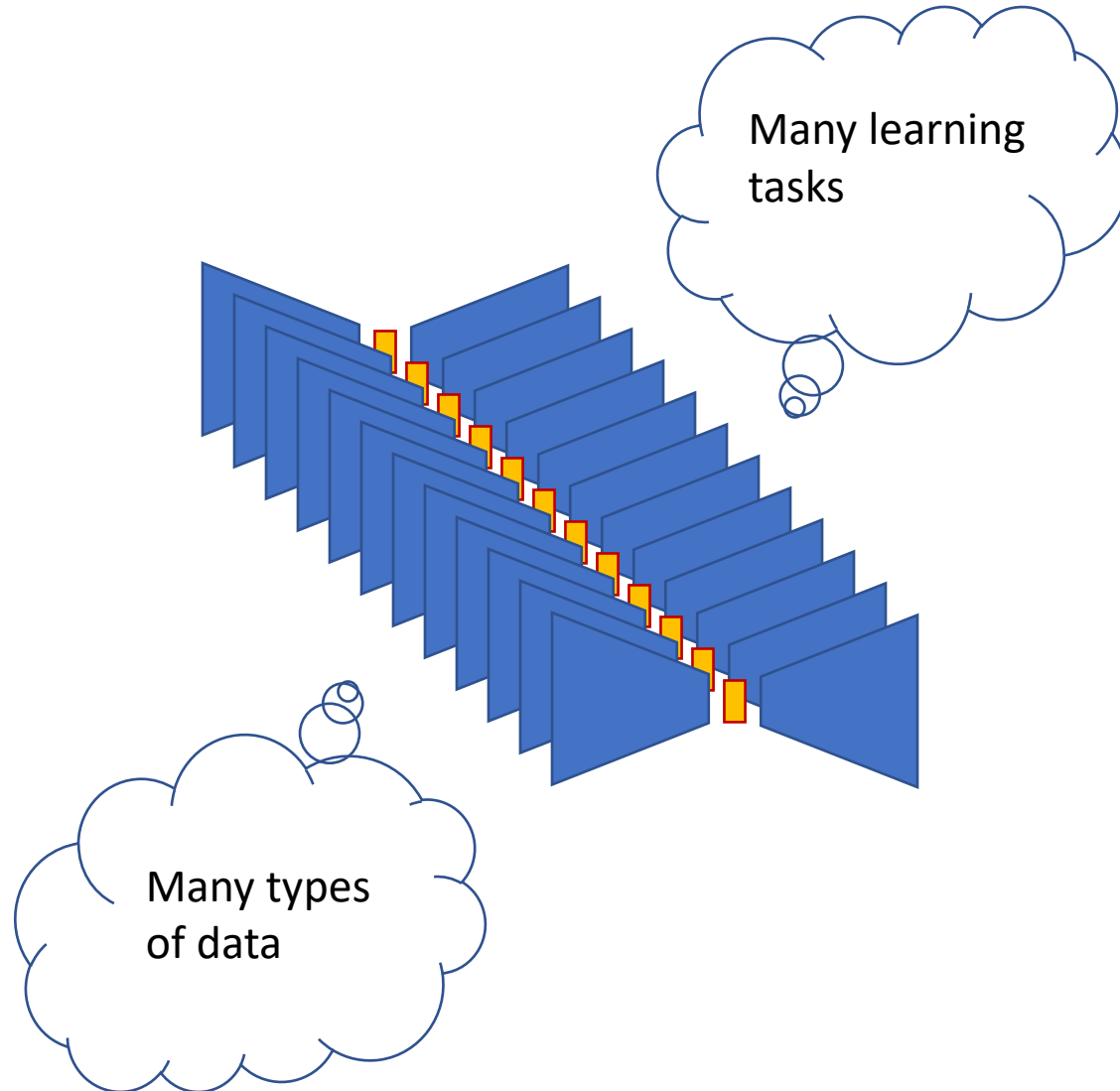


Functions as Lenses on Data



Data Representations and Latent Spaces

(Too) Many Latent Spaces



Codes

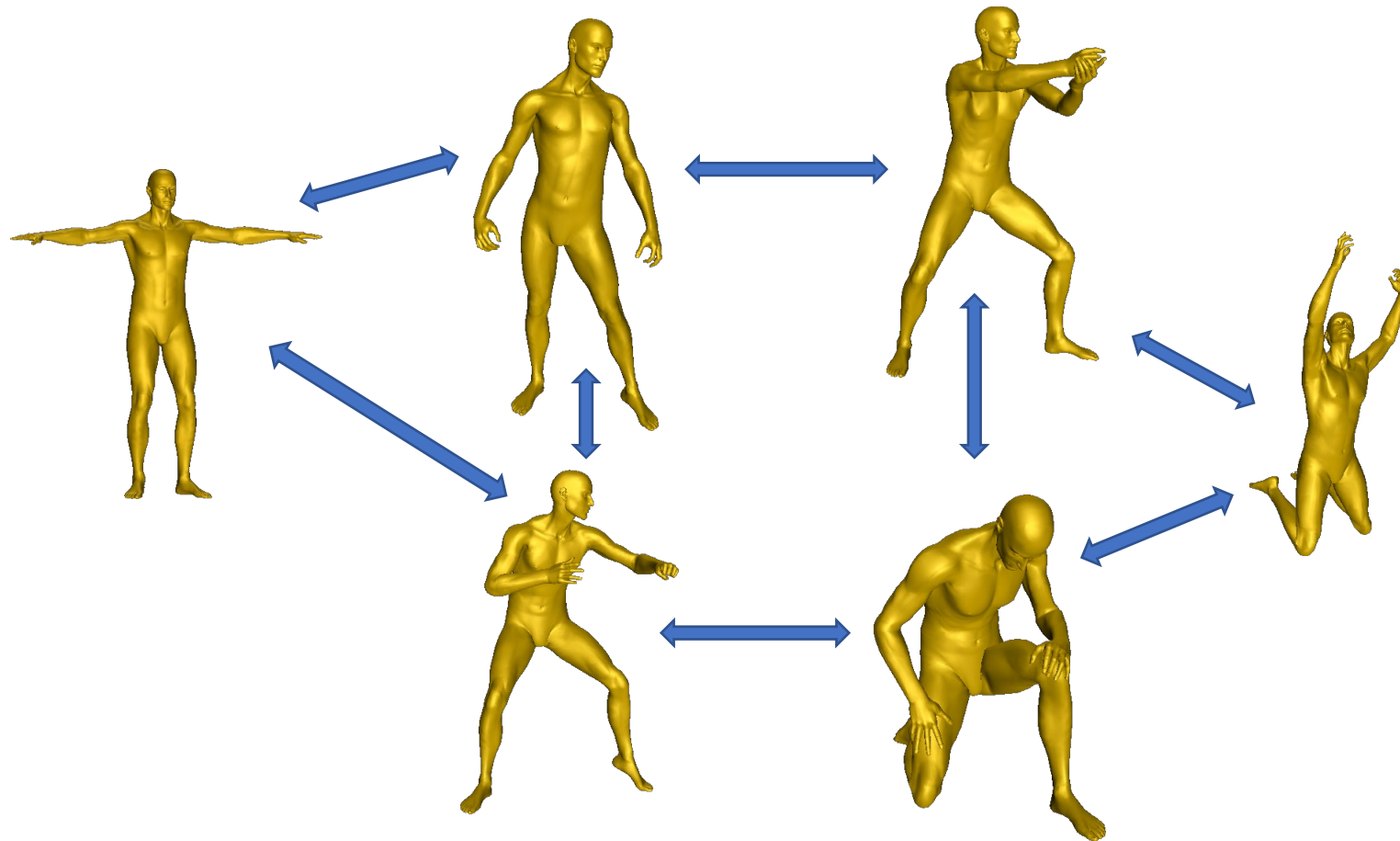
Latent vectors

Parametrizations

Representations

...

Correlated Data Sets



Correlated Tasks

RGB Image



Predicted Normals



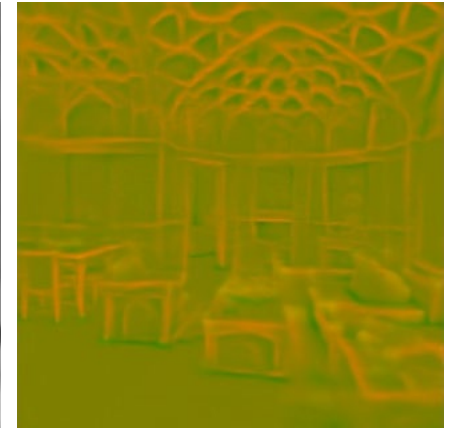
Predicted Depth



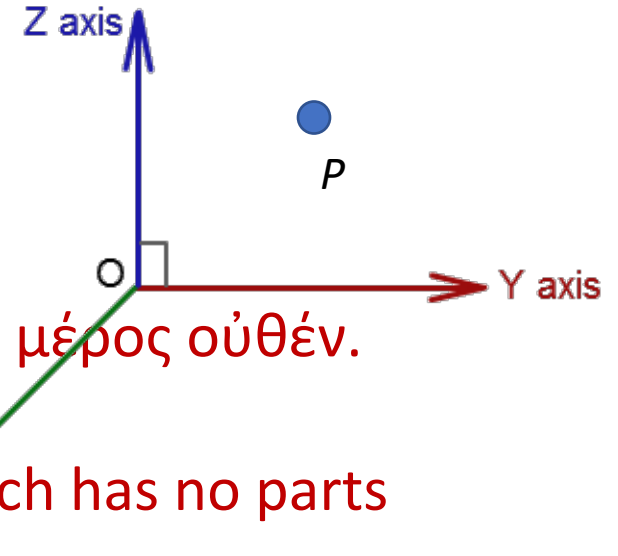
Predicted (re)Shading



Predicted Curvature

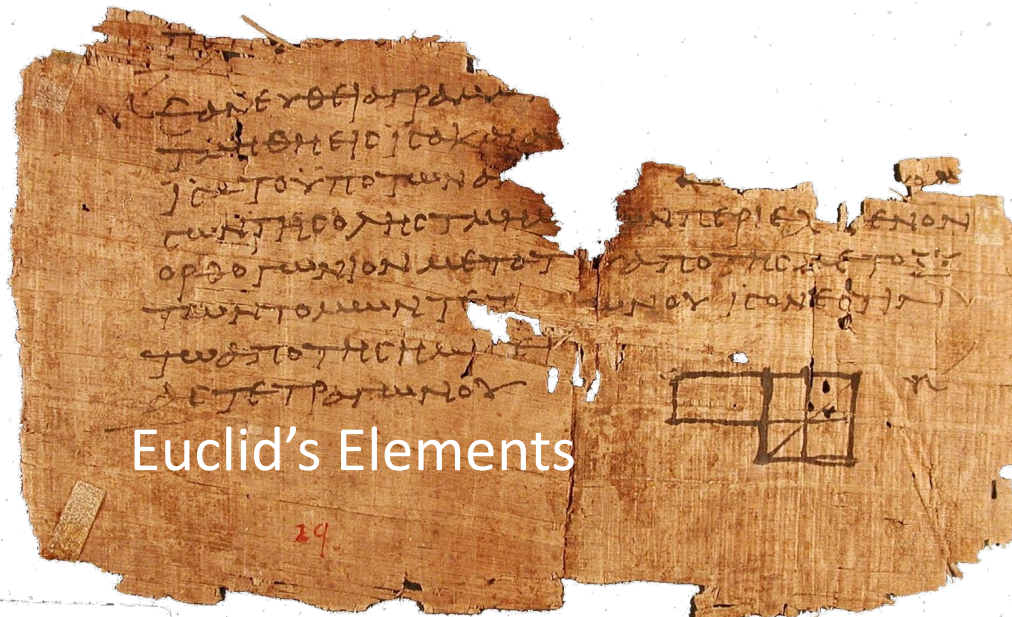


The Structure of Latent Spaces



α'. Σημεῖόν ἐστιν, οὐ μέρος οὐθέν.

a. A point is that which has no parts



Euclid's Elements

Homomorphic Encryption

Chapter 2 Homomorphic Encryption

Abstract Homomorphic encryption is a form of encryption which allows specific types of computations to be carried out on ciphertexts and generate an encrypted result which, when decrypted, matches the result of operations performed on the plaintexts. This is a desirable feature in modern communication system architectures. RSA is the first public-key encryption scheme with a homomorphic property. However, for security, RSA has to pad a message with random bits before encryption to achieve semantic security. The padding results in RSA losing the homomorphic property. To avoid padding messages, many public-key encryption schemes with various homomorphic properties have been proposed in last three decades. In this chapter, we introduce basic homomorphic encryption techniques. It begins with a formal definition of homomorphic encryption, followed by some well-known homomorphic encryption schemes.

2.1 Homomorphic Encryption Definition

In abstract algebra, a homomorphism is a structure-preserving map between two algebraic structures, such as groups.

A group is a set, G , together with an operation \circ (called the group law of G) that combines any two elements a and b to form another element, denoted $a \circ b$. If G satisfies the group axioms, the set and operation, (G, \circ) , must satisfy four requirements:

- 1. The result of the operation, $a \circ b$, is also in G .
- 2. $(a \circ b) \circ c = a \circ (b \circ c)$.
- 3. There is an identity element e in G , such that for every element a in G , $a \circ e = a$ and $e \circ a = a$.
- 4. For every element a in G , there is a unique inverse element a^{-1} in G , such that $a \circ a^{-1} = e$ and $a^{-1} \circ a = e$.



Craig Gentry

The Mathematics of Latent Codes

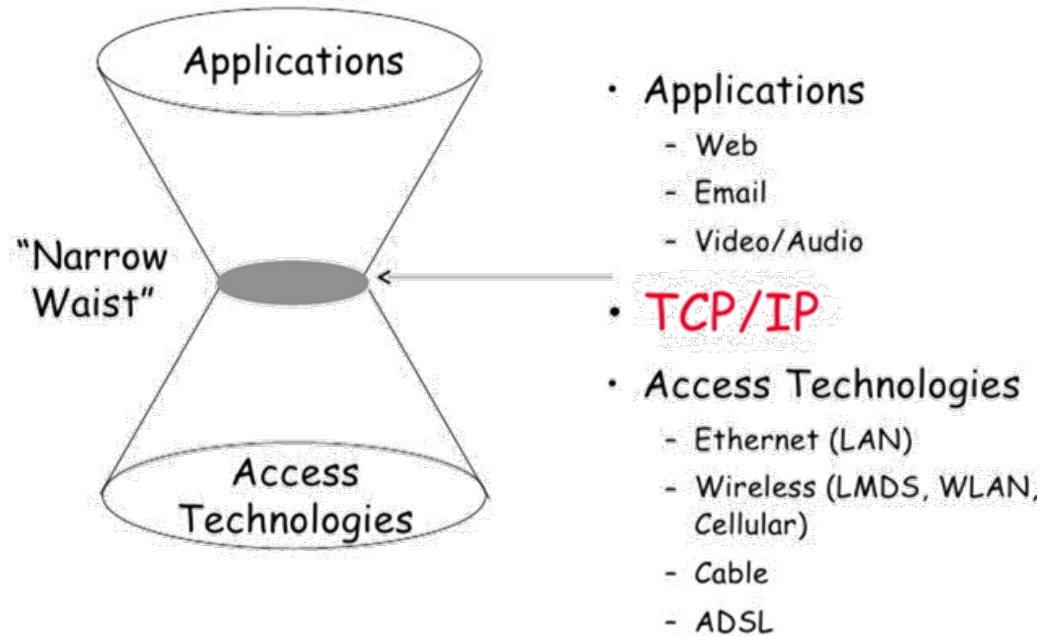
- What kinds of mathematical objects can serve well as latent space codes?
- How should such an object vary with
 - the input data?
 - the input modality?
 - the learning task?



A Shared “Narrow Neck / Waist” in Other Fields



Power distribution



Network protocol stack

[Image: R. Katz]

Relating Data Sets: Horizontal Map Networks

Horizontal Networks & Joint Learning

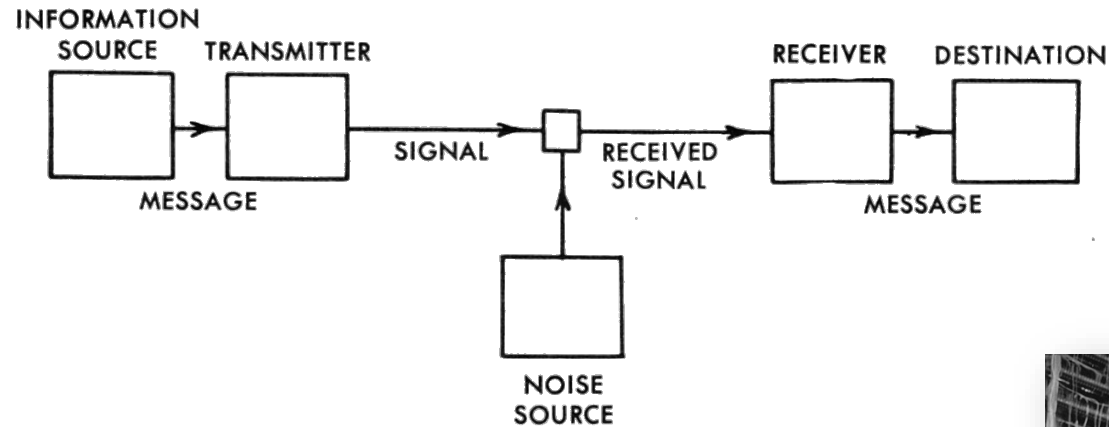


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Similarity as a communications channel



The Mathematical Theory of Communication



Claude Shannon

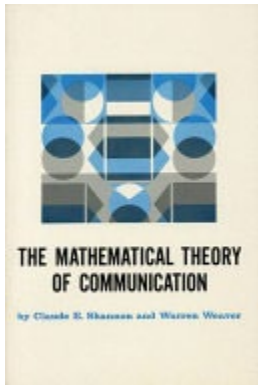
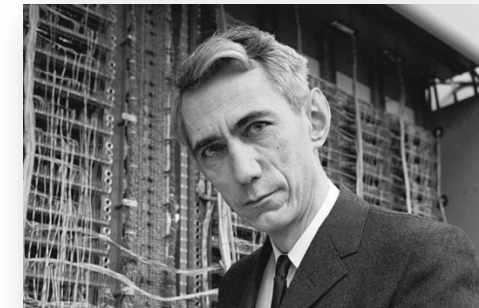


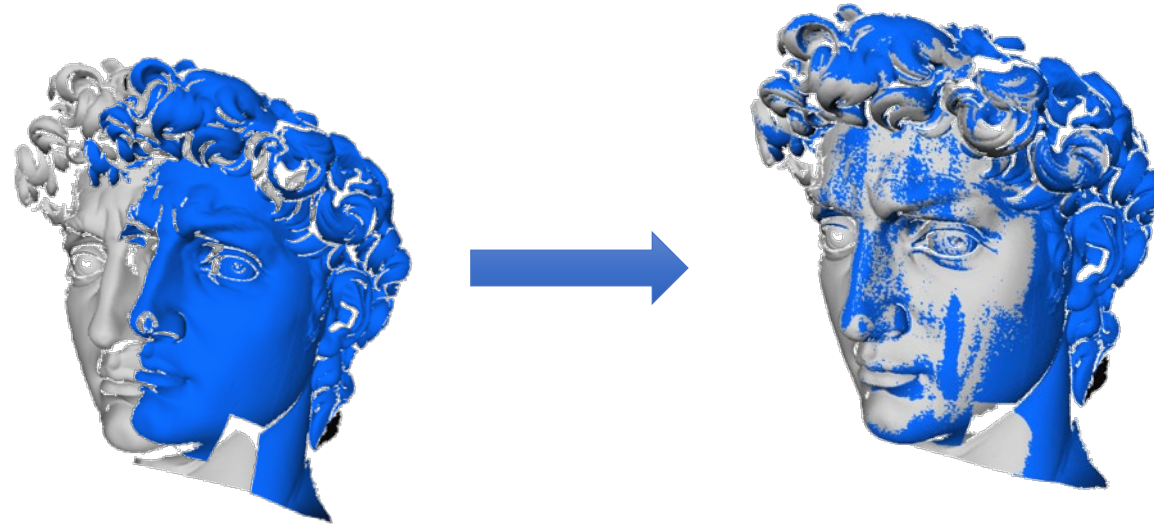
Fig. 1. — Schematic diagram of a general communication system.



Networks of Visual Data



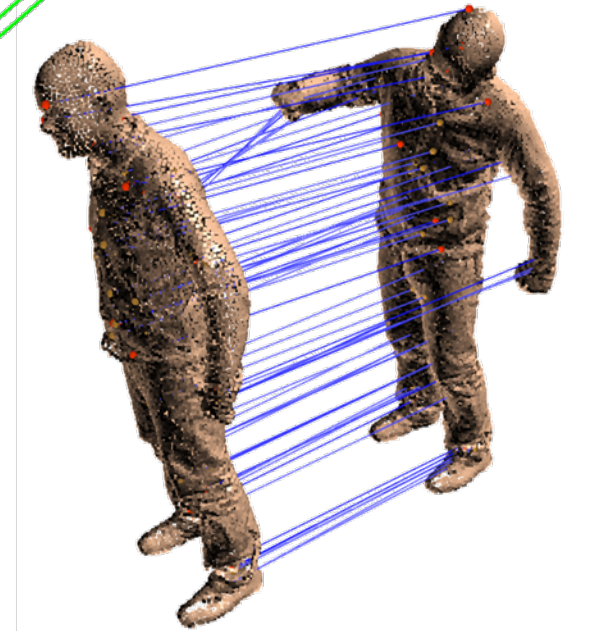
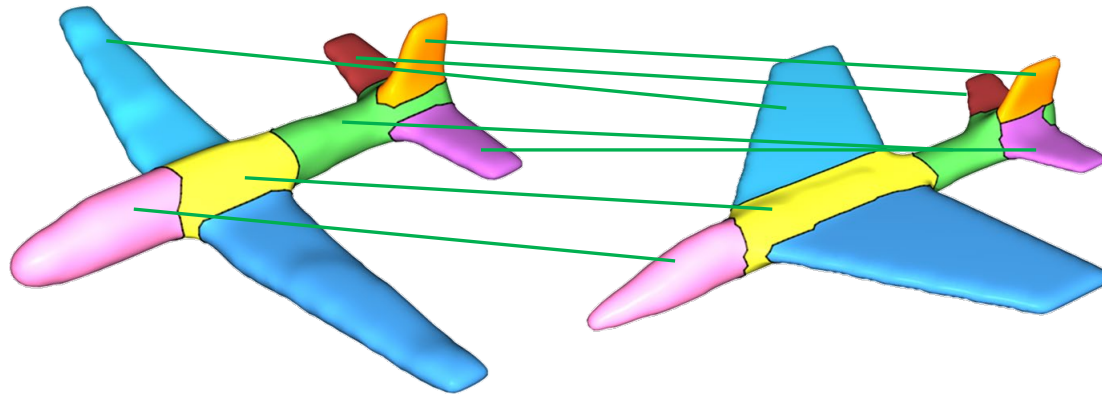
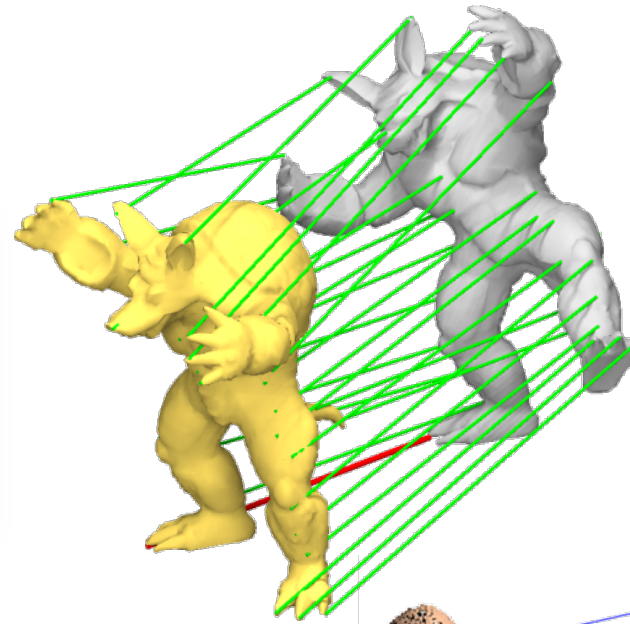
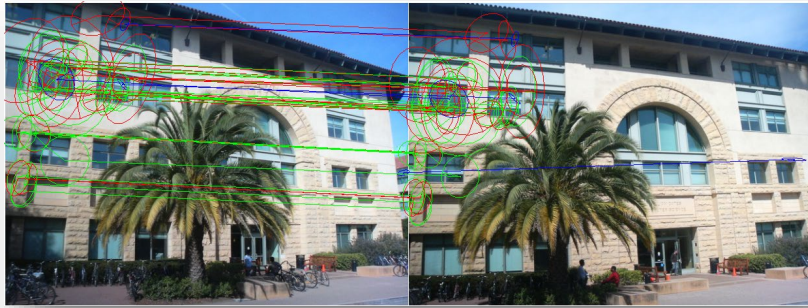
Alignment and Registration



Rigid Registration

Low-dimensional transformation group

Maps and Correspondences



Societies, or Social Networks of Data Sets

Our understanding of data can greatly benefit from extracting these relations and building relational networks.

We can exploit the relational network to

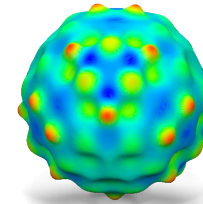
- transport information around the network
- assess the validity of operations or interpretations of data (by checking consistency against related data)
- assess the quality of the relations themselves (by checking consistency against other relations through cycle closure, etc.)
- extract shared structure among the data

Thus the network becomes the great regularizer in joint data analysis.



Functionals Over Data and Functional Maps

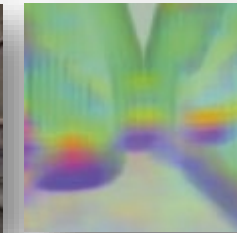
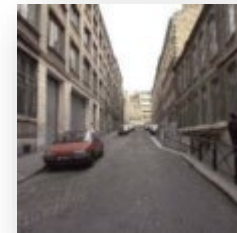
Knowledge as Functions over Data



Curvature



Parts

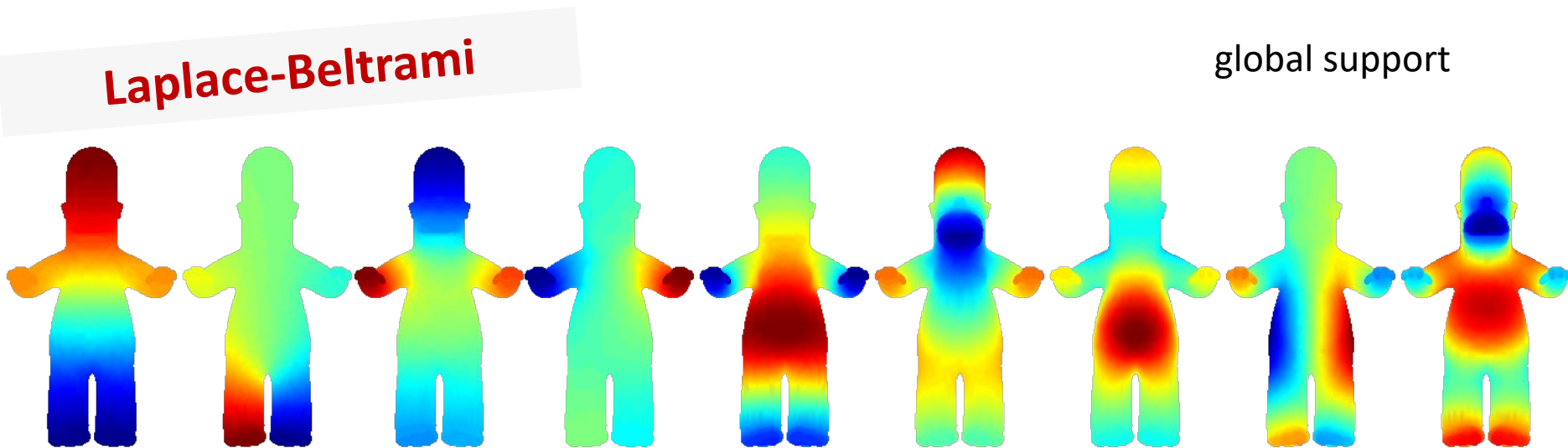
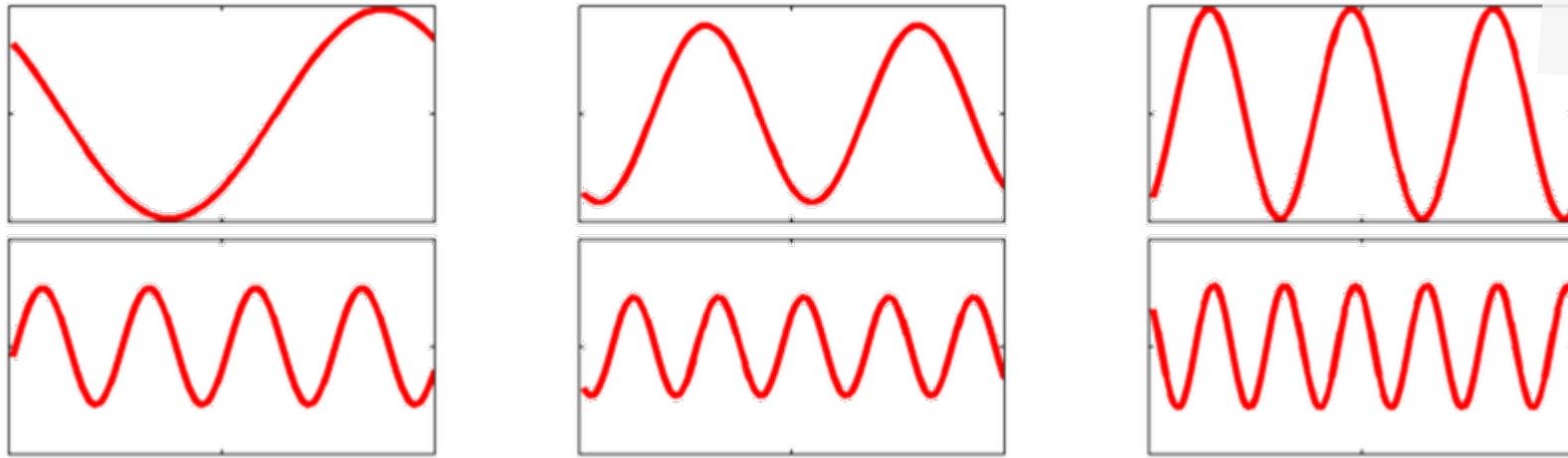


SIFT flow, C. Liu 2011

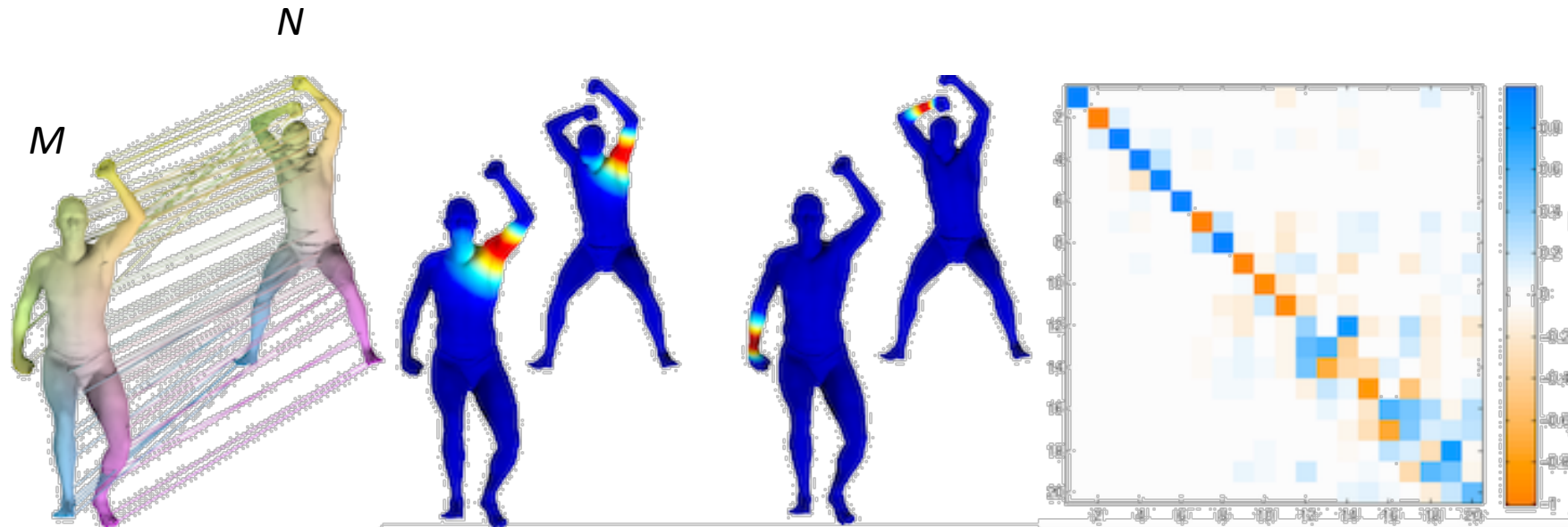


Knowledge towers over visual data: function spaces

Hierarchical Bases for a Function Space



Functional Maps

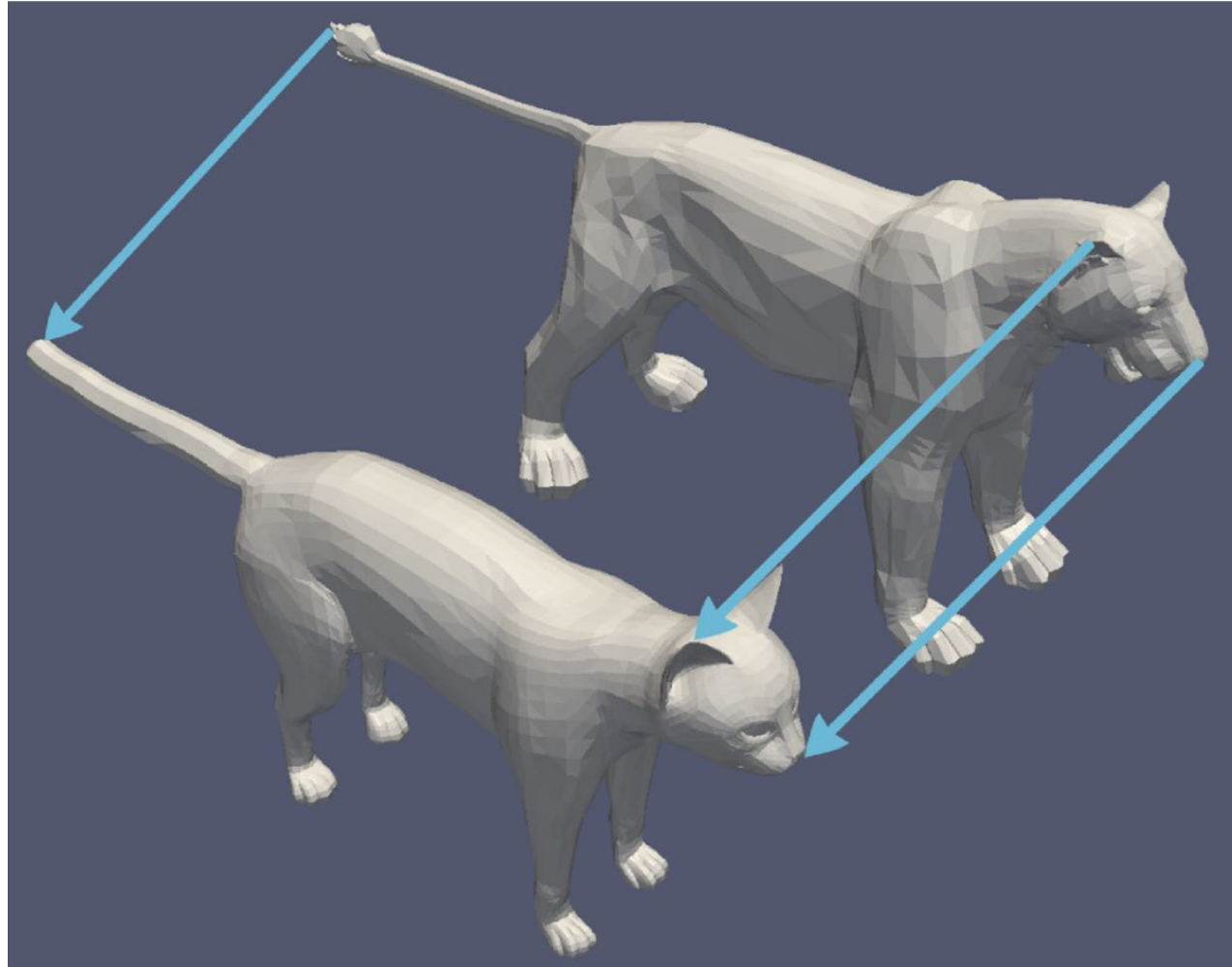


$$T : N \rightarrow M$$

$$C_T : L^2(M) \rightarrow L^2(N)$$
$$f \mapsto f \circ T$$

A contravariant functor

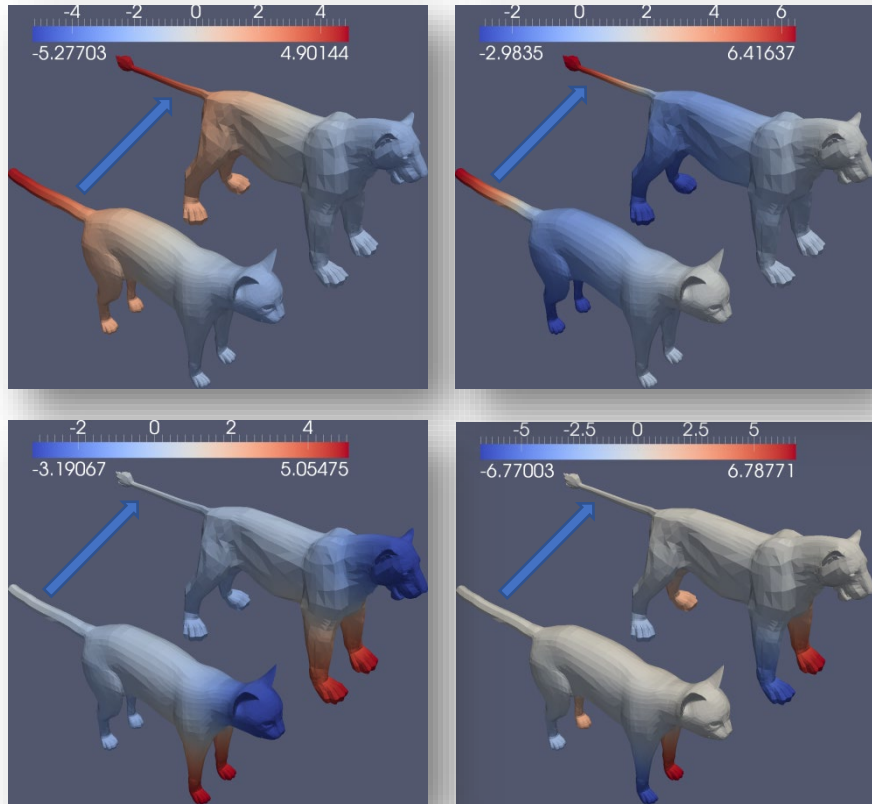
Starting from a Regular Map ϕ



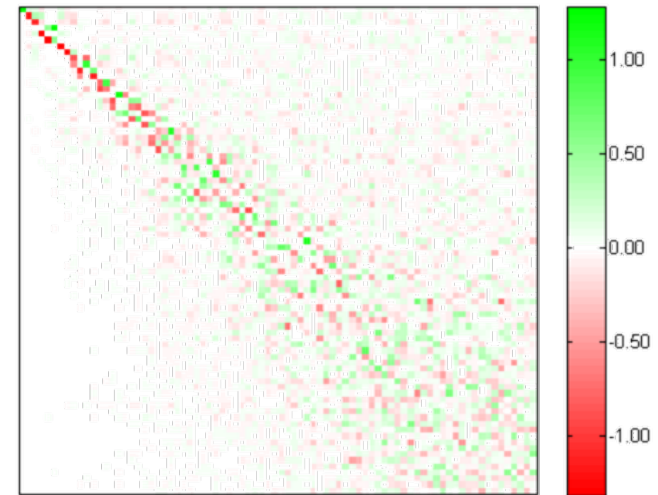
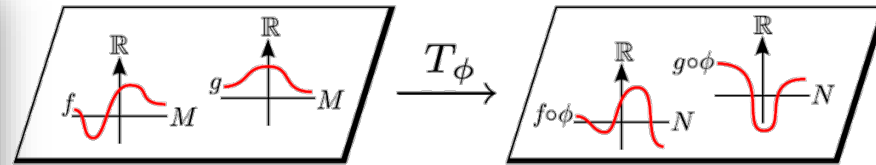
$\phi: \text{lion} \rightarrow \text{cat}$

A Contravariant Functor

from cat to lion



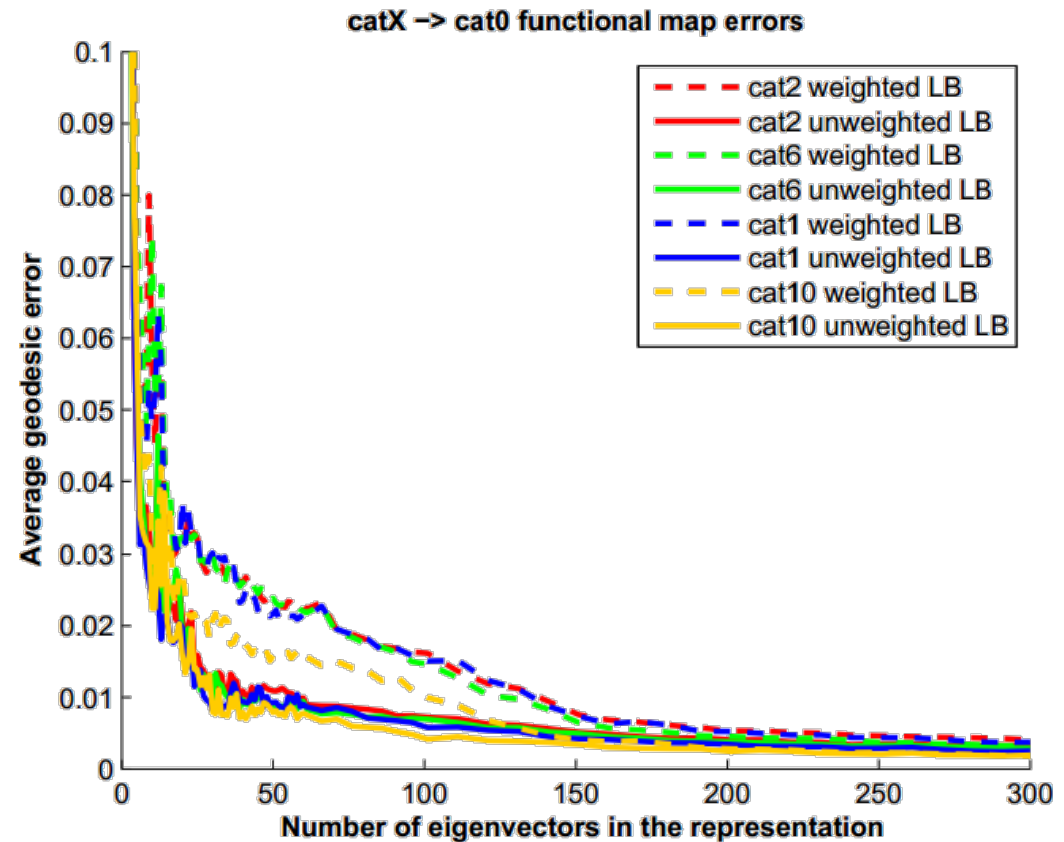
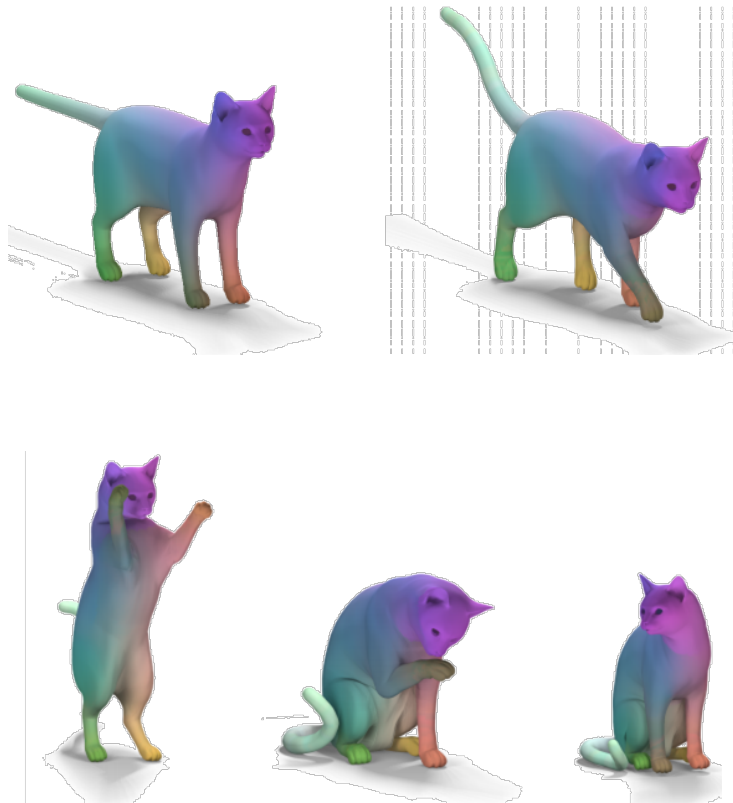
Functions on cat are transferred to lion using T_ϕ



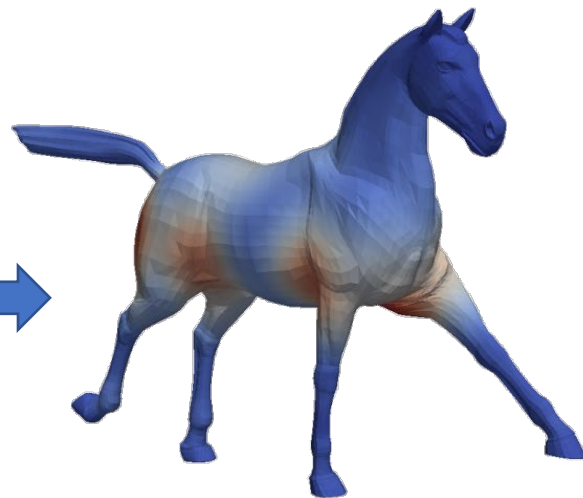
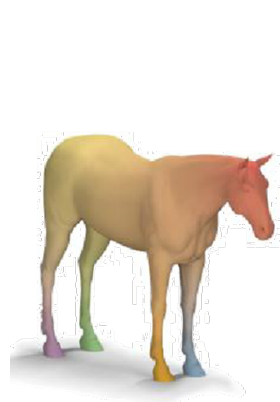
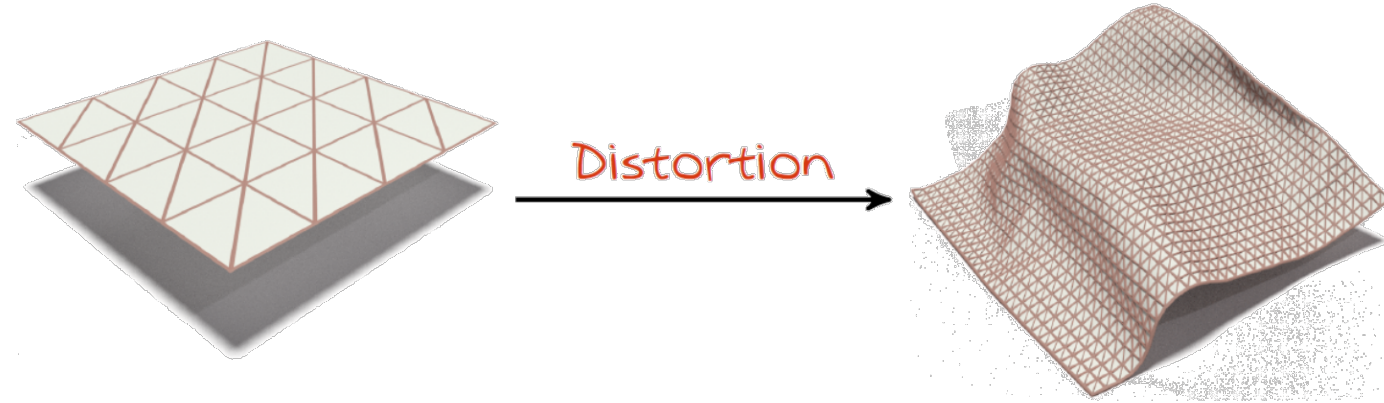
T_ϕ is a linear operator (matrix)

$$T_\phi : L^2(cat) \rightarrow L^2(lion)$$

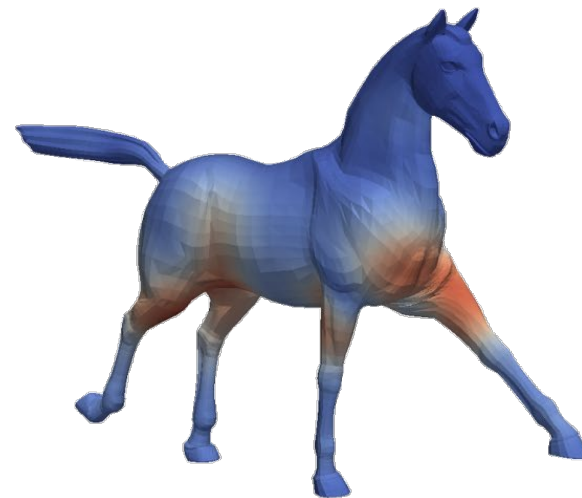
Compact Encoding of Maps as Matrices



Changes to Measure and Metric under a Map

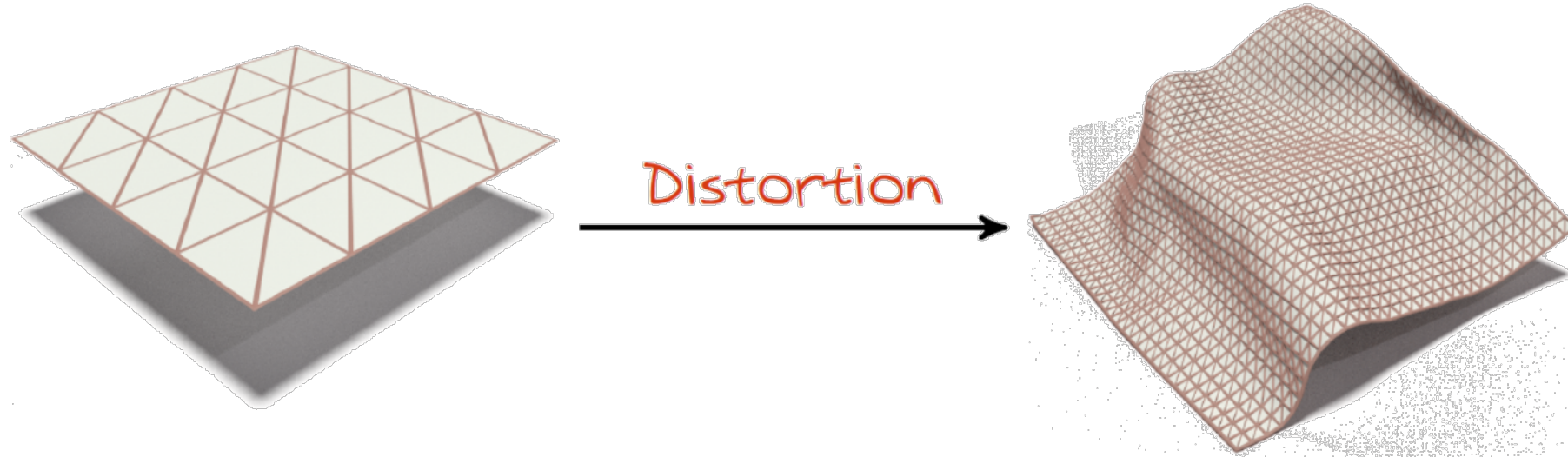


Area distortion



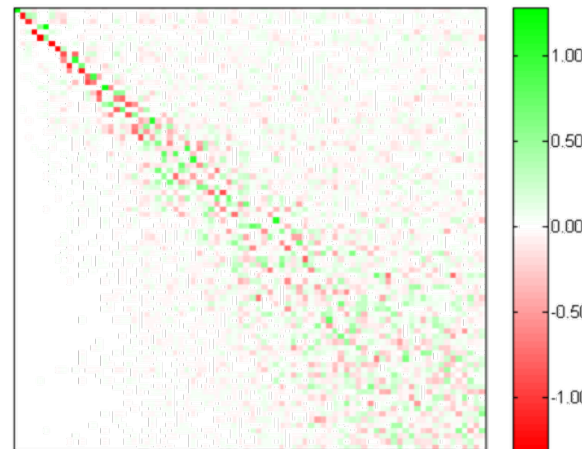
Conformal distortion

Shape Differences are a Change Recipe



A recipe encoded as a matrix:

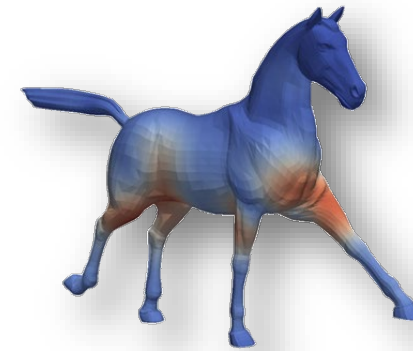
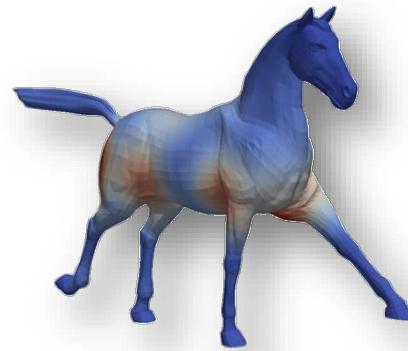
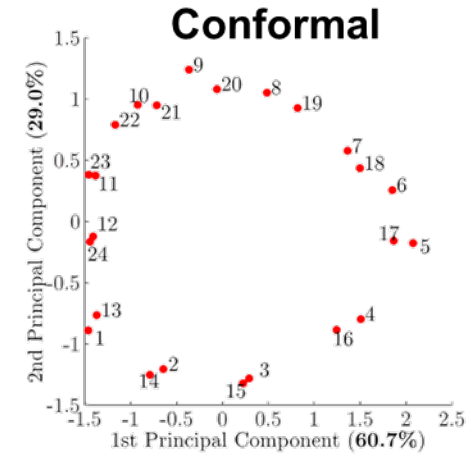
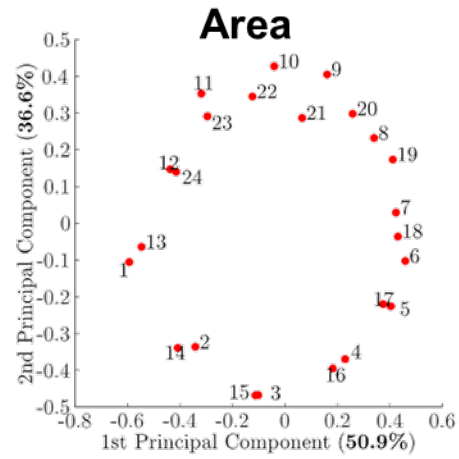
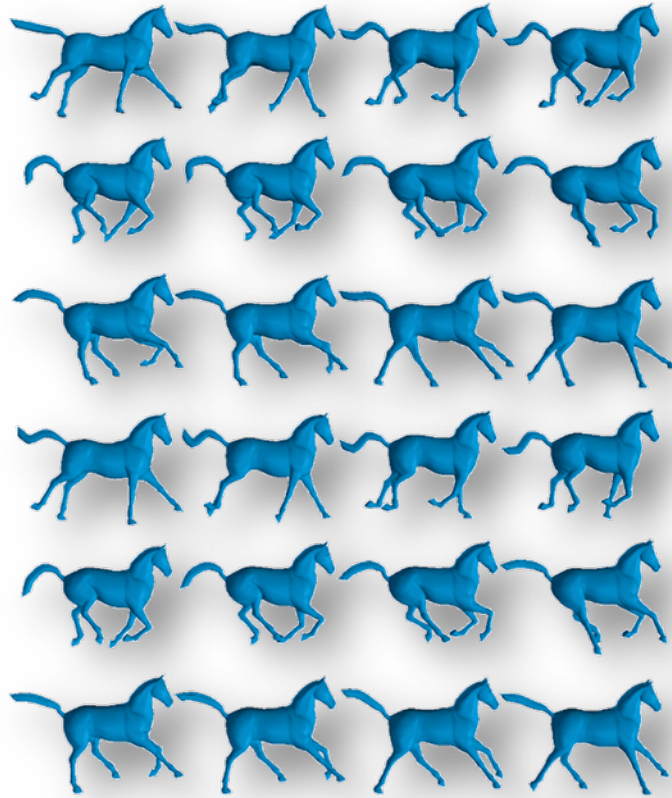
Area distortion
Conformal distortion



A novel type of latent space representation for 3D data

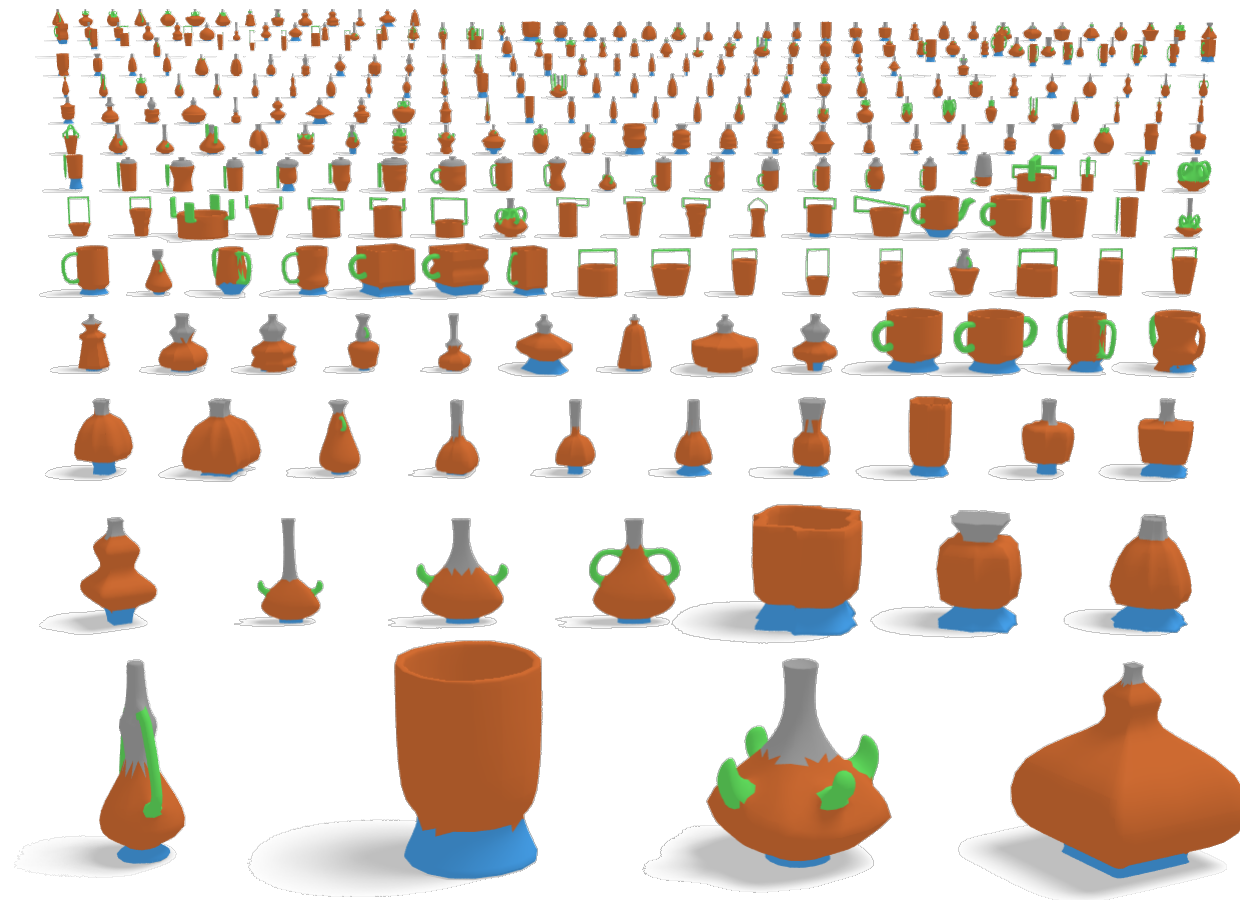
Under some conditions, lossless

The Space of Shapes

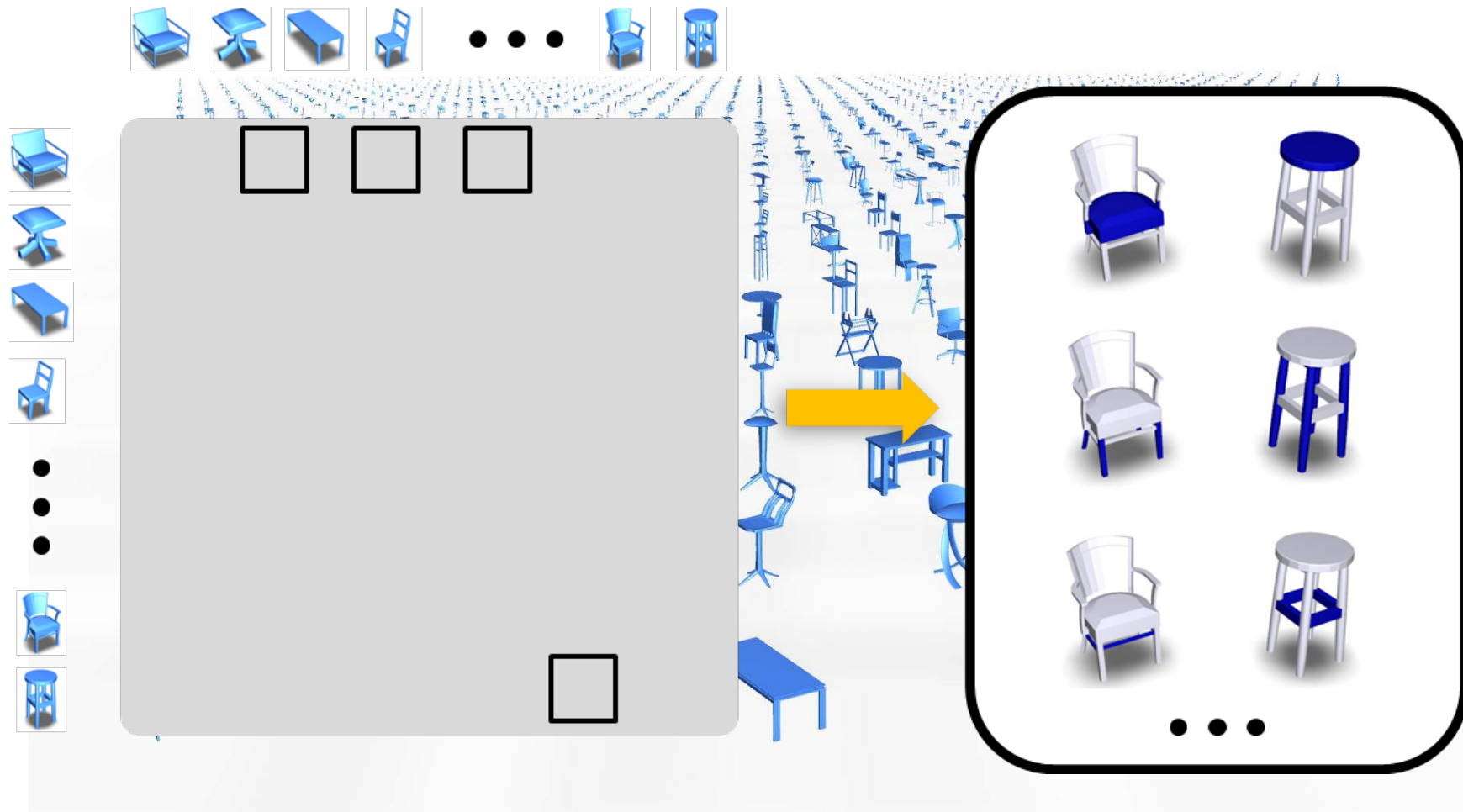


Horizontal Map Networks

Joint Analysis: Co-Segmentation

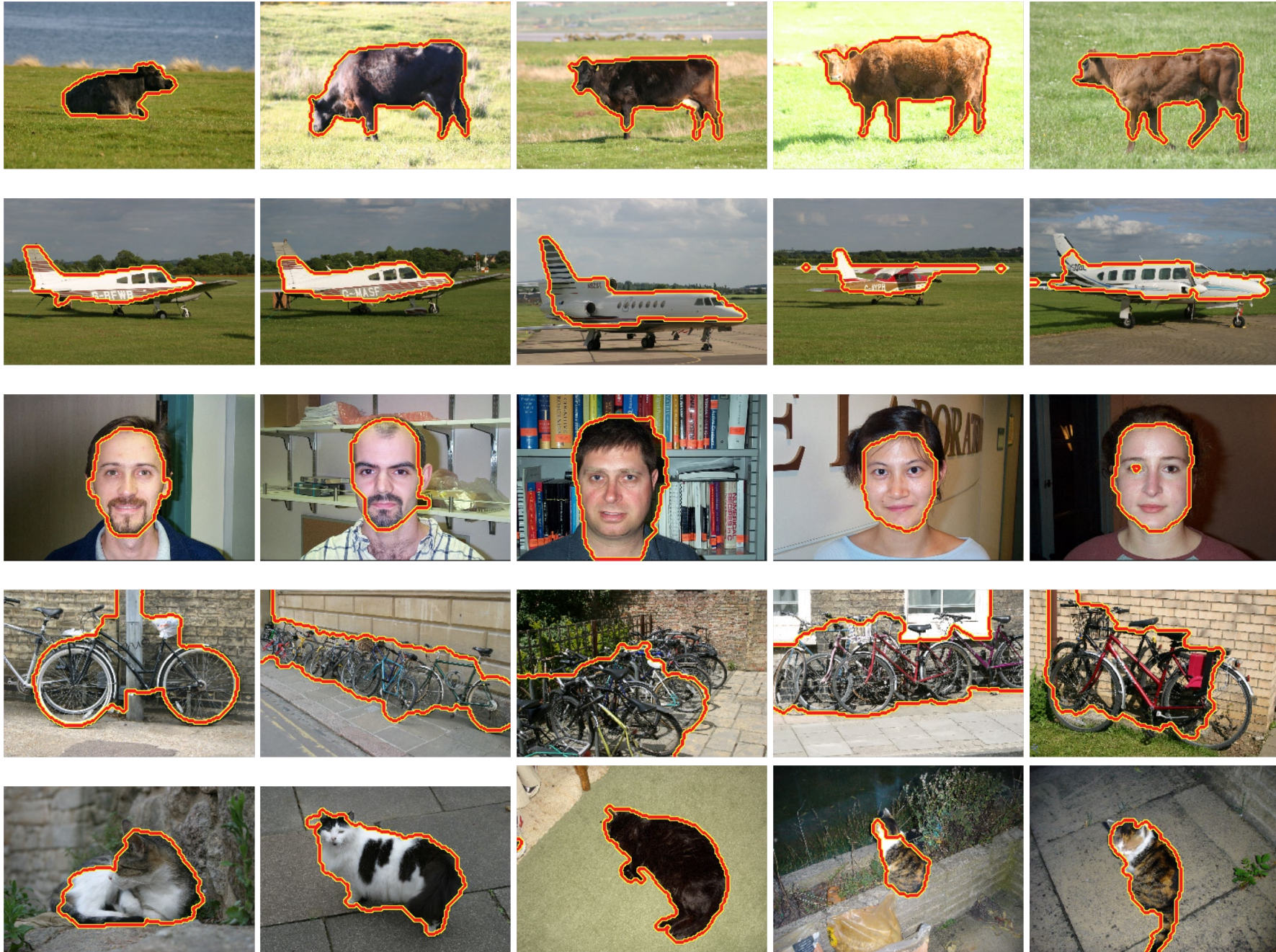


Unsupervised Structure Extraction

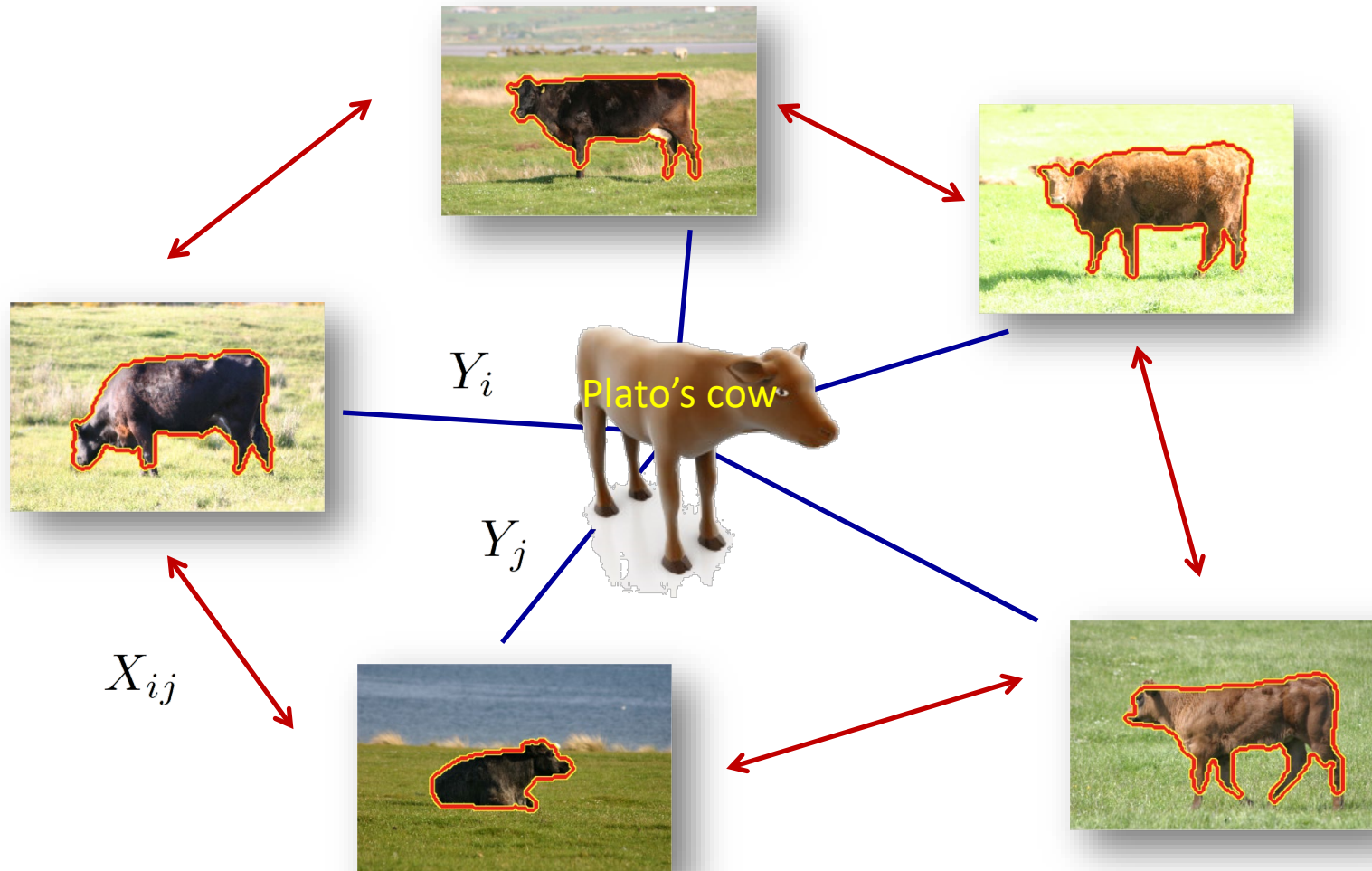


[Q. Huang, F. Wang, L. Guibas, '14]

MSRC: 5 images per class are shown

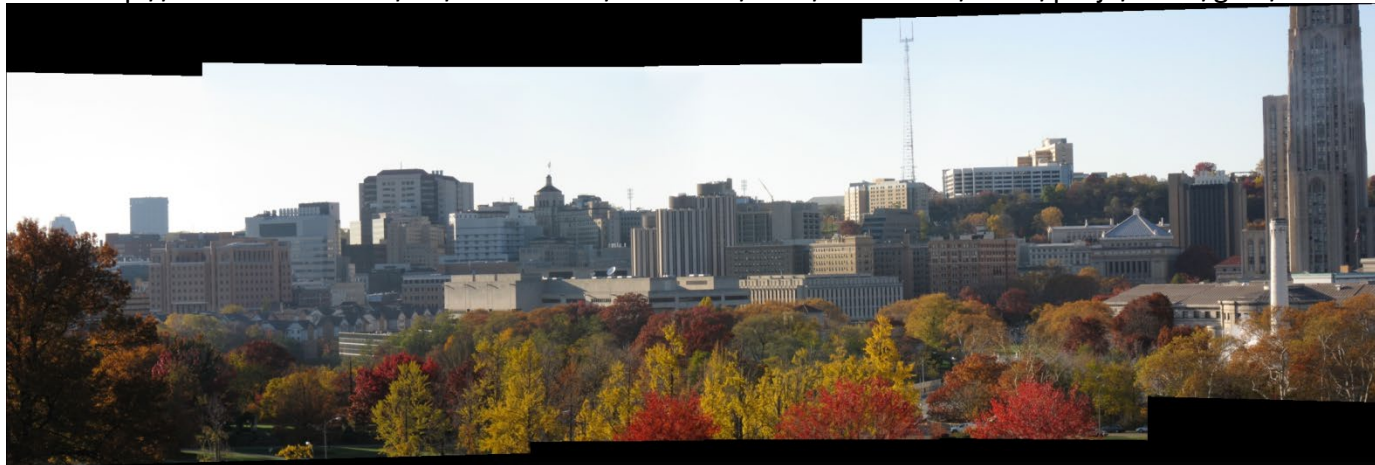


The Network is the Abstraction



Mosaicking or SLAM at the Level of Functions

<http://www.cs.cmu.edu/afs/cs.cmu.edu/academic/class/15463-f08/www/proj4/www/gme/>



robotics.ait.kyushu-u.ac.jp

Joint Learning and Information Aggregation

- Need to aggregate information:
 - *over different data sets*
 - *over different modalities (geometry, appearance, language)*
 - *over space and time*
 - *over different representations*
 - *over different predictions*
 - *over different tasks*

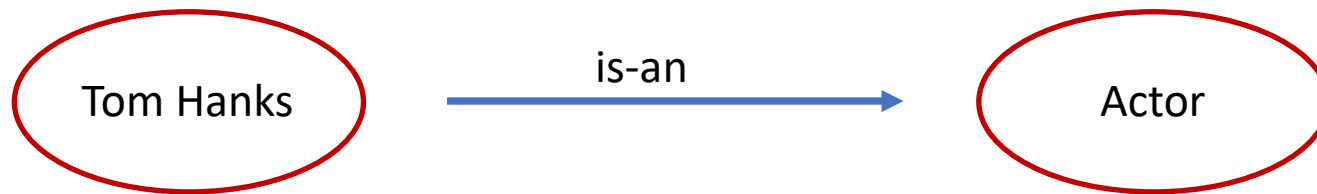
in settings where the above refer to same entities in the world – and are thus correlated



Visual Data Repositories for Storing Semantic Knowledge

Semantic Networks

- ◆ Also known as **frame networks**
- ◆ Encode semantic relations between concepts
- ◆ Often used as a form of knowledge representation
- ◆ A directed or undirected graph consisting of vertices, which represent concepts, and edges which represent concept relations

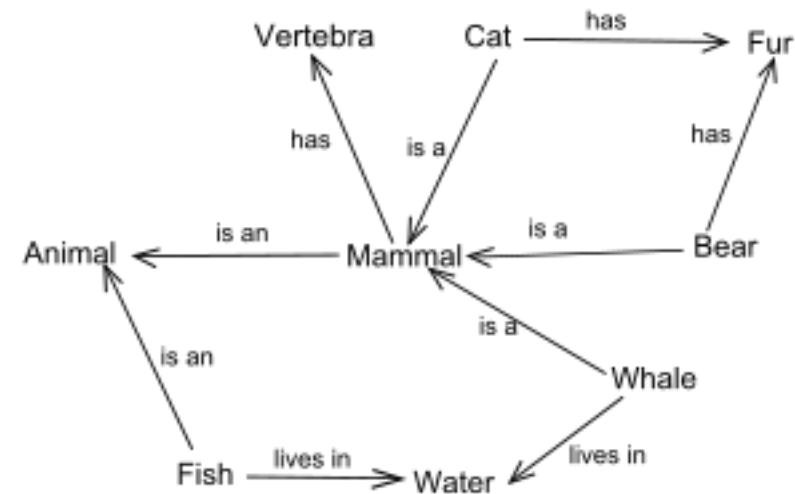


Example of a Semantic Net

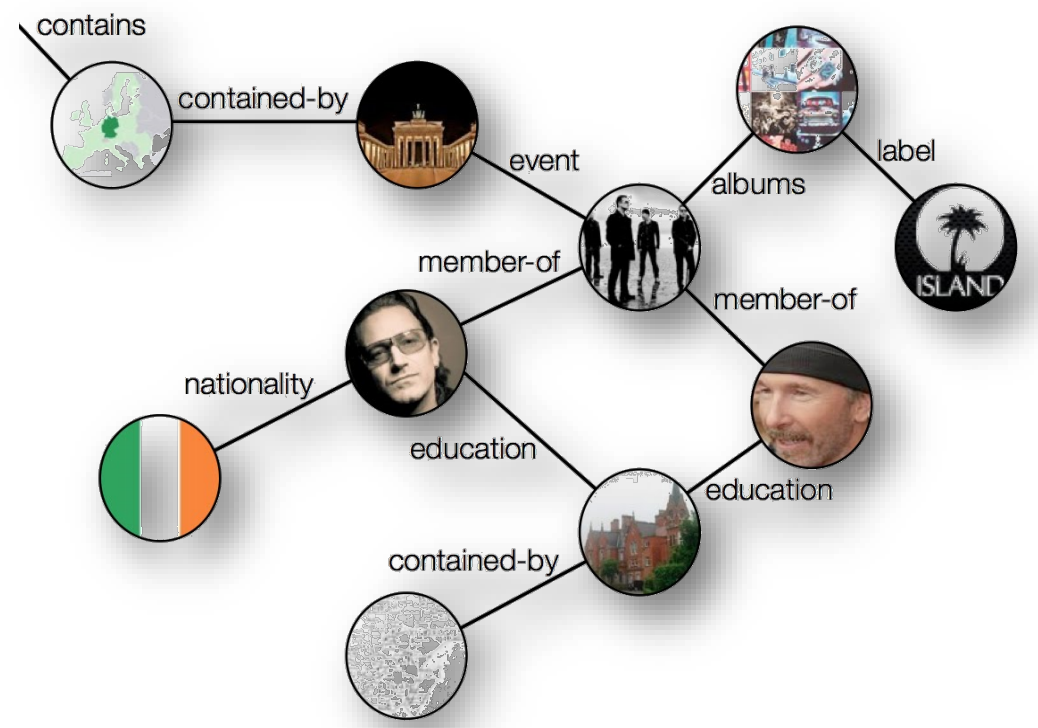
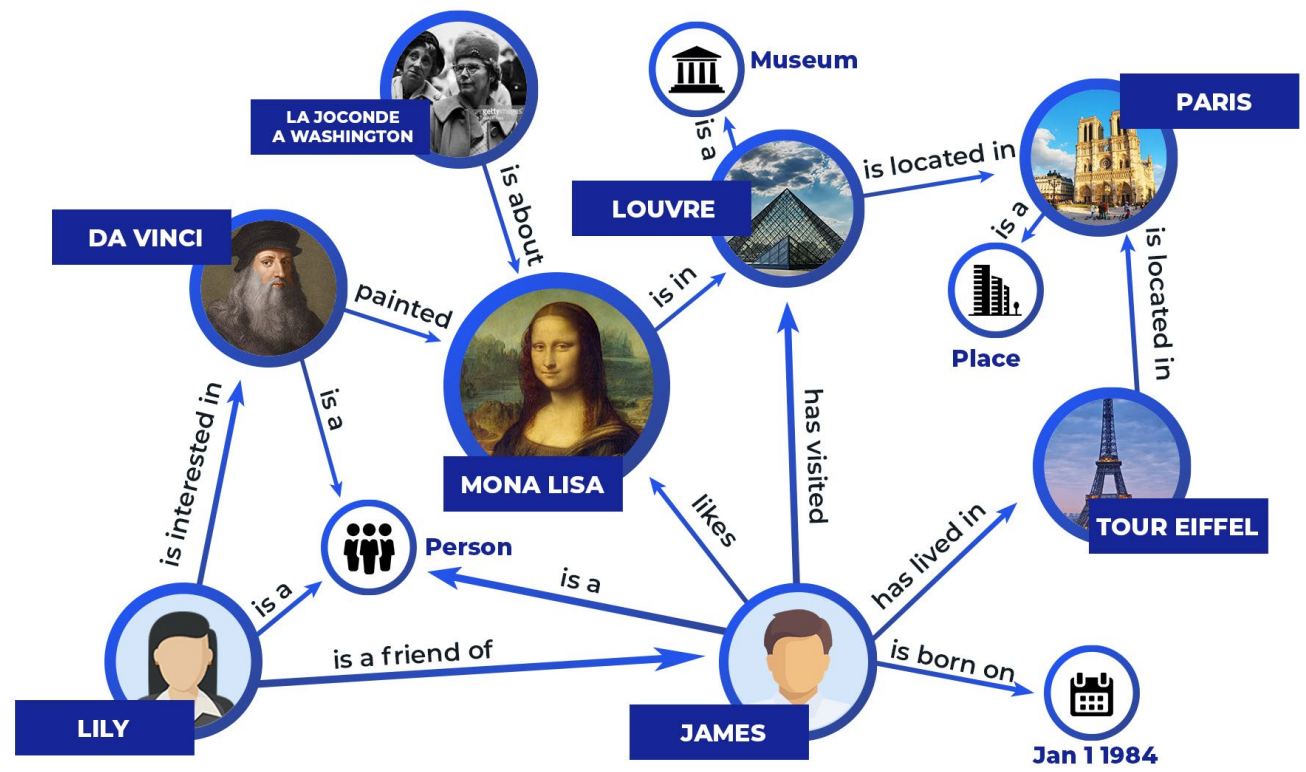
Semantic Net in Lisp

```
(defun *database* ()  
'((canary (is-a bird)  
          (color yellow)  
          (size small))  
  (penguin (is-a bird)  
           (movement swim))  
  (bird (is-a vertebrate)  
        (has-part wings)  
        (reproduction egg-laying))))
```

Graph representation



Google Knowledge Graph



What is WordNet?



Original paper
by
**[George
Miller, et al
1990]** cited
over 5,000
times

Organizes over
150,000 words
into 117,000
categories
called *synsets*.

Establishes
ontological and
lexical
relationships in
NLP and related
tasks.

WordNet

- ◆ a lexical database of English
- ◆ words -> synonym sets (synsets)

```
dog, domestic dog, Canis familiaris
=> canine, canid
=> carnivore
=> placental, placental mammal, eutherian, eutherian mammal
=> mammal
=> vertebrate, craniate
=> chordate
=> animal, animate being, beast, brute, creature, fauna
=> ...
```

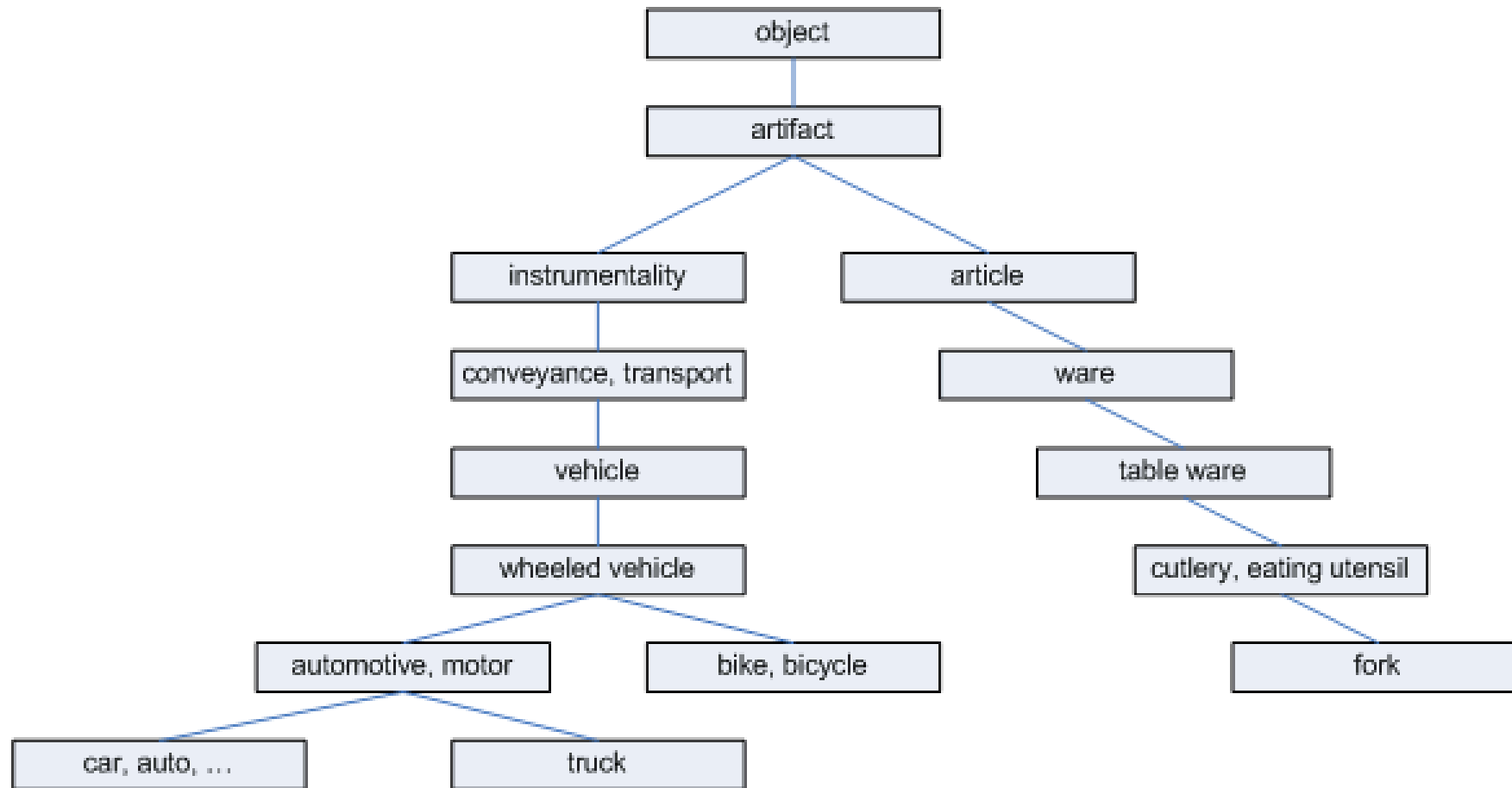
*G. A. Miller, R. Beckwith, C. D. Fellbaum, D. Gross, K. Miller. 1990.
WordNet: An online lexical database. Int. J. Lexicograph.*

WordNet

- ◆ Important relations between synsets (nouns):

Relation	Definition	Example
Hypernym	From concepts to superordinates	water ¹ → liquid
Hyponym	From concepts to subtypes	water ¹ → seawater
Has-Part	From groups to their members	water ¹ → oxygen
Part-of	From members to their groups	water ¹ → ice
Antonym	Opposites	leader → follower

Taxonomy: is-a Relationship



Partonomy: has-a Relationship

- S (n) **car, auto, automobile, machine, motorcar** (a motor vehicle with four wheels; usually propelled by an internal combustion engine) "he needs a car to get to work"
 - [direct hyponym / full hyponym](#)
 - [partonymy](#)
 - S (n) **accelerator, accelerator pedal, gas pedal, gas, throttle, gas** (a pedal that controls the throttle valve) "he stepped on the gas"
 - S (n) **air bag** (a safety restraint in an automobile; the bag inflates on collision and prevents the driver or passenger from being thrown forward)
 - S (n) **auto accessory** (an accessory for an automobile)
 - S (n) **automobile engine** (the engine that propels an automobile)
 - S (n) **automobile horn, car horn, motor horn, horn, buzzer** (a device on an automobile for making a warning noise)
 - S (n) **buffer, fender** (a cushion-like device that reduces shock due to an impact)
 - S (n) **bumper** (a mechanical device consisting of bars at either end of a vehicle to absorb shock and prevent serious damage)
 - S (n) **car door** (the door of a car)
 - S (n) **car mirror** (a mirror that the driver of a car can use)
 - S (n) **car seat** (a seat in a car)
 - S (n) **car window** (a window in a car)
 - S (n) **fender, wing** (a barrier that surrounds the wheels of a vehicle to block splashing water or mud) "in Britain they call a fender a wing"
 - S (n) **first gear, first, low gear, low** (the lowest forward gear ratio in the gear box of a motor vehicle; used to start a car moving)
 - S (n) **floorboard** (the floor of an automobile)
 - S (n) **gasoline engine, petrol engine** (an internal-combustion engine that burns gasoline; most automobiles are driven by gasoline engines)
 - S (n) **glove compartment** (compartment on the dashboard of a car)
 - S (n) **grille, radiator grille** (grating that admits cooling air to car's radiator)
 - S (n) **high gear, high** (a forward gear with a gear ratio that gives the greatest vehicle velocity for a given engine speed)
 - S (n) **hood, bonnet, cowf, cowling** (protective covering consisting of a metal part that covers the engine) "there are powerful engines under the hoods of new cars"
 - S (n) **luggage compartment, automobile trunk, trunk** (compartment in an automobile that carries luggage or shopping or tools) "he put his golf bag in the trunk"
 - S (n) **rear window** (car window that allows vision out of the back of the car)
 - S (n) **reverse, reverse gear** (the gears by which the motion of a machine can be reversed)
 - S (n) **roof** (protective covering on top of a motor vehicle)
 - S (n) **running board** (a narrow footboard serving as a step beneath the doors of some old cars)
 - S (n) **stabilizer bar, anti-sway bar** (a rigid metal bar between the front suspensions and between the rear suspensions of cars and trucks; serves to stabilize the car)
 - S (n) **sunroof, machine-roof** (an automobile roof having a sliding or raisable panel) "'sunshine-roof' is a British term for 'sunroof'"
 - S (n) **tail fin, tailfin, fin** (one of a pair of decorations projecting above the rear fenders of an automobile)
 - S (n) **third gear, third** (the third from the lowest forward ratio gear in the gear box of a motor vehicle) "you shouldn't try to start in third gear"
 - S (n) **window** (a transparent opening in a vehicle that allow vision out of the sides or back; usually is capable of being opened)



ShapeNet (>3M Models)

SHAPE NET Search Options Home About Download Statistics

chair
a seat for one person, with a support for the back; 'he put his coat over the back of the chair and sat down'
[ImageNet](#) [MetaData](#)


Choose a taxonomy:
ShapeNetCore

- airplane,aeroplane,plane(12,4501)
- aquarium,fish tank,marine museum(0,4)
- ashcan,trash can,garbage can,wastebin,ash bin(1,10)
- bag,travelling bag,travel bag,grip,suitcase(1,10)
- basket,handbasket(2,140)
- bathtub,bathing tub,bath,tub(0,932)
- bed(13,353)
- bench(5,1953)
- birdhouse(0,79)
- boat(12,1635)
- bookshelf(0,495)
- bottle(6,550)
- bowl(1,234)
- bus,autobus,coach,charabanc,double-decker,jack bus(1,10)
- cabinet(9,1644)
- camera,photographic camera(4,134)
- can,tin,tin can(2,108)
- cap(4,81)
- car,auto,automobile,machine,motorcar(18,244)
- cellular telephone,cellular phone,cellphone,cell phone(1,10)
- chair(23,7083)**
- chair(1,10)

Synset models

Displaying 1 to 40 of 7080

< 1 2 3 4 5 6 7 8 9 10 11 12 13 ... 177 >



club chair cantilever chair armchair straight chair straight chair club chair deck chair rex chair

straight chair club chair club chair swivel chair butterfly chair armchair armchair club chair

recliner cantilever chair swivel chair swivel chair armchair folding chair rocking chair club chair

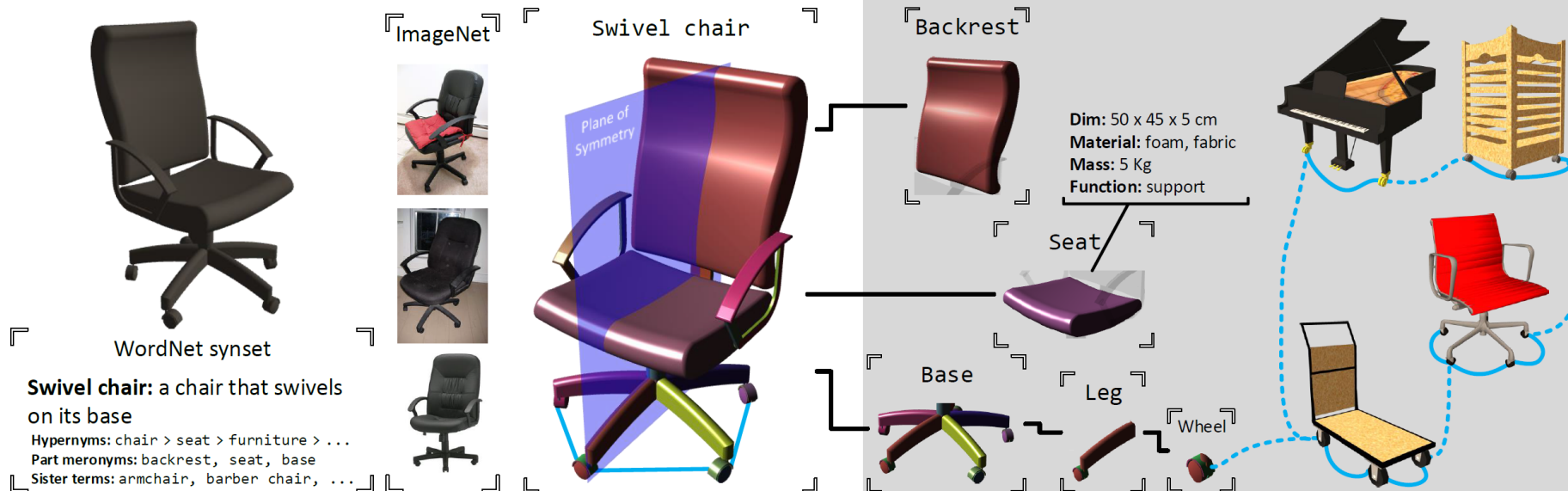
chair chair chair chair chair chair chair chair

Object Knowledge

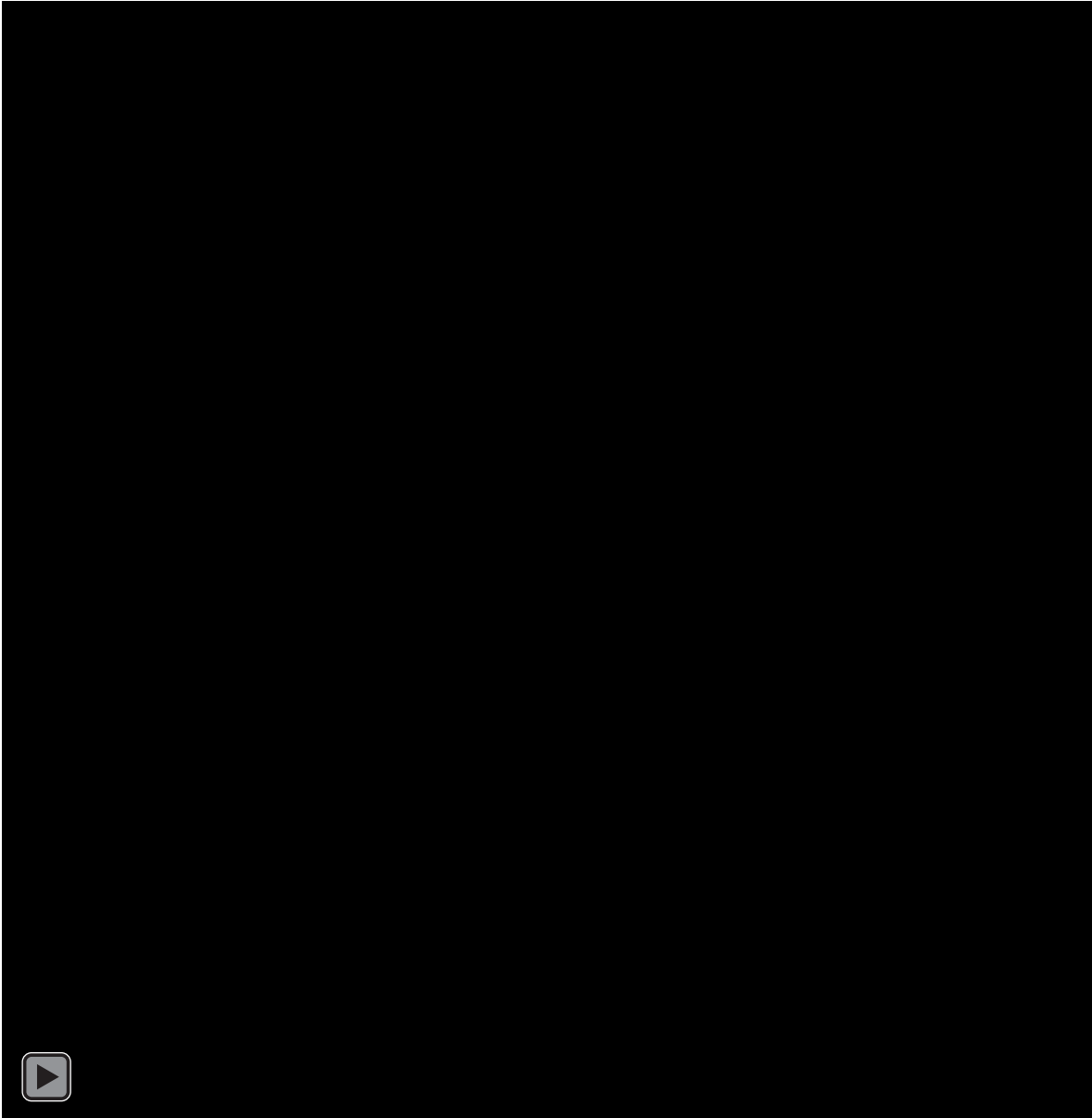
Parts, symmetries, keywords, physical properties, materials, affordances, ...



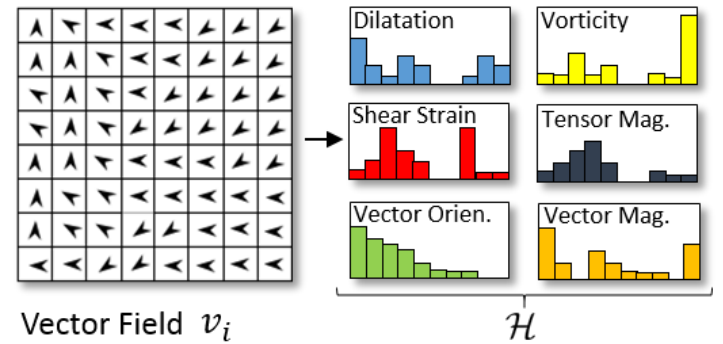
Link to WordNet Taxonomy Alignment+Symmetry Part Hierarchy Part Correspondences



Object Interaction Knowledge



Vector Field to Histograms

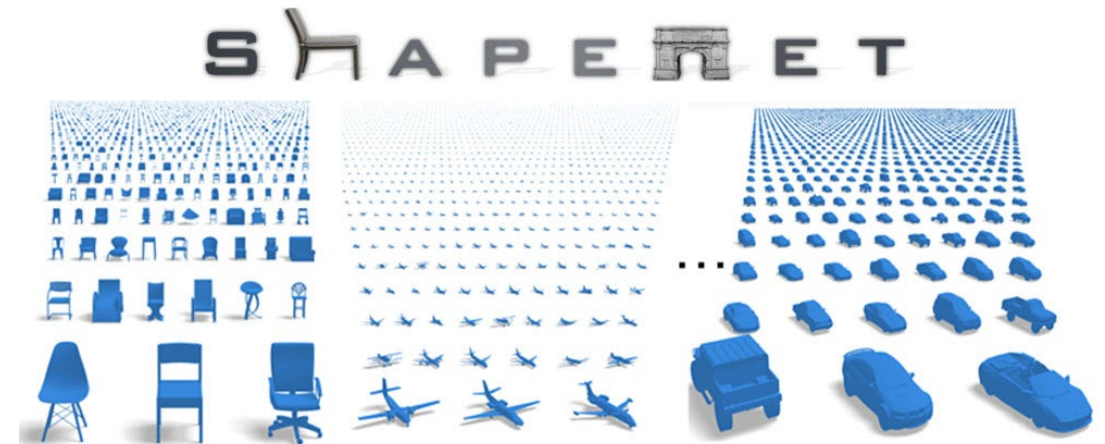


Supervised Methods

Visual DataSets

ImageNet and ShapeNet

- ◆ Explain how big visual datasets including ImageNet and ShapeNet are organized



Lecture on Wednesday, March 31

- Live lecture by Kaichun Mo will be from 10:30 – 11:50 am (of course it will be recorded)

Course Mechanics

Class Mechanics

- Two online weekly lectures
- Online office hours
- Class web site <http://cs233.stanford.edu>
<http://graphics.stanford.edu/courses/cs233-21-spring>
- Use Canvas for lecture videos
- Use Piazza (discussion group), Gradescope (for homework submissions)

Course Work

- Four assignments
 - modest programming in MATLAB/Python for three, one in JavaPlex
 - working in groups OK, up to three students
 - Use Google Cloud for Education
- An open book midterm
- Class participation
- No final

CS233 / CME 251 Class Schedule

Monday		Wednesday		Monday		Wednesday		Monday		Wednesday	
March 29		March 31		April 26		April 28		May 24		May 26	
Introduction; Geometric and topological perspective on data analysis; Data representations; Learning on point clouds and graphs; Joint data analysis. Lecture Slides: Reading:		Visual data sets: ImageNet and ShapeNet; Techniques for annotation and annotation transport. Lecture Slides: Reading:		Topological inference; the Mapper algorithm. Applications. Lecture Slides: Reading:		Representations of 3D Geometry: Voxel-Grids, Point Clouds, Meshes and Other Boundary Models, Solid Models. Lecture Slides: Reading:		Networks of shapes and images; cycle consistency; map processing and latent spaces. Lecture Slides: Reading:		Deep nets for graphs and meshes. Lecture Slides: Reading:	
April 5		April 7		May 3		May 5		May 31		June 2	
Linear algebraic techniques: principal components analysis (PCA), Kernel PCA. Lecture Slides: Reading:		Linear algebraic techniques: canonical correlation analysis (CCA). Multidimensional scaling (MDS). Lecture Slides: Reading: Homework 1 out.		Geometry processing; Laplace-Beltrami and other operators on meshes. Lecture Slides: Reading:		Rigid and non-rigid shape alignment. Global and local shape descriptors; intrinsic descriptors, heat and wave kernel signatures. Lecture Slides: Reading: Homework 2 due. Homework 3 out.		Memorial day holiday -- no class		Encoding shape differences and shape variability. Class summary. Lecture Slides: Reading: Homework 4 due Friday, June 4.	
April 12		April 14		May 10		May 12					
Graph methods; spectral approaches, graph Laplacians, Laplacian embeddings, spectral clustering. Lecture Slides: Reading:		Non-linear dimensionality reduction: locally linear embeddings, Laplacian eignemaps, Isomap, t-SNE. Lecture Slides: Reading:		Class Midterm		Geometric deep learning; Volumetric and multi-view CNNs for 3D geometry Lecture Slides: Reading:					
April 19		April 21		May 17		May 19					
Computational topology: topology review, complexes, homology groups. Lecture Slides: Reading:		Persistent homology, barcodes and persistence diagrams. Reading: Lecture Slides: Homework 1 due. Homework 2 out.		Deep nets for pointclouds and applications to classification and segmentation. Lecture Slides: Reading:		Functional spaces and functional maps, variations; map visualization. Lecture Slides: Reading: Homework 3 due. Homework 4 out.					

Key Course Goals

- Cover basic tools for geometric and topological data analysis, both supervised and unsupervised
- Present mathematical ways to encode and transfer knowledge about the data
- Present methods for joint data analysis – benefiting from the “wisdom of the collection”

Data Has Shape



That's All

