

# CS233, CME251: Geometric and Topological Data Analysis

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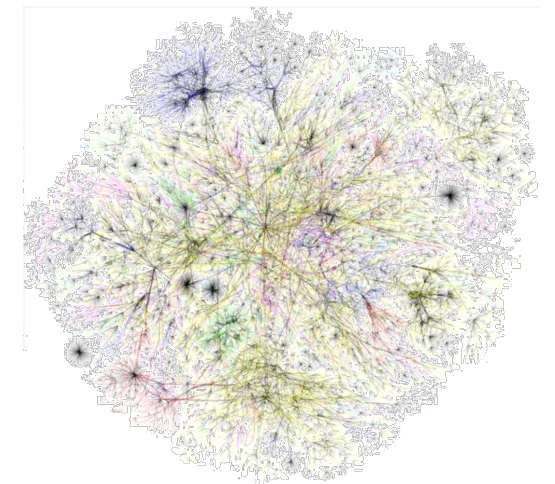
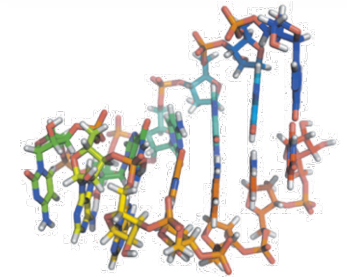
Lecture 1  
28 March 2022



# Introduction

# Big Data Era

- Data from many kinds of sensors
- Data from simulations
- Data from the activities of individuals on the internet



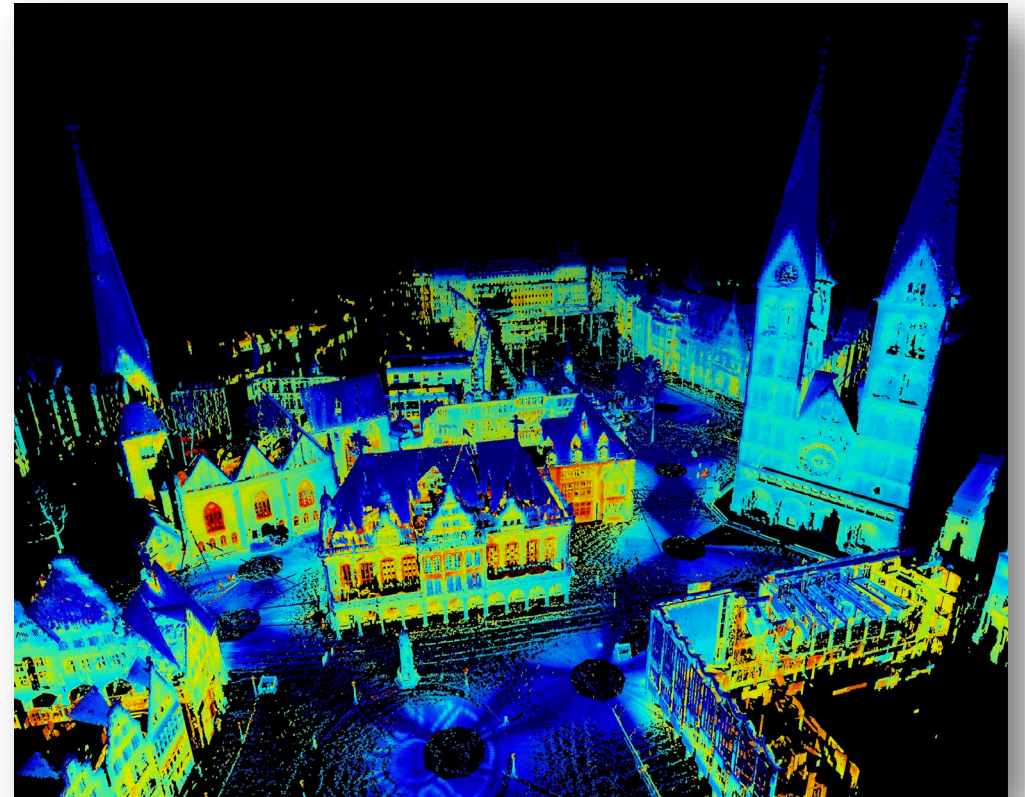
“Data Science”

# Data Sets of Geometric Character

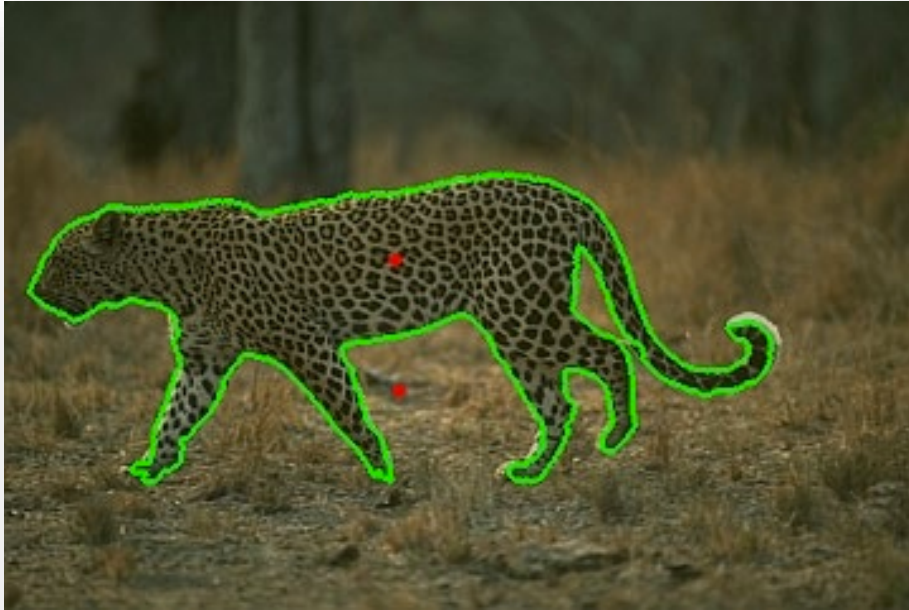


2D vehicle GPS traces (Beijing)

3D city scans (Bremen)



# Semantics Requires Geometry

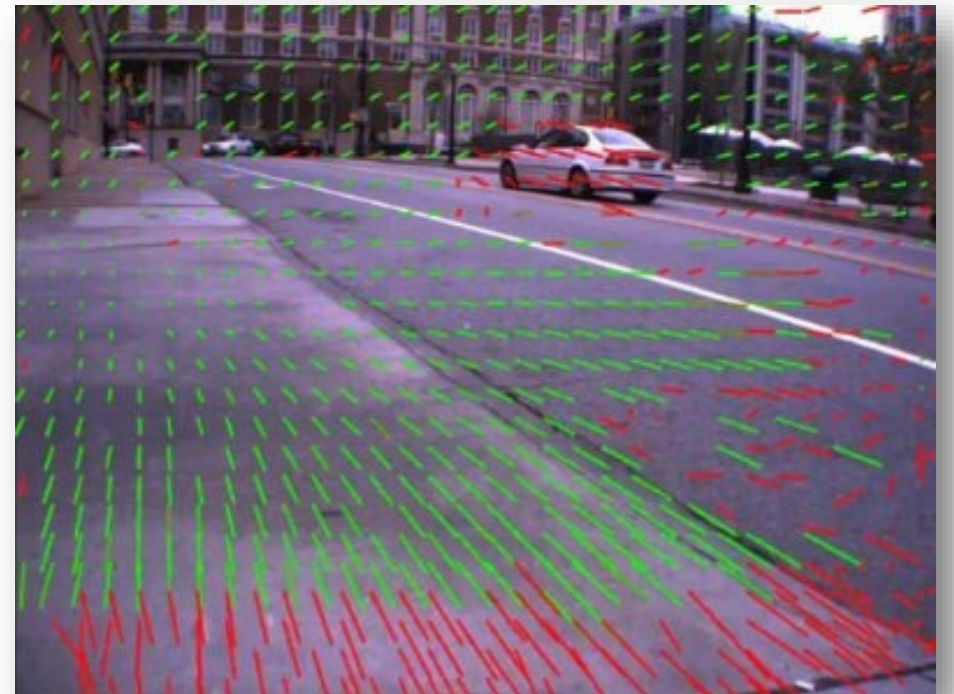


Object recognition and foreground extraction

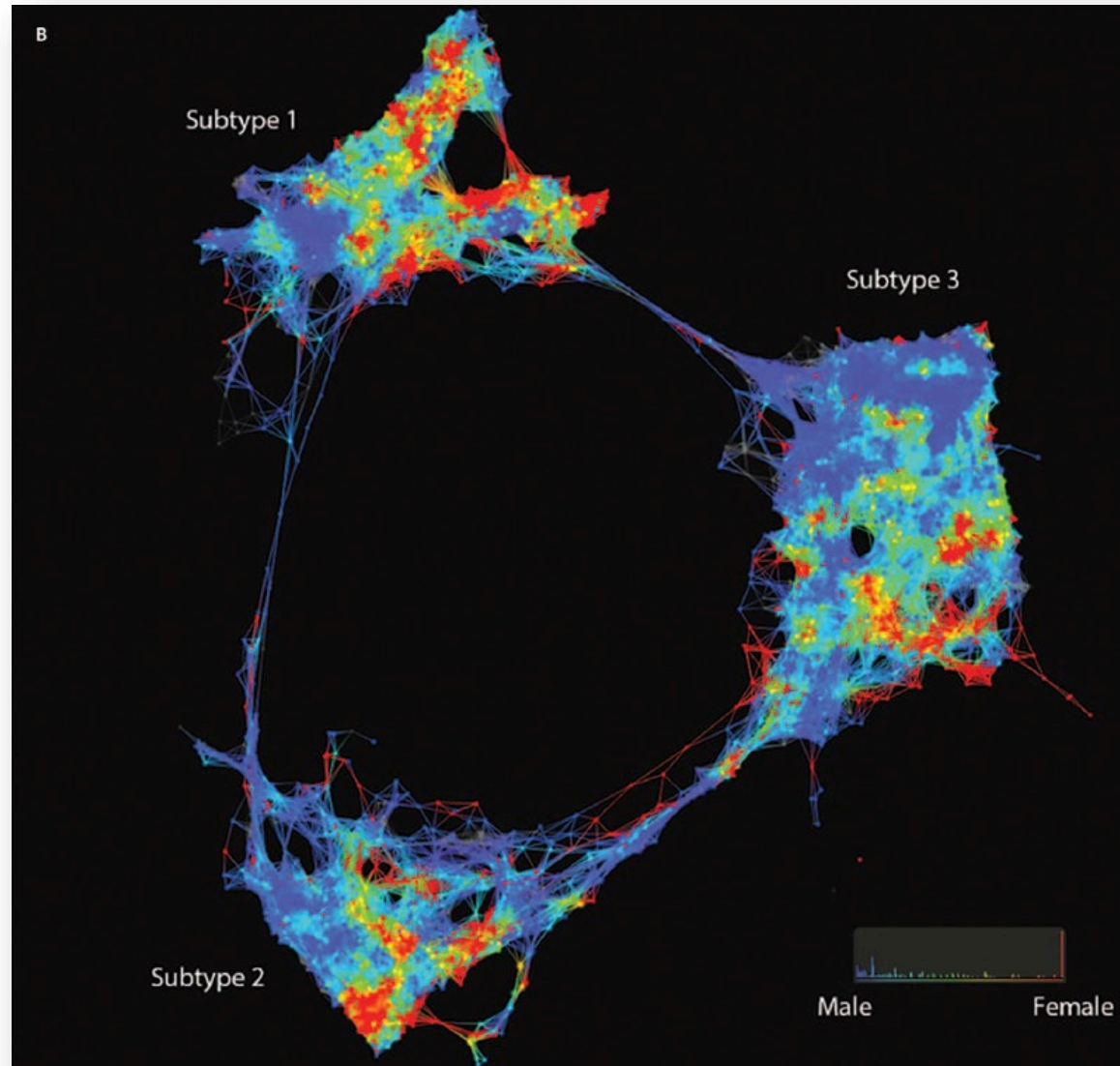
Segmentation involves geometry

Motion involves geometry

Optical flow

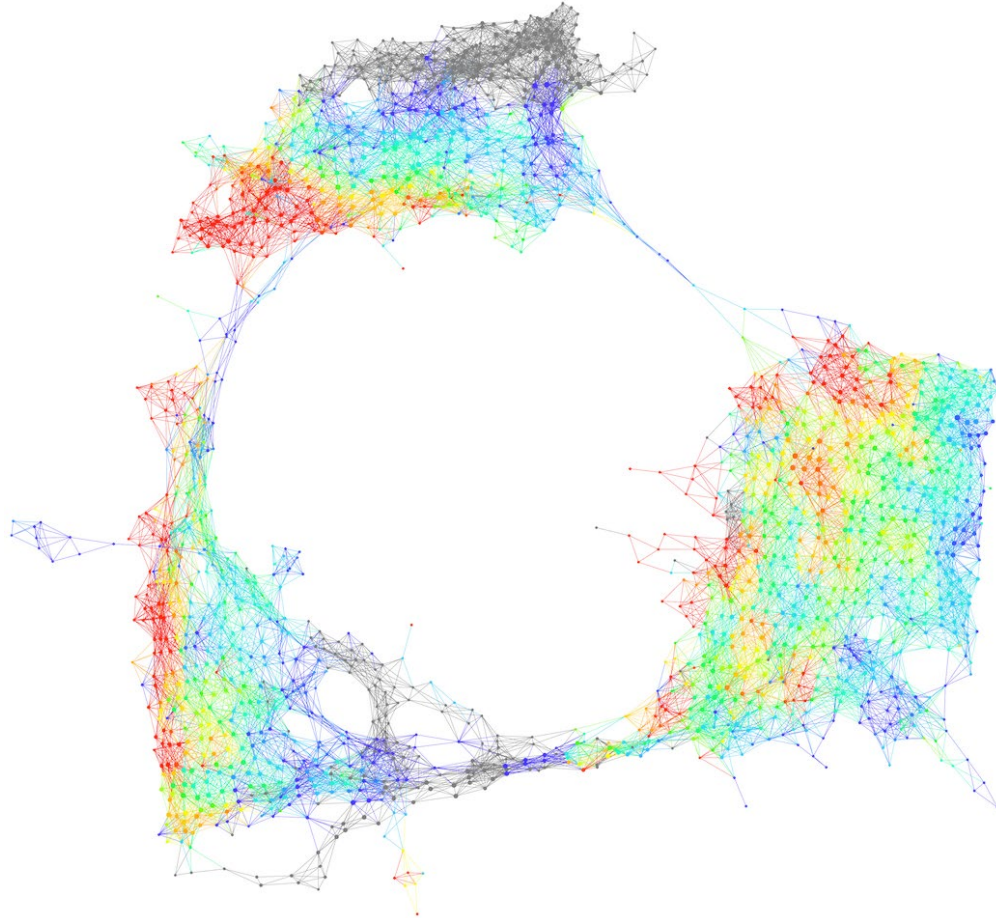


# Non-Geometric Data



Diabetes II  
subtypes

# Data Has Shape

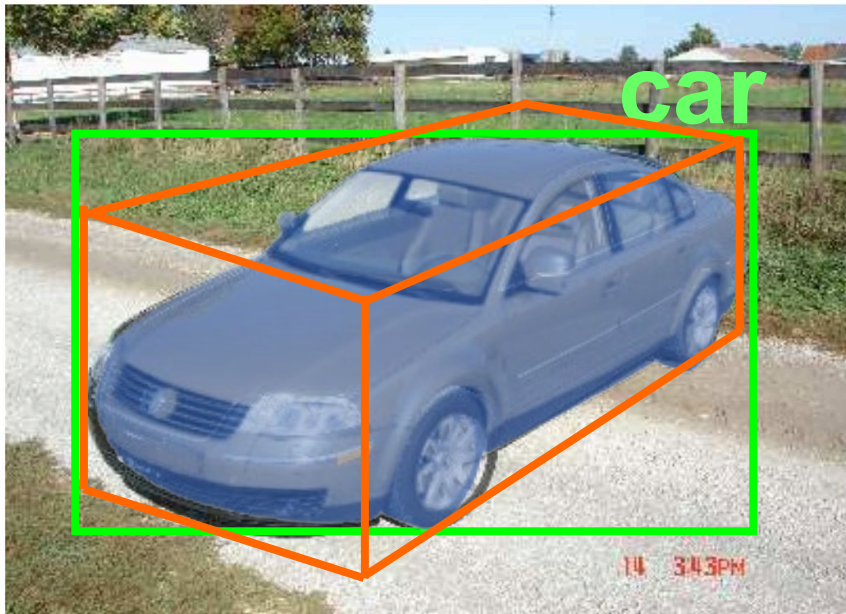


# The Geometry of Data or the Data of Geometry

geometric data analysis

# The Role of Supervision in Geometric ML

- The success of deep learning has to a large extent been made possible by huge annotated data sets and the availability of large amounts of computing power
- Yet, in many settings, obtaining quality training data at scale from human annotation is challenging – especially in geometric or 3D settings



ImageNet



# Towards Reduced or No Supervision

- Thus it is important to consider learning protocols and architectures that reduce, minimize, or eliminate supervision
  - Transfer learning, semi-supervised learning, few-shot learning, unsupervised learning
- **Joint learning** takes a more holistic approach towards the “social aspects” of learning
  - data, training or testing, have correlations
  - learning tasks have correlations
  - representations of data have correlations

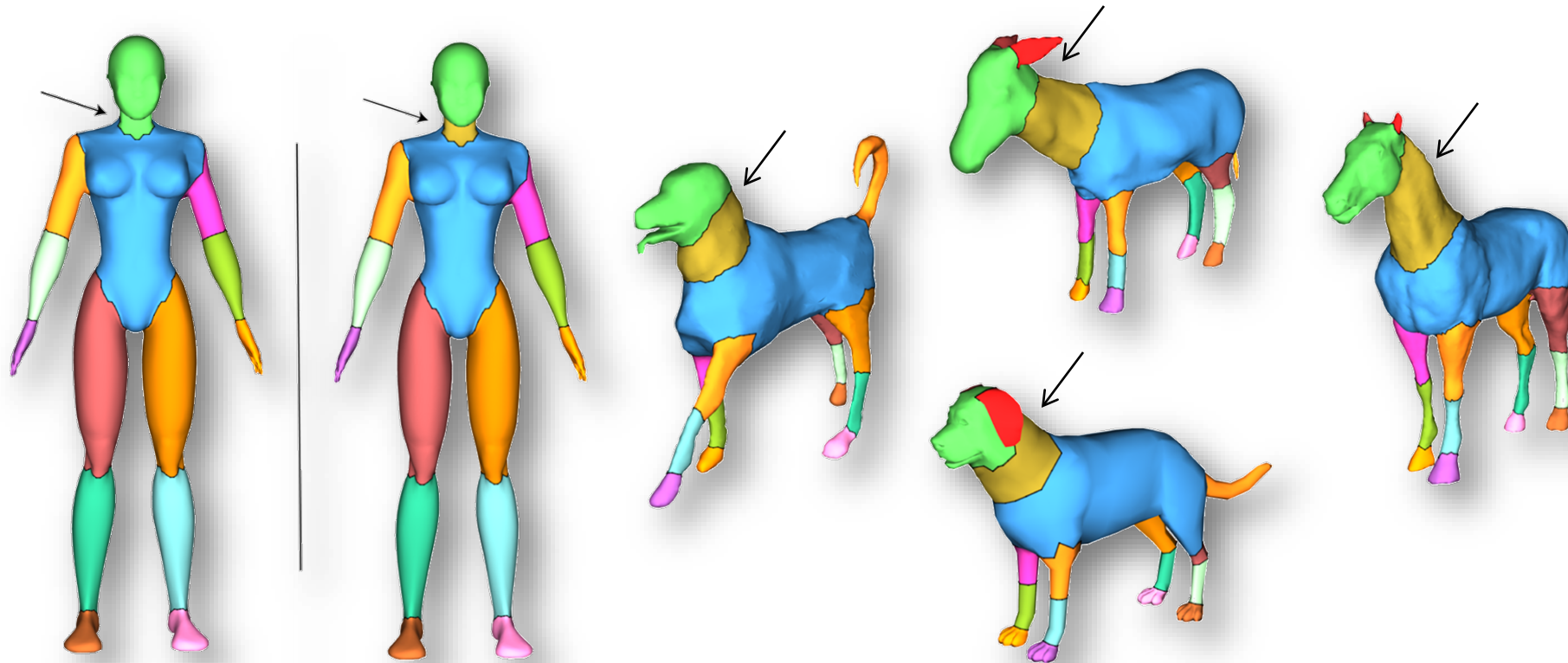


ImageNet



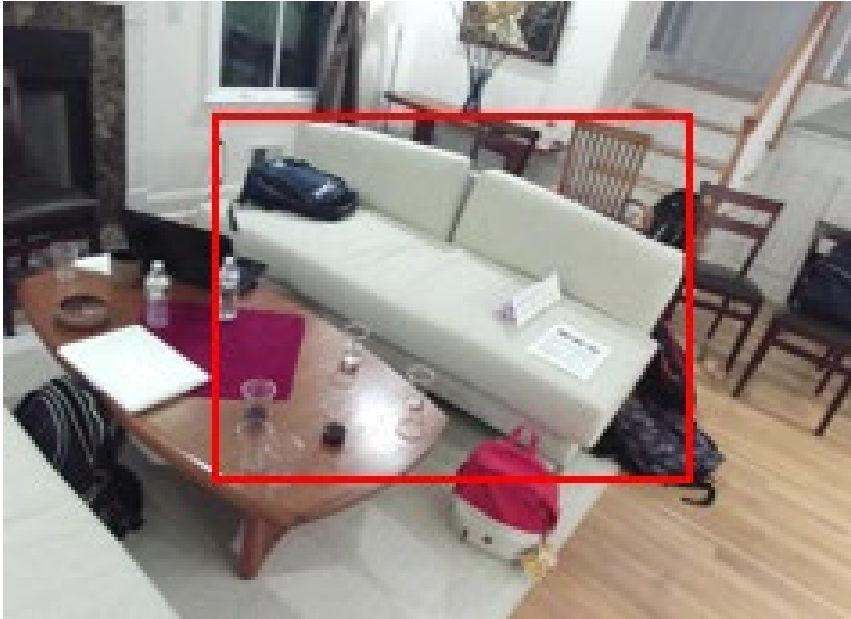
# Unsupervised Segmentation: “Wisdom of the Crowd”

- The interpretation of a particular piece of geometric data is deeply influenced by our interpretation of other related data

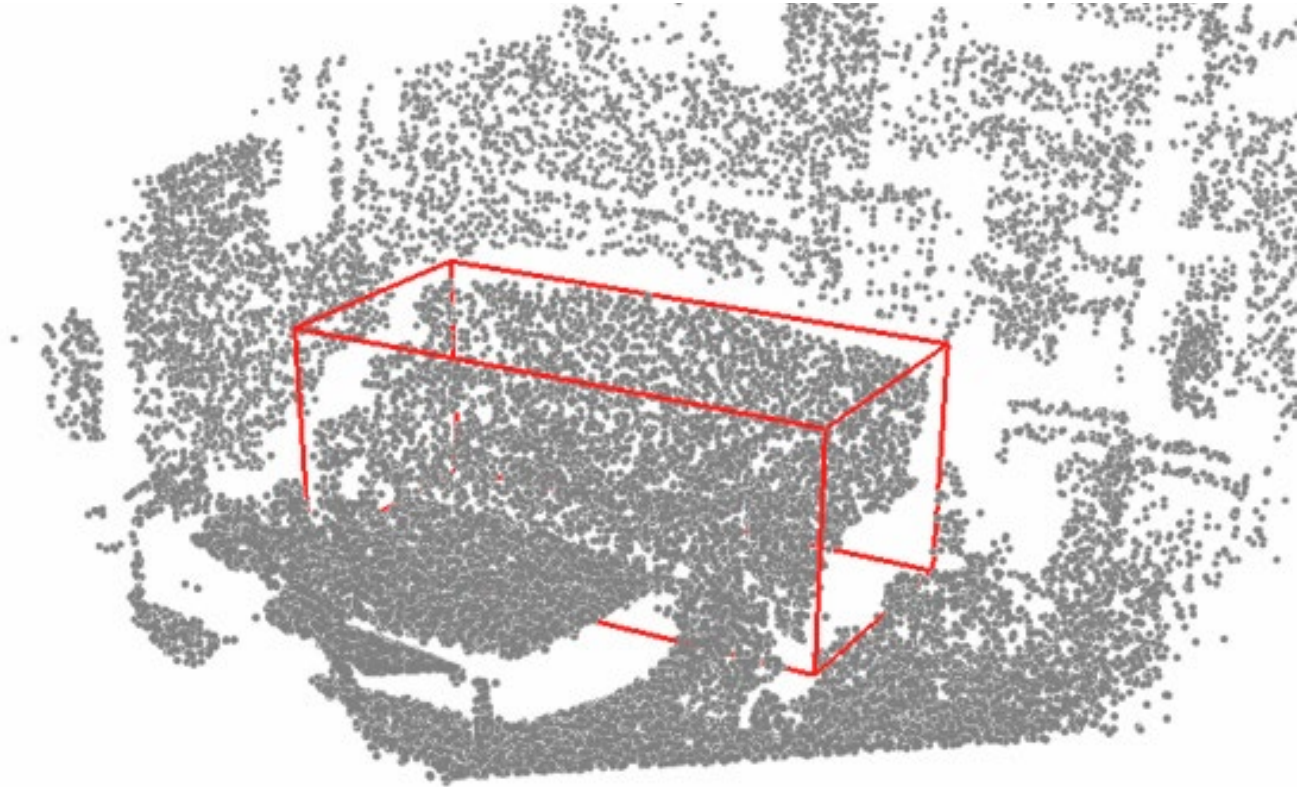


3D Segmentation

# Data Integration Across Multiple Modalities



- + High resolution
- + Dense coverage
- - Subject to many imaging artifacts



- + Absolute depth and scale
- - Sparse, low rez

# Joint Learning

- Need to aggregate information:
  - over different data sets
  - over different modalities (geometry, appearance, language)
  - over space and time
  - over different representations
  - over different predictions
  - over different tasks

*in settings where the above refer to same entities in the world – and are thus correlated*



# The Challenge: Information Transportation and Aggregation

- Aggregation needs
  - the ability to *transport information*
  - the ability to ignore *nuisance factors*
  - to ability to have a common language
    - *information representation consistency*
- Some powerful tools
  - voting mechanisms
  - factorization and canonicalization
  - path invariance and loop closure in transport



Tower of Babel

# Look at Data from Many View Points

- Algebraic
- Graph theory based
- Geometric
- Topological



Getting to the semantics of data

# CS233 Key Course Goals

- Cover basic tools for **geometric and topological data analysis**, both supervised and unsupervised
- Discuss mathematical ways, based on geometry and topology, to **encode and transfer knowledge** about the data
- Introduce methods for **joint data analysis and joint machine learning** – benefiting from the “wisdom of the collection”
- Challenge: a diversity of tools ... LA, ML, Stat, optimization, geometry processing, computer vision, algebraic topology ...

# Prerequisites / Overlaps

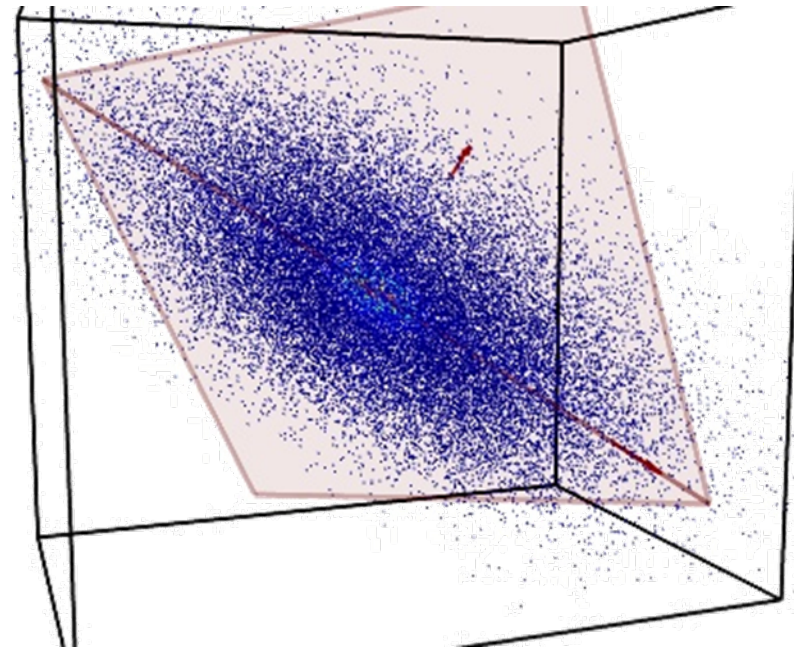
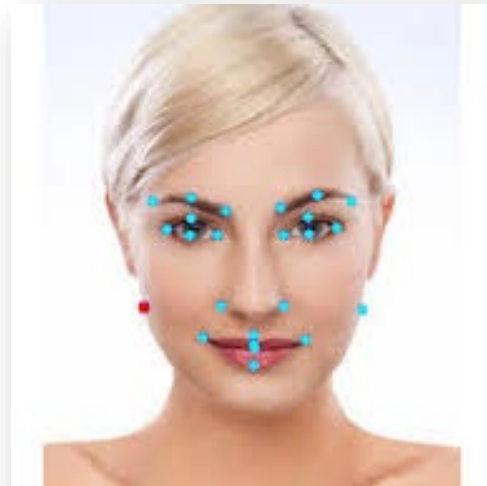
- Presumes some knowledge of linear algebra, optimization, algorithmic thinking, basic geometry ...
- As compared to CS229, STATS216, greater emphasis on diverse, irregular data types (e.g., point clouds, graphs, meshes) and unsupervised methods, in addition to the geometry / topology angle
- Data sets will be primarily visual/geometric (images, 2D/3D point clouds, meshes, or CAD models)

# Unsupervised Methods

## The Linear Space View of Data

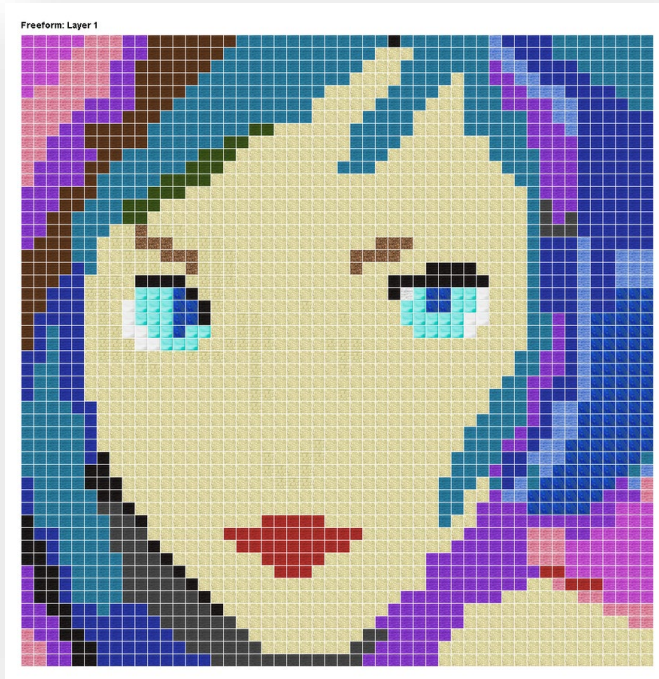
# Embedding Data into a Euclidean Space

- Attributes or features can be used to map data to a Euclidean space

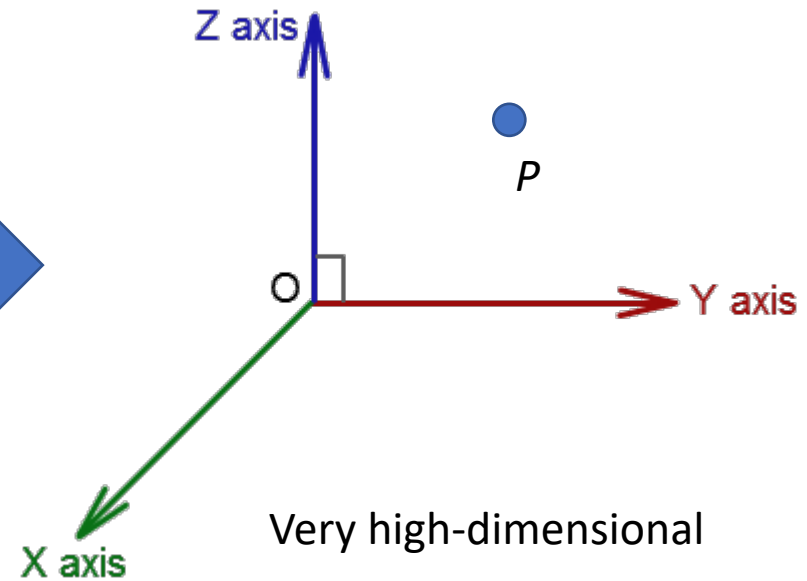


# Direct and Indirect Embeddings

◆ Input



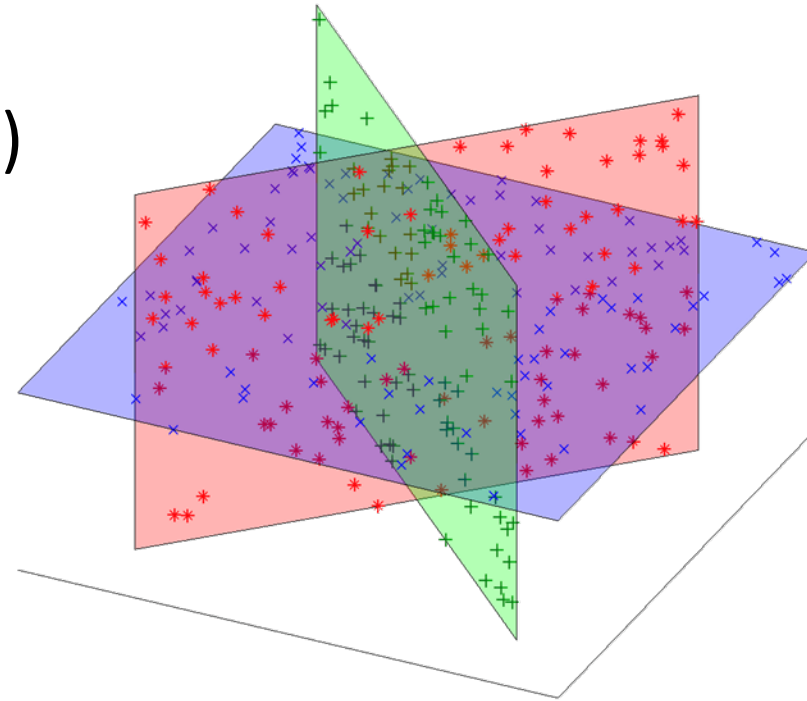
◆ Latent code



# Linear Space Methods

- Principal components analysis (PCA)
- Canonical correlation analysis (CCA)

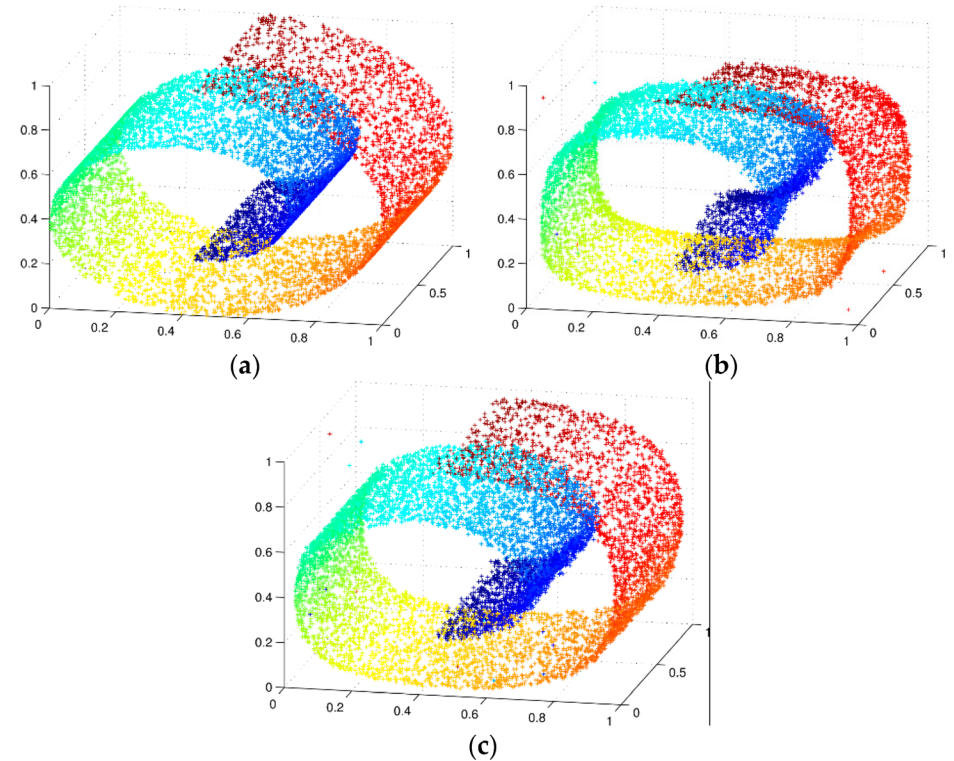
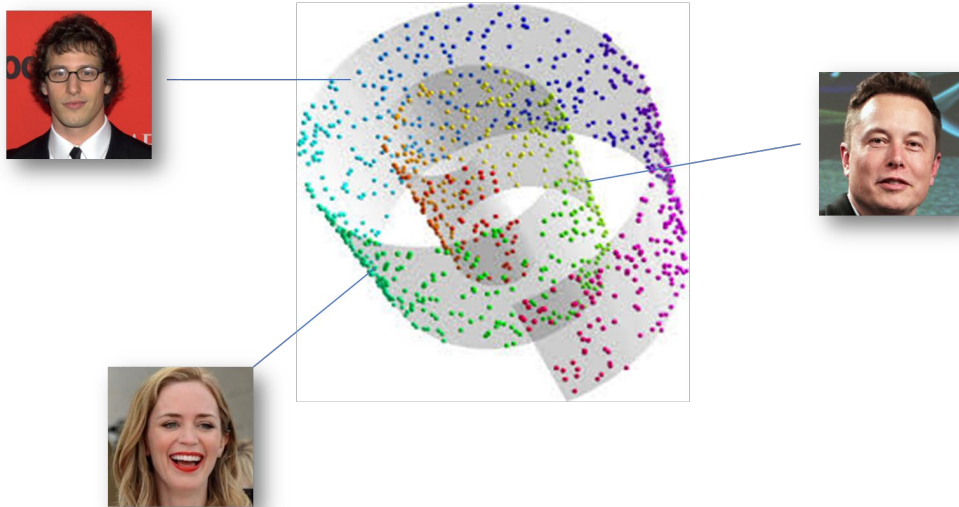
Dimensionality reduction



Low-d data (intrinsic dimension) living on a linear subspace,  
inside a high-d space (extrinsic dimension)

# Data as Points on a Manifold

- Non-linear dimensionality reduction
- Low-d data inside high-d space may lie on a non-flat manifold



Isomap, locally linear embeddings, Laplacian eigenmaps, t-SNE

# Supervised Methods, Deep Learning

# Machine Learning

## ◆ Traditional Programming

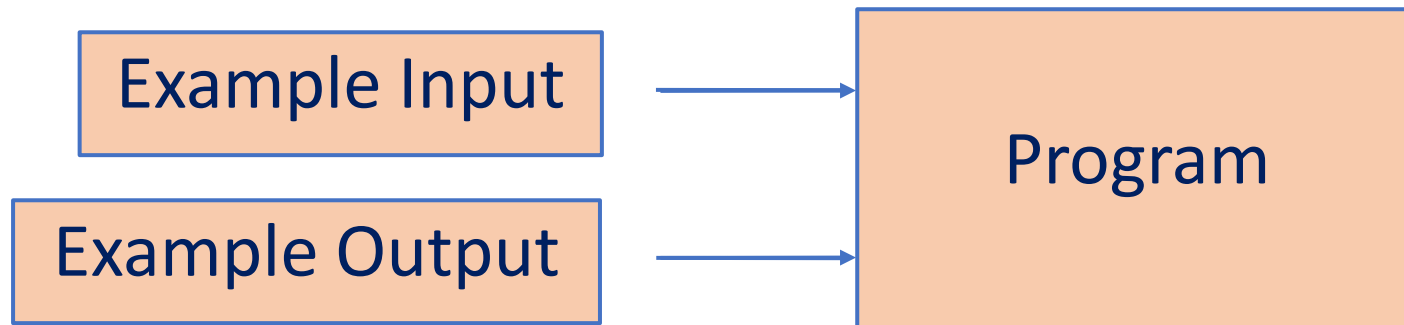


# Machine Learning

## ◆ Traditional Programming



## ◆ Machine Learning



# Machine Learning

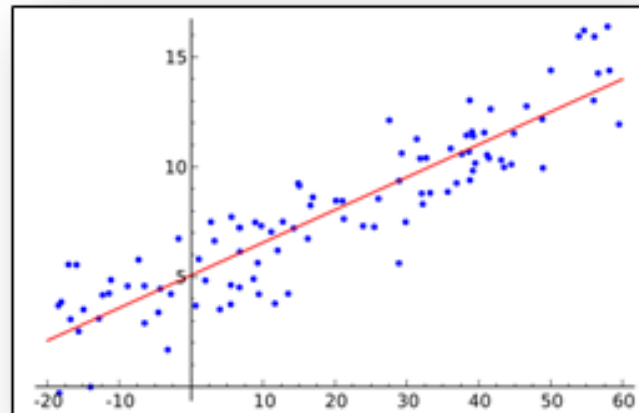
Parametrized by many  
**learnable** parameters

$f(\$

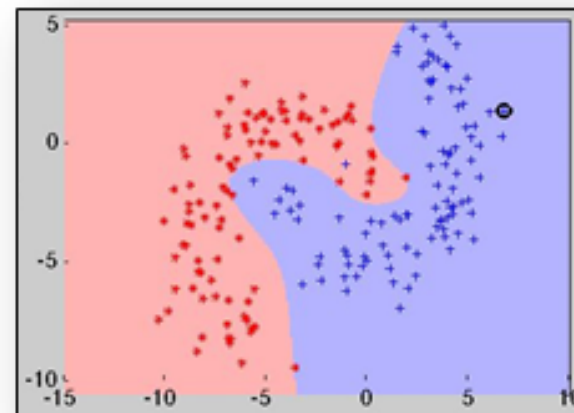


$)=cat$

Model Fitting



Regression

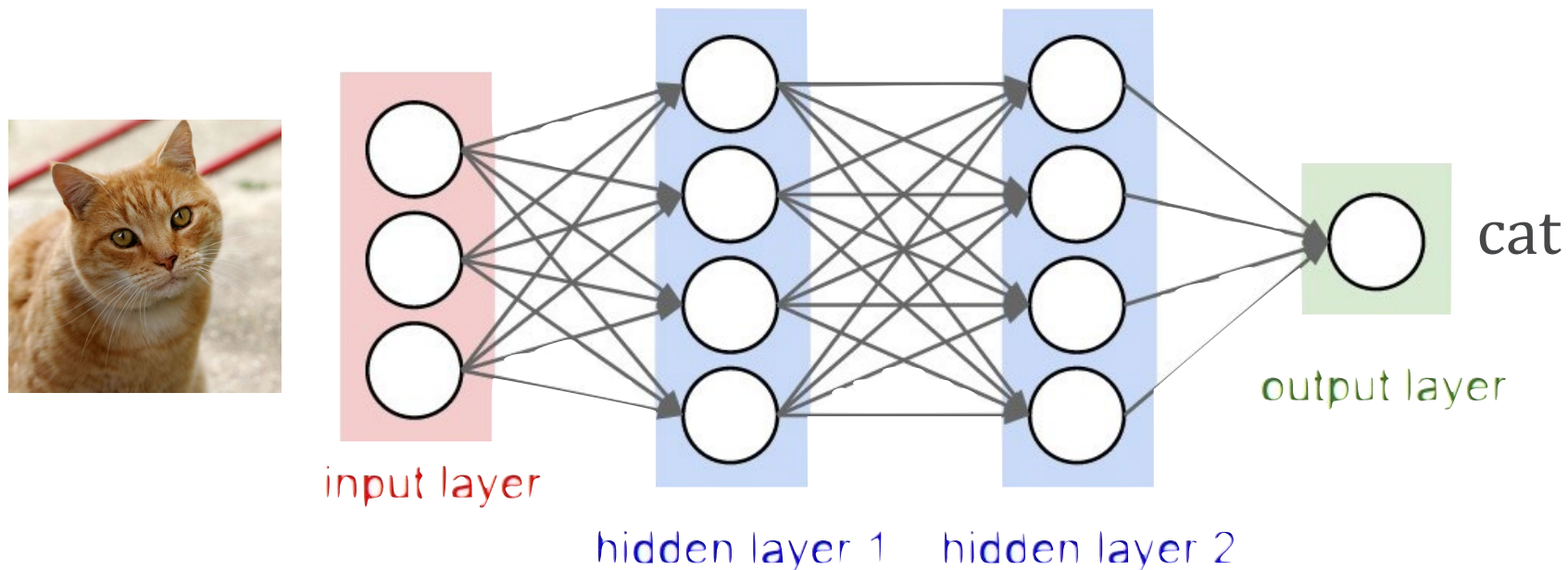


Classification

# Deep Learning

- ◆ Deep learning allows computational models that are composed of **multiple processing layers to learn representations of data** with **multiple levels of abstraction**.

*Deep Learning by Y. LeCun et al. Nature 2015*

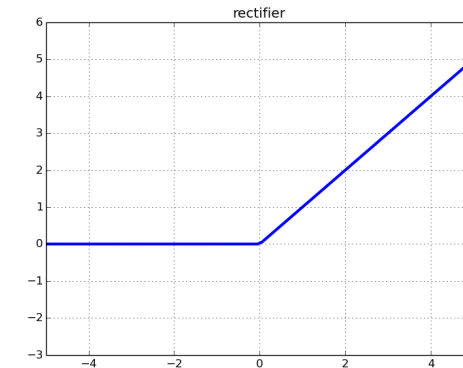
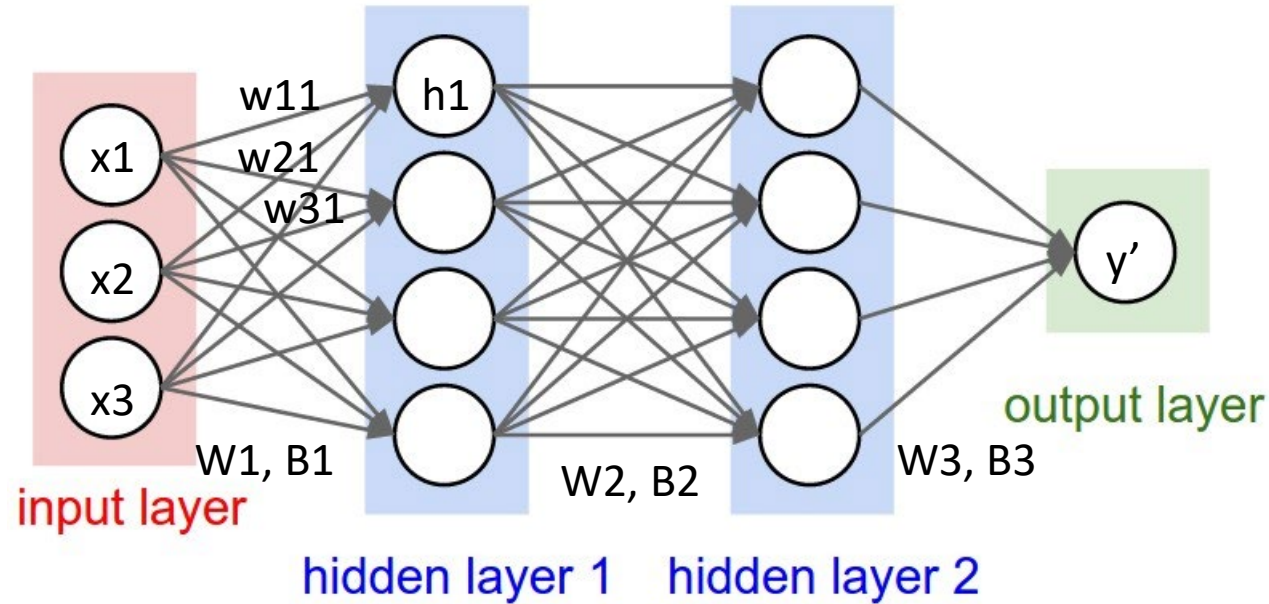


# Inside Neural Networks

$$h1 = f(w11 * x1 + w21 * x2 + w31 * x3 + b1)$$

$f$ : non-linear activation function

$$\text{ReLU}(x) = \begin{cases} 0 & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$

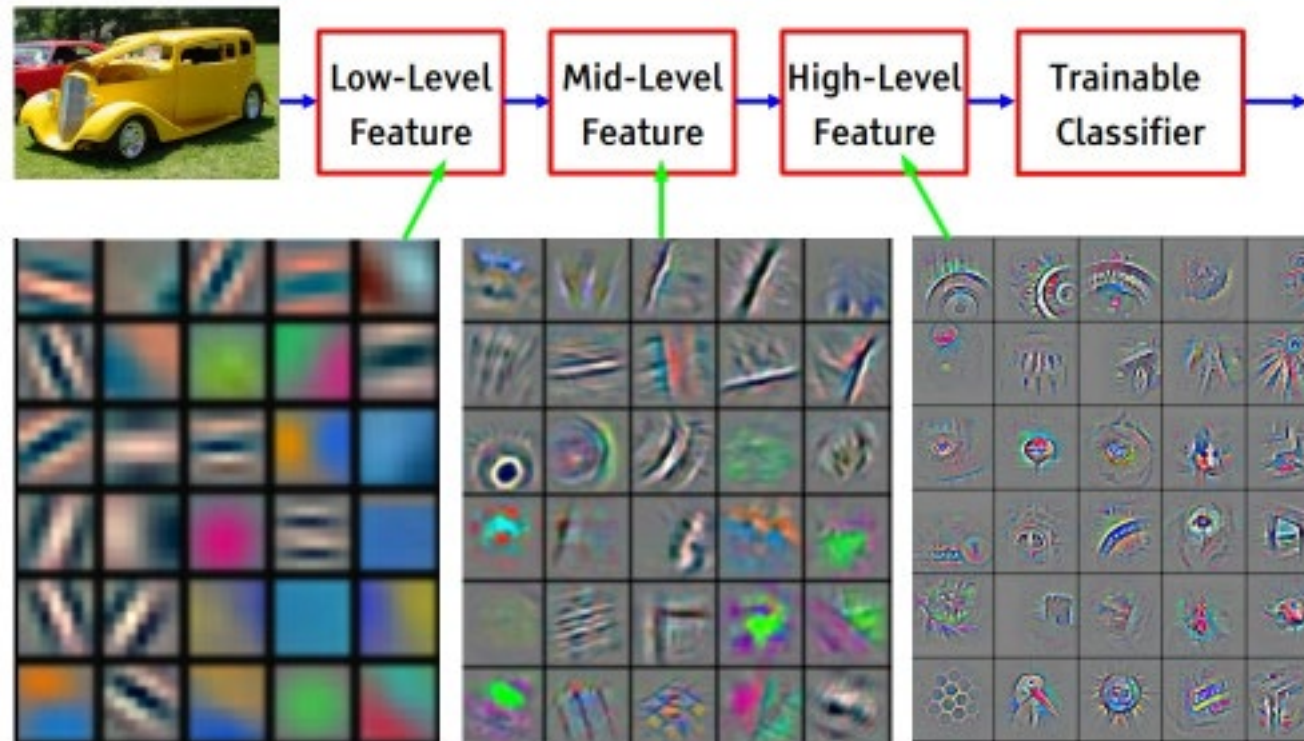


**Model:** Multi-Layer Perceptron (MLP)

$$y' = W_3 f(W_2 f(W_1 x + b_1) + b_2) + b_3$$

# Neural Networks as Feature Extractors

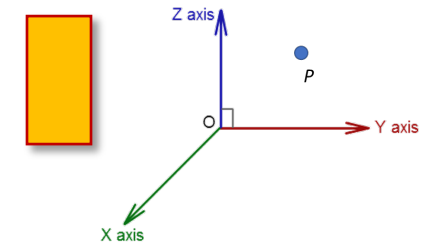
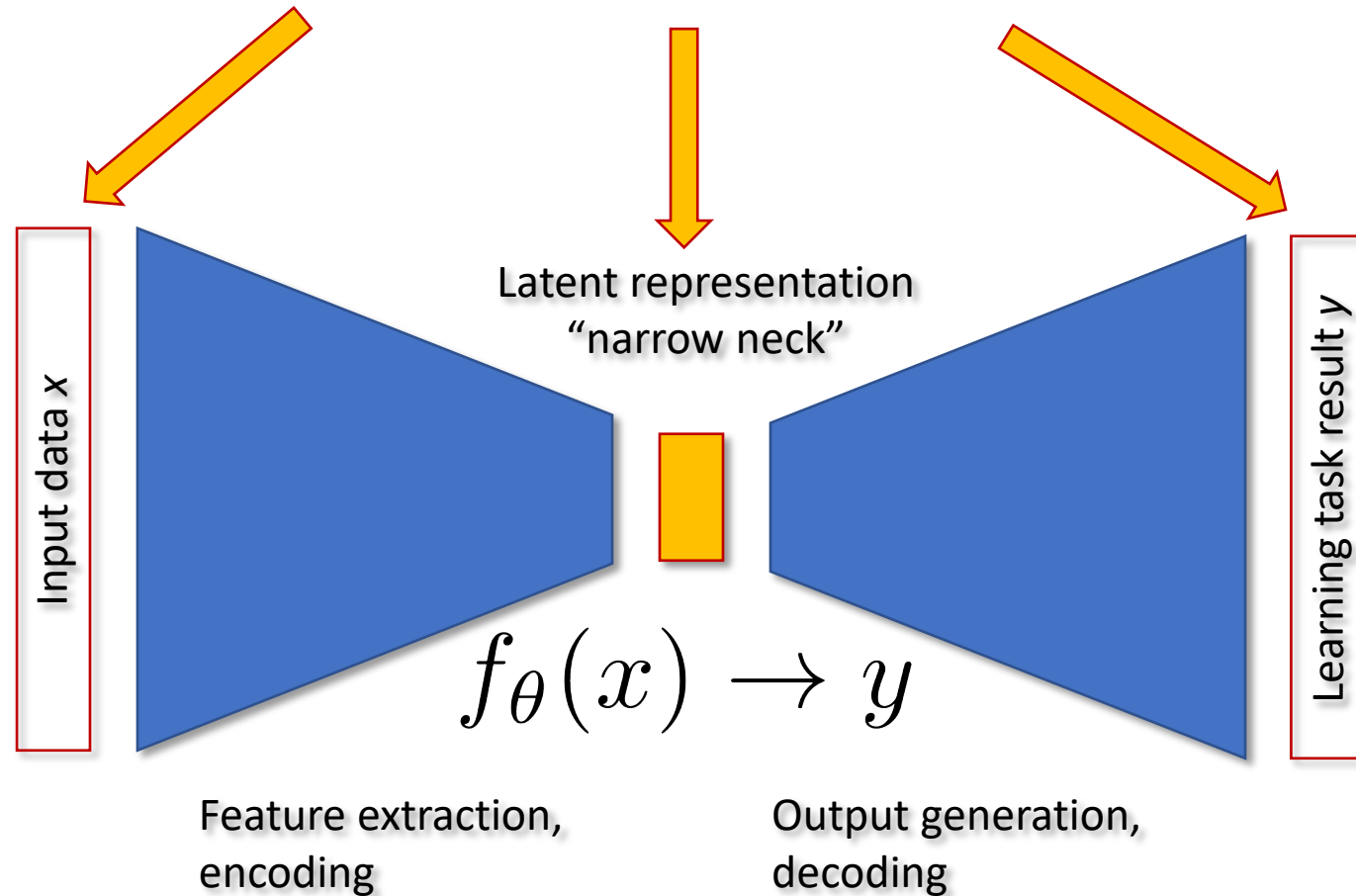
- ◆ Neural networks extract powerful features from data



Feature visualization of convolutional net trained on ImageNet from [Zeiler & Fergus 2013]

Image Credits: Yan LeCun

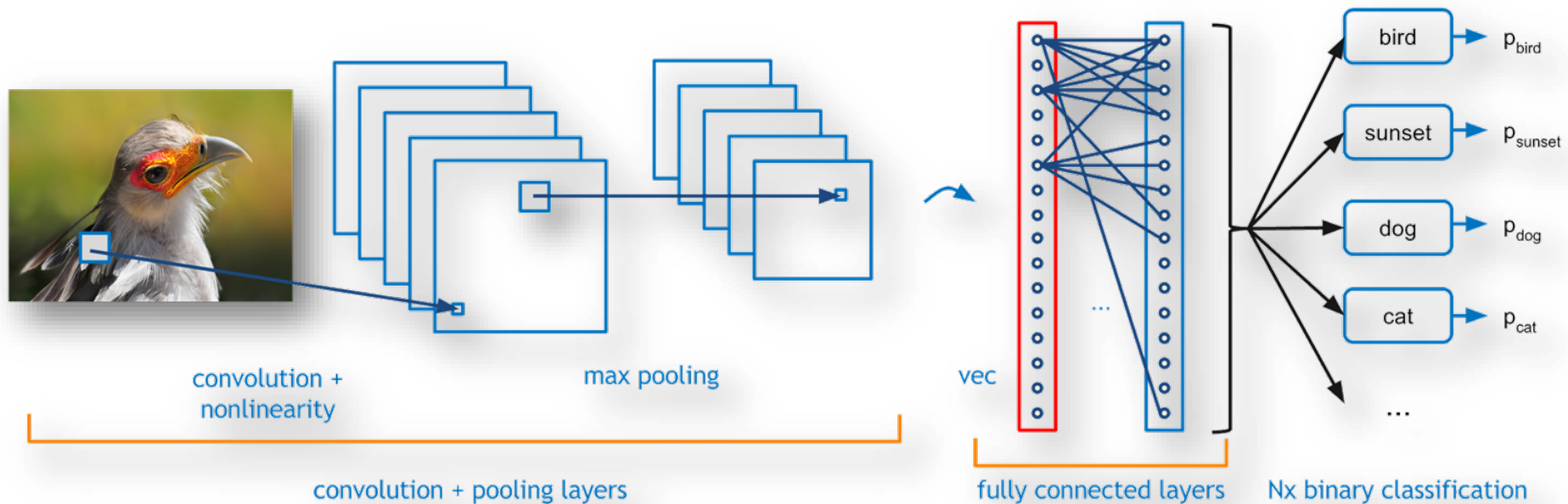
# Trainable Feed-Forward Learning Pipelines



A latent code (= a point in a Euclidean feature space) acts as a low-d proxy for high-d input data, w.r.t. a learning task.

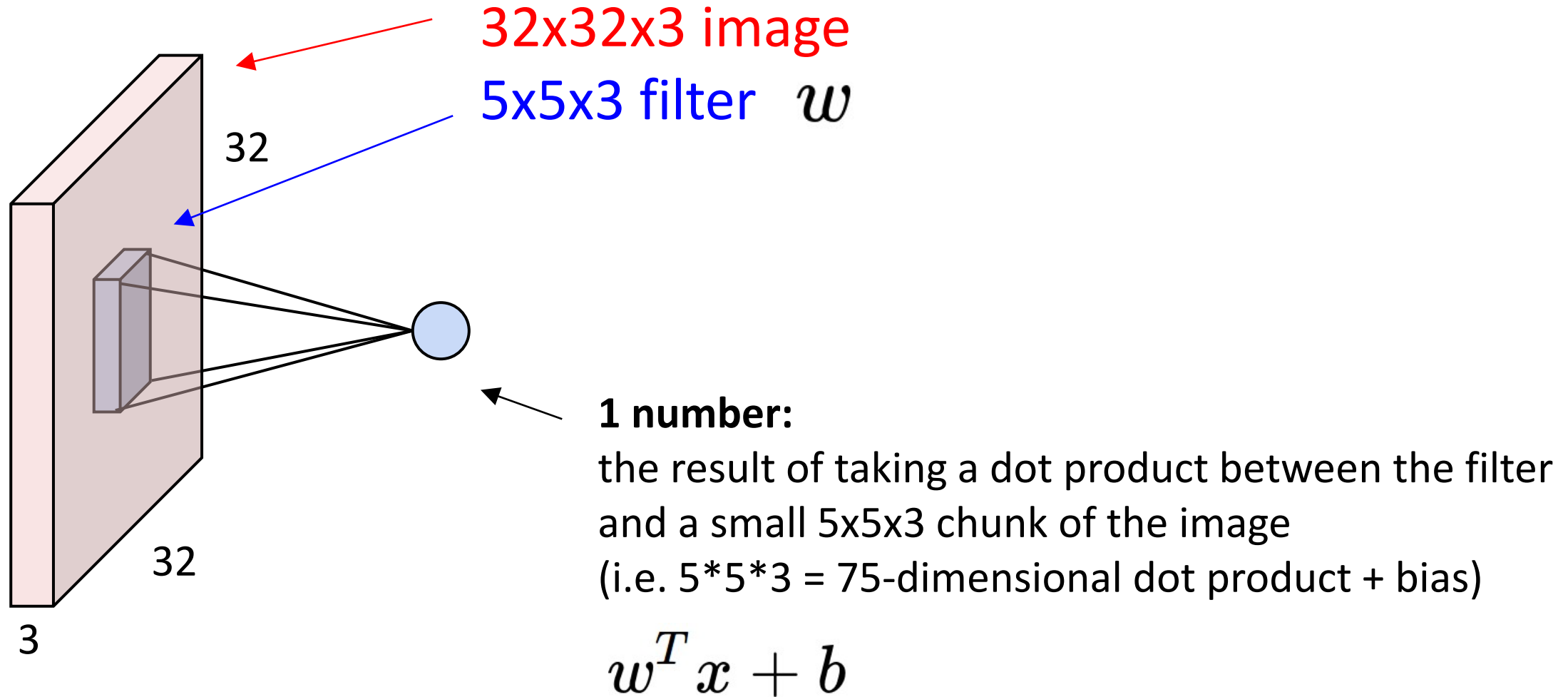
# Image Deep Learning Networks

- Feed forward networks

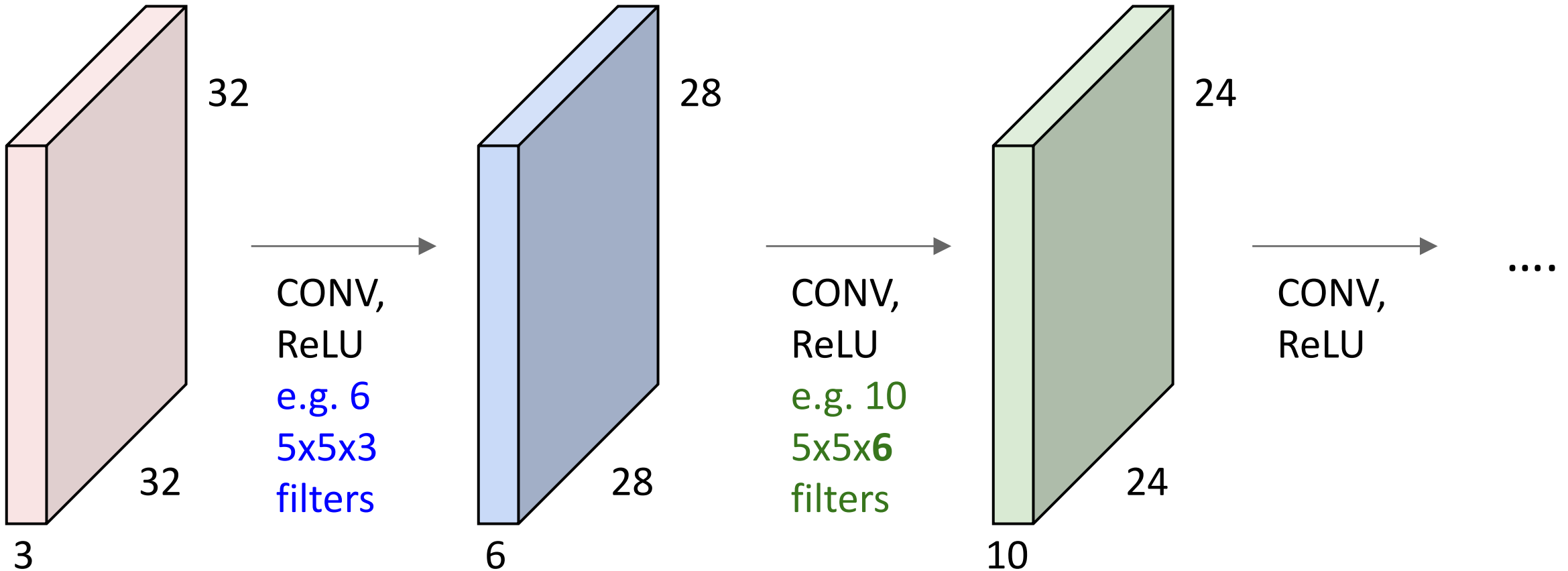


Classification

# Convolutional Layers



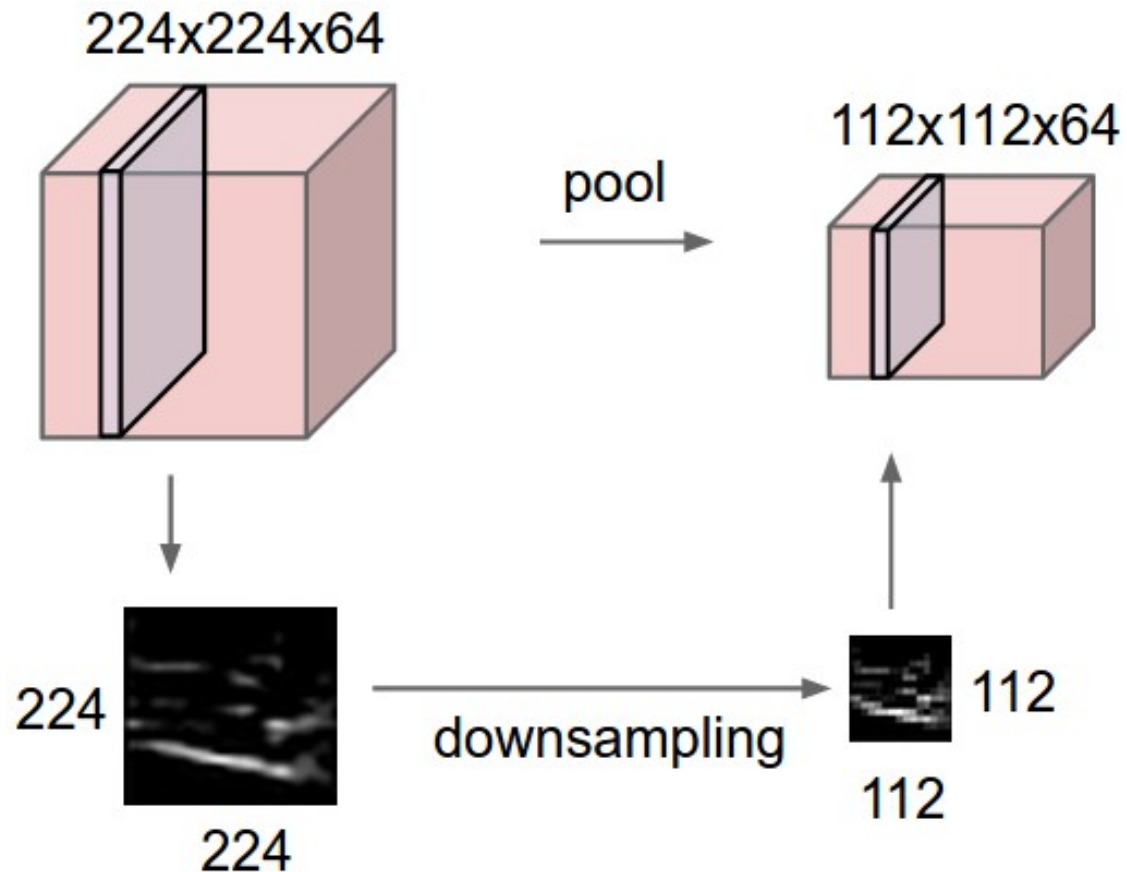
# Convolutional Architectures



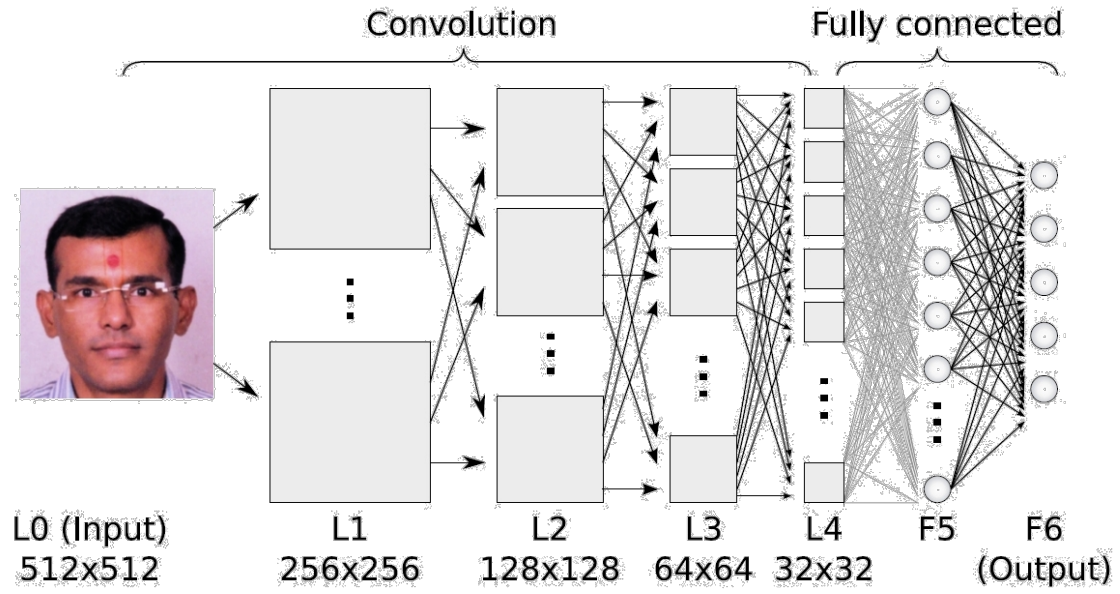
ConvNets are a sequence of convolutional layers, interspersed with activation functions

# Pooling Layer

- makes the representations smaller and more manageable
- operates over each activation map independently:



# Data Abstraction Across Layers

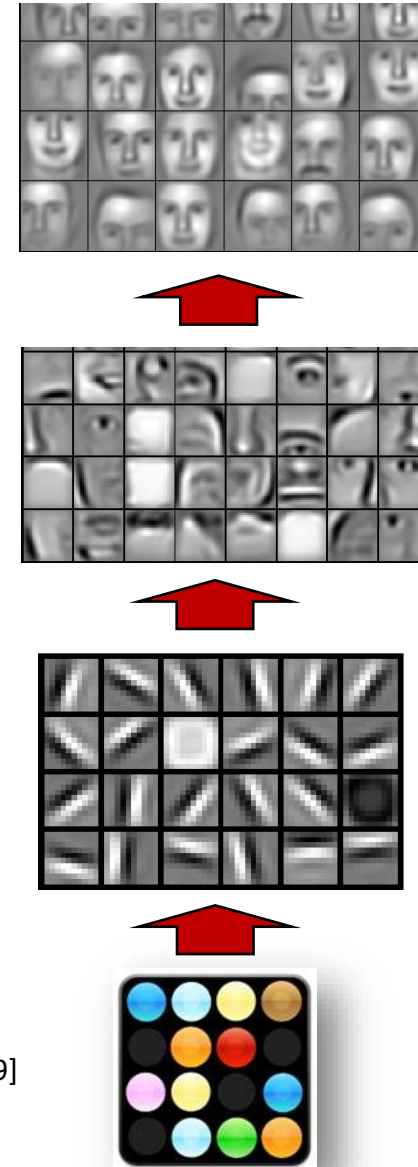


[Makwana, 2016]

Data-driven feature learning at ascending abstraction layers

“Vertical” networks

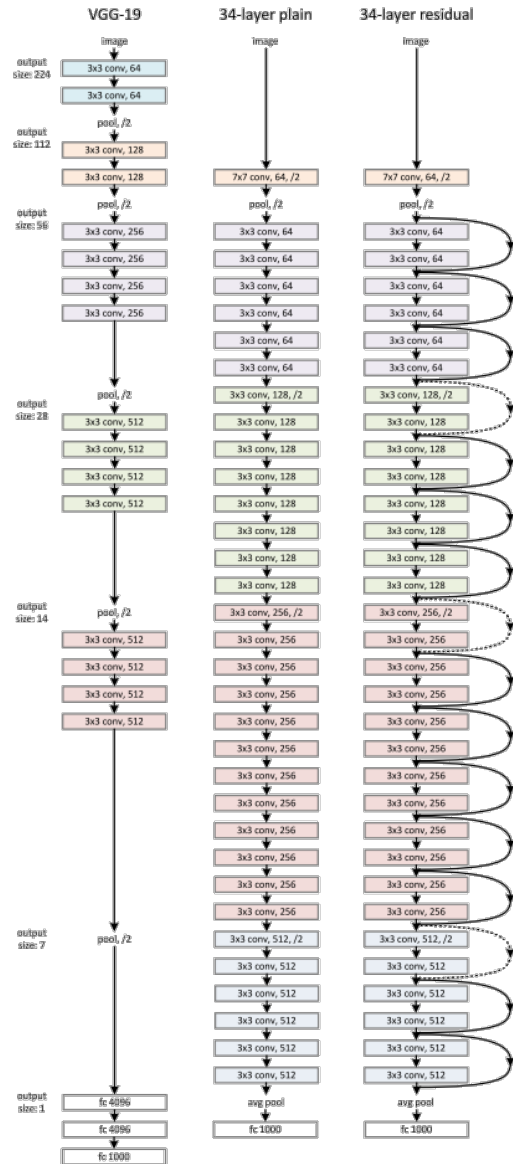
[Lee et al., 2009]



# Success Made Possible By ....

[He et al., 2015]

Novel deep architectures



Plenty of annotated data

Lots of computing power



# Transformers

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## Attention Is All You Need

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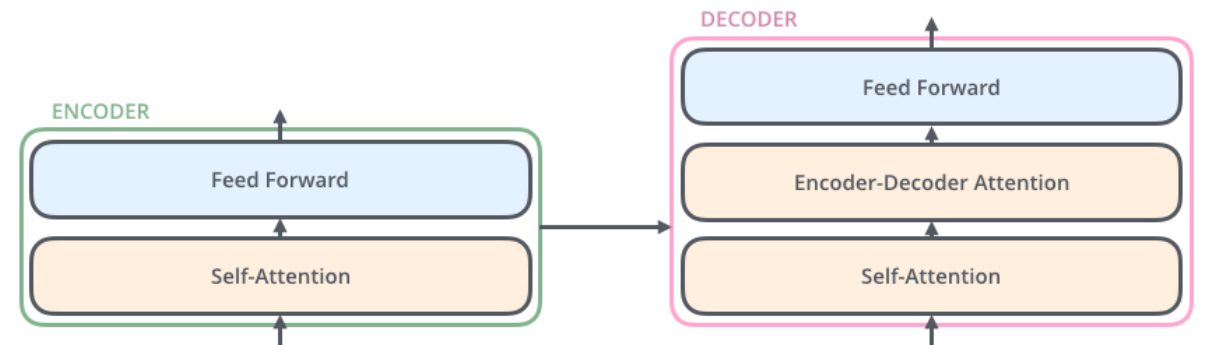
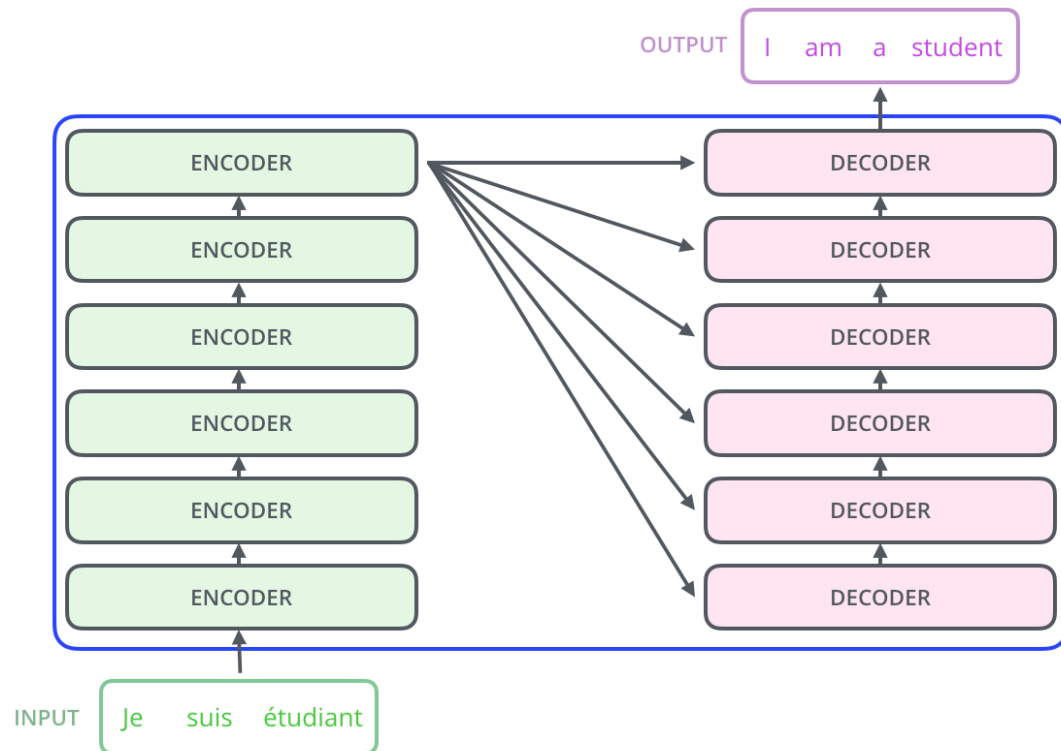
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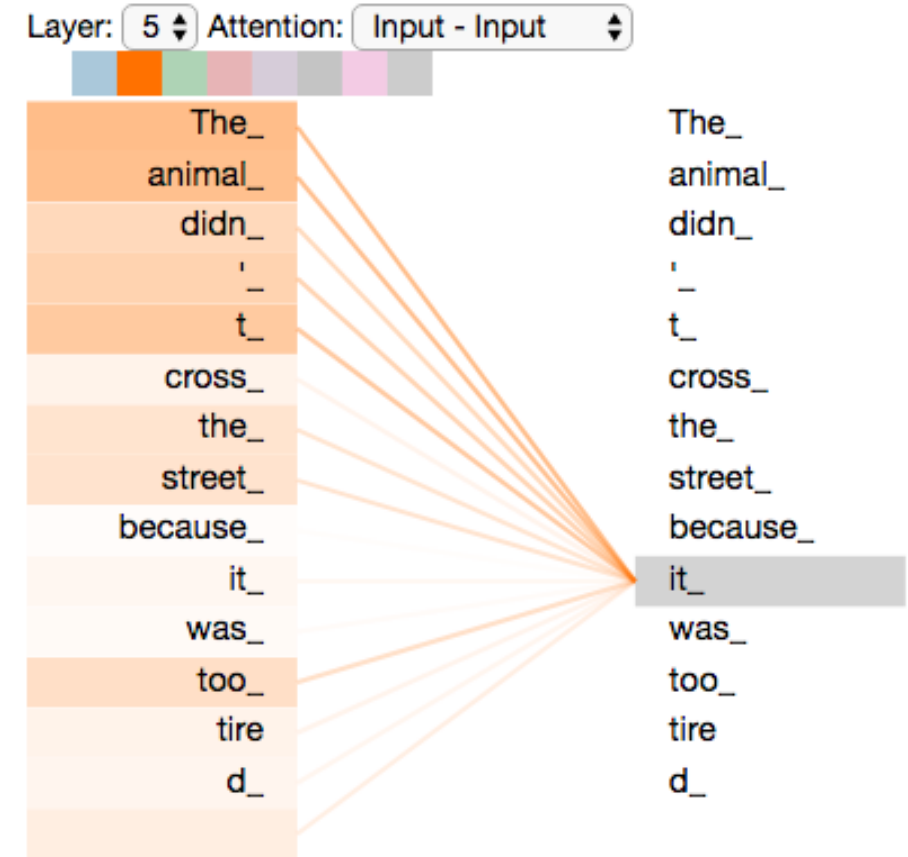
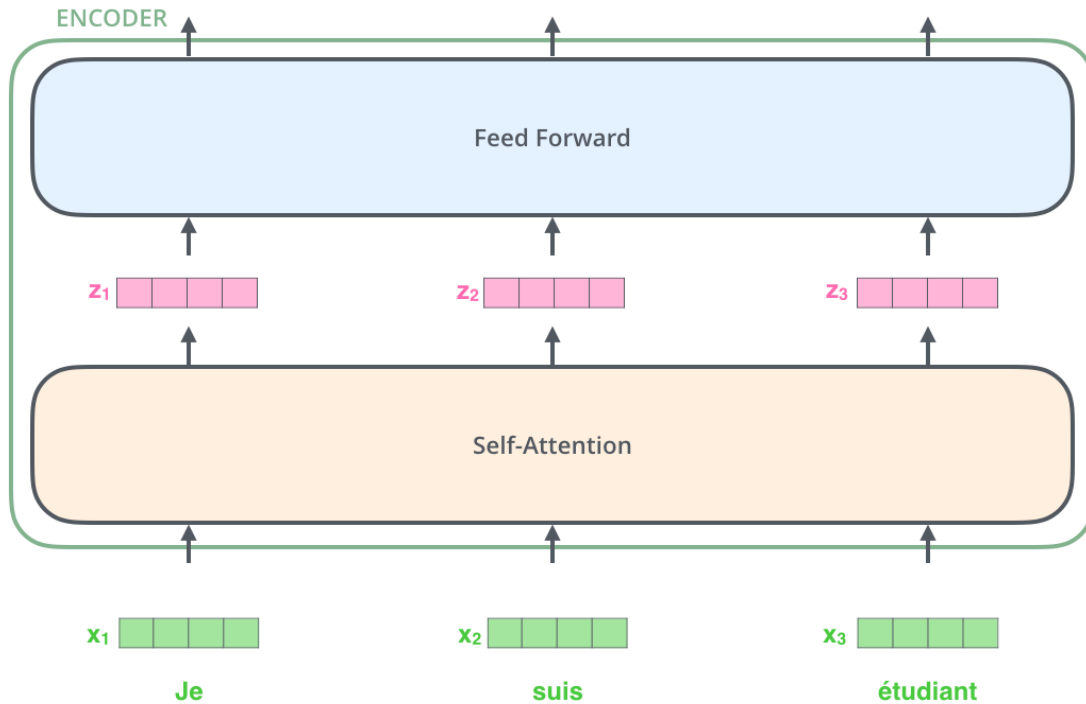


# Transformers



Credits: Jay Alammam's blog  
(<http://jalammar.github.io/illustrated-transformer/>)

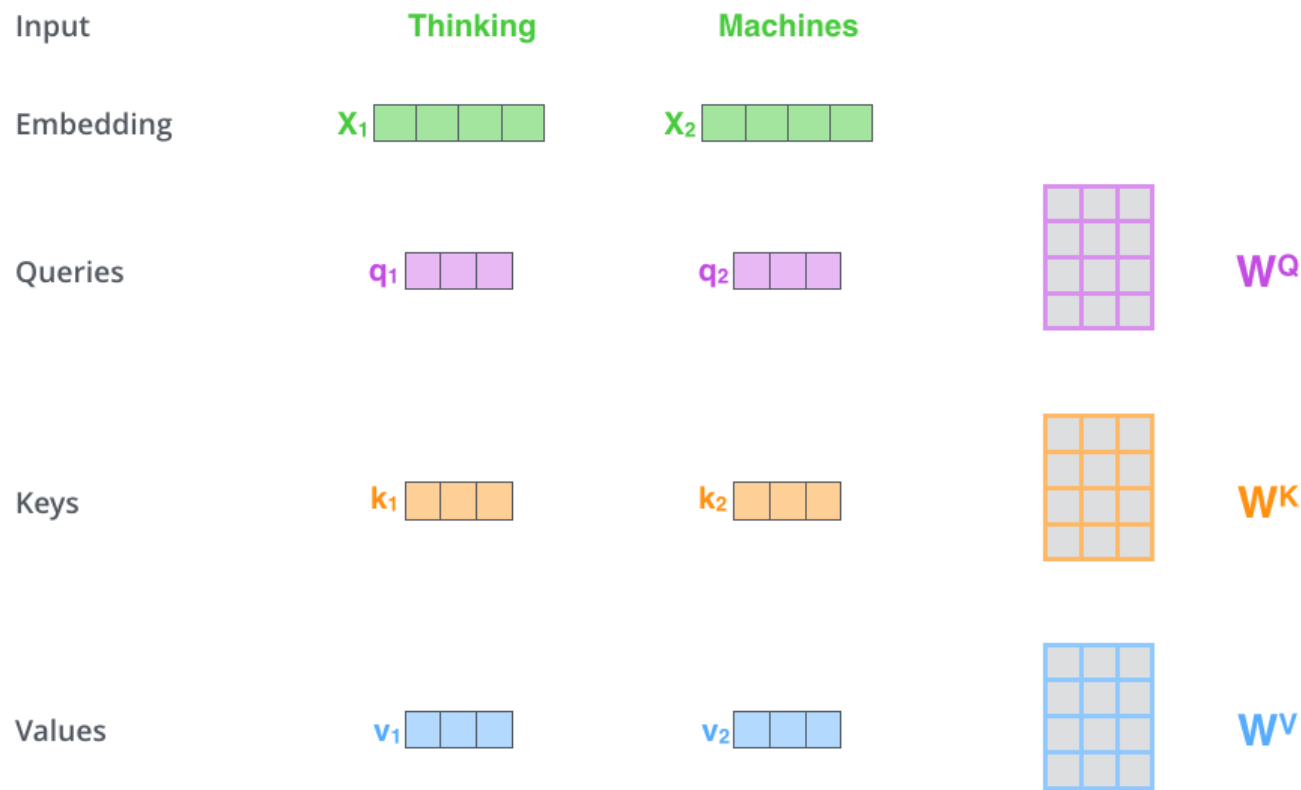
# Transformers for NLP: Attention Mechanism



Credits: Jay Alammar's blog

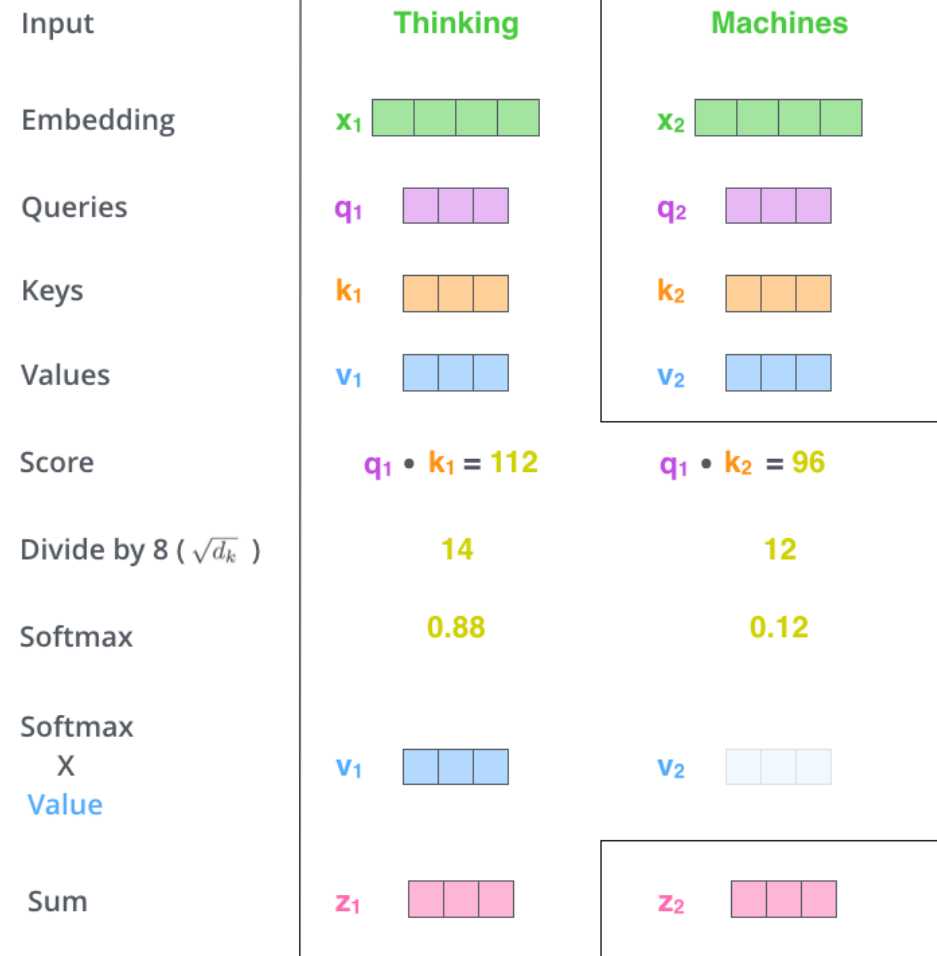
(<http://jalammar.github.io/illustrated-transformer/>)

# A Closer Look at Self-Attention



$$Q = W^Q \times X; \quad K = W^K \times X; \quad V = X^V \times X$$

$$Attention(Q, K, V) = softmax\left(\frac{QK^T}{\sqrt{d_k}}\right)V$$



Credits: Jay Alammar's blog  
[\(http://jalammar.github.io/illustrated-transformer/\)](http://jalammar.github.io/illustrated-transformer/)

# Visual Transformer (ViT)

**Encoder:**  $\mathbf{c} = T_w(\mathbf{z})$

where  $\mathbf{z}$  is spatial CNN features

$T_w(\cdot)$  is the transformer encoder

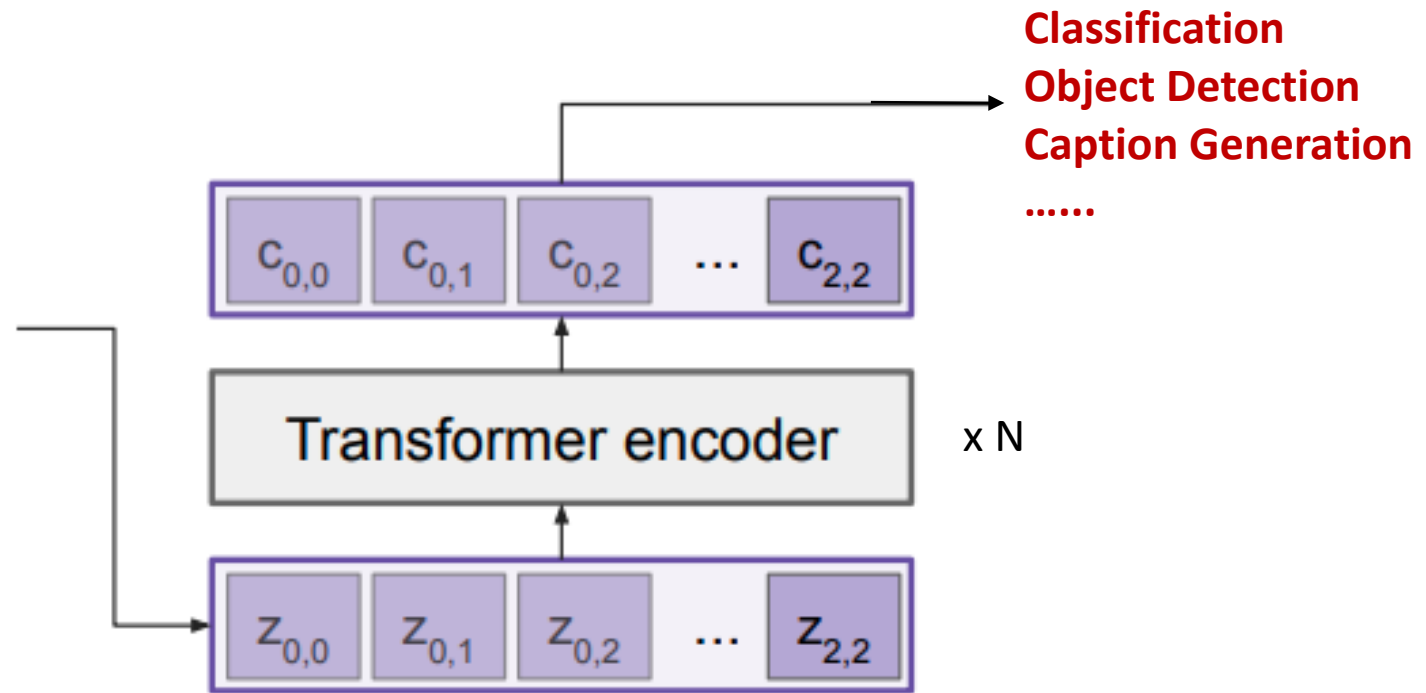


$z_{0,0}$	$z_{0,1}$	$z_{0,2}$
$z_{1,0}$	$z_{1,1}$	$z_{1,2}$
$z_{2,0}$	$z_{2,1}$	$z_{2,2}$

Features:  
H x W x D

Extract spatial features from a pretrained CNN

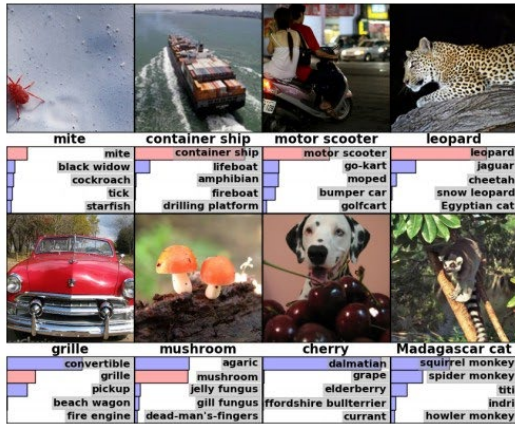
The general attention layer is a new type of layer (like Convolutional layer) that can be used to design new neural network architectures.



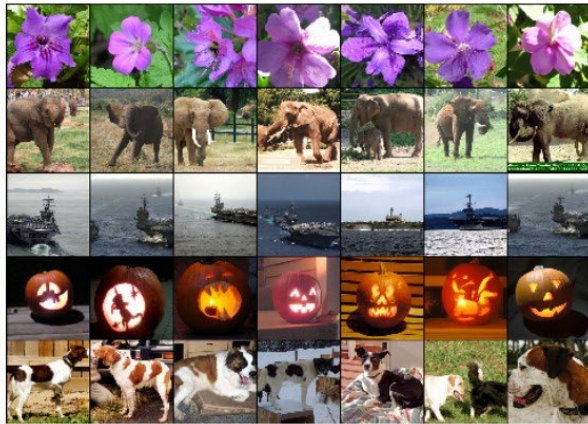
Flattened into tokens, but with positional encodings

# Deep Nets are Everywhere

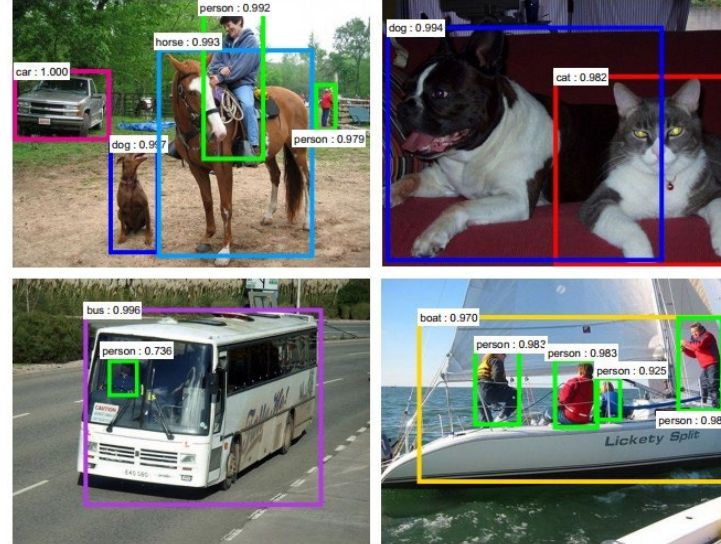
## Classification



## Retrieval



## Detection



## Text processing

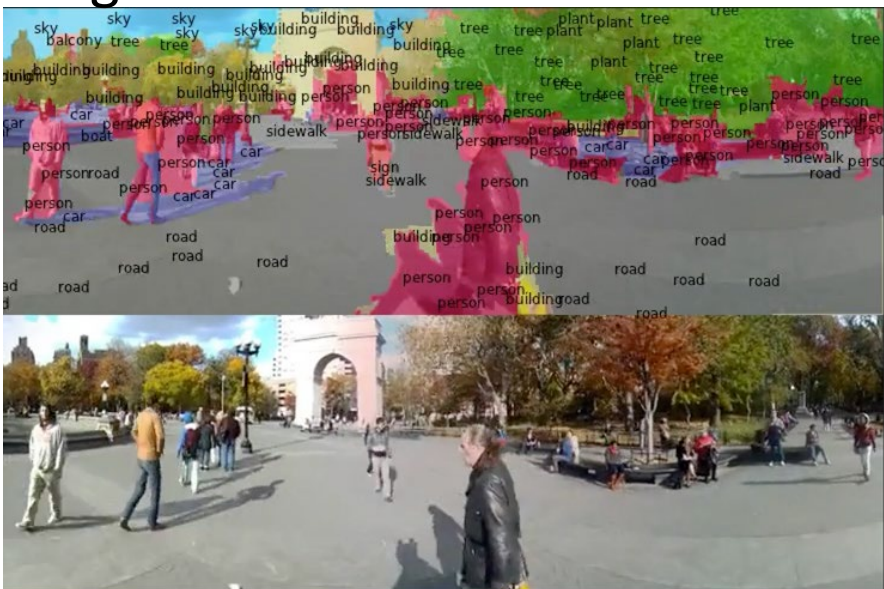
I taught this movie on the Sci-Fi channel recently. It actually turned out to be pretty decent as far as B-list horror/science films go. Two guys (one naive and one head) descend on a road trip to find a weird, but have the worst possible luck when it comes to a freaky, make-shift task force. Instead of playing cat and mouse with them. Things are further complicated when they pick up a ridiculously worthless hitchhiker. What makes this film unique is that the combination of comedy and terror actually work in this movie, unlike so many others. The two guys are likable enough and there are some good character-driven scenes. Nice pacing and comic timing make this movie more than passable for the horror/fanfic buff. **Definitely worth checking out.**

I just saw this on a local independent station in the New York City area. The cast showed promise but when I saw the director, George Constanin, I became suspicious. And sure enough, it was every bit as bad, every bit as pointless and stupid as every George Constanin movie I ever saw. He's like a stupid man's Michael Bay - with all the wildness that scoblese promises. There's no point to the conspiracy, no burning issues that urge the competitors on. We are left to ourselves to connect the dots from one bit of graffiti on various walls in the film to the next. Thus, the current budget crisis, the war in Iraq, Islamic extremism, the fate of social security, 47 million Americans without health care, stagnating wages, and the death of the middle class are all subsumed by the sheer terror of graffiti. A truly, stunningly idiotic film.

Graphics is far from the best part of the game. **This is the number one best TI game in the series!** Next to Underground. It deserves strong love. It is an insane game. There are massive levels, massive unlockable characters... it's just a massive game. **Make your money on this game. This is the kind of money that it makes!** And even though graphics suck, that doesn't make a game good. Actually, the graphics were good at the time. Today the graphics are crap. WHO CARES? As they say in Canada, This is the fun game, see. (You get to go to Canada in THPS3) Well, I don't know if they say that, but they might. who knows. Well, Canadian people do. Wait a minute, I'm getting off topic. This game rocks. Buy it, play it, enjoy it, love it. It's PURE BILLIANCE.

The first was good and original. I was a not bad horror/comedy movie. So I heard a second one was made and I had to watch it. What really makes this movie work is Judd Nelson's character and the sometimes clever script. **A pretty good script for a person who wrote the Final Destination films and the direction was okay.** Sometimes there's scenes where it looks like it was filmed using a home video camera with a grainy look. Great made-for-TV movie. **It was worth the effort and probably worth buying just to get that nice eerie feeling and watch Judd Nelson's Stanley doing what he does best!** I suggest newcomers to watch the first one before watching the sequel just so you'll have an idea what Stanley is like and get a little history background.

## Segmentation



## Control

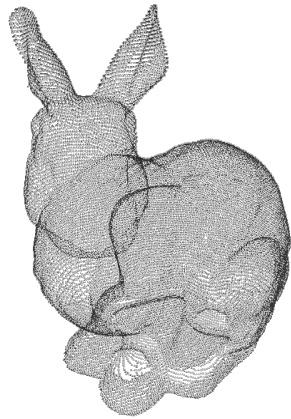


## Image captioning

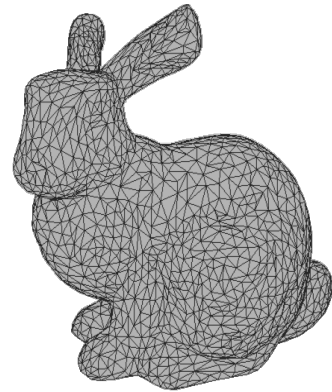
## Speech processing

# Learning on Irregular Data

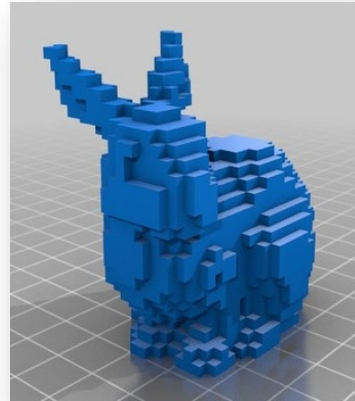
# Multiple Representations for 3D Geometric Data



Point Cloud



Mesh



Volumetric



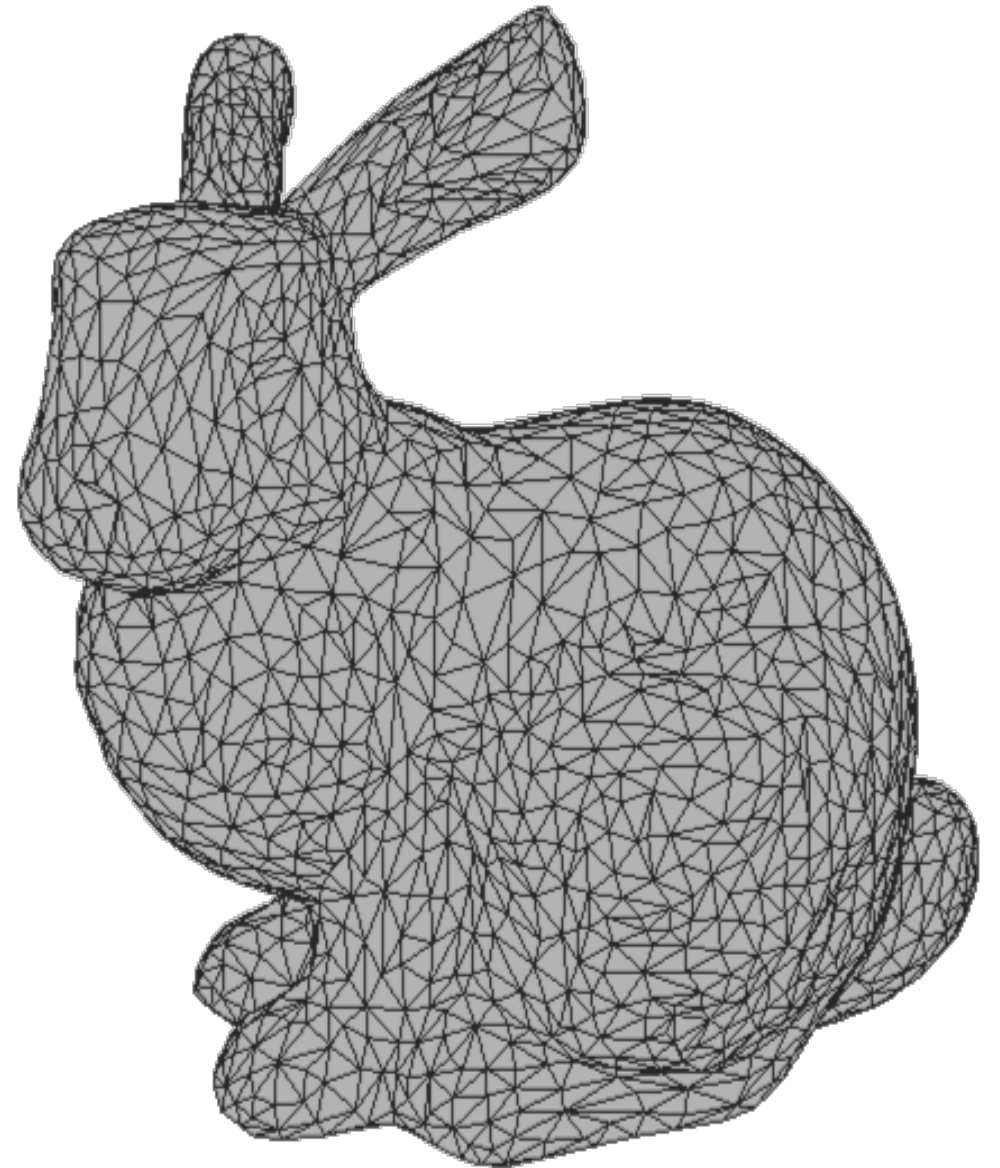
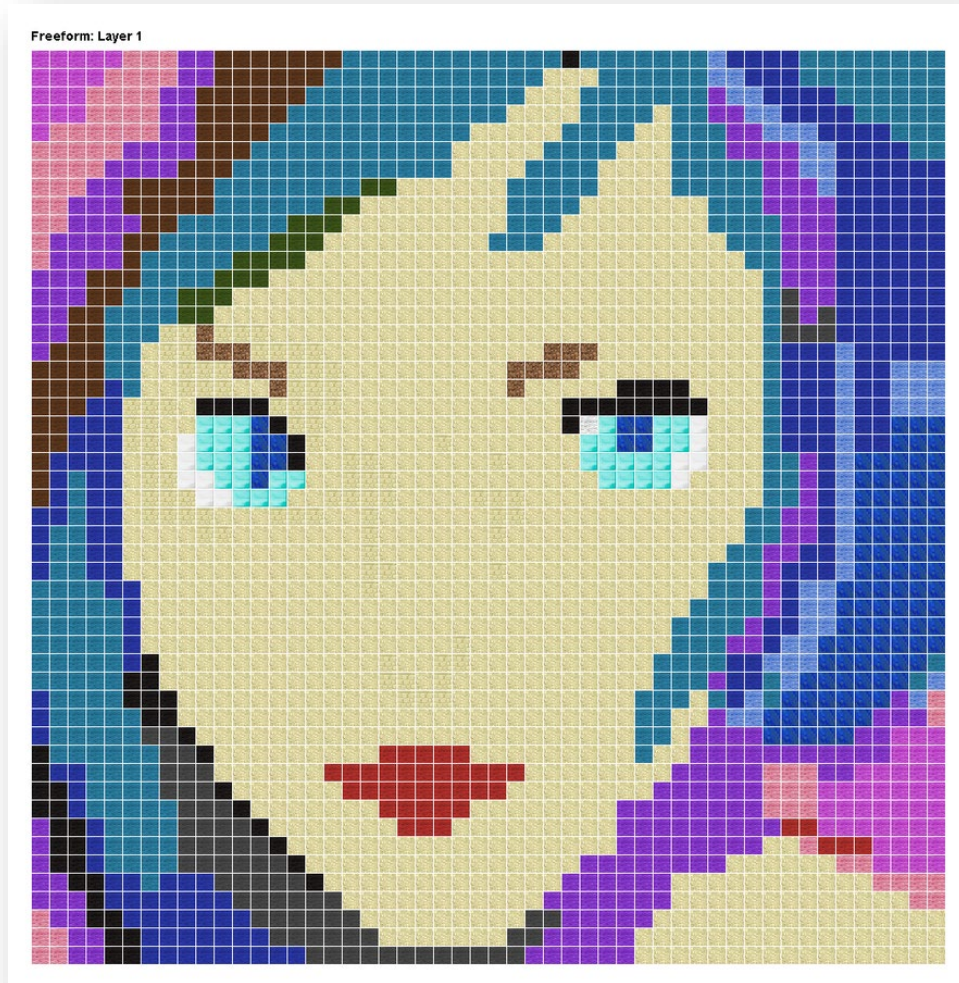
Projected View

RGB(D)

...

These are irregular representations

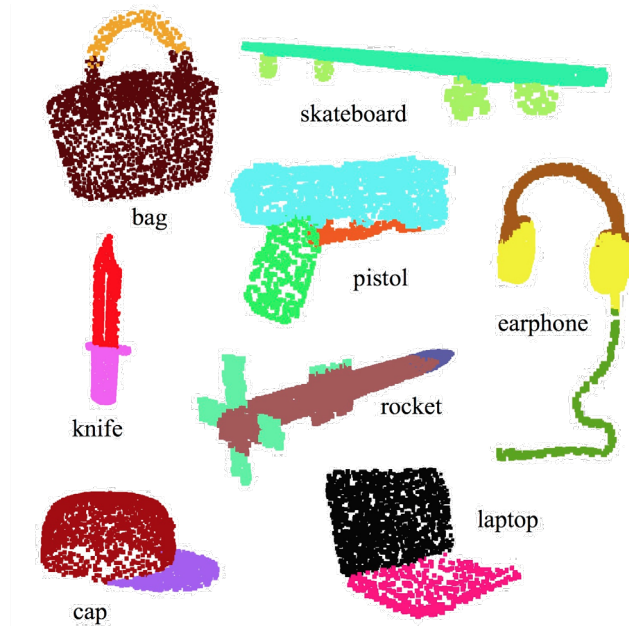
# Regular vs Irregular



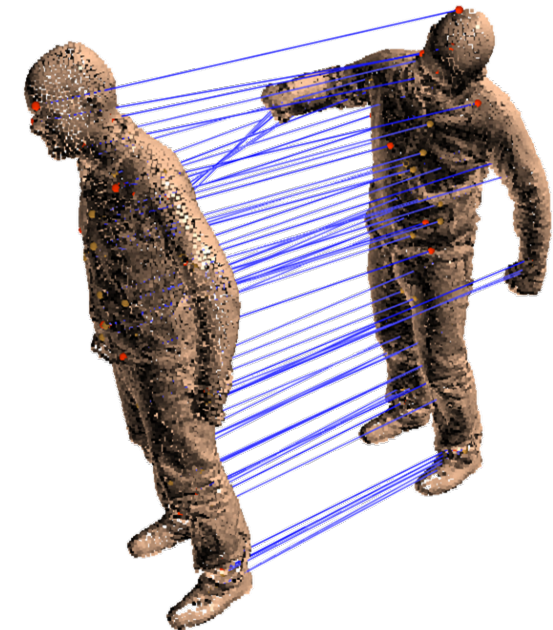
# Deep Learning for 3D Geometry Analysis



Classification

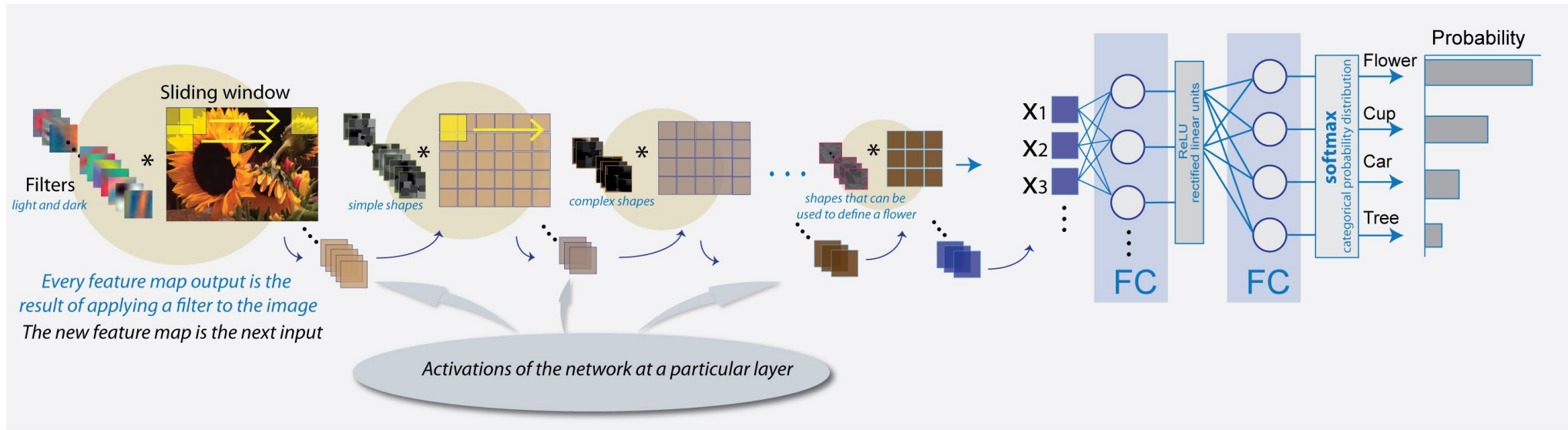
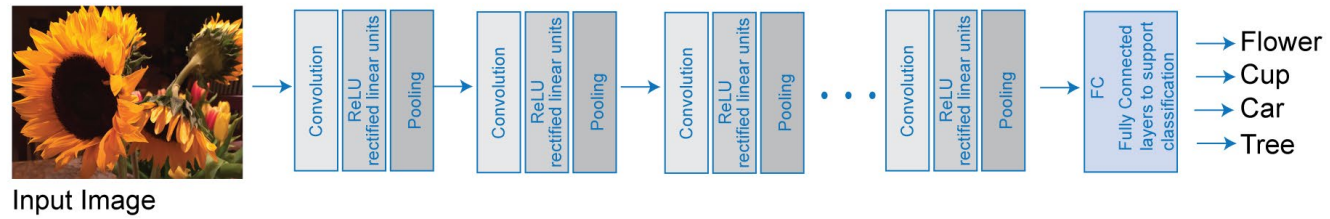


Segmentation/Parsing  
(object/scene)



Correspondences

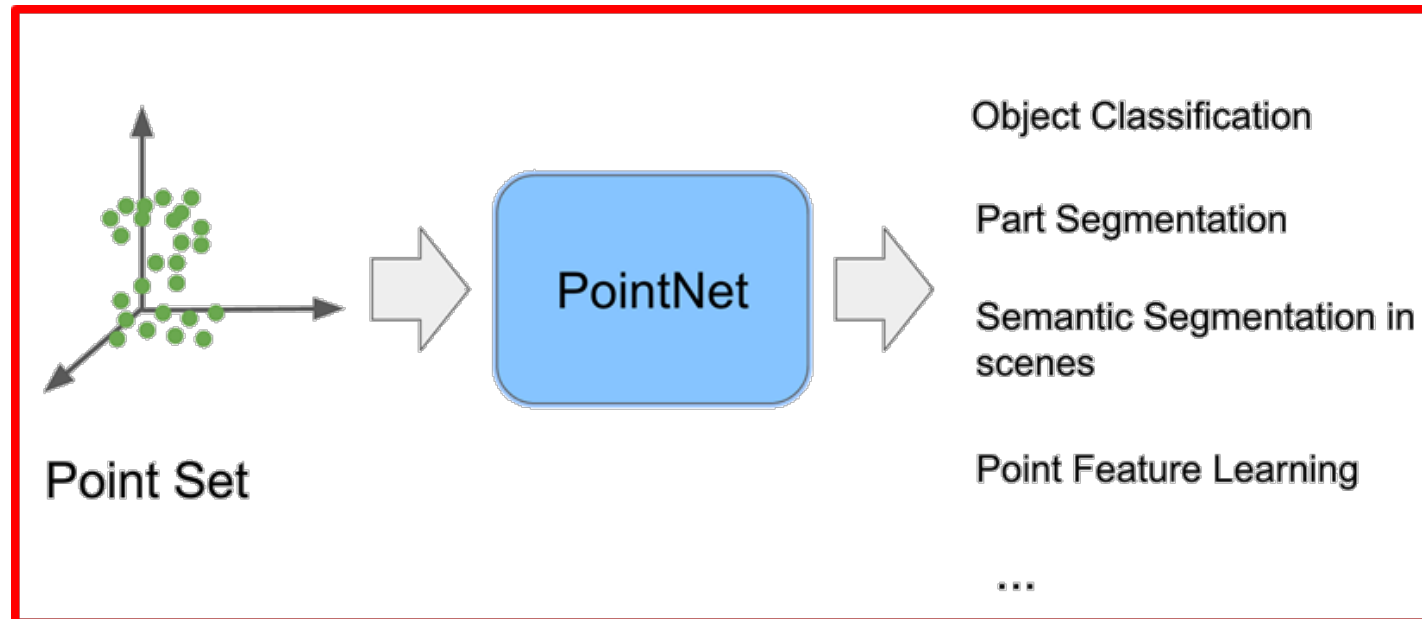
# Convolutional Networks



Convolutions require regular array type inputs for weight sharing and other optimizations

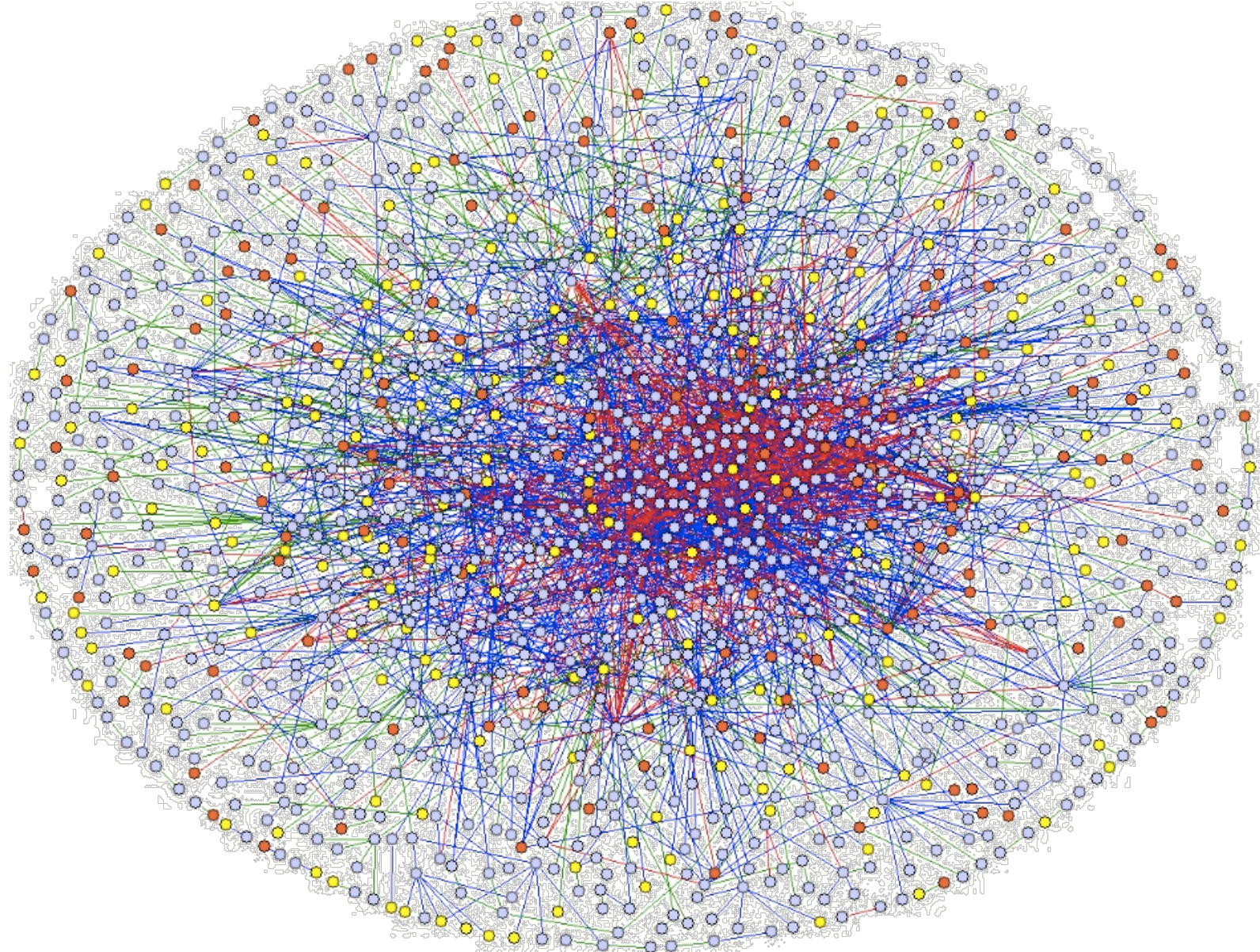
# PointNet: Working Directly with Point Cloud Data

- Goal: design a NN architecture that can work directly with point clouds
- Must deal with unstructured, unordered data

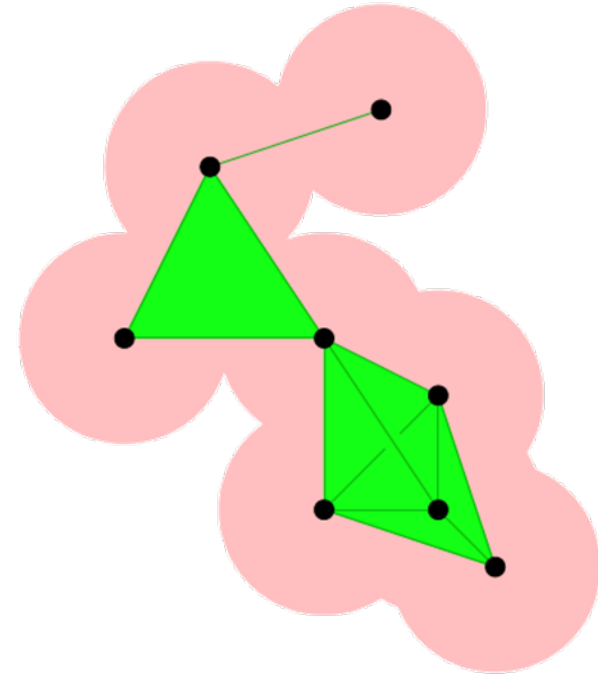
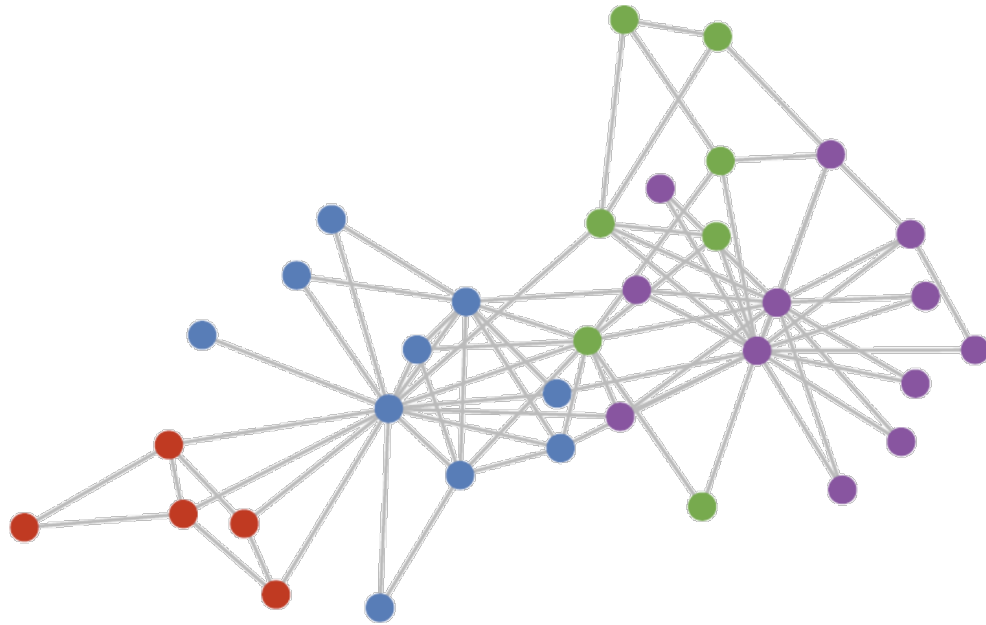


# The Graph View of Data

# A Graph View of Data

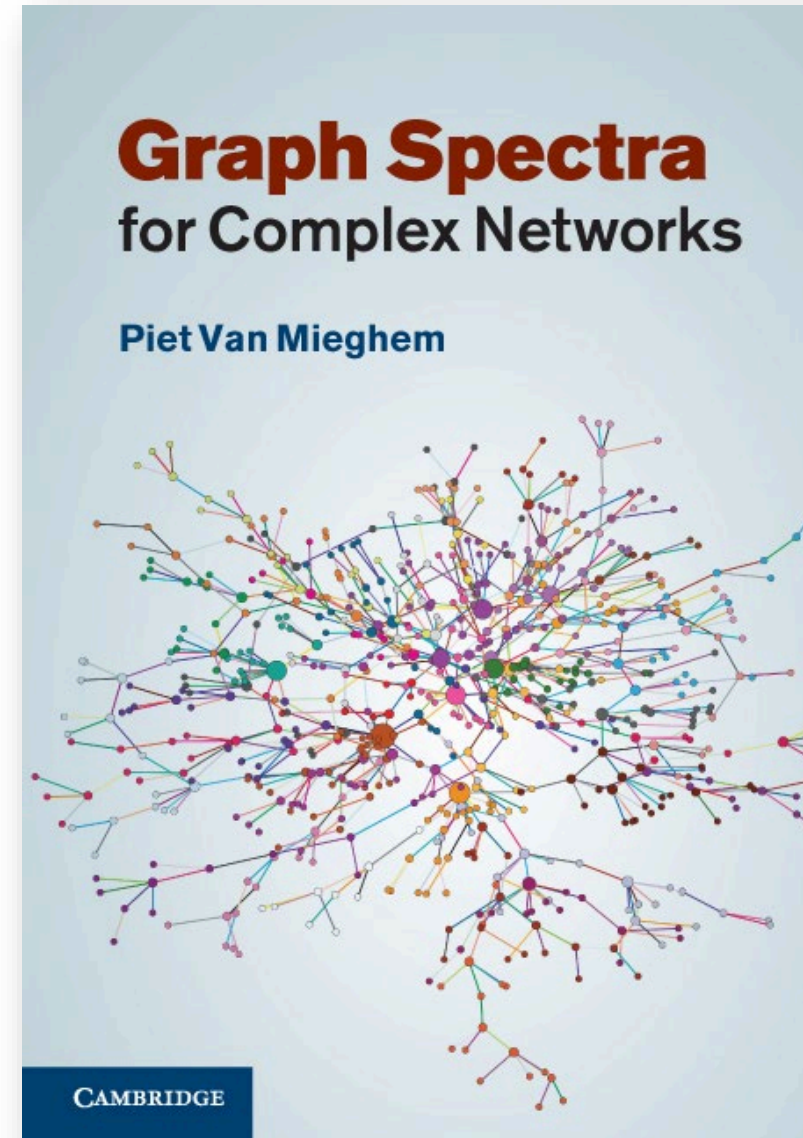


# Graphs and Simplicial Complexes



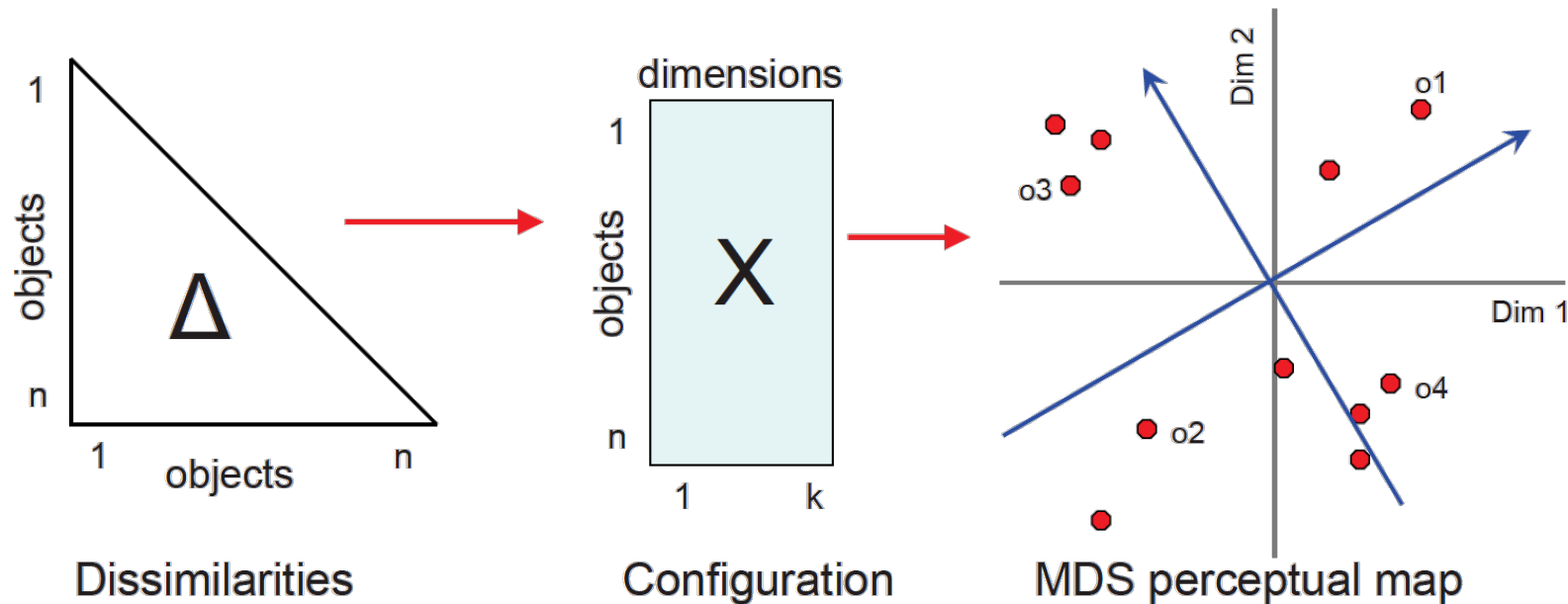
# Spectral Methods in Graph Theory

- Linking the graph-theoretic and linear algebraic view of data.



# Multidimensional Scaling (MDS)

- A “distance preserving” embedding of the data into a Euclidean space
  - Sometimes distances are observed directly (e.g., similarity ratings)
  - Sometimes they can be calculated from a data table (e.g., Euclidean distances, correlations)

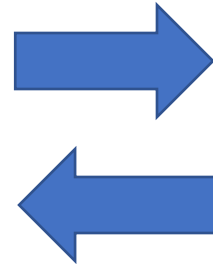


# Topological Data Analysis (TDA)

# From Data to Algebraic Objects

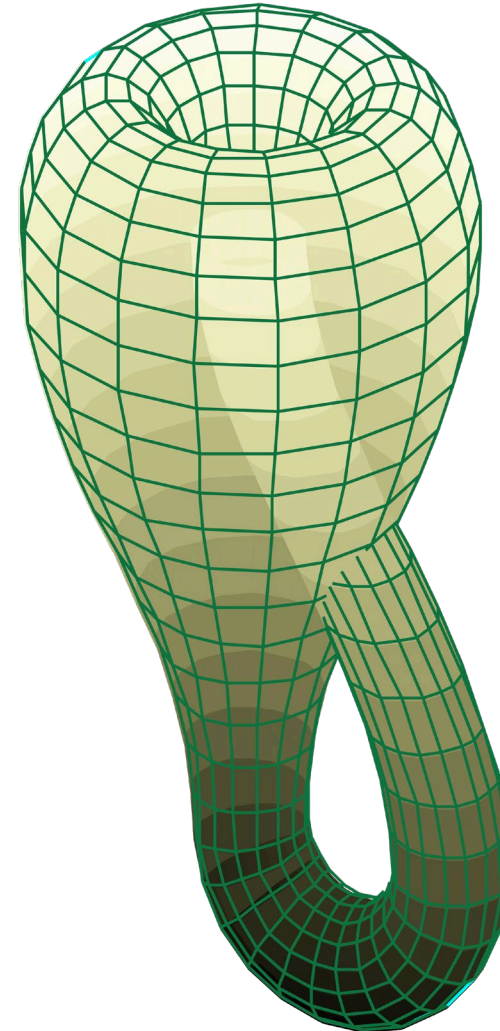


Algebraic entities as  
data descriptors

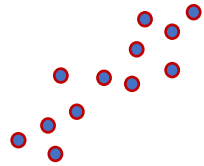


# Computational Topology

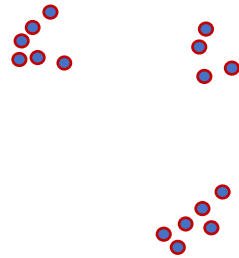
- Topology is the branch of mathematics that does not take distances too seriously. G. Carlsson
- Large distances (aka “similarity metrics” are often suspect ...



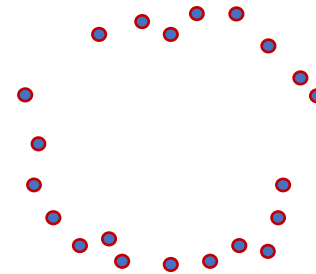
# In TDA, Sampled Spaces: “The Shape of Data”



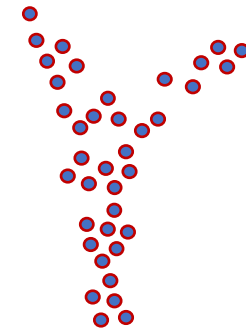
Regression



Cluster



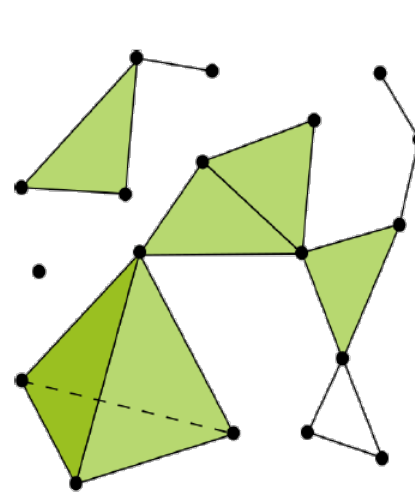
Loop



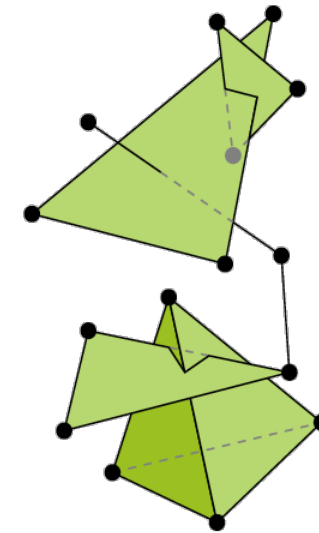
Flared

# Simplicial Complexes

- A **simplicial complex**  $K$  is a finite set of simplices such that
  1.  $\sigma \in K, \tau \leq \sigma \Rightarrow \tau \in K$ ,
  2.  $\sigma, \sigma' \in K \Rightarrow \sigma \cap \sigma' \leq \sigma, \sigma'$  or  $\sigma \cap \sigma' = \emptyset$ .
- The **dimension** of  $K$  is  $\dim K = \max\{\dim \sigma \mid \sigma \in K\}$ .
- The **vertices** of  $K$  are the zero-simplices in  $K$ .
- A simplex is **principal** if it has no proper coface in  $K$ .



(left) an example

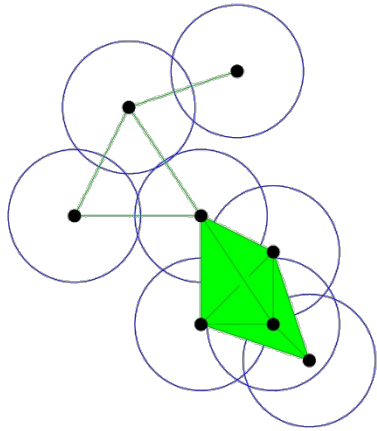


(right) a non example

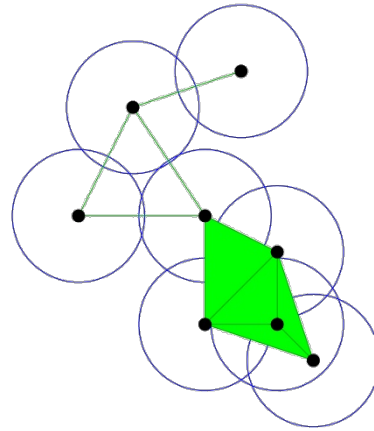
# Complexes via Proximity

Must choose which simplices to introduce

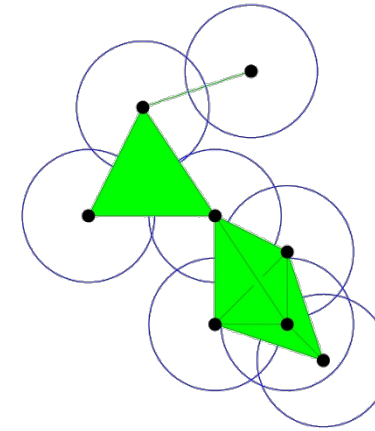
Čech



Alpha



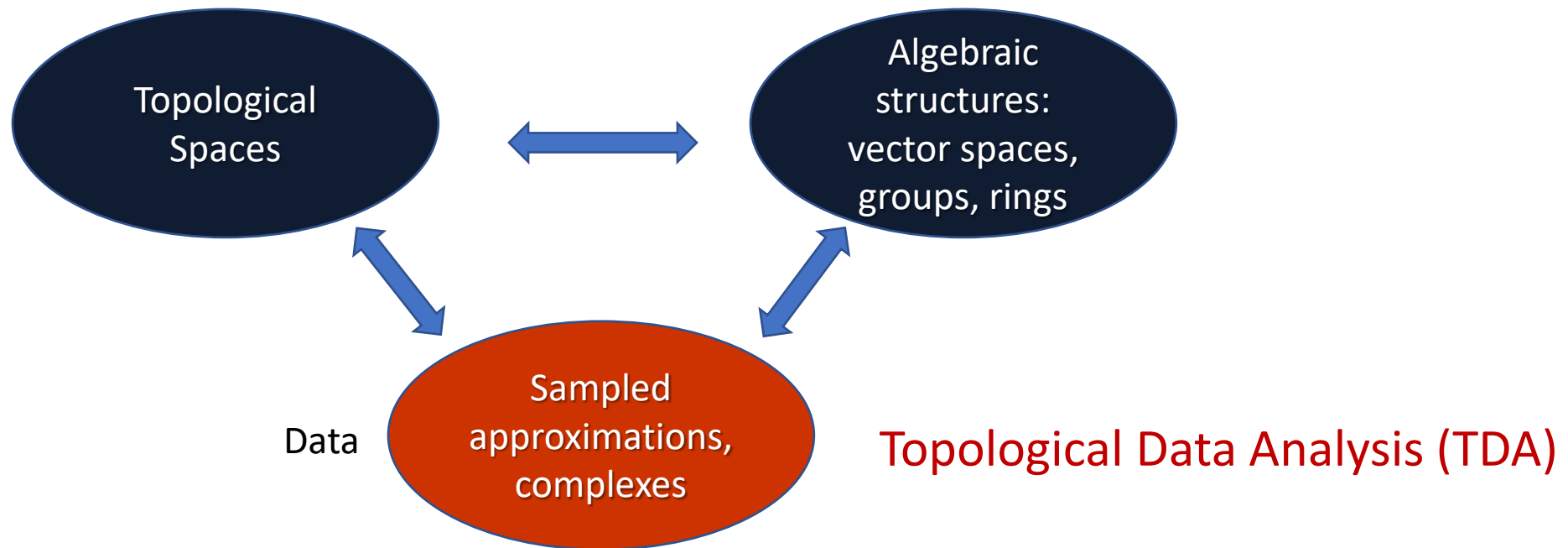
Rips



Combinatorial complexes provide discrete representations of the underlying space

# Algebraic Topology and Topology Inference

- Unlike geometry, topology studies the **global** structure of spaces
- Getting to this structure via **algebra** algebraic topology

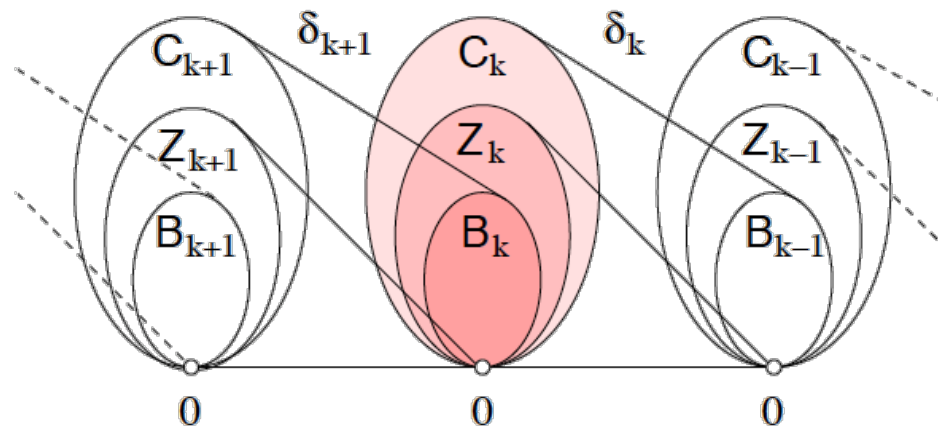


# Algebraic Topology: Simplicial Homology

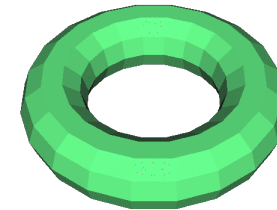
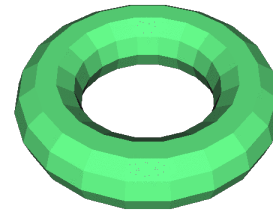
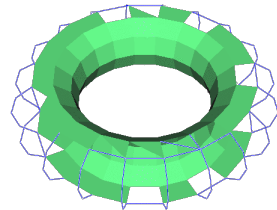
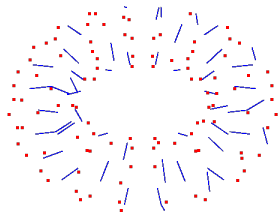
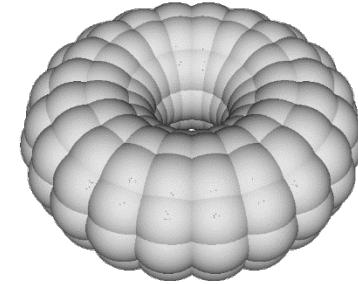
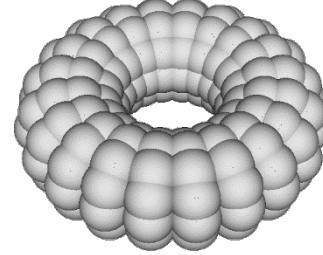
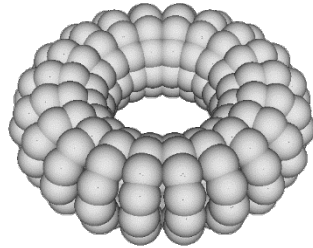
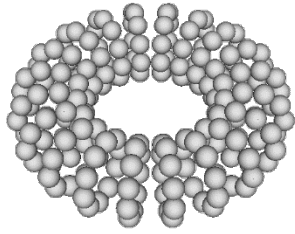
- The  $k$ th homology group is

$$H_k = Z_k / B_k = \ker \partial_k / \text{im } \partial_{k+1}.$$

- If  $z_1 = z_2 + B_k$ ,  $z_1, z_2 \in Z_k$ , we say  $z_1$  and  $z_2$  are **homologous**
- $z_1 \sim z_2$ .



# A Multiscale View of Data



$\beta_0 = 150$   
 $\beta_1 = 0$

$\beta_0 = 1$   
 $\beta_1 = 37$

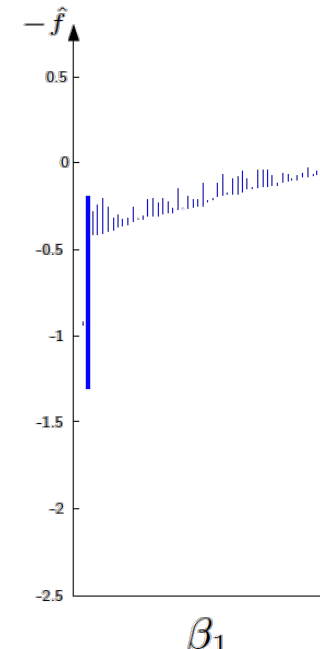
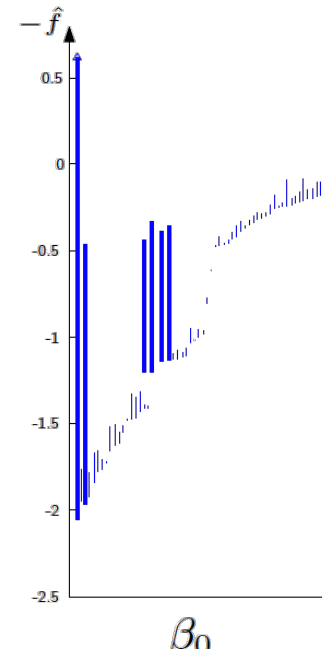
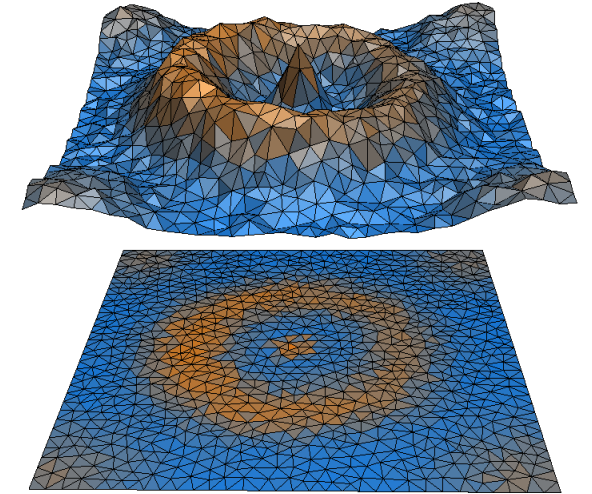
$\beta_0 = 1$   
 $\beta_1 = 2$

$\beta_0 = 1$   
 $\beta_1 = 1$

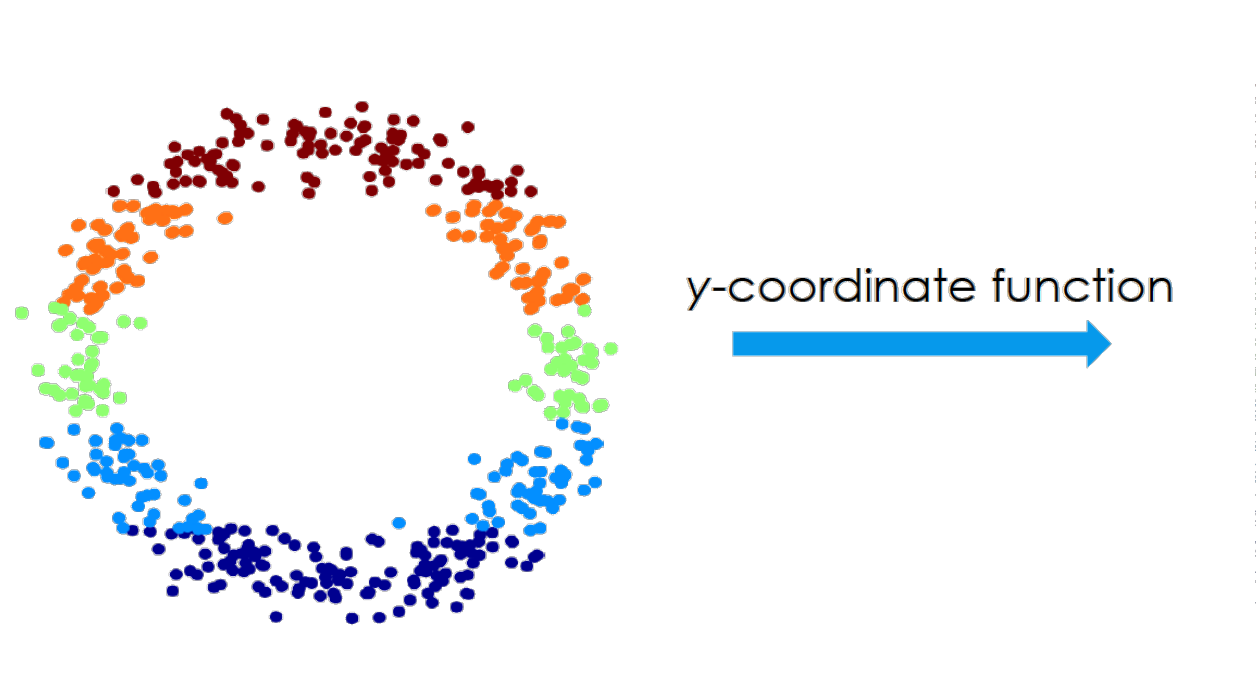


# Persistent Homology

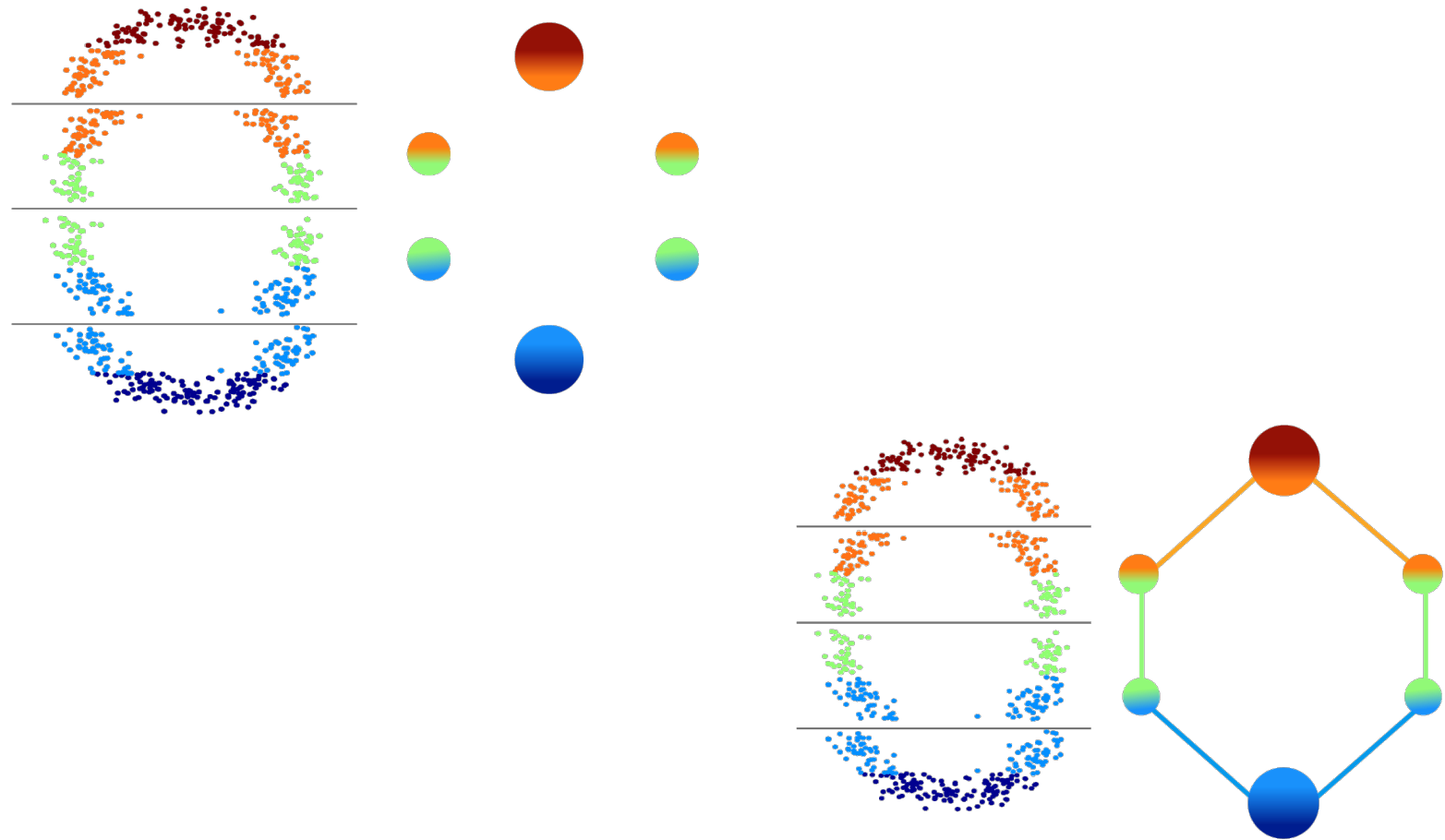
- Persistence of topological features
- Barcodes and persistence diagrams
- Data embeddings into non-Euclidean spaces



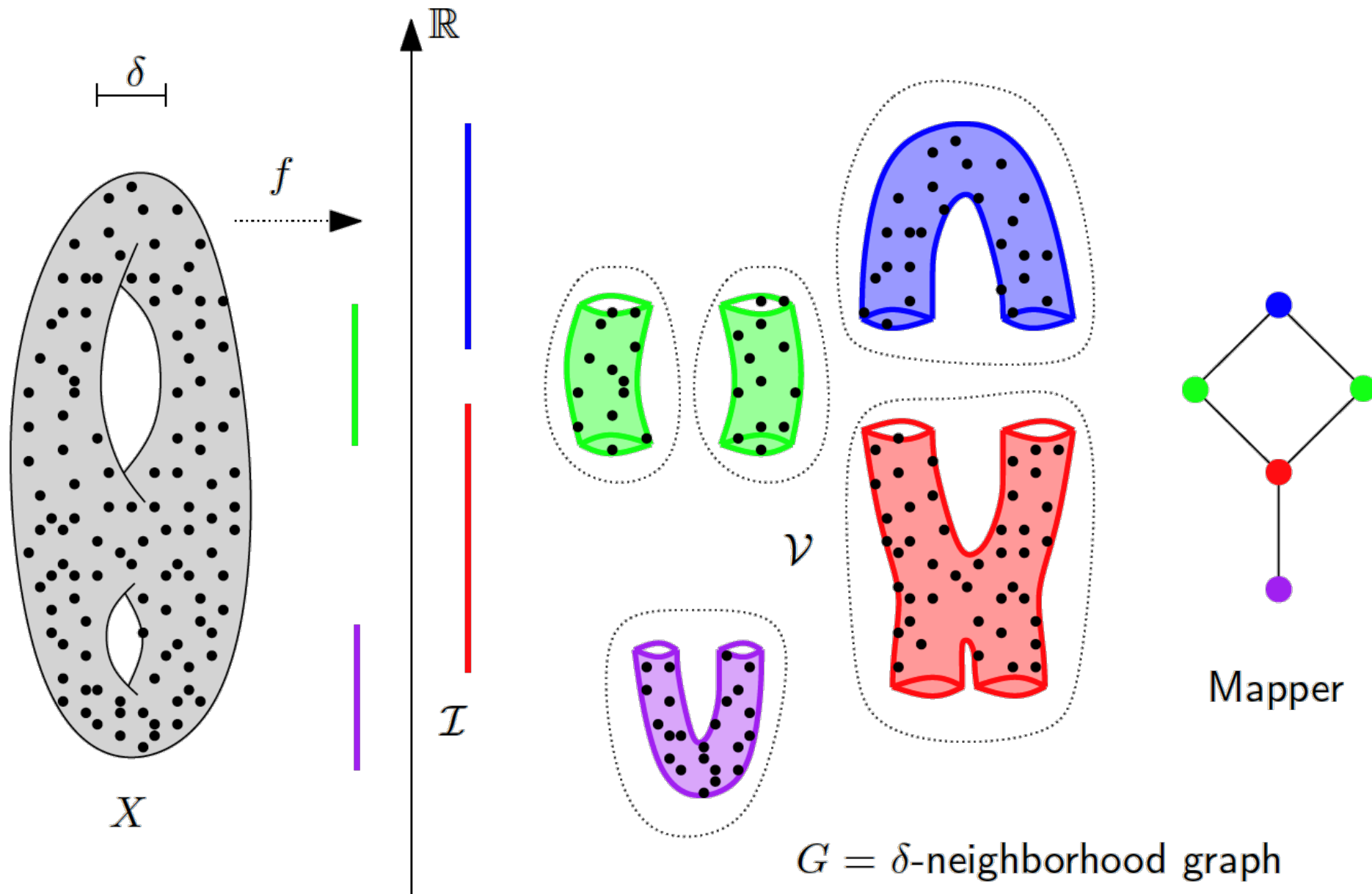
# Functions on the Data as Filters



# Complexes via Function Filters

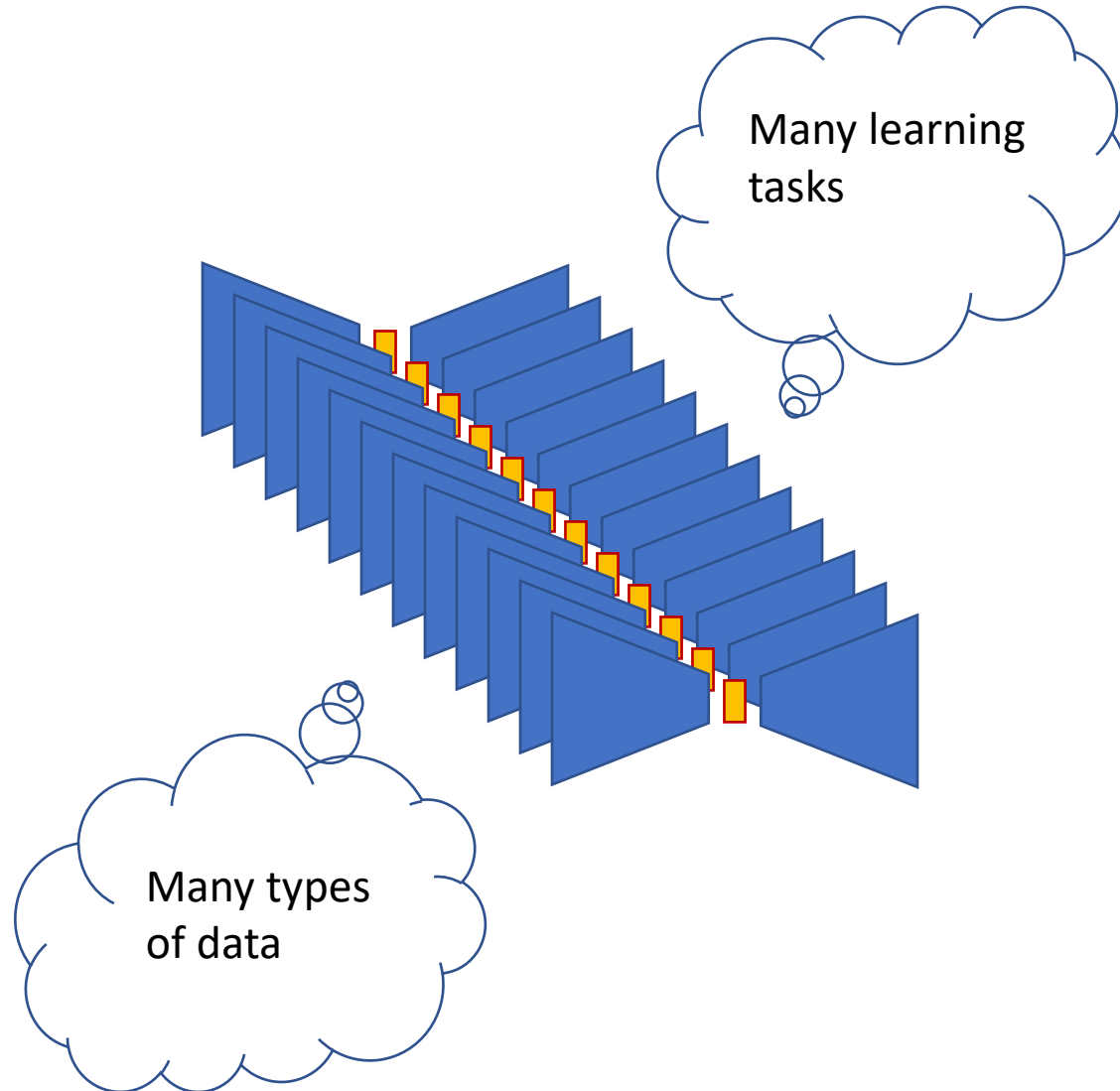


# Functions as Lenses on Data



# Data Representations and Latent Spaces

# (Too) Many Latent Spaces



Codes

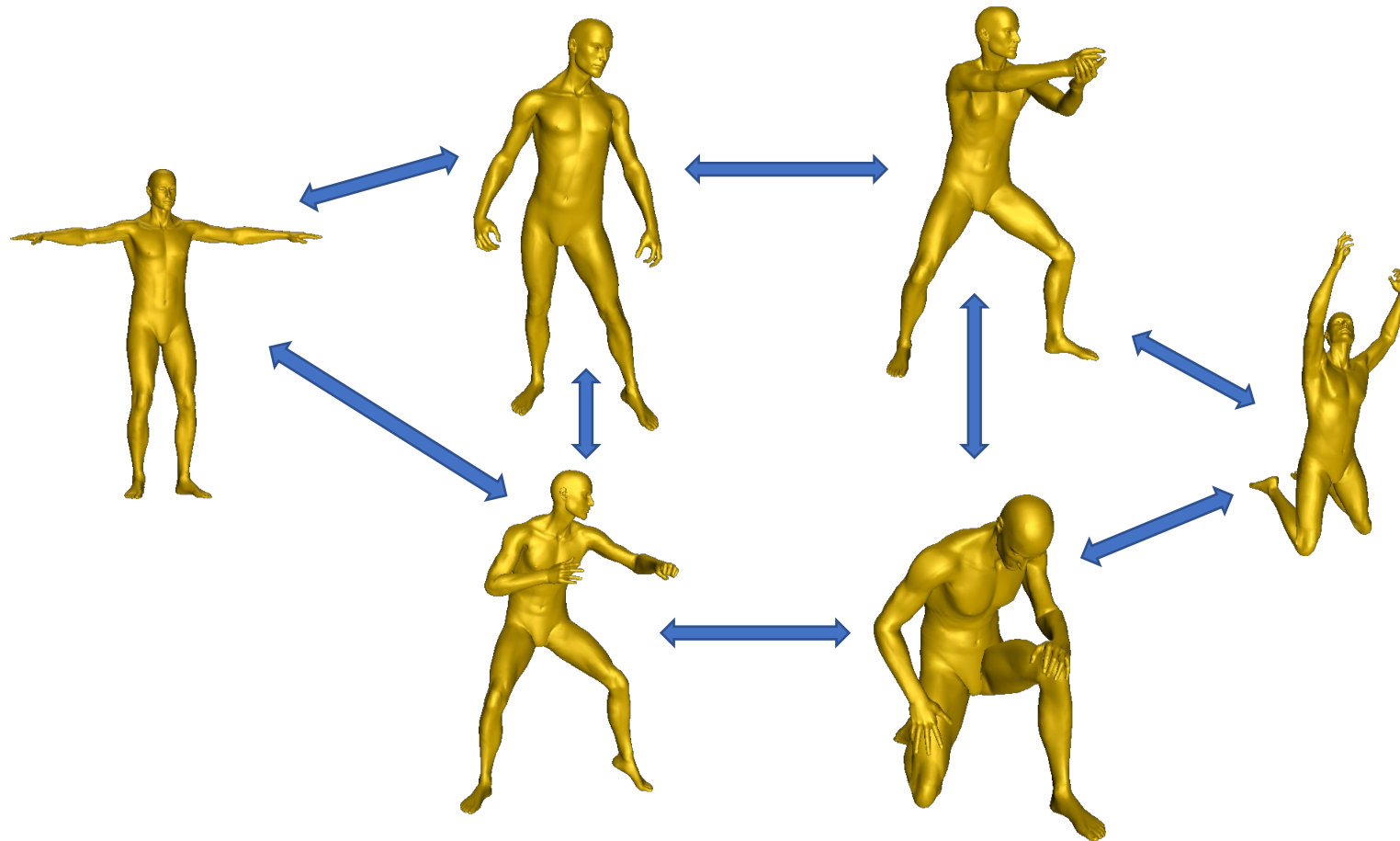
Latent vectors

Parametrizations

Representations

...

# Correlated Data Sets



# Correlated Tasks

RGB Image



Predicted Normals



Predicted Depth



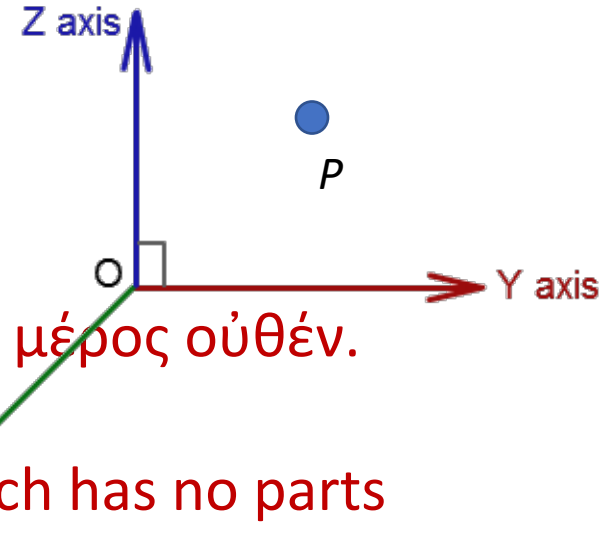
Predicted (re)Shading



Predicted Curvature

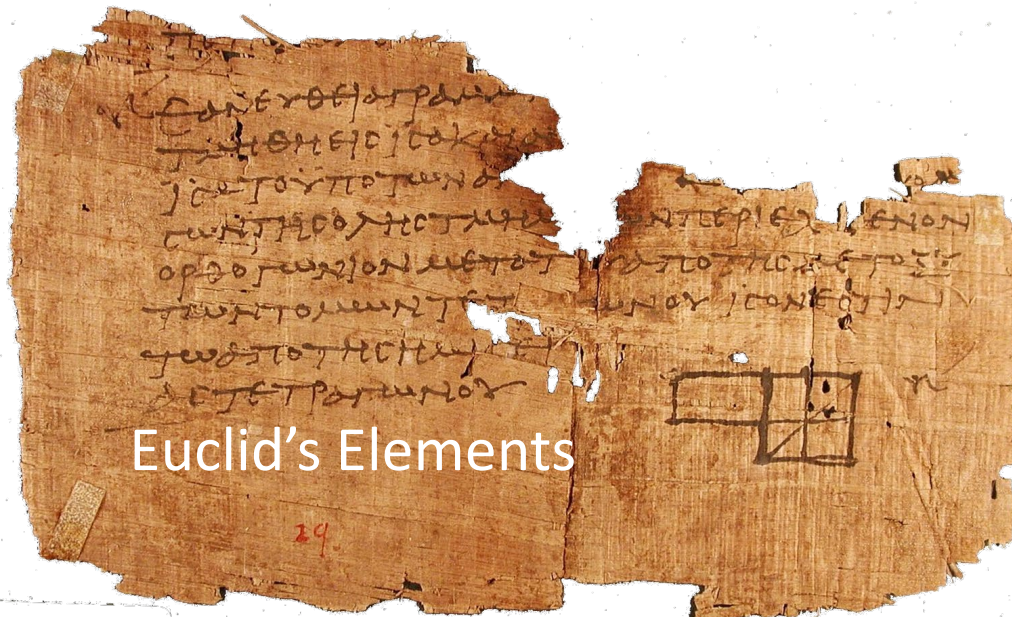


# The Structure of Latent Spaces



α'. Σημεῖόν ἐστιν, οὐ μέρος οὐθέν.

a. A point is that which has no parts



Euclid's Elements

# The Mathematics of Latent Codes

- What kinds of mathematical objects can serve well as latent space codes?
- How should such an object vary with
  - the input data?
  - the input modality?
  - the learning task?



# Homomorphic Encryption

## Chapter 2 Homomorphic Encryption

**Abstract** Homomorphic encryption is a form of encryption which allows specific types of computations to be carried out on ciphertexts and generate an encrypted result which, when decrypted, matches the result of operations performed on the plaintexts. This is a desirable feature in modern communication system architectures. RSA is the first public-key encryption scheme with a homomorphic property. However, for security, RSA has to pad a message with random bits before encryption to achieve semantic security. The padding results in RSA losing the homomorphic property. To avoid padding messages, many public-key encryption schemes with various homomorphic properties have been proposed in last three decades. In this chapter, we introduce basic homomorphic encryption techniques. It begins with a formal definition of homomorphic encryption, followed by some well-known homomorphic encryption schemes.

### 2.1 Homomorphic Encryption Definition

In abstract algebra, a homomorphism is a structure-preserving map between two algebraic structures, such as groups.

A group is a set,  $G$ , together with an operation  $\circ$  (called the group law of  $G$ ) that combines any two elements  $a$  and  $b$  to form another element, denoted  $a \circ b$ . If  $G$  satisfies the group axioms, the set and operation,  $(G, \circ)$ , must satisfy four requirements:

- 1. The result of the operation,  $a \circ b$ , is also in  $G$ .
- 2.  $(a \circ b) \circ c = a \circ (b \circ c)$ .
- 3. For every element  $a$  in  $G$ , there is a unique element  $e$  such that  $a \circ e = a$  and  $e \circ a = a$ .
- 4. For every element  $a$  in  $G$ , there is a unique element  $a^{-1}$  such that  $a \circ a^{-1} = e$  and  $a^{-1} \circ a = e$ .



Craig Gentry

# Relating Data Sets: Horizontal Map Networks

# Horizontal Networks & Joint Learning

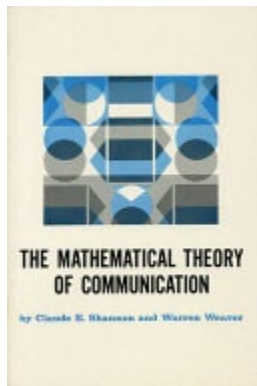
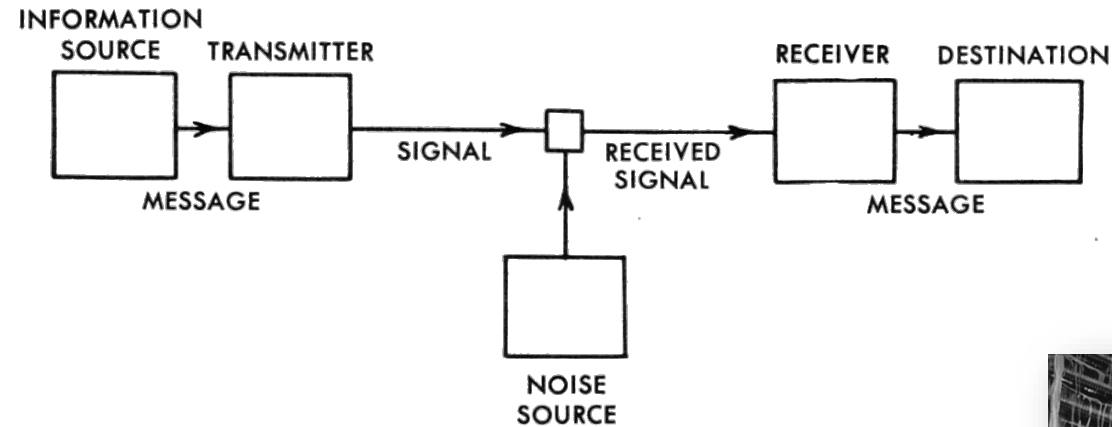


34

Similarity as a communications channel



*The Mathematical Theory of Communication*



Claude Shannon

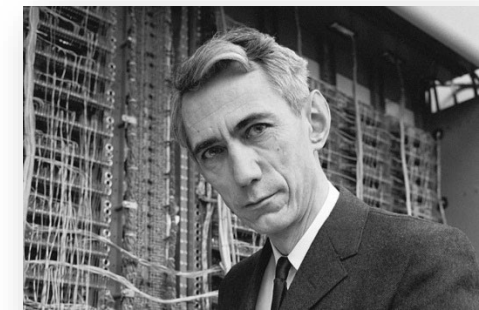
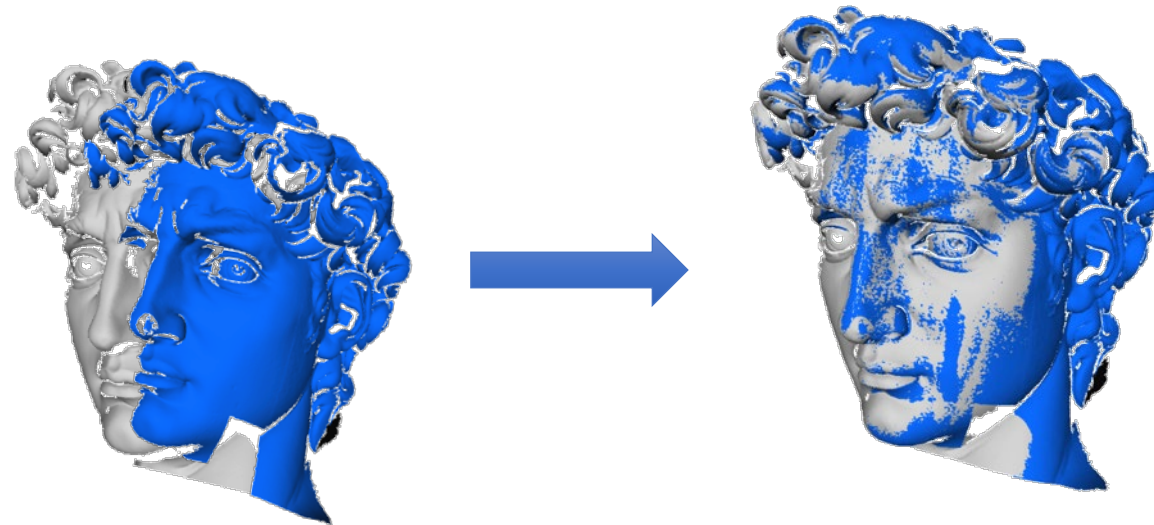


Fig. 1. — Schematic diagram of a general communication system.

# Networks of Visual Data



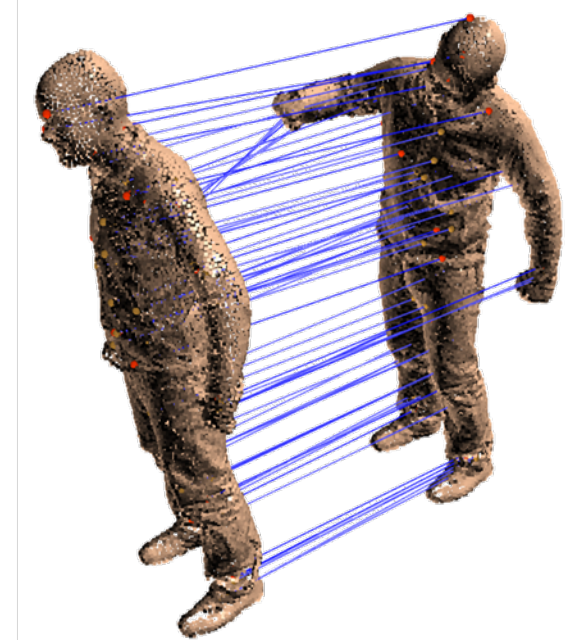
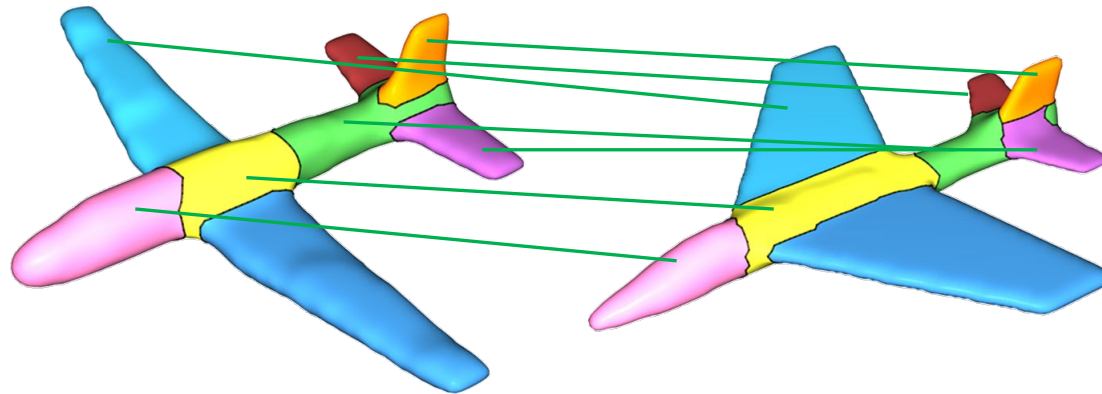
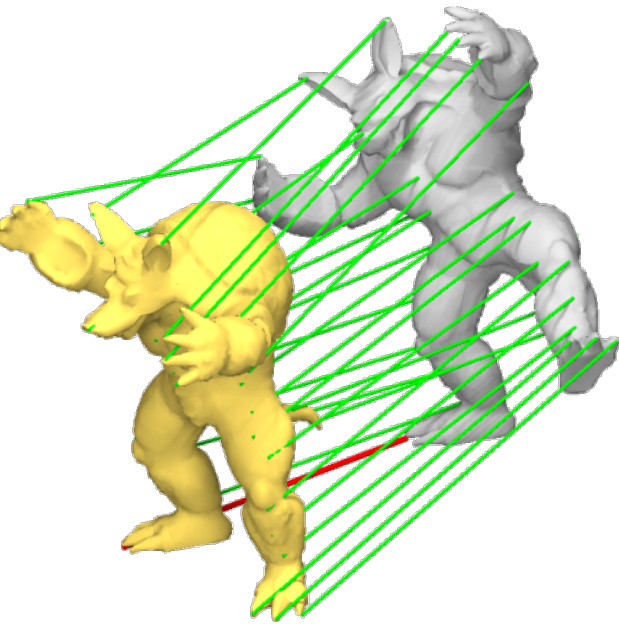
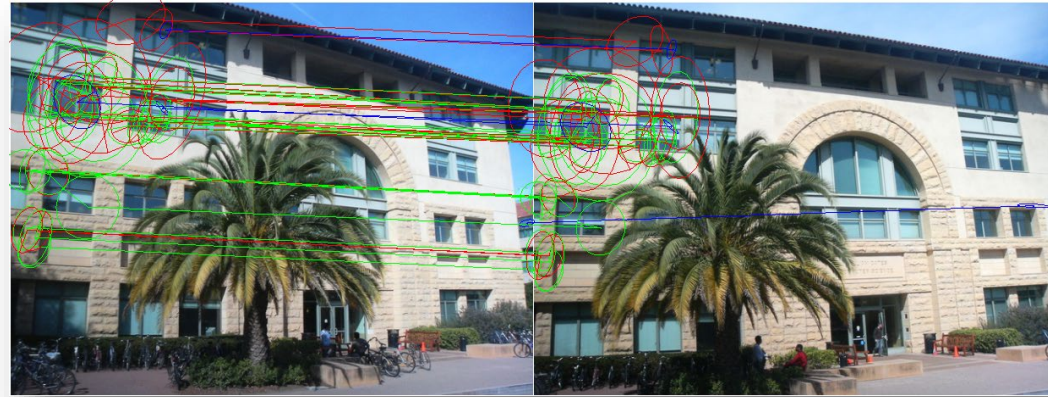
# Alignment and Registration



Rigid Registration

Low-dimensional transformation group

# Maps and Correspondences





# Societies, or Social Networks of Data Sets

Our understanding of data can greatly benefit from extracting these relations and building relational networks.

We can exploit the relational network to

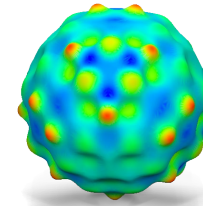
- transport information around the network
- assess the validity of operations or interpretations of data (by checking consistency against related data)
- assess the quality of the relations themselves (by checking consistency against other relations through cycle closure, etc.)
- extract shared structure among the data

Thus the network becomes an important regularizer in joint data analysis.



# Functionals Over Data and Functional Maps

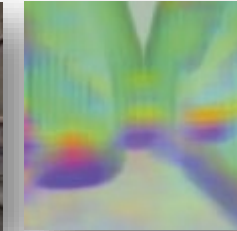
# Knowledge as Functions over Data



Curvature



Parts

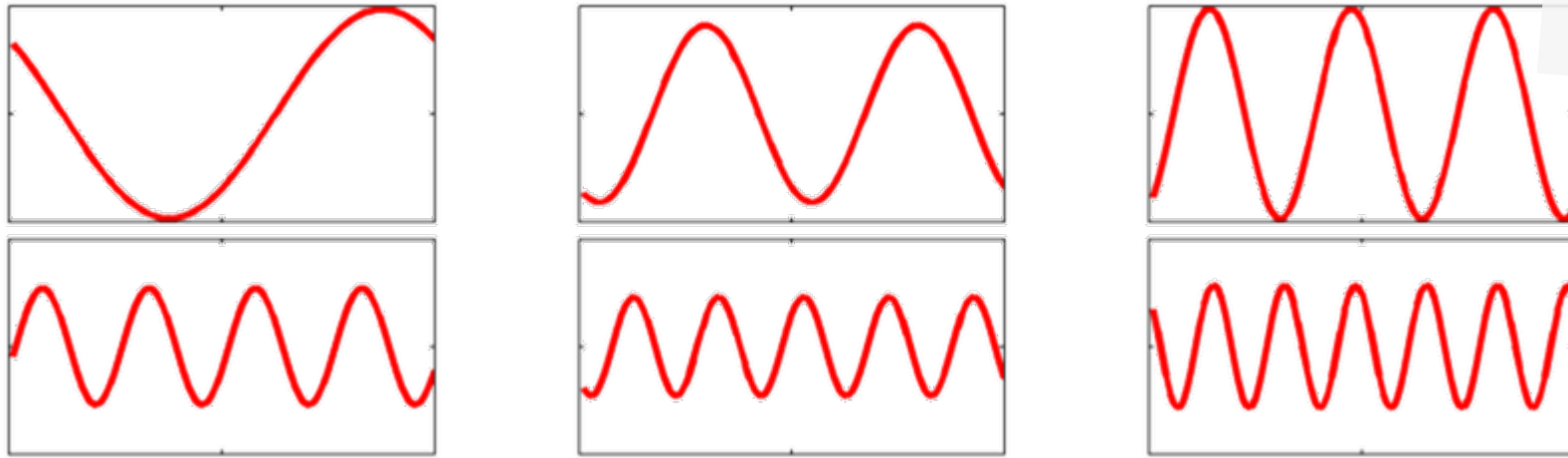


SIFT flow, C. Liu 2011



Knowledge towers over visual data: function spaces

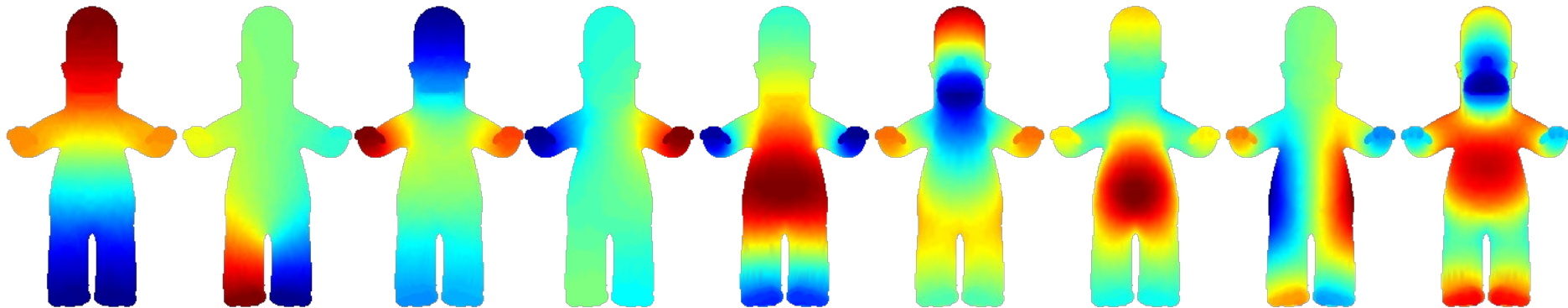
# Hierarchical Bases for a Function Space



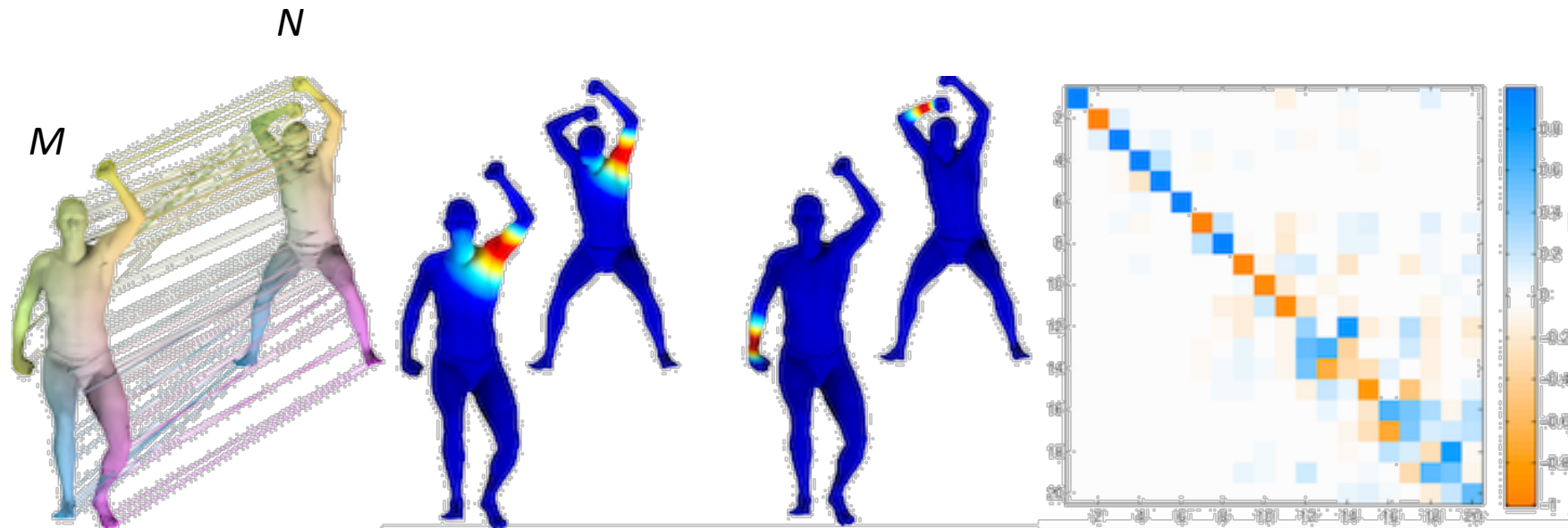
Fourier

Laplace-Beltrami

global support



# Functional Maps

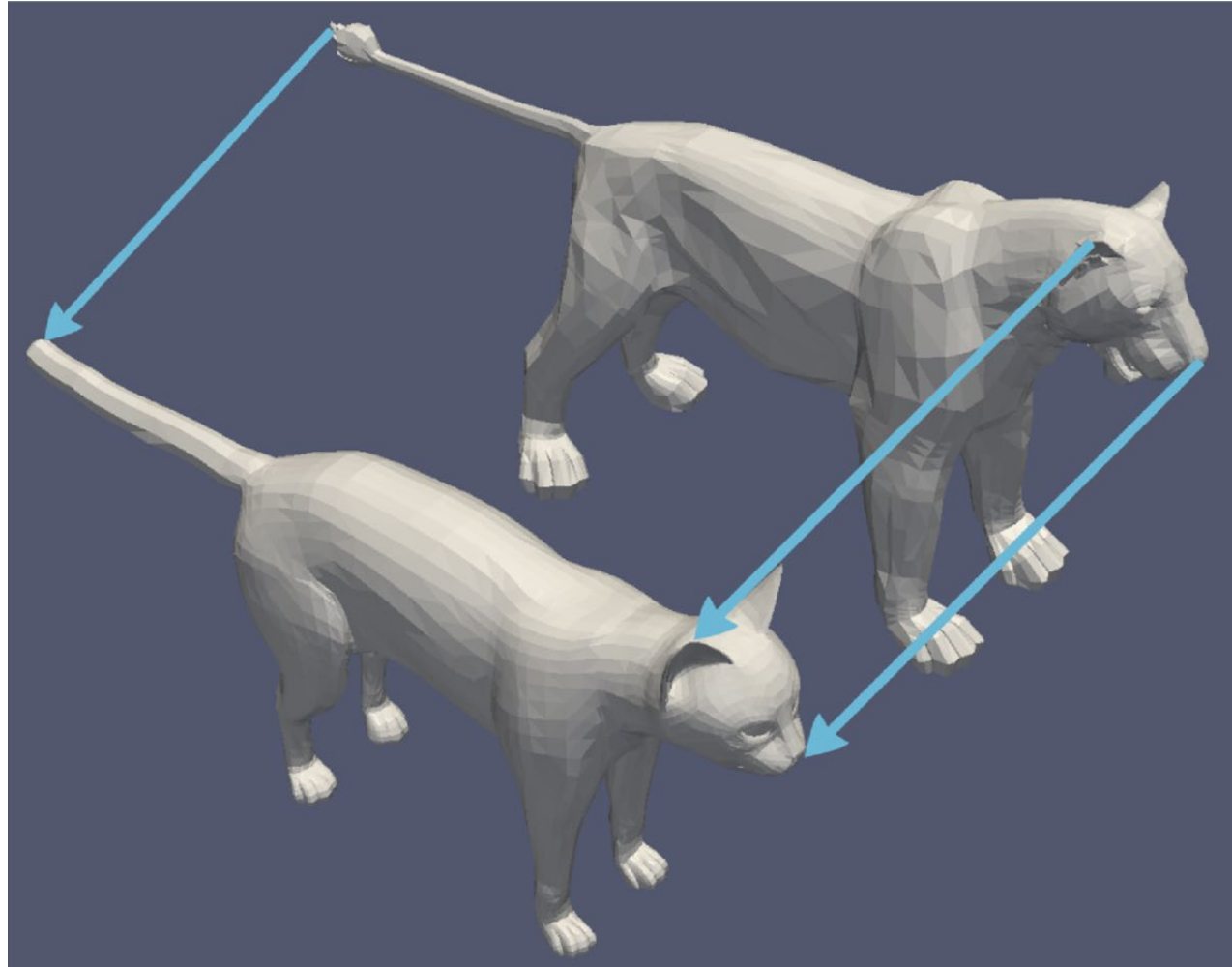


$$T : N \rightarrow M$$

$$C_T : L^2(M) \rightarrow L^2(N)$$
$$f \mapsto f \circ T$$

A contravariant functor

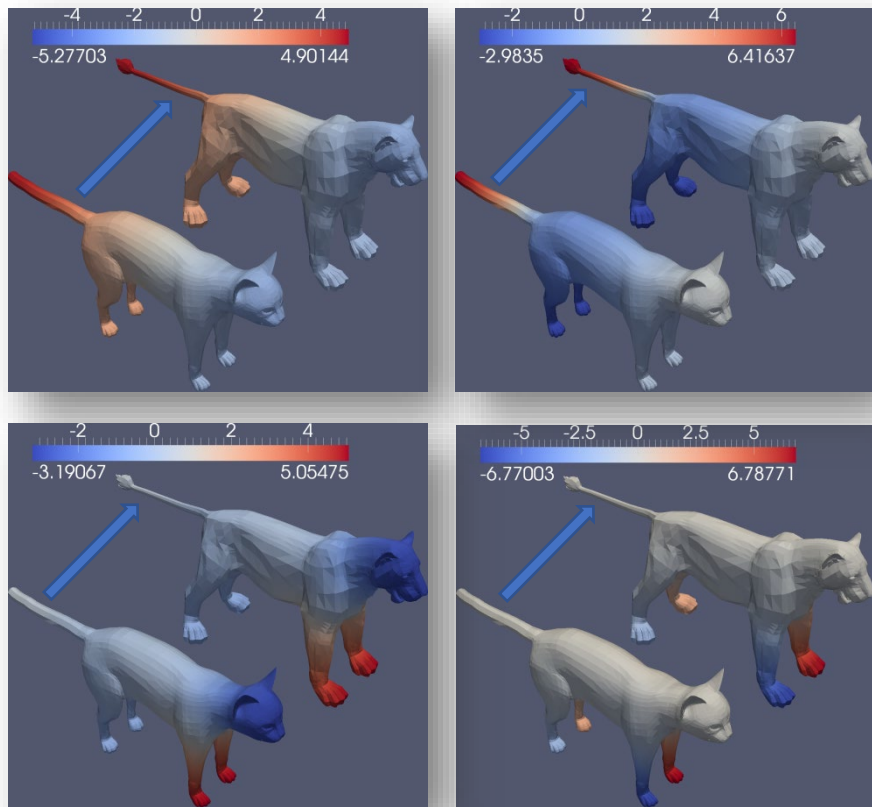
# Starting from a Regular Map $\phi$



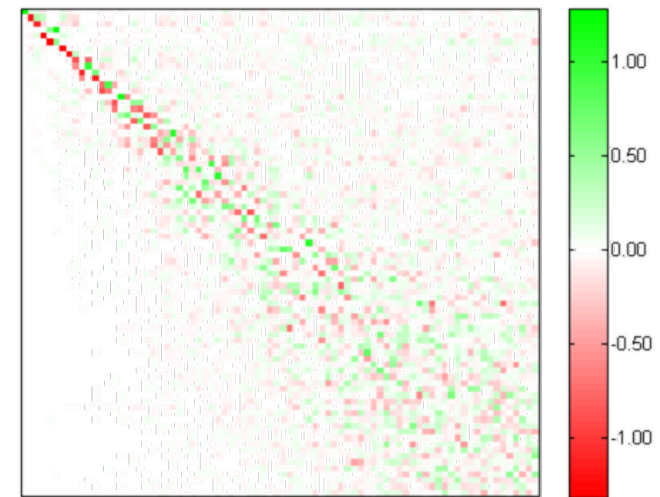
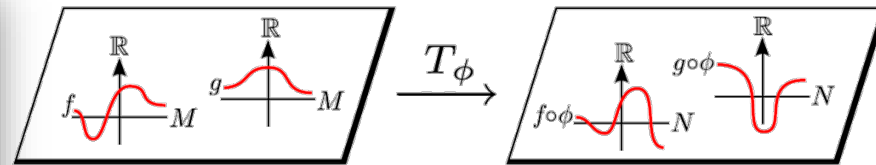
$\phi: \text{lion} \rightarrow \text{cat}$

# A Contravariant Functor

from cat to lion



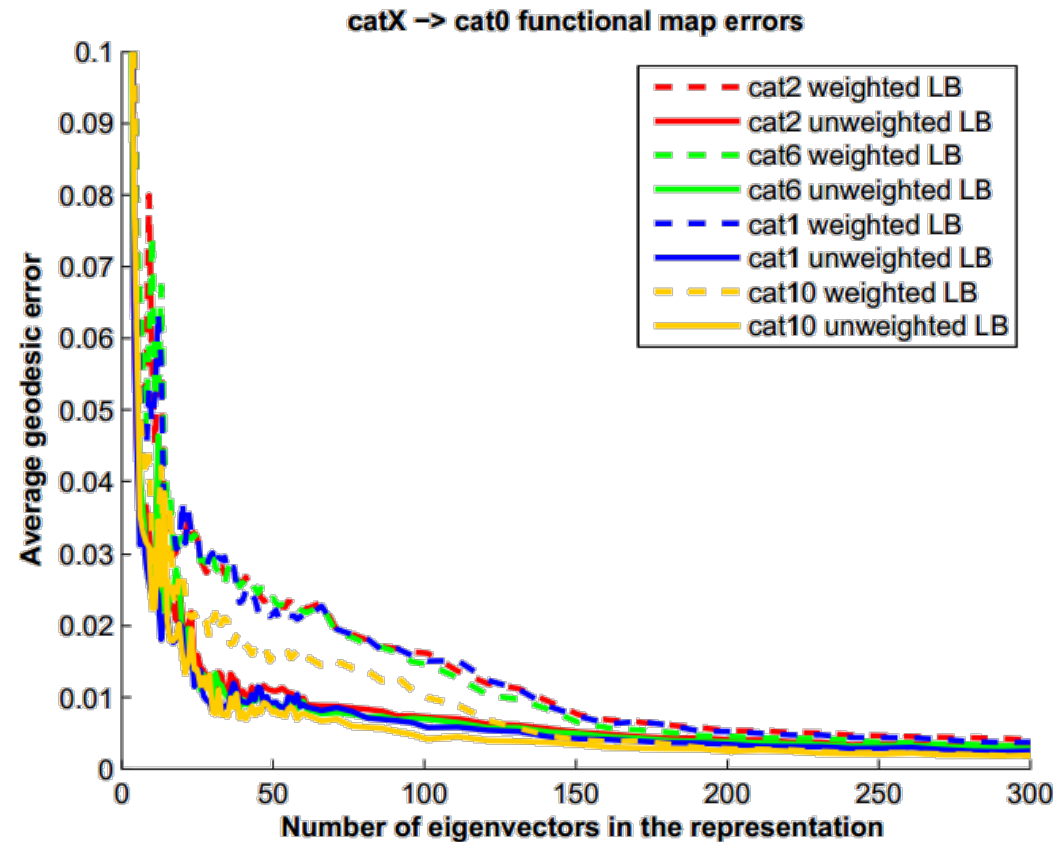
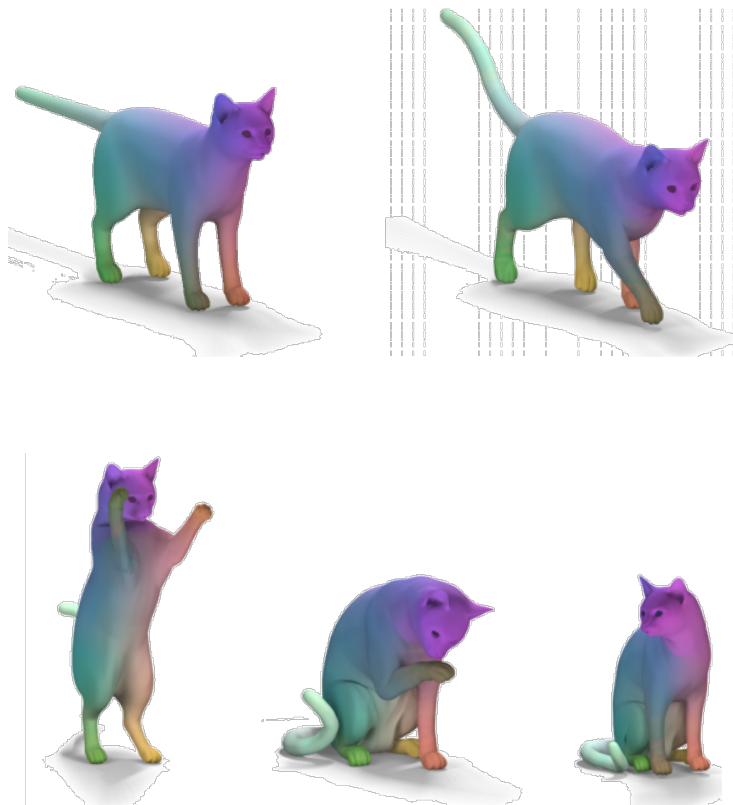
Functions on cat are transferred to lion using  $T_\phi$



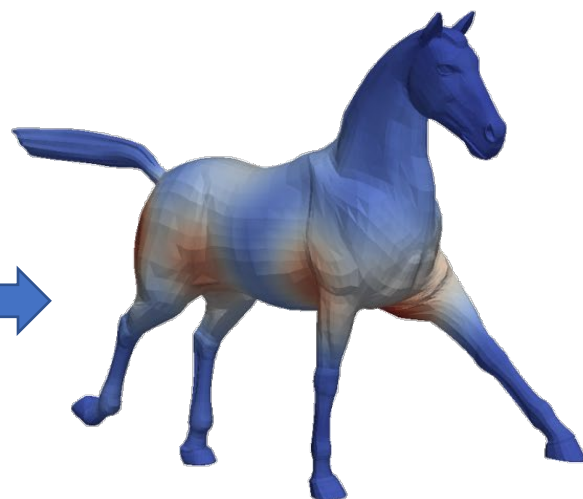
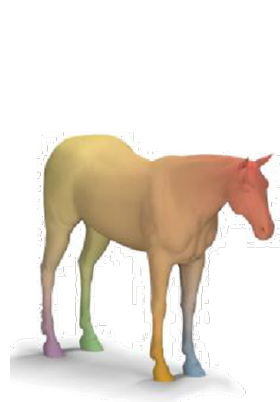
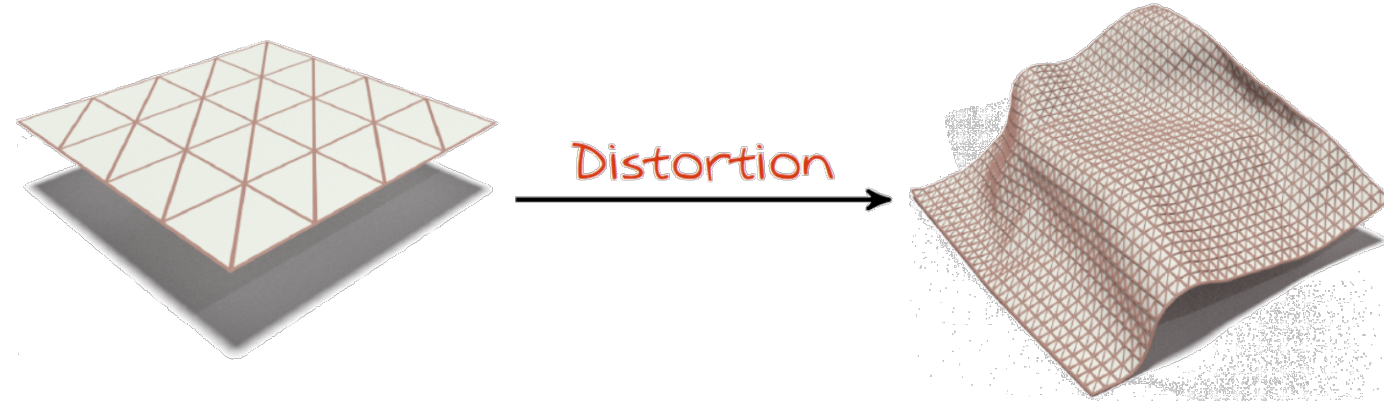
$T_\phi$  is a linear operator (matrix)

$$T_\phi : L^2(cat) \rightarrow L^2(lion)$$

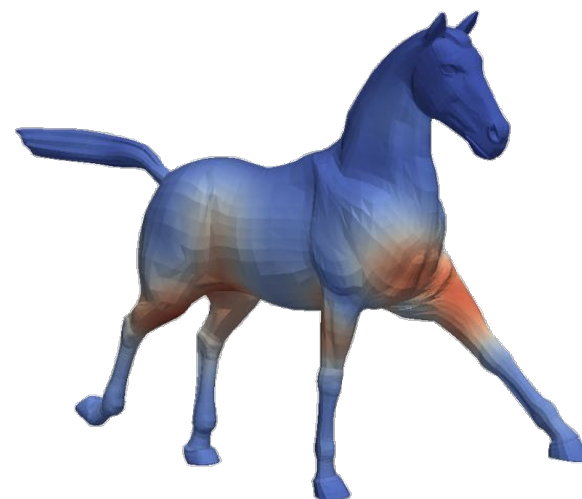
# Compact Encoding of Maps as Matrices



# Changes to Measure and Metric under a Map

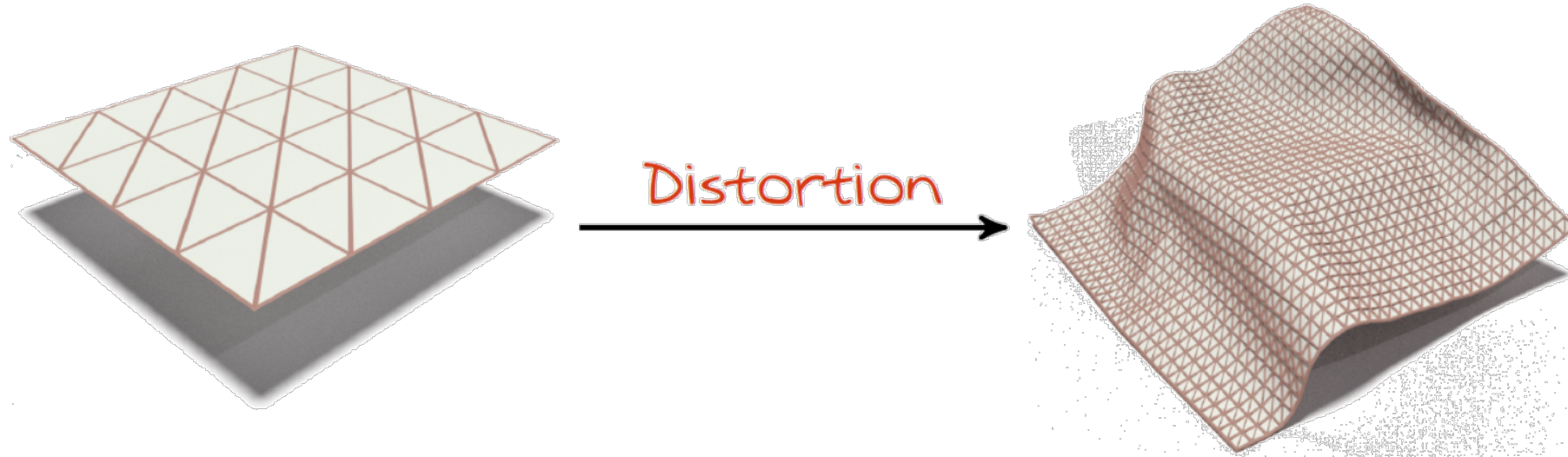


Area distortion



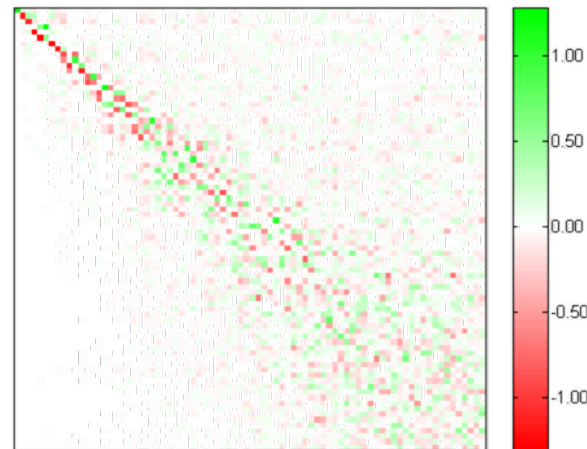
Conformal distortion

# Shape Differences are a Change Recipe



A recipe encoded as a matrix:

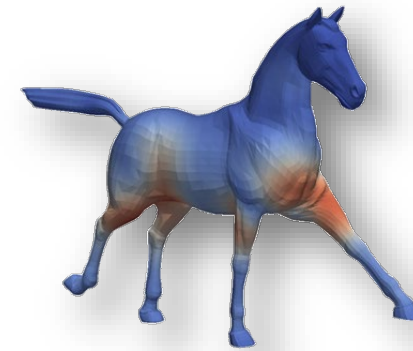
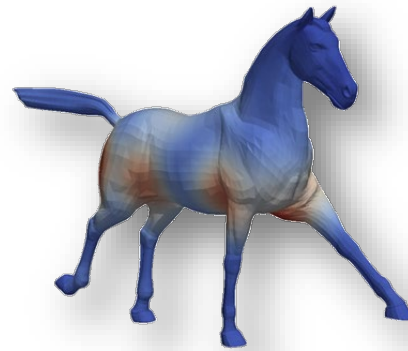
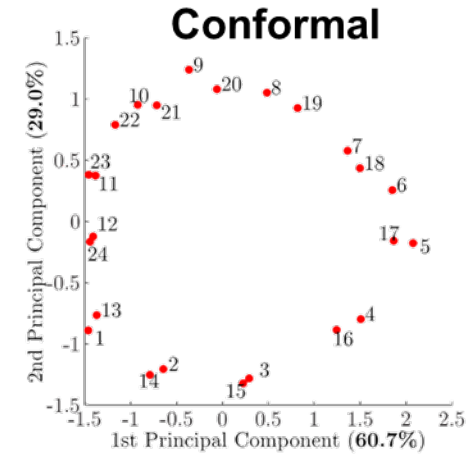
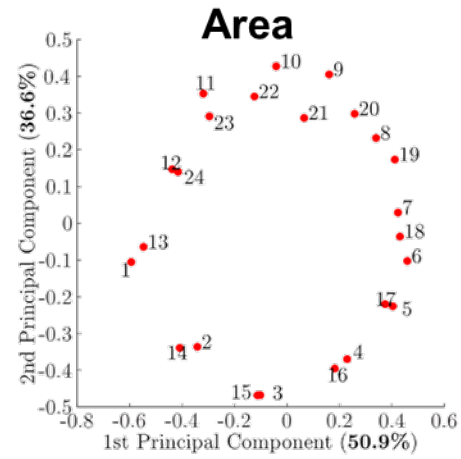
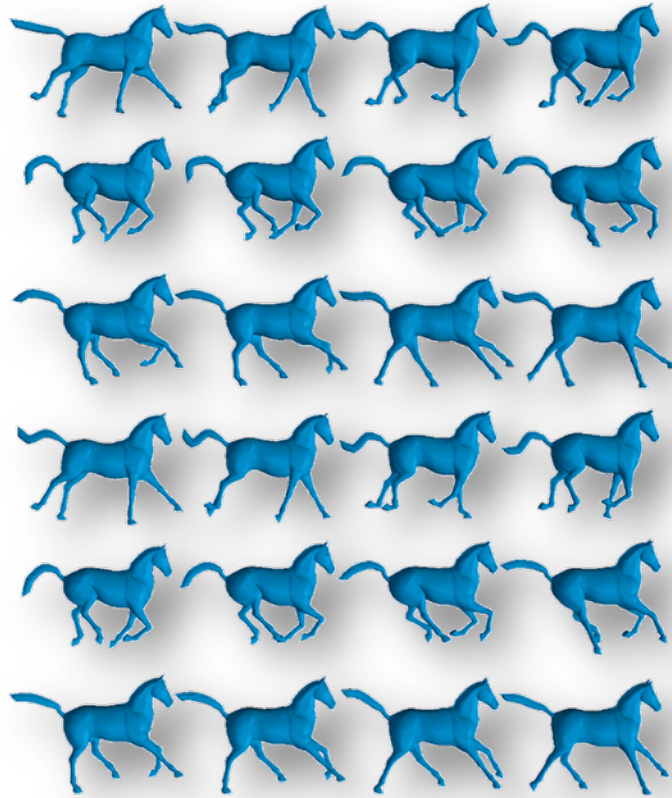
Area distortion  
Conformal distortion



A novel type of latent space representation for 3D data

Under some conditions, lossless

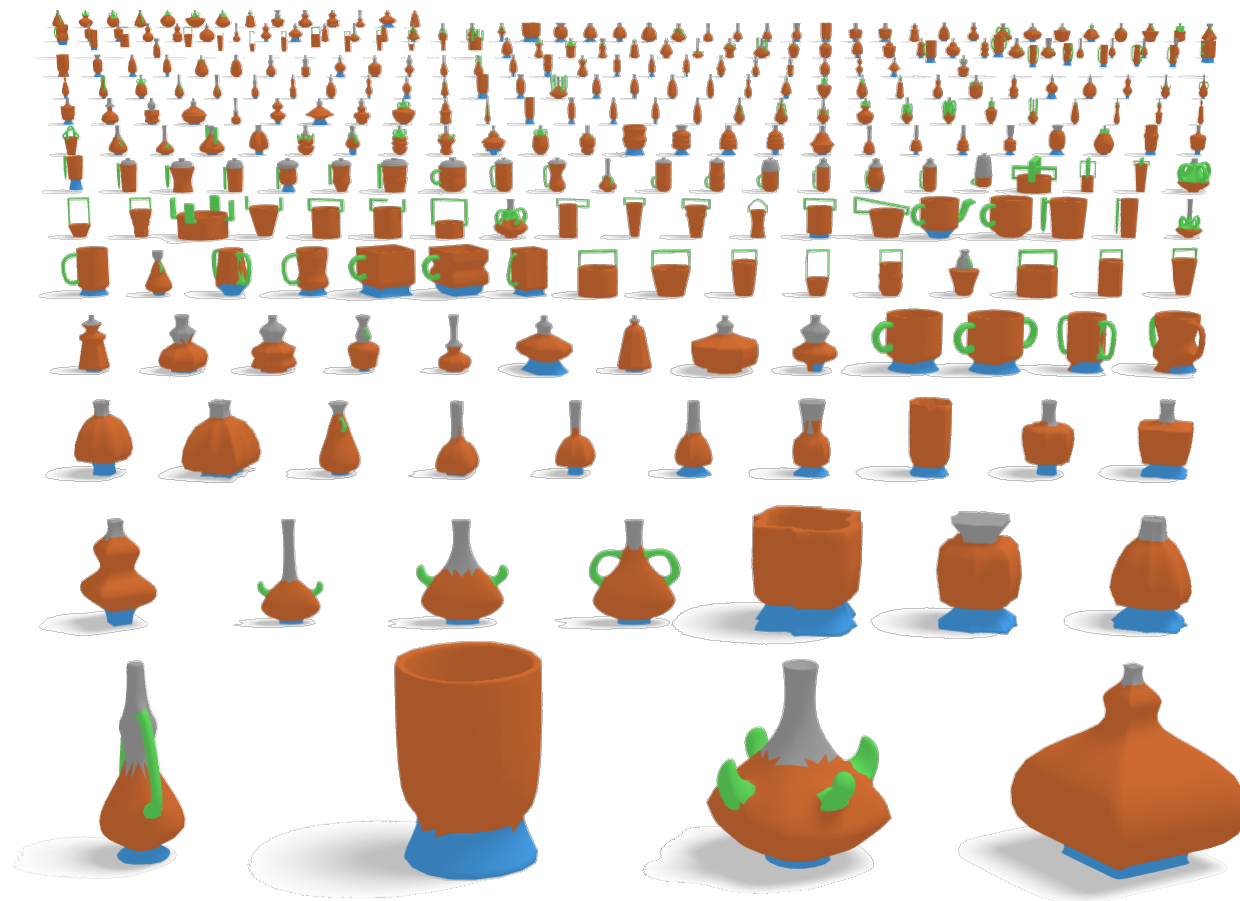
# The Space of Shapes



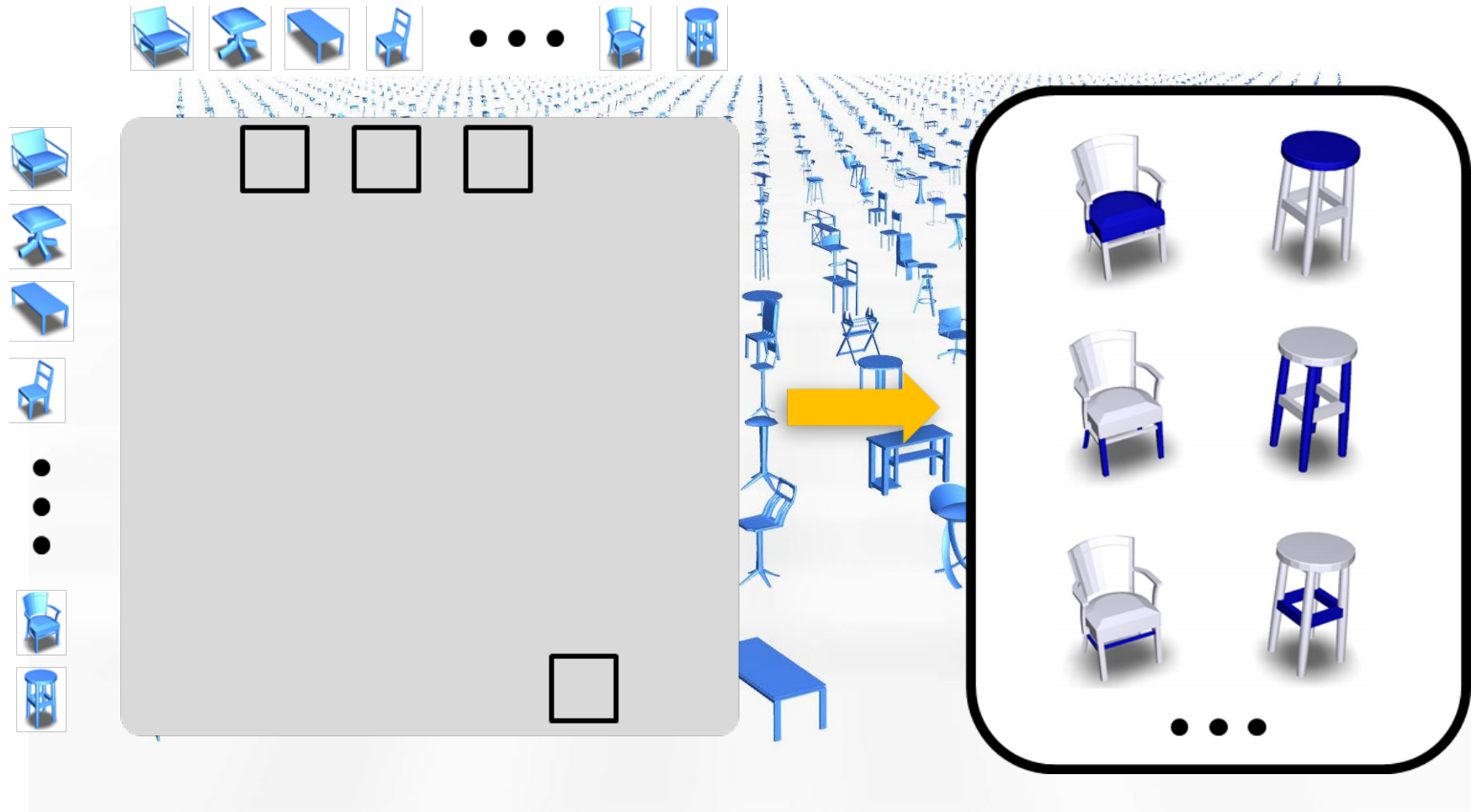
# Horizontal Map Networks



# Joint Analysis: Co-Segmentation

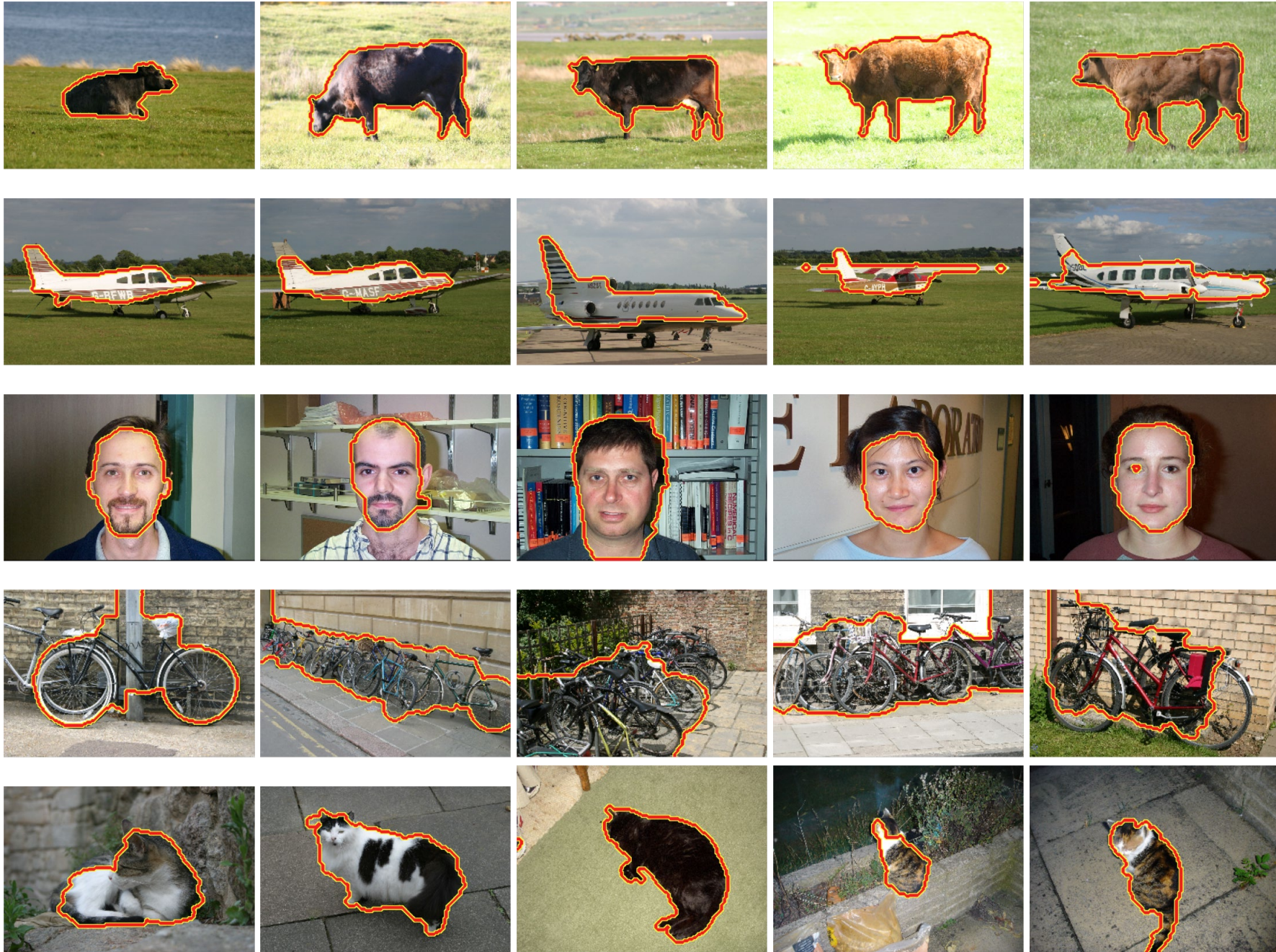


# Unsupervised Structure Extraction

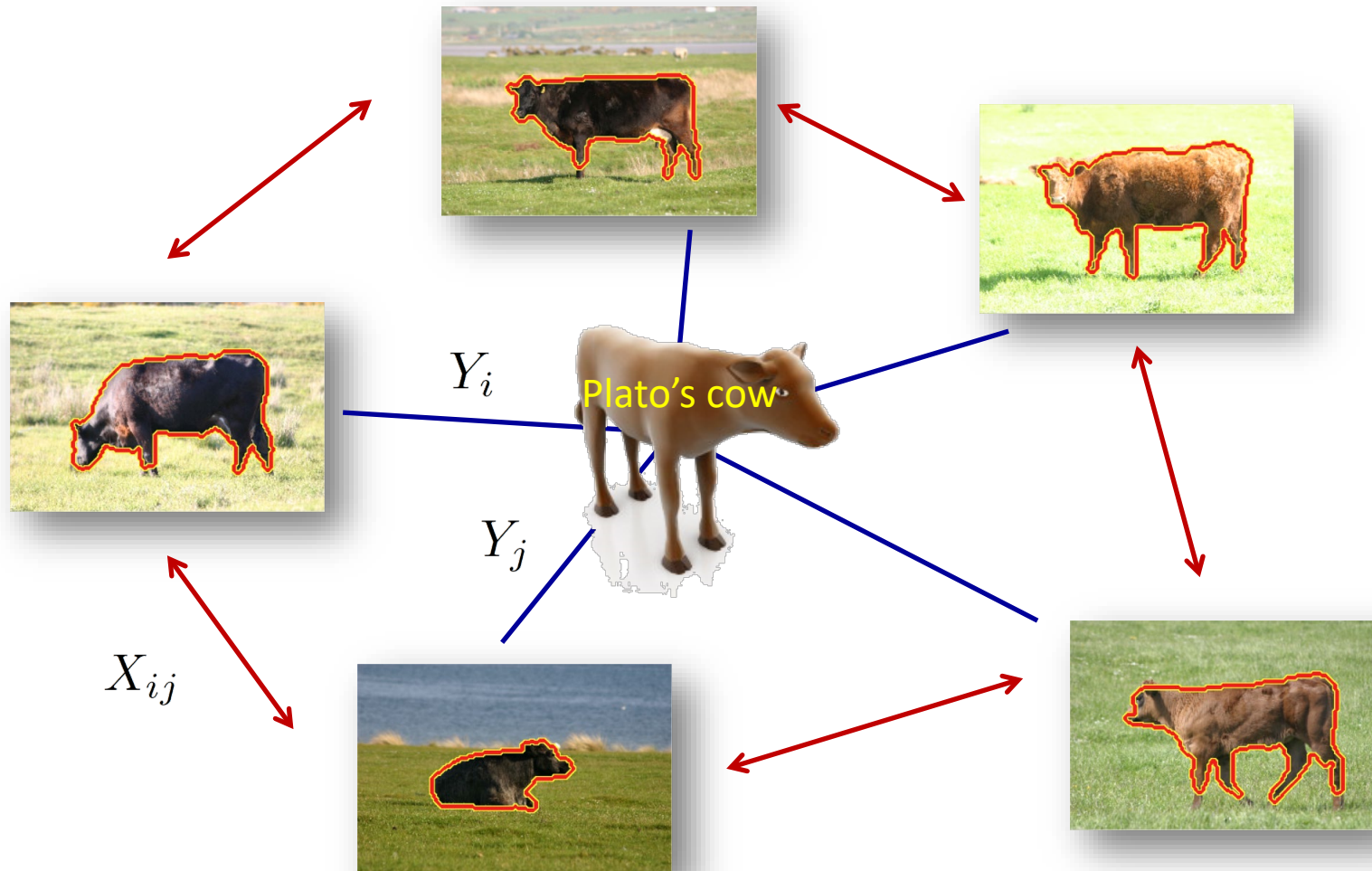


[Q. Huang, F. Wang, L. Guibas, '14]

MSRC: 5 images per class are shown

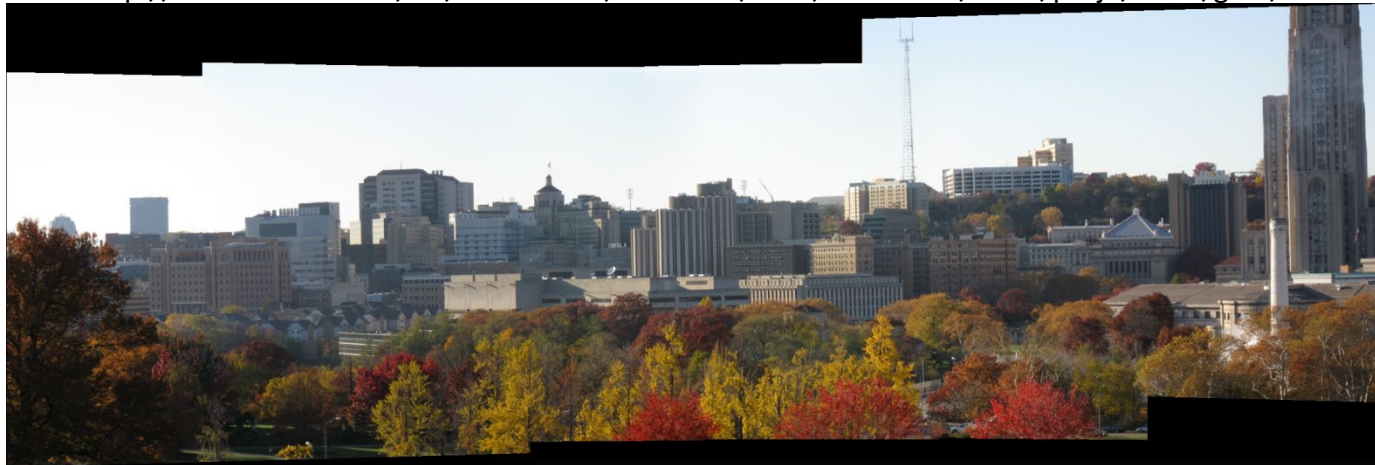


# The Network is the Abstraction



# Mosaicking or SLAM at the Level of Functions

<http://www.cs.cmu.edu/afs/cs.cmu.edu/academic/class/15463-f08/www/proj4/www/gme/>



robotics.ait.kyushu-u.ac.jp

# Joint Learning and Information Aggregation

- Need to aggregate information:
  - *over different data sets*
  - *over different modalities (geometry, appearance, language)*
  - *over space and time*
  - *over different representations*
  - *over different predictions*
  - *over different tasks*

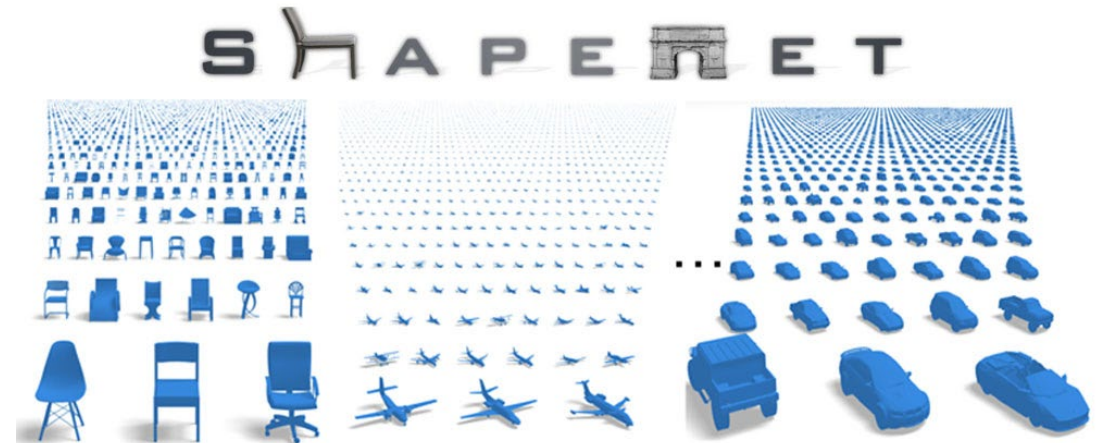
in settings where the above refer to same entities in the world – and are thus correlated



# Visual Data Repositories for Storing Semantic Knowledge

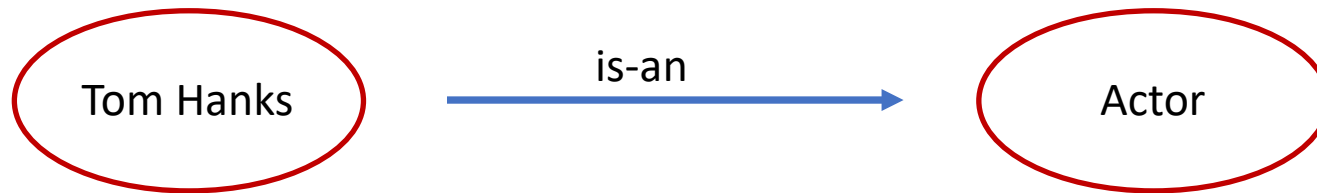
# ImageNet and ShapeNet

- ◆ Explain how big visual datasets including ImageNet and ShapeNet are organized



# Semantic Networks

- ◆ Also known as **frame networks**
- ◆ Encode semantic relations between concepts
- ◆ Often used as a form of knowledge representation
- ◆ A directed or undirected graph consisting of vertices, which represent concepts, and edges which represent concept relations

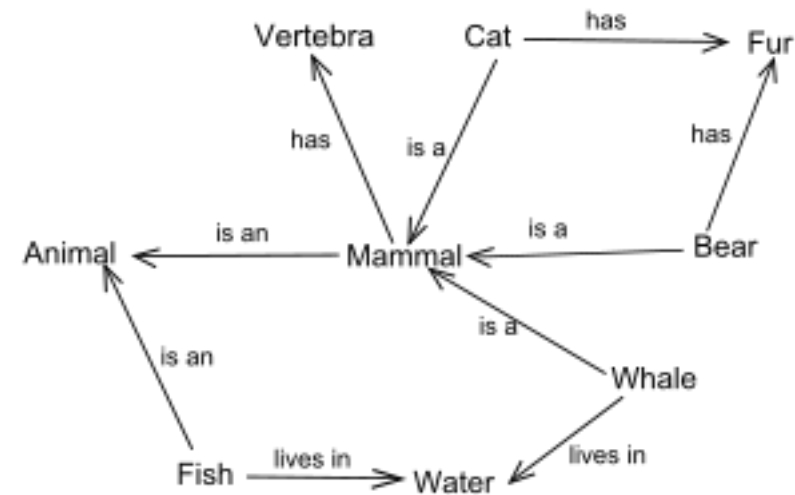


# Example of a Semantic Net

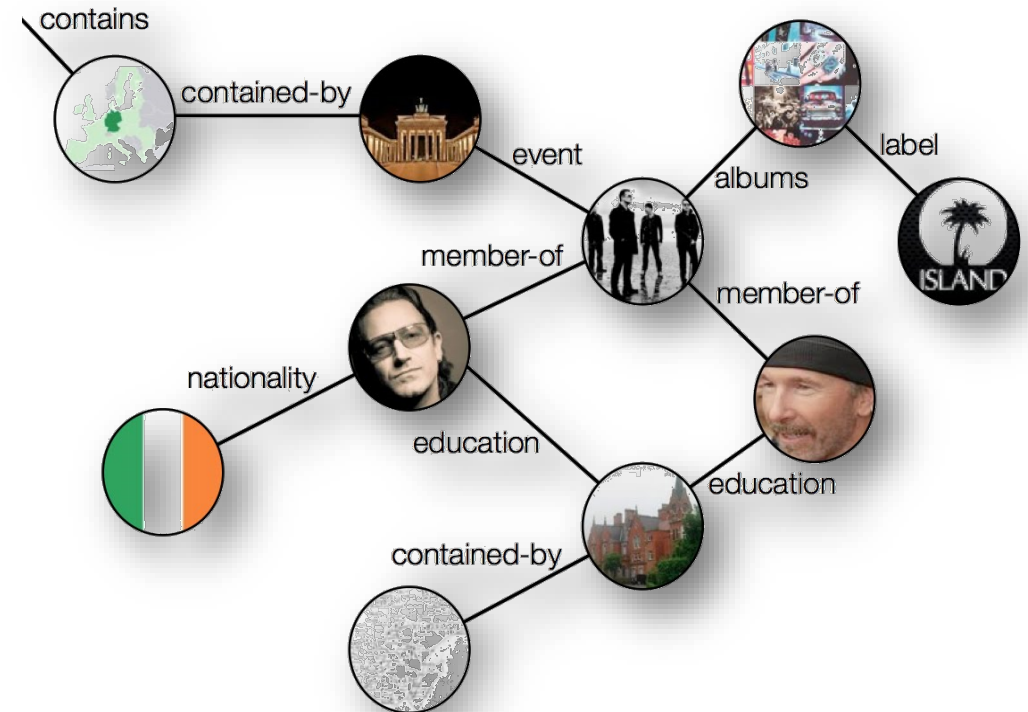
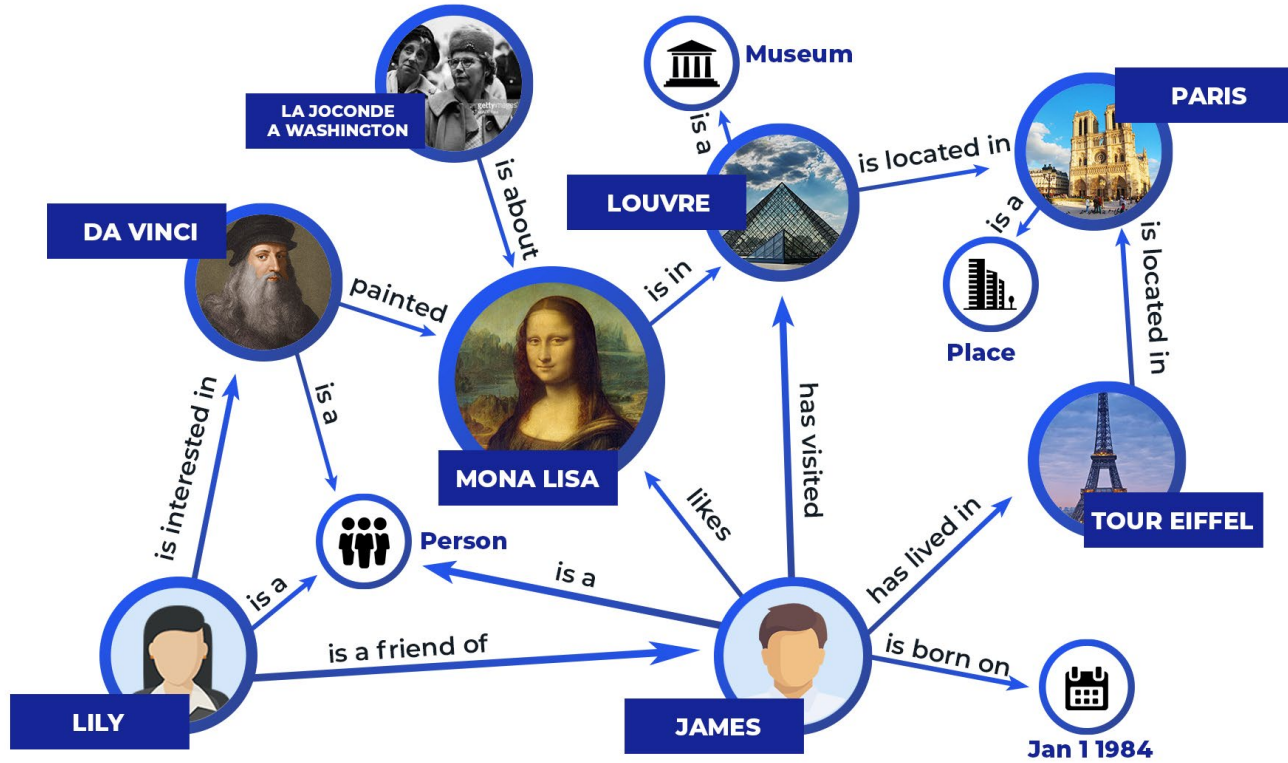
## Semantic Net in Lisp

```
(defun *database* ()  
'((canary (is-a bird)  
          (color yellow)  
          (size small))  
  (penguin (is-a bird)  
           (movement swim))  
  (bird (is-a vertebrate)  
        (has-part wings)  
        (reproduction egg-laying))))
```

## Graph representation



# Google Knowledge Graph



## What is WordNet?



Original paper  
by  
**[George  
Miller, et al  
1990]** cited  
over 5,000  
times

Organizes over  
150,000 words  
into 117,000  
categories  
called *synsets*.

Establishes  
ontological and  
lexical  
relationships in  
NLP and related  
tasks.

# WordNet

- ◆ a lexical database of English
- ◆ words -> synonym sets (synsets)

```
dog, domestic dog, Canis familiaris
=> canine, canid
=> carnivore
=> placental, placental mammal, eutherian, eutherian mammal
=> mammal
=> vertebrate, craniate
=> chordate
=> animal, animate being, beast, brute, creature, fauna
=> ...
```

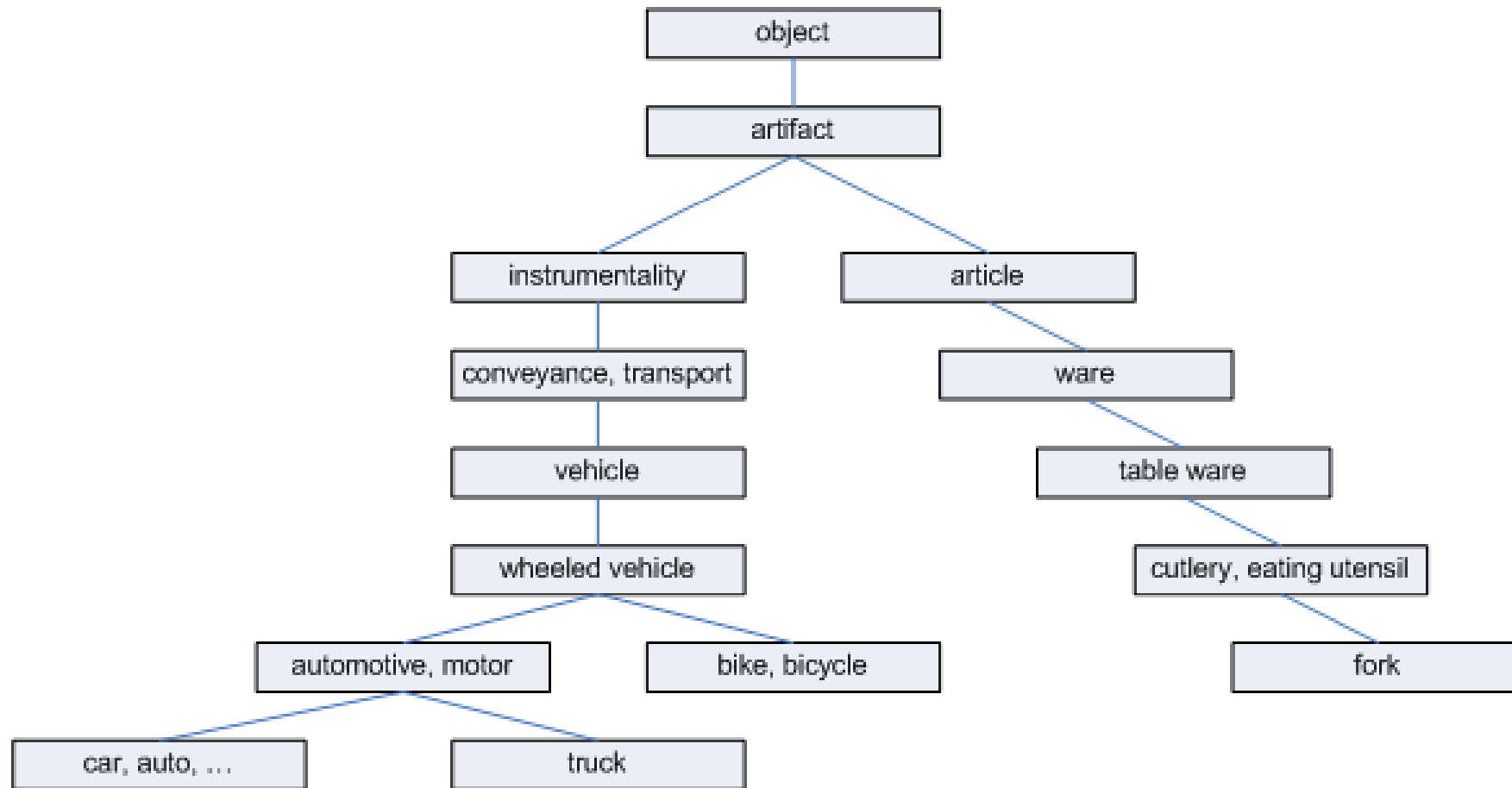
*G. A. Miller, R. Beckwith, C. D. Fellbaum, D. Gross, K. Miller. 1990.  
WordNet: An online lexical database. Int. J. Lexicograph.*

# WordNet

- ◆ Important relations between synsets (nouns):

Relation	Definition	Example
Hypernym	From concepts to superordinates	water <sup>1</sup> → liquid
Hyponym	From concepts to subtypes	water <sup>1</sup> → seawater
Has-Part	From groups to their members	water <sup>1</sup> → oxygen
Part-of	From members to their groups	water <sup>1</sup> → ice
Antonym	Opposites	leader → follower

# Taxonomy: is-a Relationship



# Partonomy: has-a Relationship

- S (n) **car, auto, automobile, machine, motorcar** (a motor vehicle with four wheels; usually propelled by an internal combustion engine) "he needs a car to get to work"
  - [direct hyponym / full hyponym](#)
  - [partonymy](#)
    - S (n) **accelerator, accelerator pedal, gas pedal, gas, throttle, gas** (a pedal that controls the throttle valve) "he stepped on the gas"
    - S (n) **air bag** (a safety restraint in an automobile; the bag inflates on collision and prevents the driver or passenger from being thrown forward)
    - S (n) **auto accessory** (an accessory for an automobile)
    - S (n) **automobile engine** (the engine that propels an automobile)
    - S (n) **automobile horn, car horn, motor horn, horn, buzzer** (a device on an automobile for making a warning noise)
    - S (n) **buffer, fender** (a cushion-like device that reduces shock due to an impact)
    - S (n) **bumper** (a mechanical device consisting of bars at either end of a vehicle to absorb shock and prevent serious damage)
    - S (n) **car door** (the door of a car)
    - S (n) **car mirror** (a mirror that the driver of a car can use)
    - S (n) **car seat** (a seat in a car)
    - S (n) **car window** (a window in a car)
    - S (n) **fender, wing** (a barrier that surrounds the wheels of a vehicle to block splashing water or mud) "in Britain they call a fender a wing"
    - S (n) **first gear, first, low gear, low** (the lowest forward gear ratio in the gear box of a motor vehicle; used to start a car moving)
    - S (n) **floorboard** (the floor of an automobile)
    - S (n) **gasoline engine, petrol engine** (an internal-combustion engine that burns gasoline; most automobiles are driven by gasoline engines)
    - S (n) **glove compartment** (compartment on the dashboard of a car)
    - S (n) **grille, radiator grille** (grating that admits cooling air to car's radiator)
    - S (n) **high gear, high** (a forward gear with a gear ratio that gives the greatest vehicle velocity for a given engine speed)
    - S (n) **hood, bonnet, cowl, cowling** (protective covering consisting of a metal part that covers the engine) "there are powerful engines under the hoods of new cars"
    - S (n) **luggage compartment, automobile trunk, trunk** (compartment in an automobile that carries luggage or shopping or tools) "he put his golf bag in the trunk"
    - S (n) **rear window** (car window that allows vision out of the back of the car)
    - S (n) **reverse, reverse gear** (the gears by which the motion of a machine can be reversed)
    - S (n) **roof** (protective covering on top of a motor vehicle)
    - S (n) **running board** (a narrow footboard serving as a step beneath the doors of some old cars)
    - S (n) **stabilizer bar, anti-sway bar** (a rigid metal bar between the front suspensions and between the rear suspensions of cars and trucks; serves to stabilize the car)
    - S (n) **sunroof, sunshade-roof** (an automobile roof having a sliding or raisable panel) "'sunshade-roof' is a British term for 'sunroof'"
    - S (n) **tail fin, tailfin, fin** (one of a pair of decorations projecting above the rear fenders of an automobile)
    - S (n) **third gear, third** (the third from the lowest forward ratio gear in the gear box of a motor vehicle) "you shouldn't try to start in third gear"
    - S (n) **window** (a transparent opening in a vehicle that allow vision out of the sides or back; usually is capable of being opened)





# ShapeNet (>3M Models)

SHAPE NET Search Options Home About Download Statistics

**chair**  
a seat for one person, with a support for the back; 'he put his coat over the back of the chair and sat down'  
[ImageNet](#) [MetaData](#)


Choose a taxonomy:  
ShapeNetCore

- airplane,aeroplane,plane(12,4501)
- aquarium,fish tank,marine museum(0,4)
- ashcan,trash can,garbage can,wastebin,ash bin(1,10)
- bag,traveling bag,travelling bag,grip,suitcase(1,10)
- basket,handbasket(2,140)
- bathtub,bathing tub,bath,tub(0,932)
- bed(13,353)
- bench(5,1953)
- birdhouse(0,79)
- boat(12,1635)
- bookshelf(0,495)
- bottle(6,550)
- bowl(1,234)
- bus,autobus,coach,charabanc,double-decker,jack bus(1,10)
- cabinet(9,1644)
- camera,photographic camera(4,134)
- can,tin,tin can(2,108)
- cap(4,81)
- car,auto,automobile,machine,motorcar(18,244)
- cellular telephone,cellular phone,cellphone,cell phone(1,10)
- chair(23,7083)**
- chair(1,10)

Synset models

Displaying 1 to 40 of 7080

< 1 2 3 4 5 6 7 8 9 10 11 12 13 ... 177 >



club chair cantilever chair armchair straight chair straight chair club chair deck chair rex chair

straight chair club chair club chair swivel chair butterfly chair armchair armchair club chair

recliner cantilever chair swivel chair swivel chair armchair folding chair rocking chair club chair

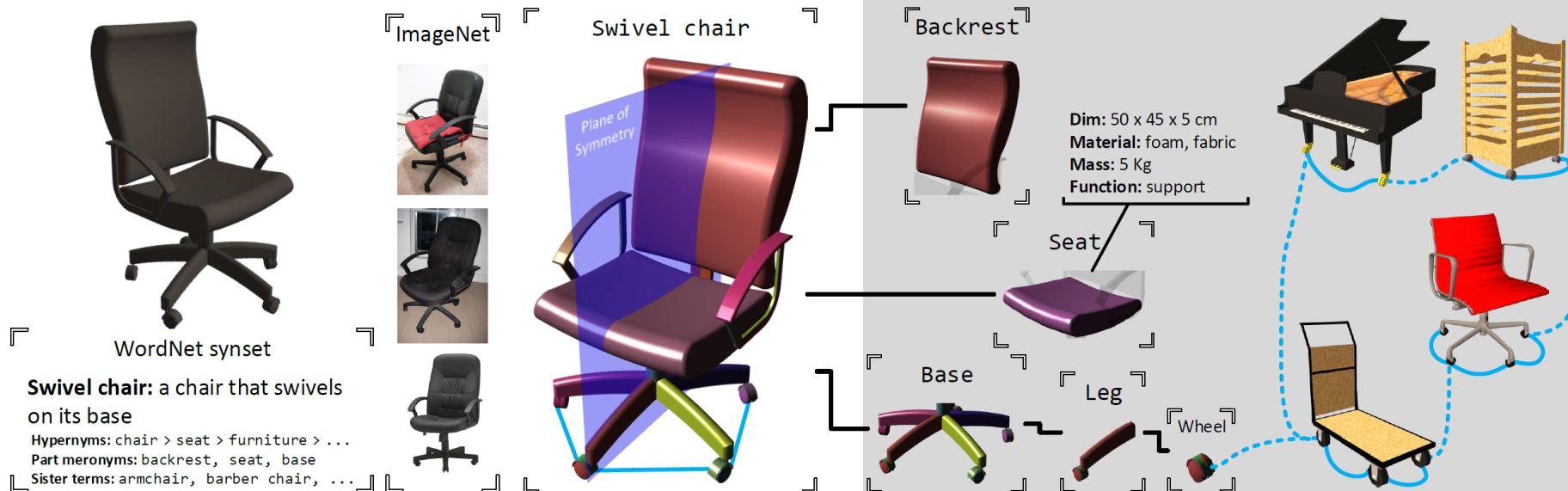
chair chair chair chair chair chair chair chair

# Object Knowledge

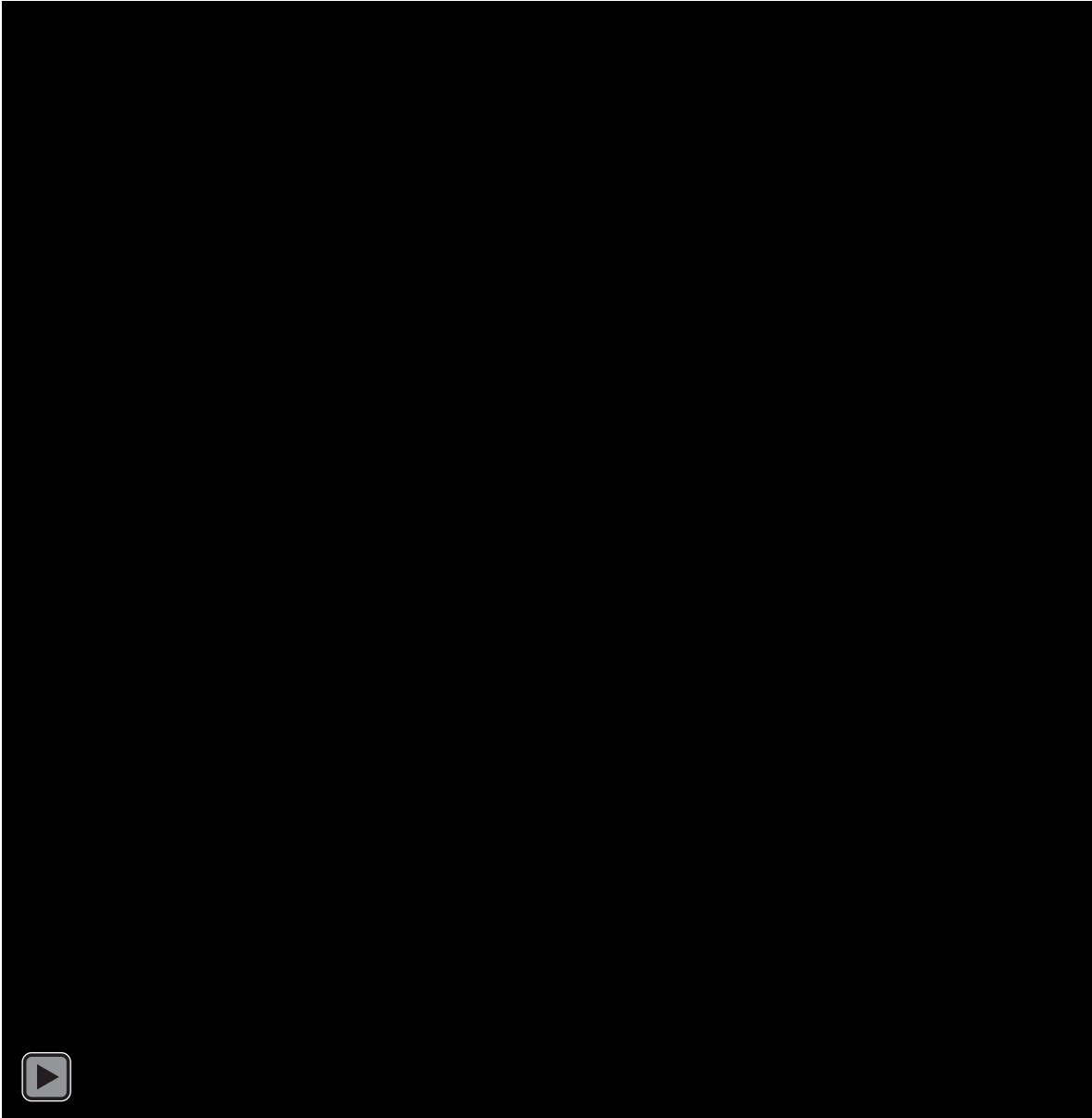
Parts, symmetries, keywords, physical properties, materials, affordances, ...



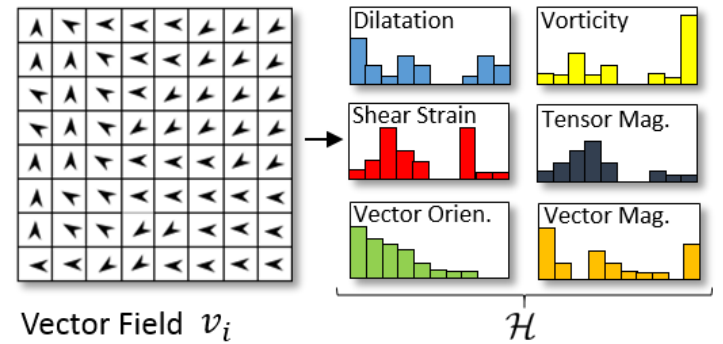
Link to WordNet Taxonomy   Alignment+Symmetry   Part Hierarchy   Part Correspondences



# Object Interaction Knowledge



Vector Field to Histograms



# The Team

# The Principals

- Leonidas (Leo) Guibas (CS & EE)
  - Instructor
- Davis Rempe (CS)
  - Course Assistant
- Despoina Paschalidou (CS)
  - Course Assistant
- Carrie Petersen (CS)
  - Admin



Davis



Leo



Despoina



Carrie

# Course Mechanics

# CS233 / CME 251 Class Schedule

Monday	Wednesday
<p><b>March 28</b></p> <p>Introduction; Geometric and topological perspective on data analysis; Data representations; Learning on point clouds and graphs; Joint data analysis.</p> <p>Lecture Slides:</p> <p>Reading:</p>	<p><b>March 30</b></p> <p>Linear algebraic techniques: principal components analysis (PCA), Kernel PCA.</p> <p>Lecture Slides:</p> <p>Reading:</p>
<p><b>April 4</b></p> <p>Visual data sets: ImageNet and ShapeNet; Techniques for annotation and annotation transport.</p> <p>Lecture Slides:</p> <p>Reading:</p>	<p><b>April 6</b></p> <p>Linear algebraic techniques: canonical correlation analysis (CCA). Multidimensional scaling (MDS).</p> <p>Lecture Slides:</p> <p>Reading:</p> <p>Homework 1 out.</p>
<p><b>April 11</b></p> <p>Graph methods; spectral approaches, graph Laplacians, Laplacian embeddings, spectral clustering.</p> <p>Lecture Slides:</p> <p>Reading:</p>	<p><b>April 13</b></p> <p>Non-linear dimensionality reduction: locally linear embeddings, Laplacian eigensmaps, Isomap, t-SNE.</p> <p>Lecture Slides:</p> <p>Reading:</p>
<p><b>April 18</b></p> <p>Computational topology: topology review, complexes, homology groups.</p> <p>Lecture Slides:</p> <p>Reading:</p>	<p><b>April 20</b></p> <p>Persistent homology, barcodes and persistence diagrams.</p> <p>Lecture Slides:</p> <p>Reading:</p> <p>Homework 1 due. Homework 2 out.</p>

Monday	Wednesday
<p><b>April 25</b></p> <p>Topological inference; the Mapper algorithm. Applications.</p> <p>Lecture Slides:</p> <p>Reading:</p>	<p><b>April 27</b></p> <p>Representations of 3D Geometry: Voxel-Grids, Point Clouds, Meshes and Other Boundary Models, Solid Models.</p> <p>Lecture Slides:</p> <p>Reading:</p>
<p><b>May 2</b></p> <p>Geometry processing; Laplace-Beltrami and other operators on meshes.</p> <p>Lecture Slides:</p> <p>Reading:</p>	<p><b>May 4</b></p> <p>Rigid and non-rigid shape alignment. Global and local shape descriptors; intrinsic descriptors, heat and wave kernel signatures.</p> <p>Lecture Slides:</p> <p>Reading:</p> <p>Homework 2 due. Homework 3 out.</p>
<p><b>May 9</b></p> <p>Class Midterm</p>	<p><b>May 11</b></p> <p>Geometric deep learning; Volumetric and mesh CNNs for 3D geometry</p> <p>Lecture Slides:</p> <p>Reading:</p>
<p><b>May 16</b></p> <p>Deep nets for pointclouds and applications to classification and segmentation.</p> <p>Lecture Slides: <a href="#">Pointnets</a></p> <p>Reading:</p>	<p><b>May 18</b></p> <p>Functional spaces and functional maps, variations; map visualization.</p> <p>Lecture Slides:</p> <p>Reading:</p> <p>Homework 3 due. Homework 4 out.</p>

# CS233 / CME 251 Class Schedule

Monday		Wednesday	
May 23		May 25	
Networks of shapes and images; cycle consistency; map processing and latent spaces. Lecture Slides: Reading:		Encoding shape differences and shape variability. Lecture Slides: Reading:	
May 30		June 1	
Memorial day holiday -- no class		Class summary. Lecture Slides: Reading: Homework 4 due.	

# Class Mechanics

- Two weekly lectures
- Online and after class office hours
- Class web site <http://cs233.stanford.edu>  
<http://graphics.stanford.edu/courses/cs233-22-spring>
- Use Piazza (discussion group), Gradescope (for homework submissions)

# Course Work: Assignments

- **Assignment 1:** Principal Components Analysis (PCA) and Canonical Correlation analysis (CCA)
- **Assignment 2:** Computational Topology - Simplicial Complexes, Homology, Persistence, Mapper Algorithm
- **Assignment 3:** Geometry Correspondences, Shape Matching, Multi-way Alignments
- **Assignment 4:** Deep Learning with 3D Point-Clouds

# Course Work

- Four assignments
  - modest programming in MATLAB/Python for three, one in JavaPlex
  - working in groups OK, up to three students
  - Use Google Cloud for Education coupons for deep net training
- A midterm
- Class participation
- No final
- Special consideration for those taking the course credit / no credit

# CS233 Key Course Goals

- Cover basic tools for **geometric and topological data analysis**, both supervised and unsupervised
- Discuss mathematical ways, based on geometry and topology, to **encode and transfer knowledge** about the data
- Introduce methods for **joint data analysis and joint machine learning** – benefiting from the “wisdom of the collection”
- Challenge: a diversity of tools ... LA, ML, Stat, optimization, geometry processing, computer vision, algebraic topology ...

# Data Has Shape



# That's All

