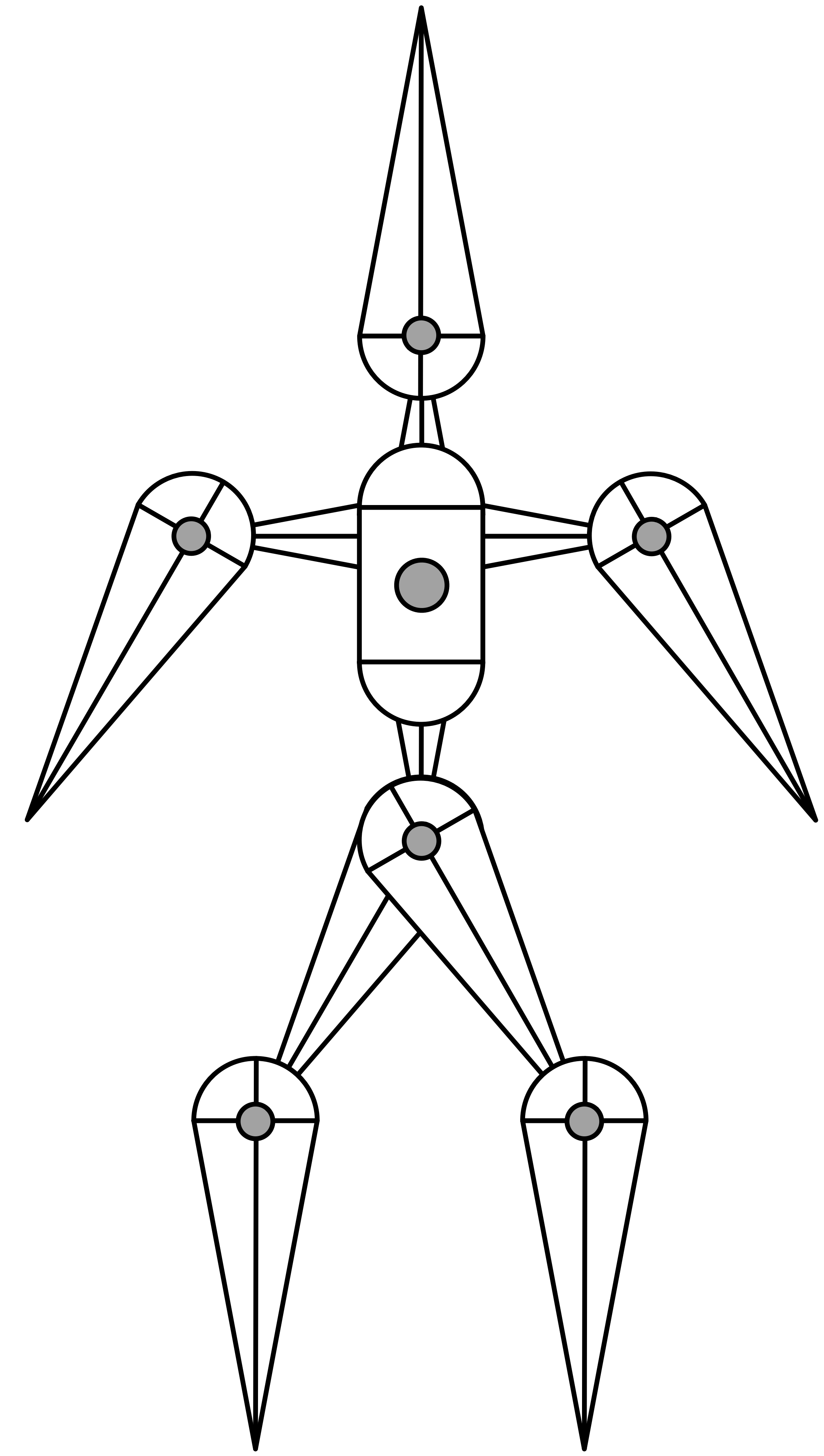
Kinematics and Motion Capture

Interactive Computer Graphics
Stanford CS248, Spring 2018

Today

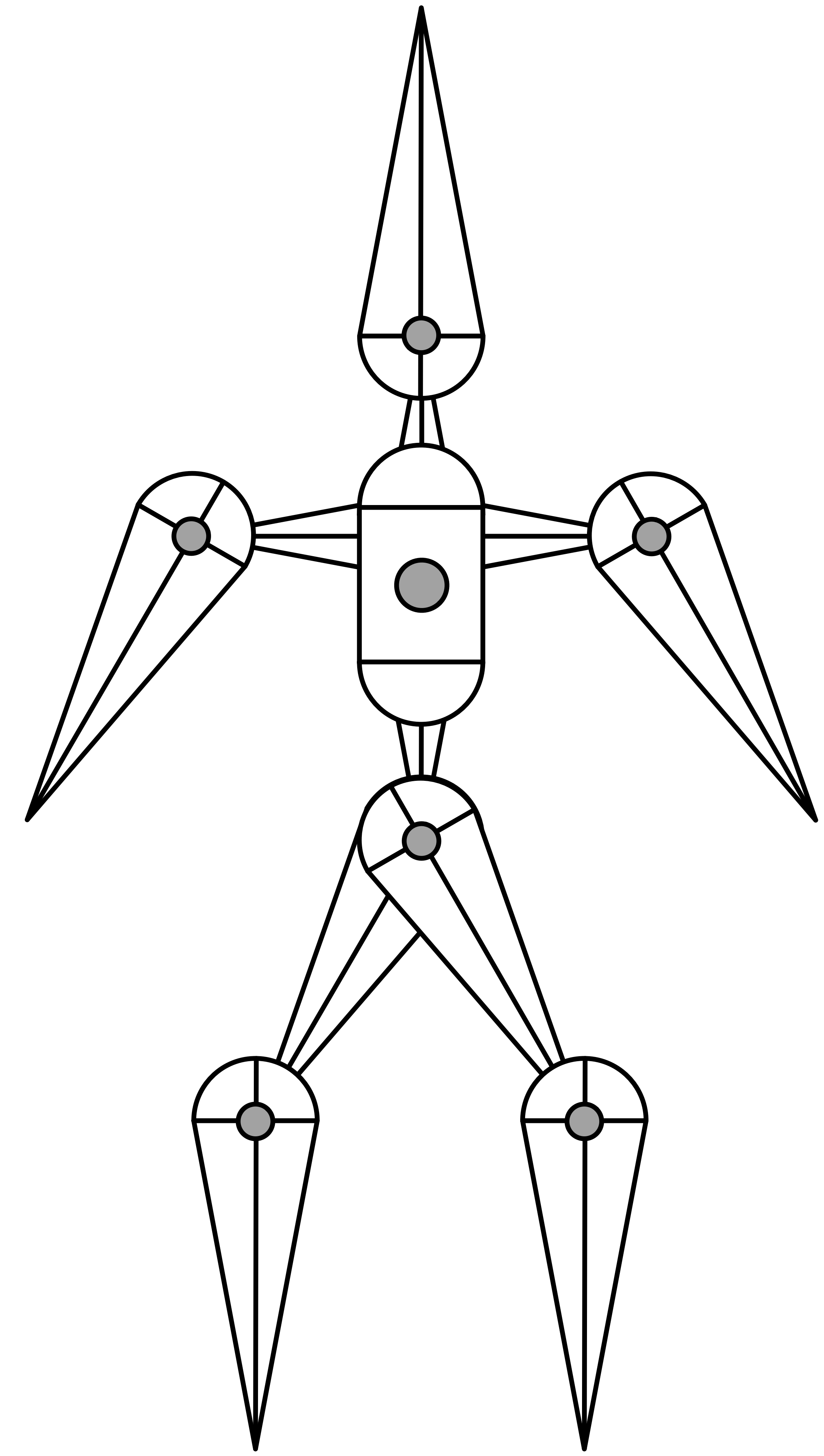
- **KINEMATICS: we are going to describe how objects move, without considering the underlying forces that generate that motion**



Forward kinematics

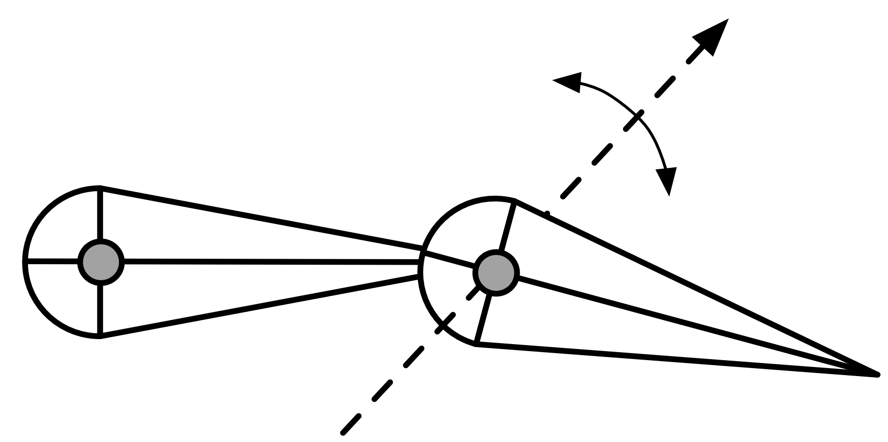
Articulated skeleton

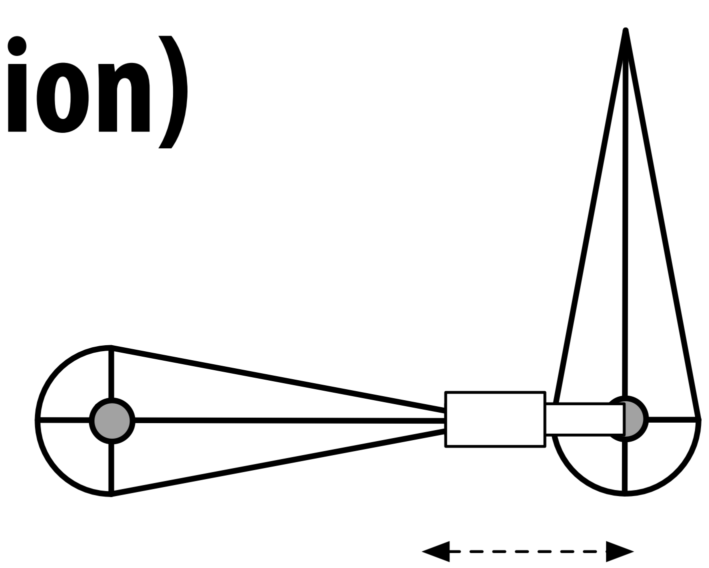
- Topology (what's connected to what)
- Geometric relations from joints
- Tree structure (in absence of loops)



Joint types

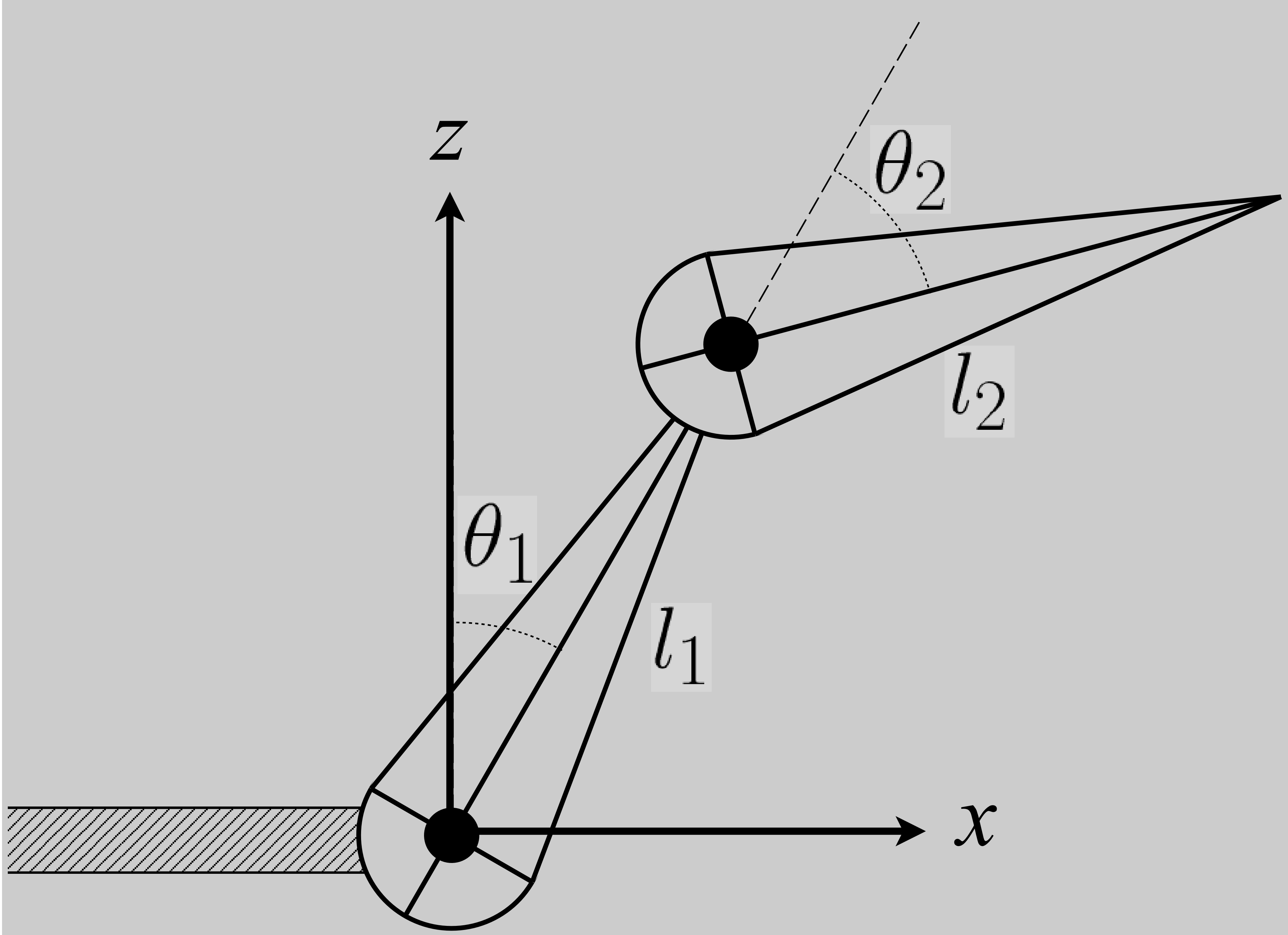
- Pin (1D rotation)
- Ball (2D rotation)
- Prismatic joint (translation)



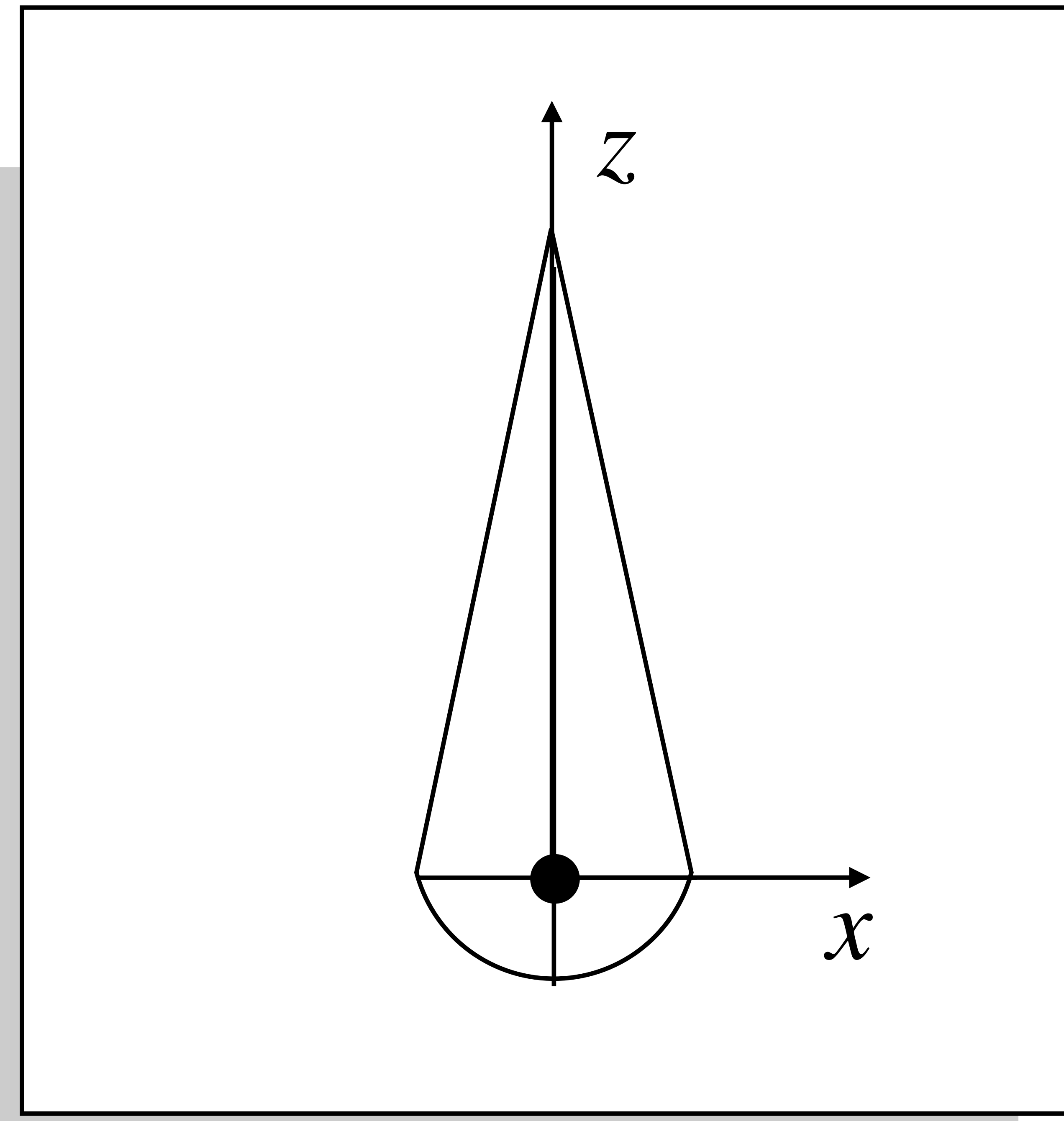


Forward kinematics

Example: simple two segment arm in 2D

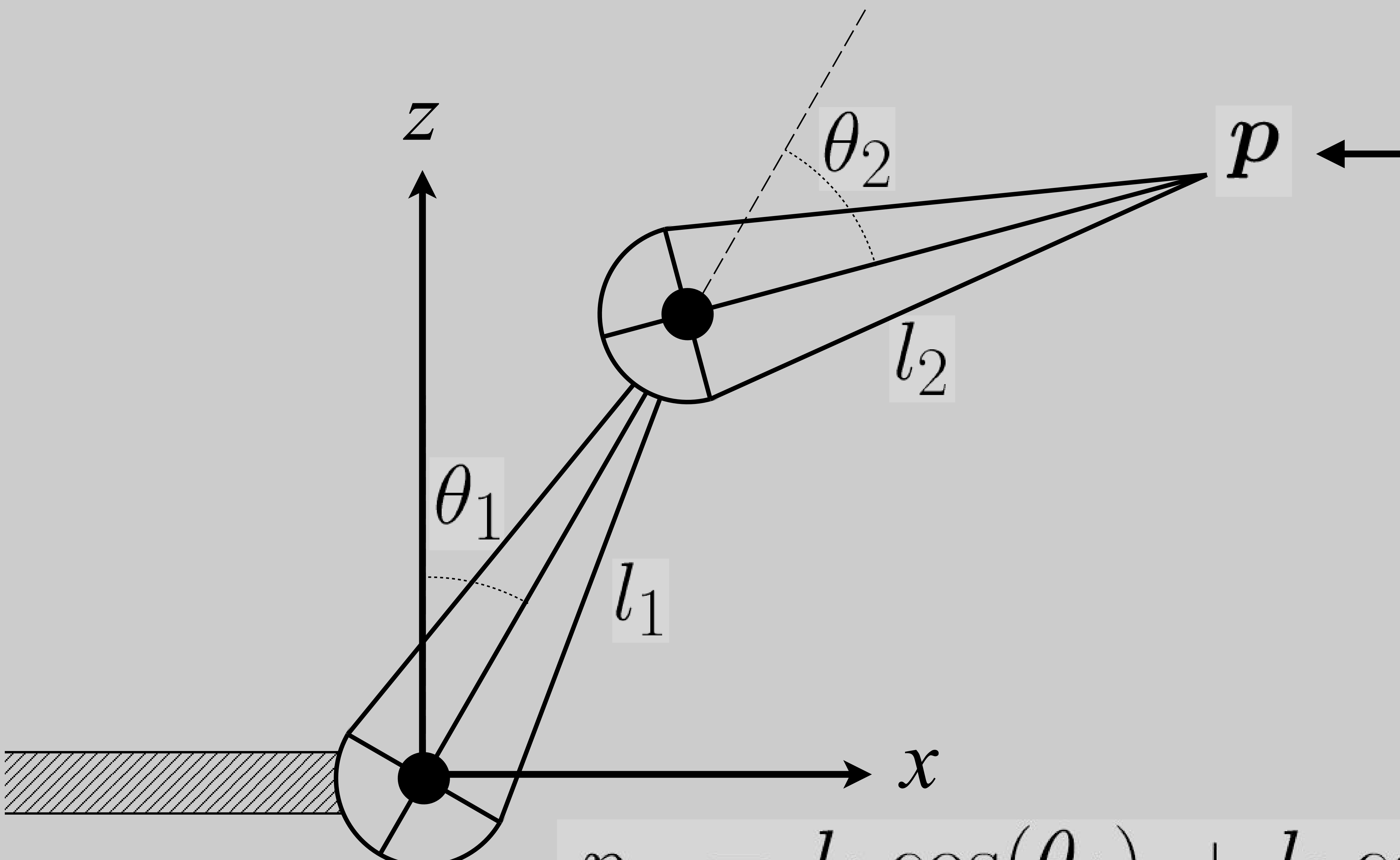


Object space position of part



Forward kinematics

Animator provides angles, and computer determines position *p* of end-effector



To transform point *p* with object space representation (0, l_2) in world space:

Rotate by θ_2

Translate by (0, l_1)

Rotate by θ_1

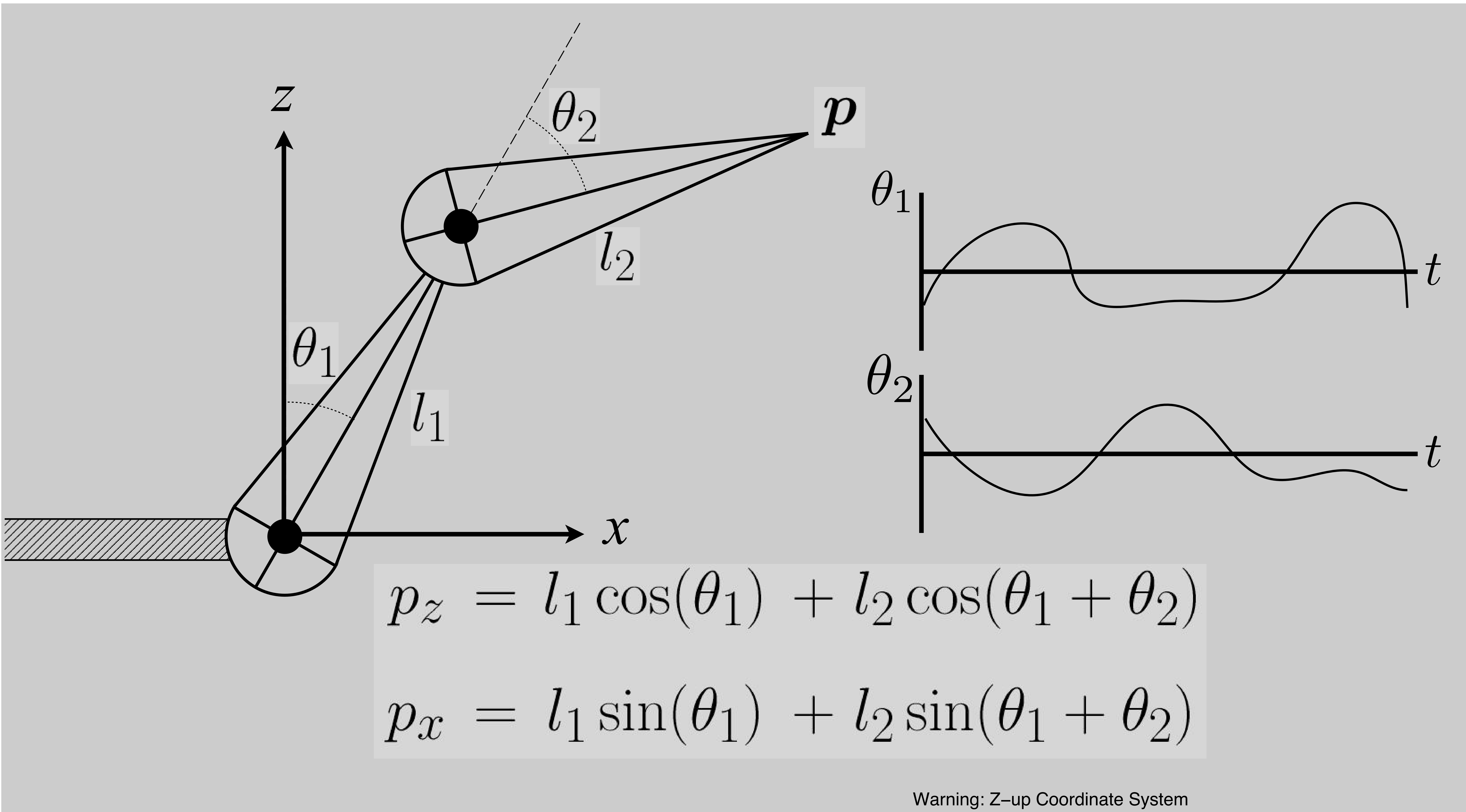
$$p_z = l_1 \cos(\theta_1) + l_2 \cos(\theta_1 + \theta_2)$$

$$p_x = l_1 \sin(\theta_1) + l_2 \sin(\theta_1 + \theta_2)$$

Warning: Z–up Coordinate System

Forward kinematics

Animation is described as angle parameter values as a function of time: $\theta_1(t), \theta_2(t)$

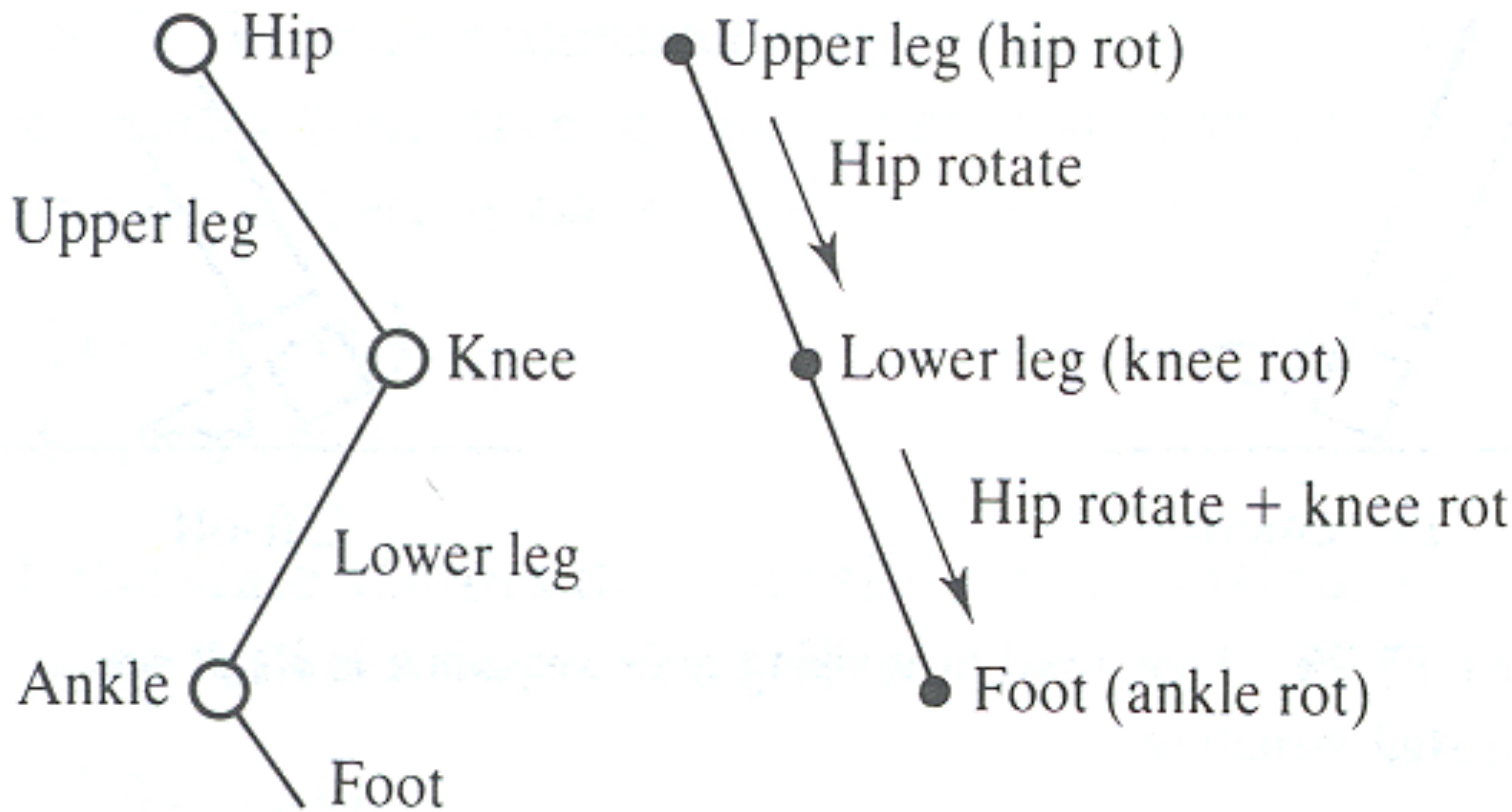


$$p_z = l_1 \cos(\theta_1) + l_2 \cos(\theta_1 + \theta_2)$$

$$p_x = l_1 \sin(\theta_1) + l_2 \sin(\theta_1 + \theta_2)$$

Example: walk cycle

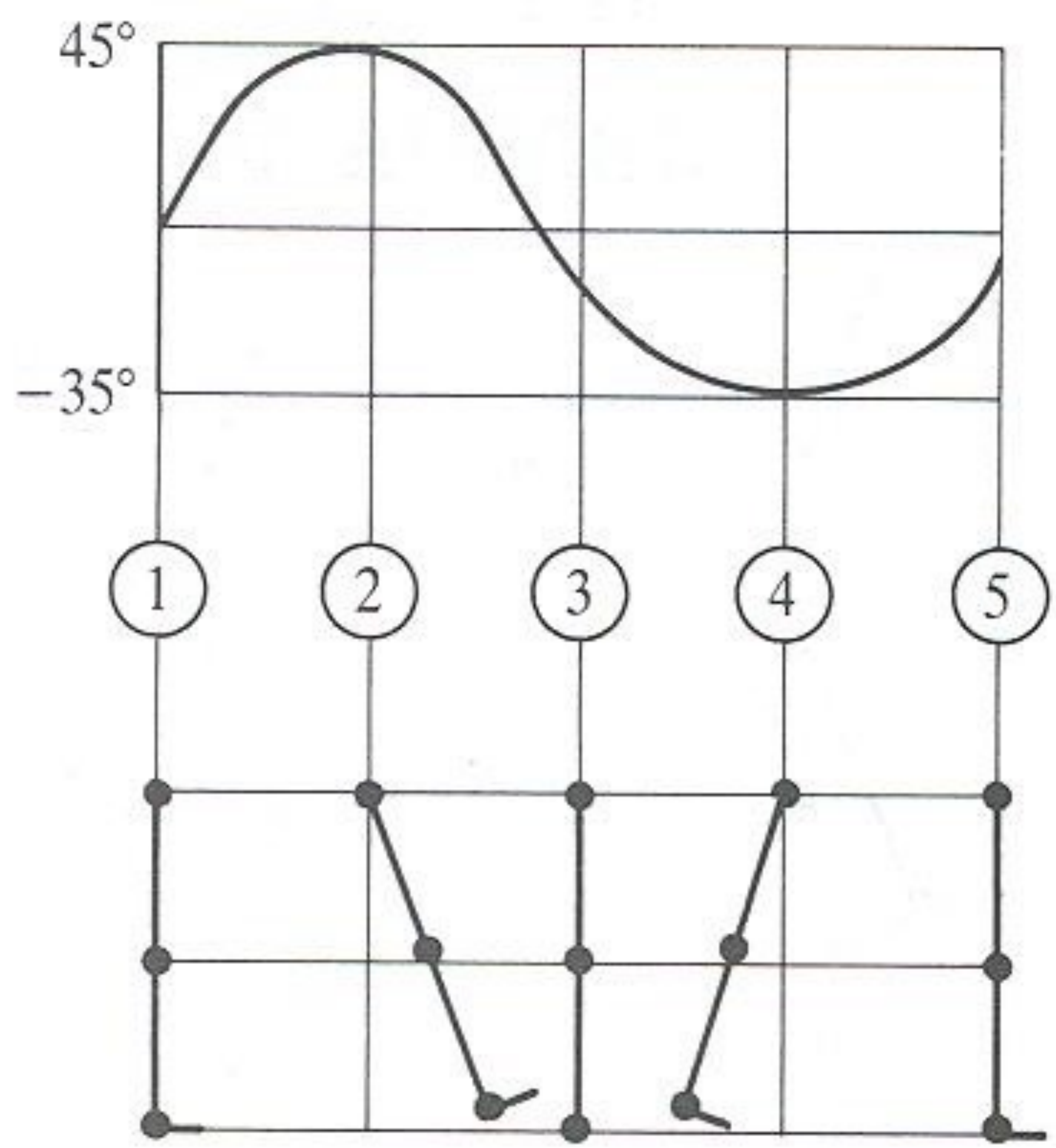
Articulated leg:



Watt & Watt

Example: walk cycle

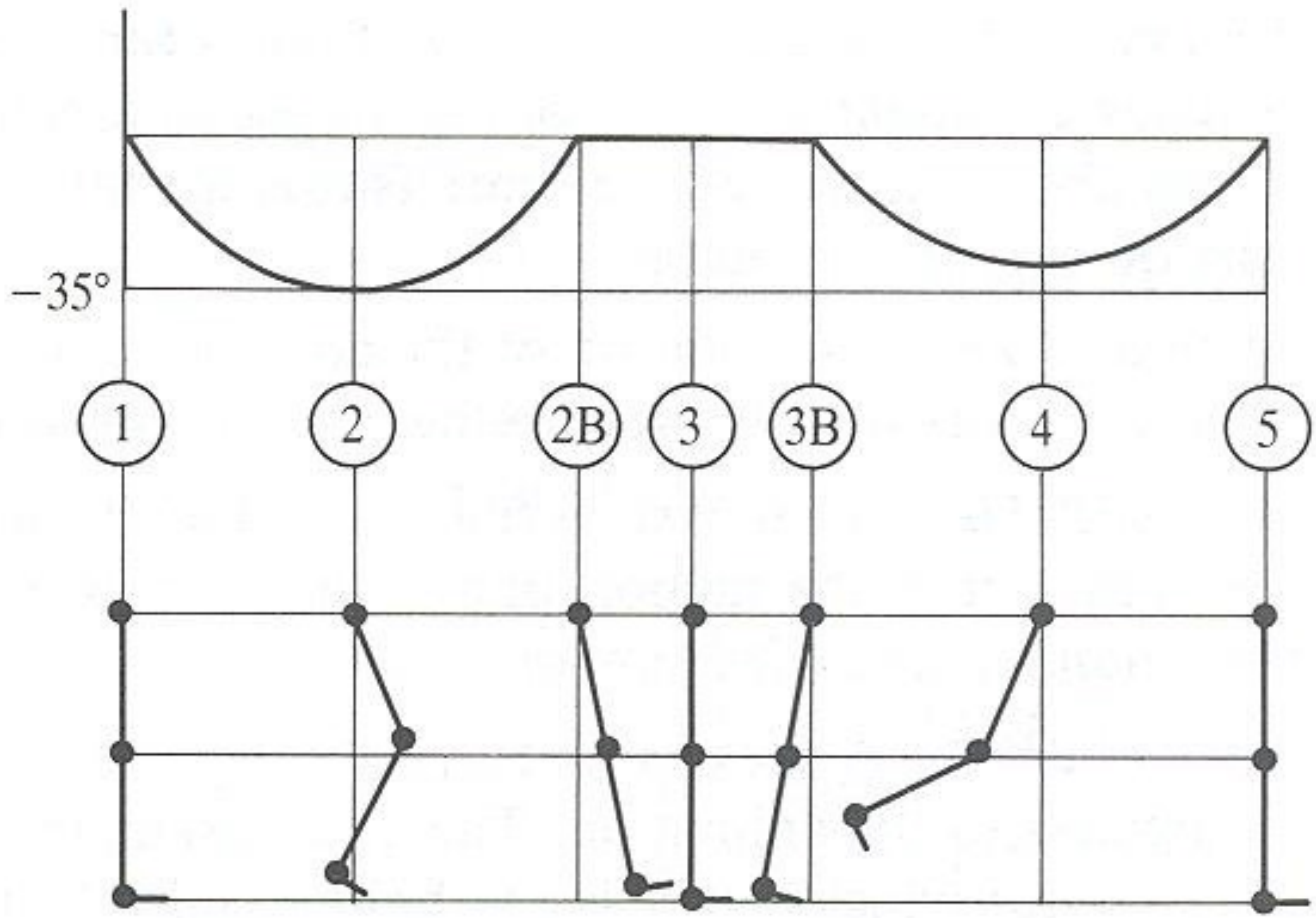
Hip joint angle



Watt & Watt

Example: walk cycle

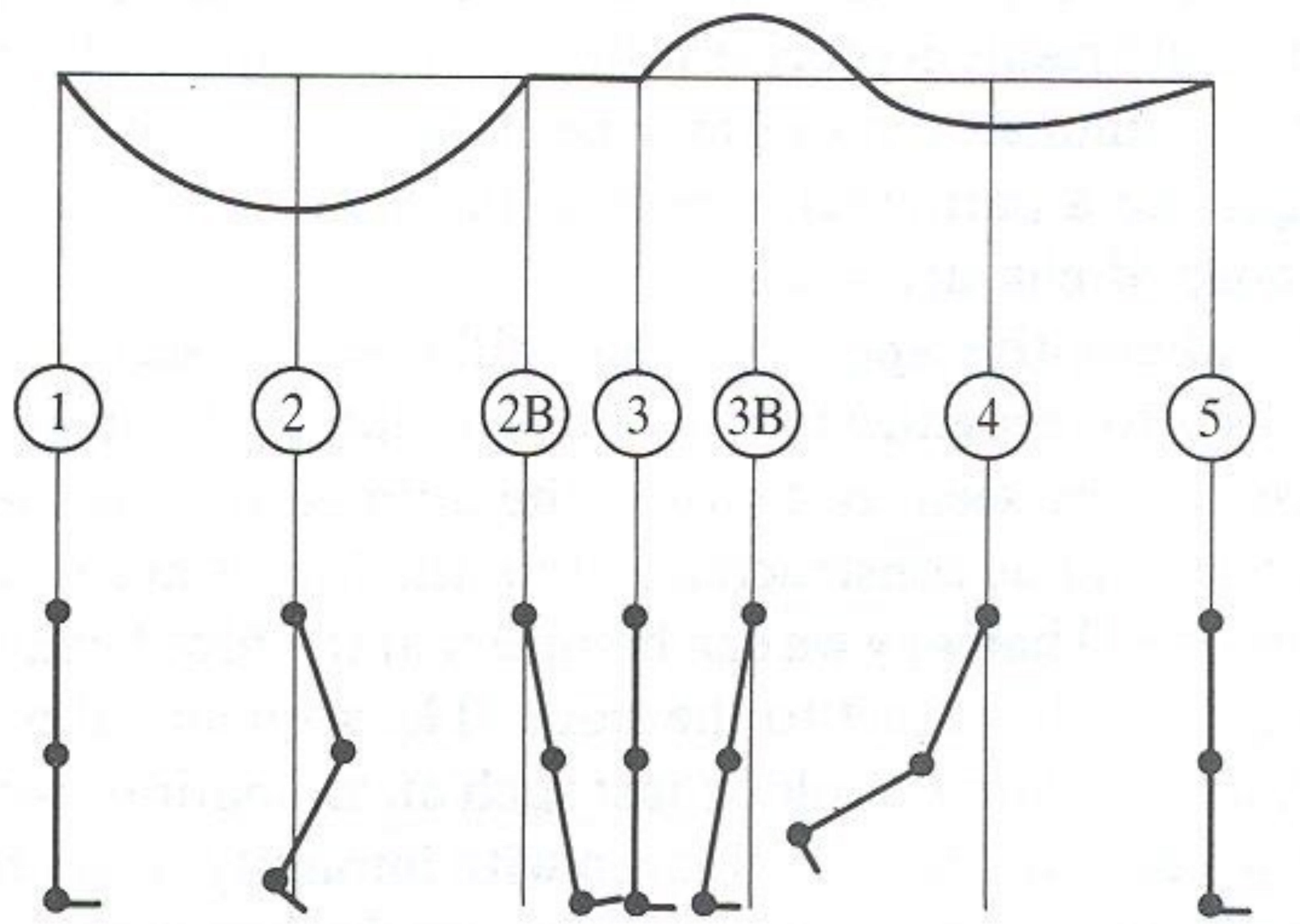
Knee joint angle



Watt & Watt

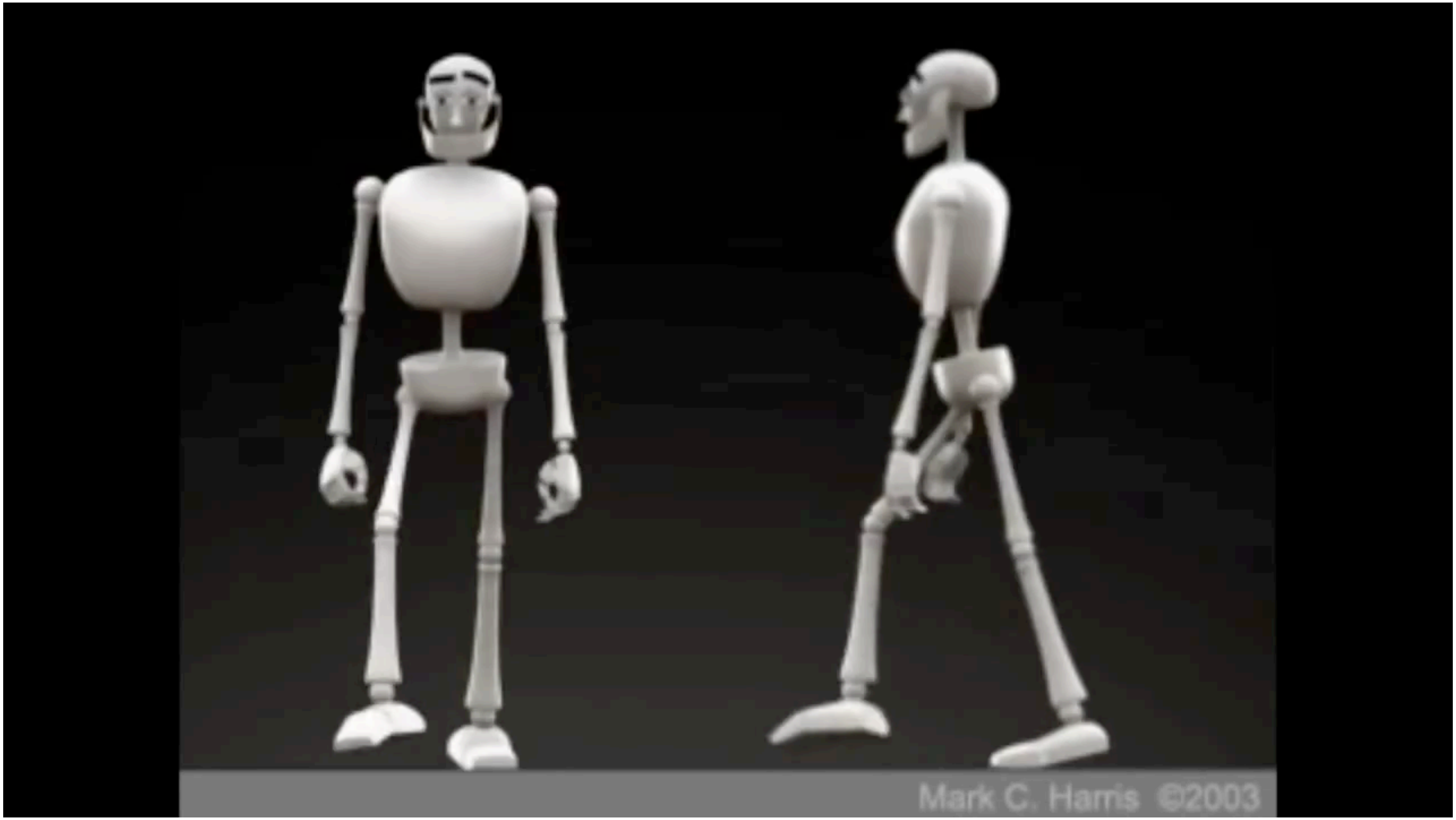
Example: walk cycle

Ankle joint angle

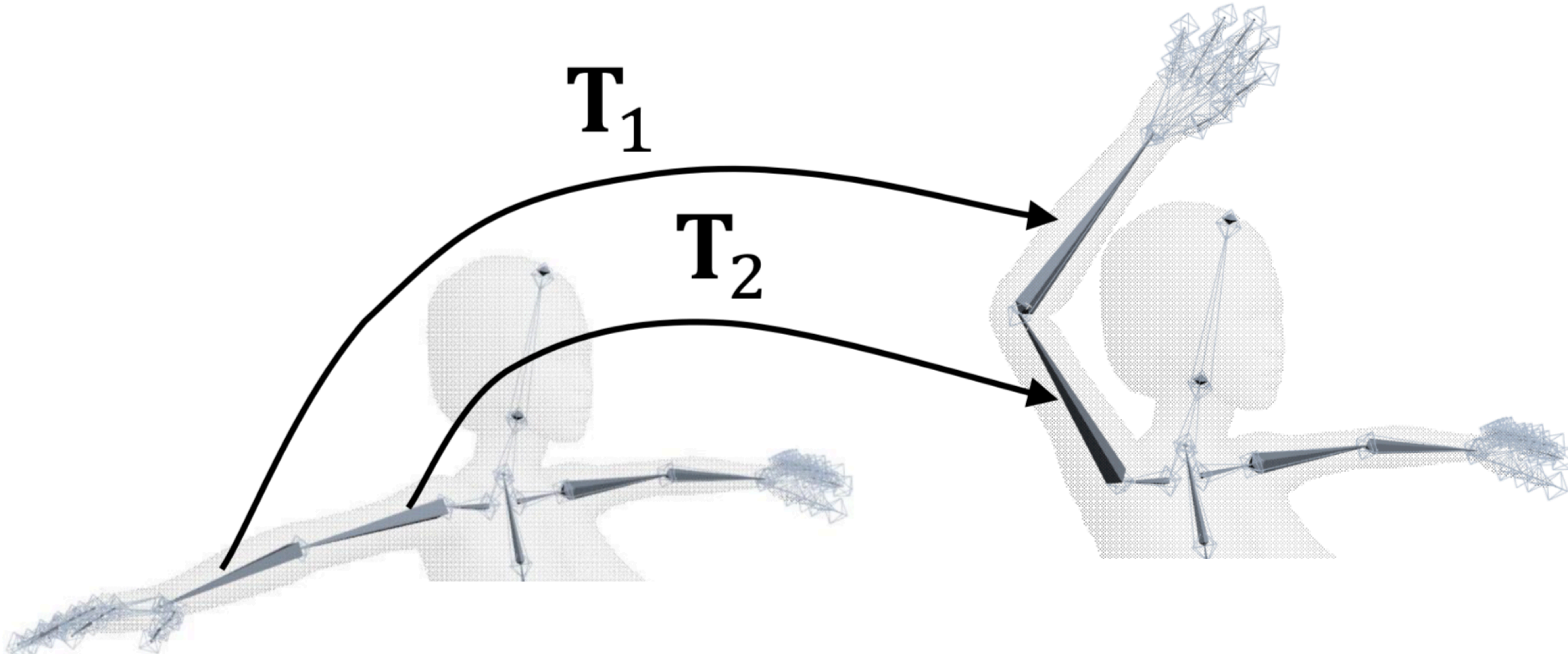


Watt & Watt

Example: walk cycle



Skinning: how to transform mesh vertices according to skeleton transforms



Skeleton joint transforms: T_1, T_2

Image credit: Ladislav Kavan

Vertex *i* on mesh

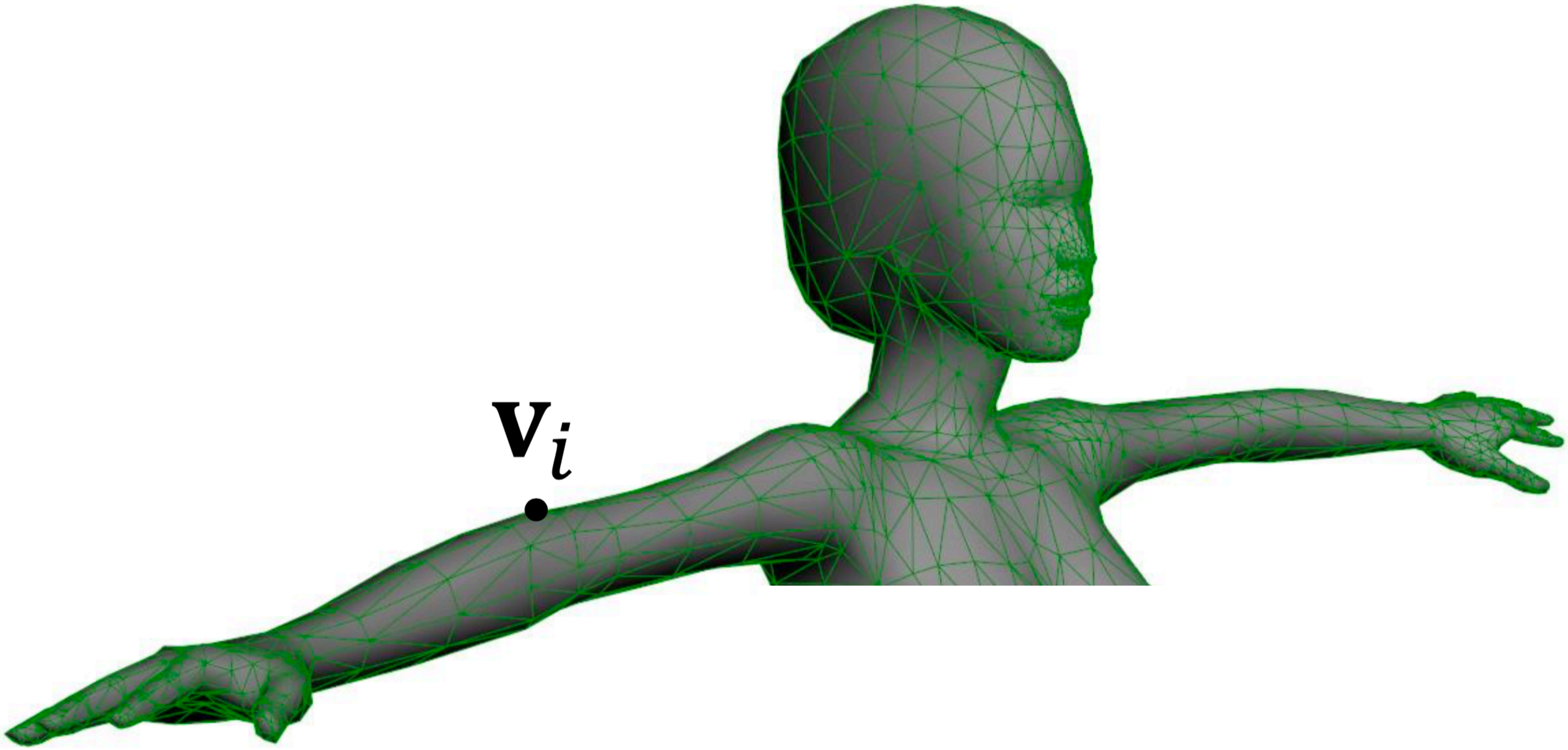
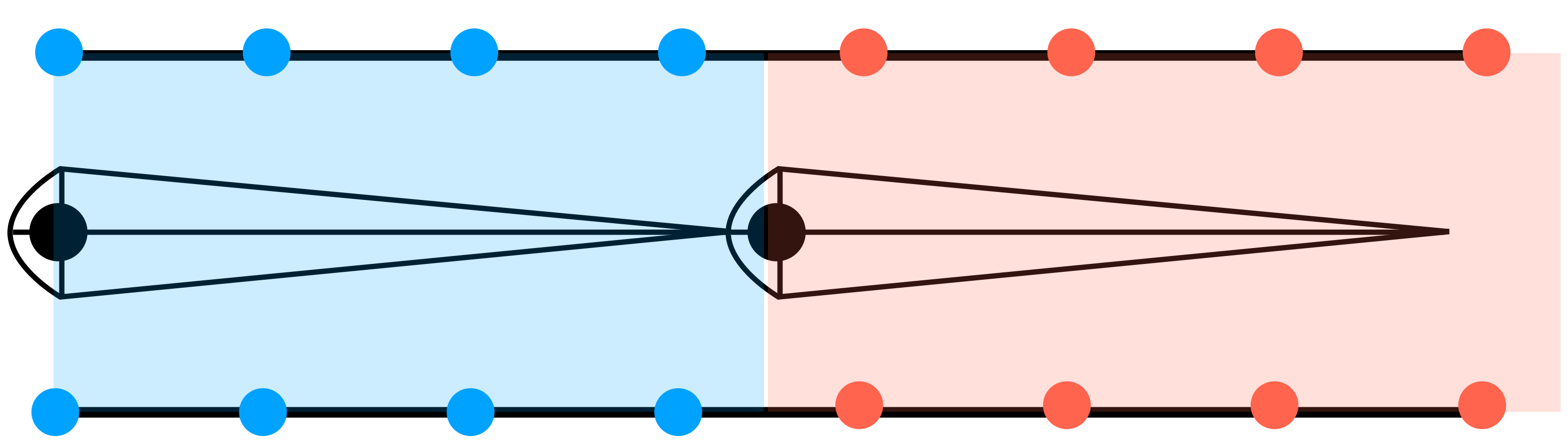


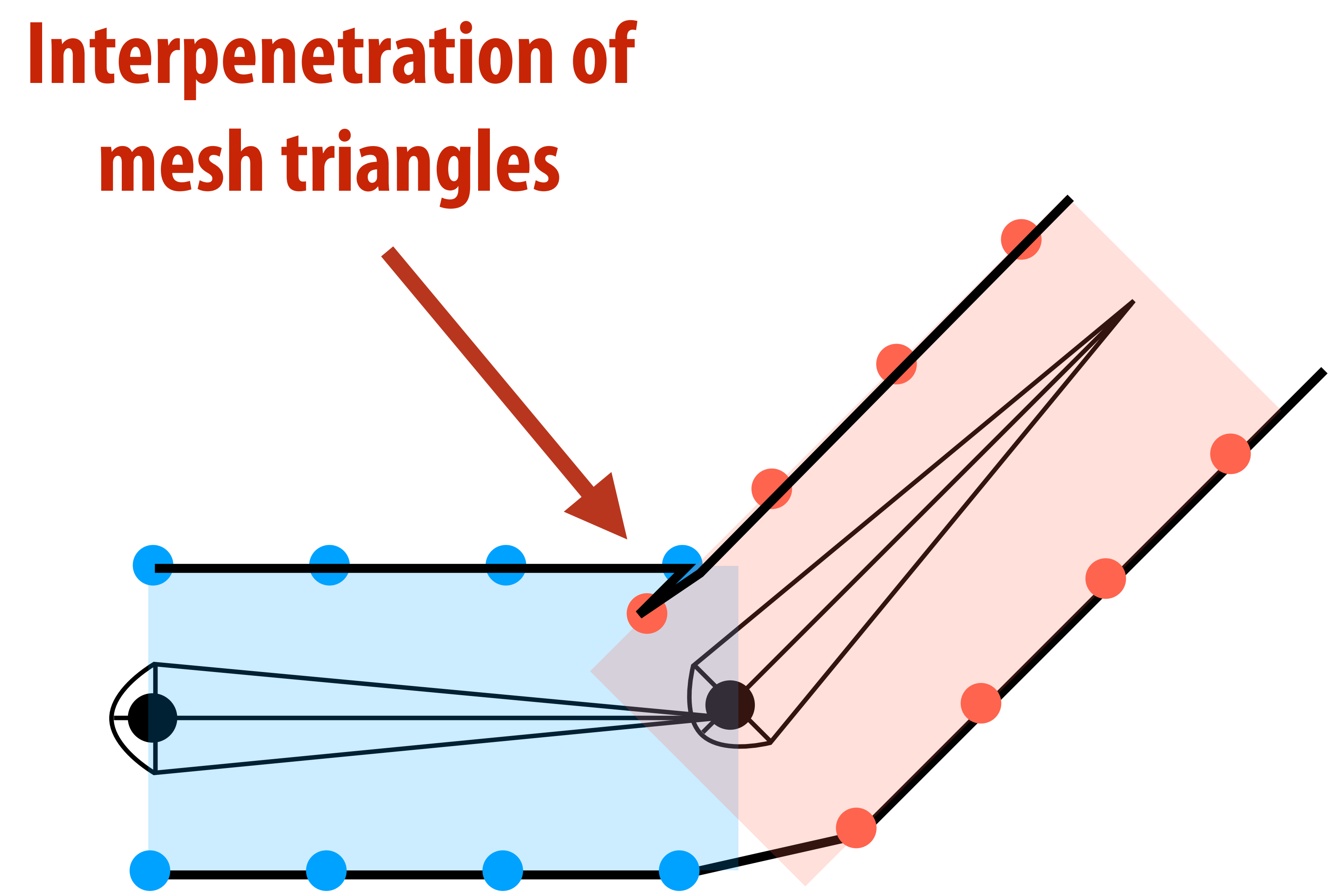
Image credit: Ladislav Kavan

Rigid body skinning

- One idea: transform mesh vertices according to transform for nearby skeleton joint



Original pose

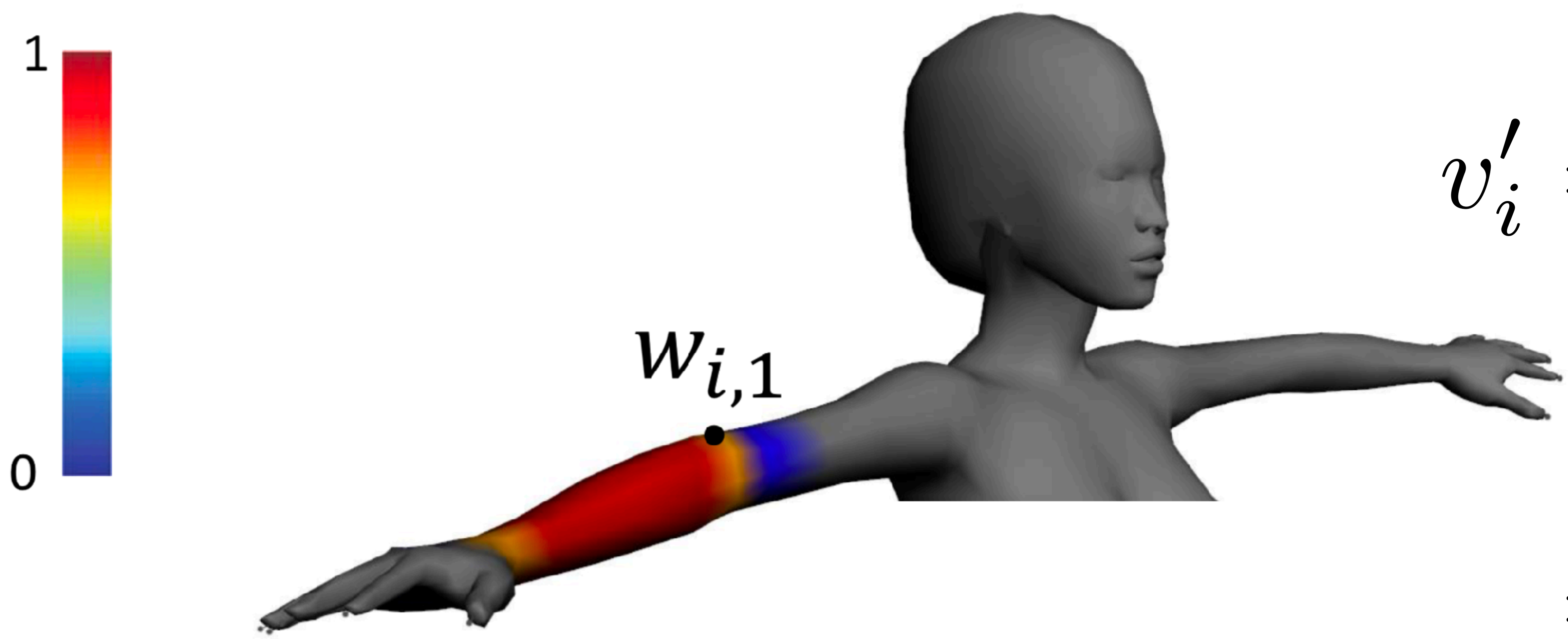


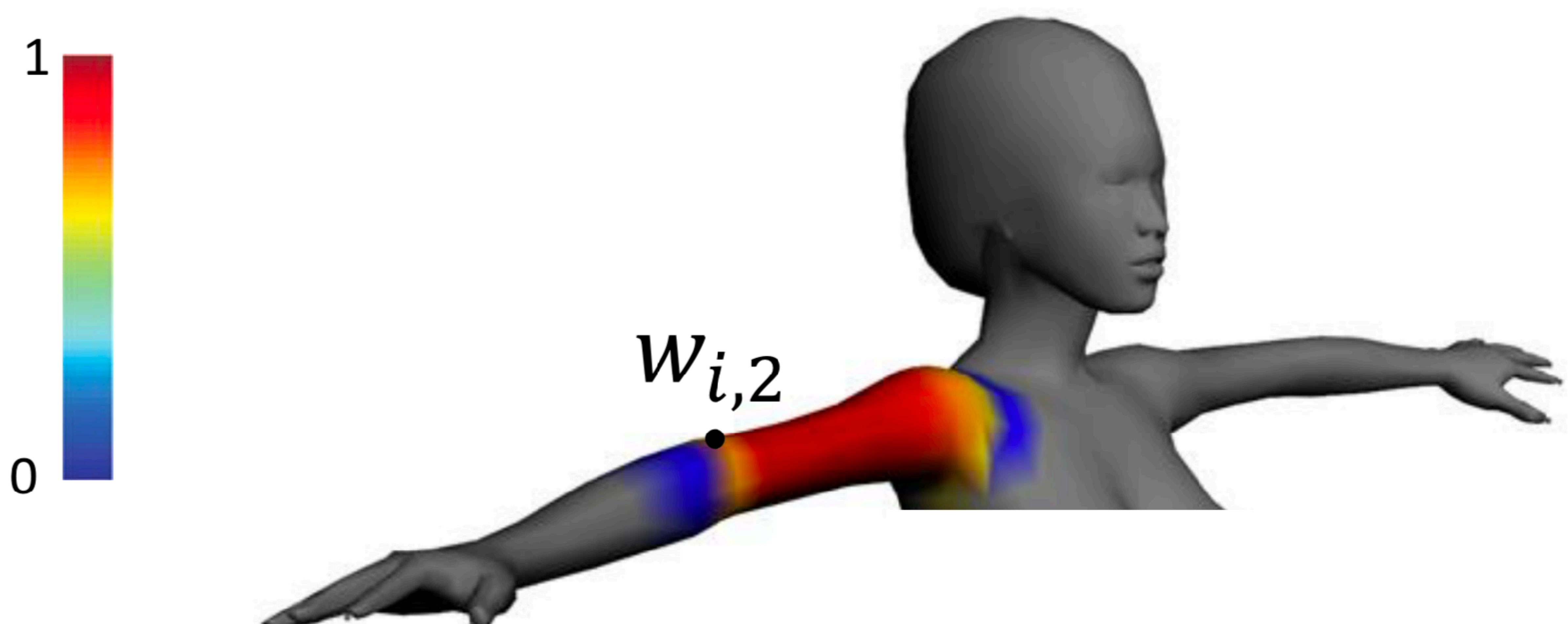
Vertices transforms according to corresponding joint transform (notice surface interpenetration)

Linear blend skinning *

Mesh vertices transformed by *linear combination* of nearby joint transforms

Very common technique for character animation in games





$$v_i' = \sum_j^N w_{ij} T_j v_i$$

$$= \left(\sum_j^N w_{ij} T_j \right) v_i$$

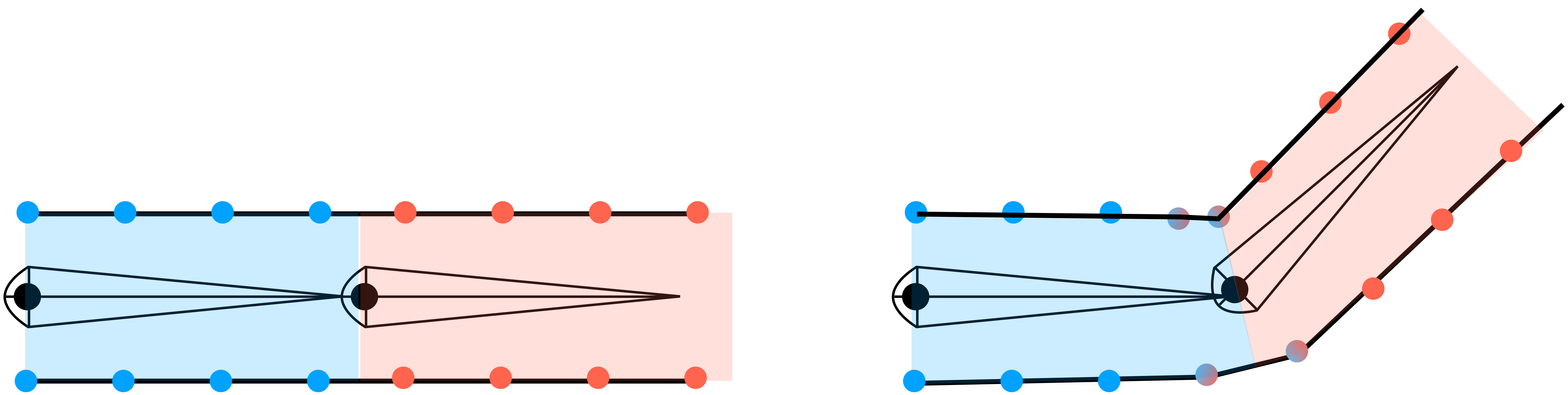
v_i = rest object space vertex position
T_j = transform for bone j
w_{ij} = weight of bone j on vertex i
N = number of bones

Image credit: Ladislav Kavan

*** Also called "matrix palette skinning" or "skeletal subspace deformation" (SSD)**

Linear blend skinning

- Transform mesh vertices according to linear combination of transforms for nearby skeleton joint

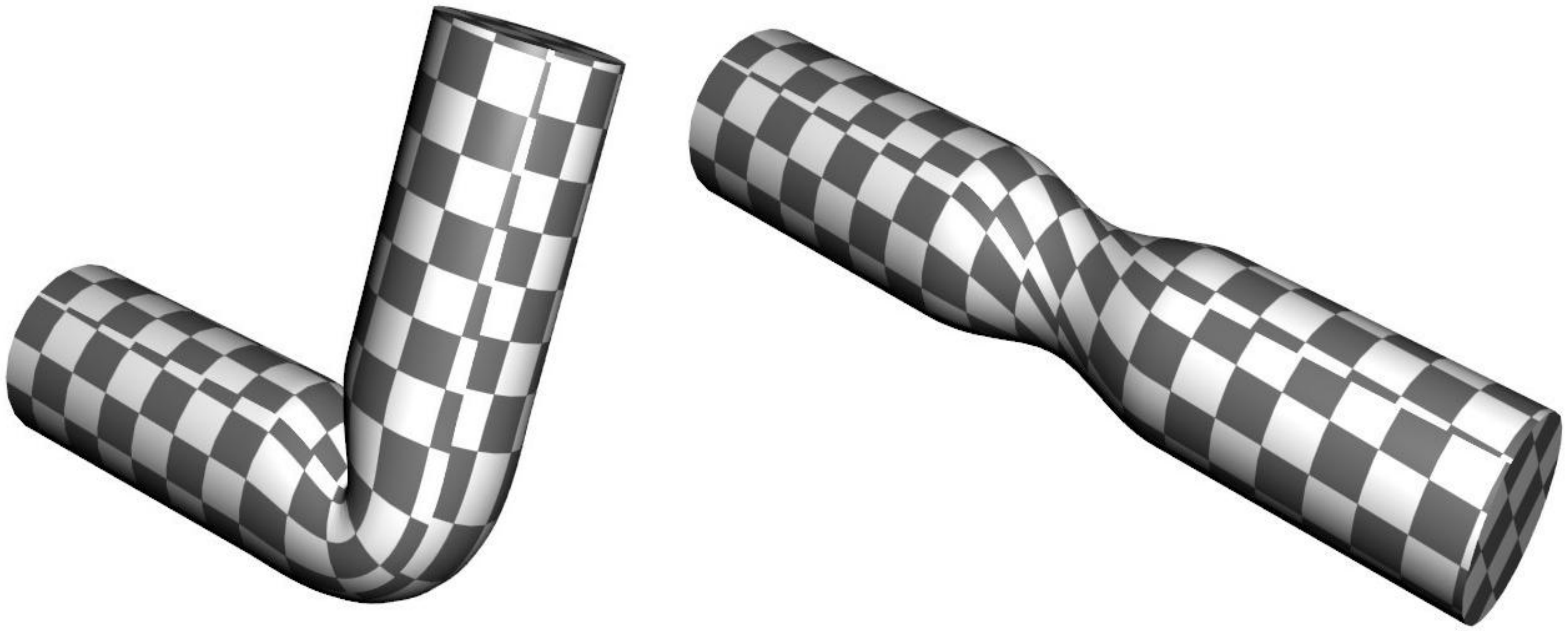


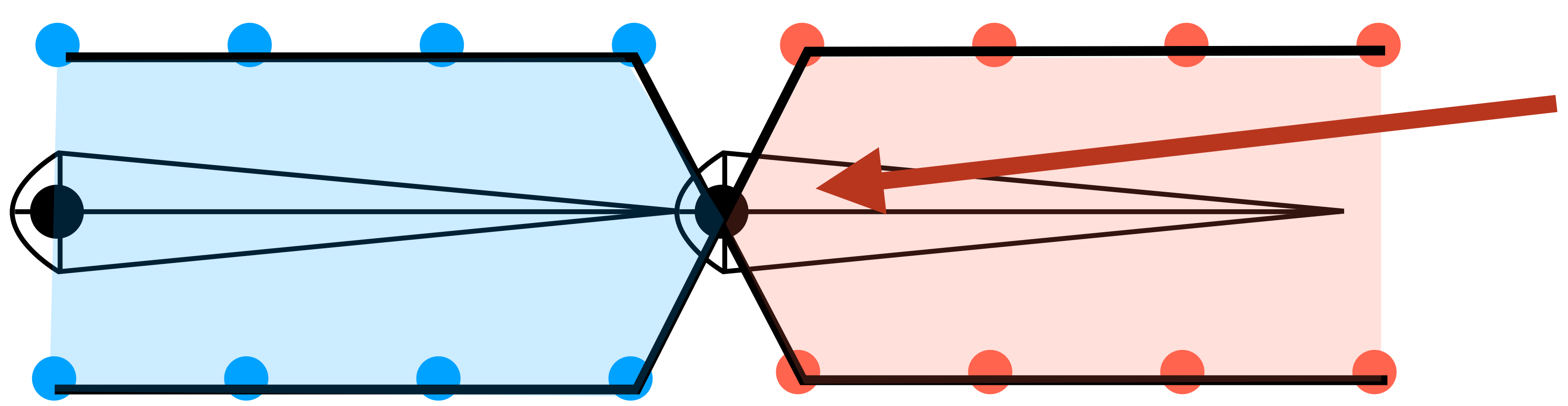
Original pose

After transform

Shortcomings of linear blend skinning

- Loss of volume under large transformations





Many more advanced solutions in literature: dual-quaternion skinning, joint-based deformers, etc.

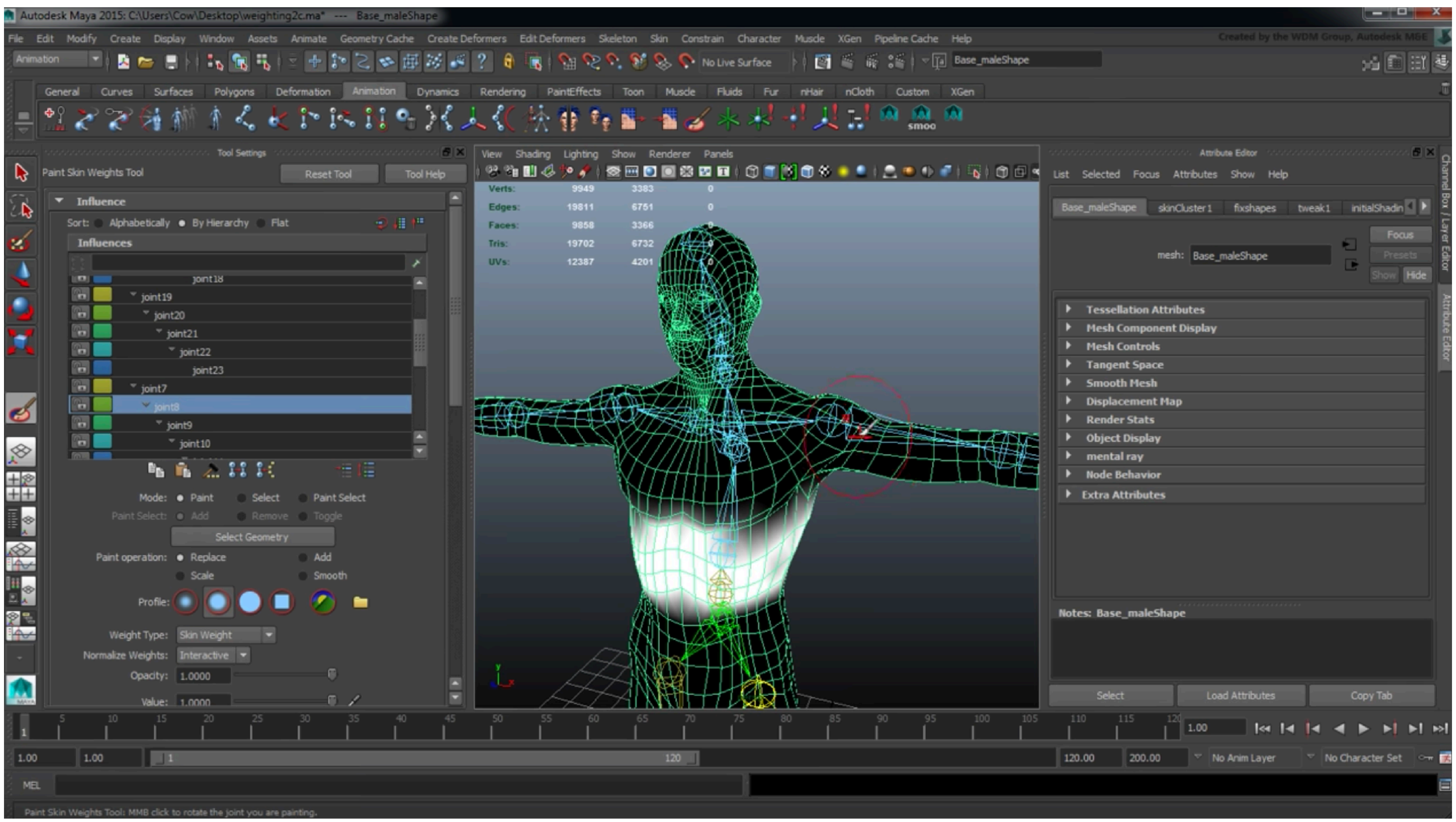
Image credit: Jacka et al.

Skinning example

Courtesy Matthew Lailler via Keenan Crane via Ren Ng

Rigging

- "Rigging" is the process of attaching a set of animation controls to a mesh
  - In the case of linear blend skinning: it is attaching a skeleton to the mesh (e.g., setting per vertex blend weights)



Example: artist painting vertex blend weights directly on mesh in Maya

Different ways to obtain joint angles

- **Hand animate values (as discussed above)**
- **Measure angles from a performance via motion capture**
- **Solve for angles based on higher-level goal (optimization)**

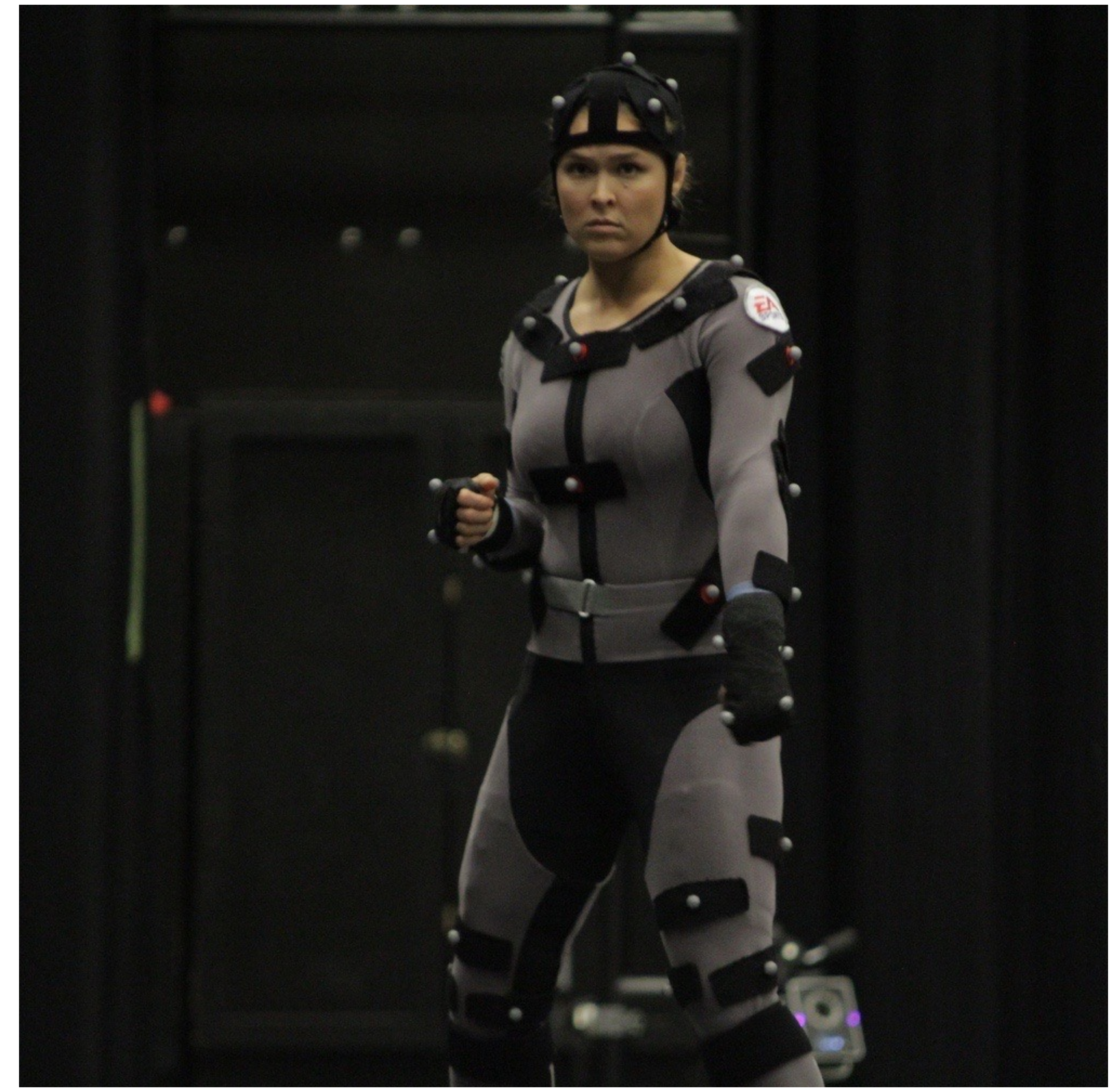
Motion Capture

Motion capture

- Data-driven approach to creating animation sequences
  - Record real-world performances (e.g. person executing an activity)
  - Extract pose as a function of time from the data collected

Motion capture room for ShaqFu

Optical motion capture

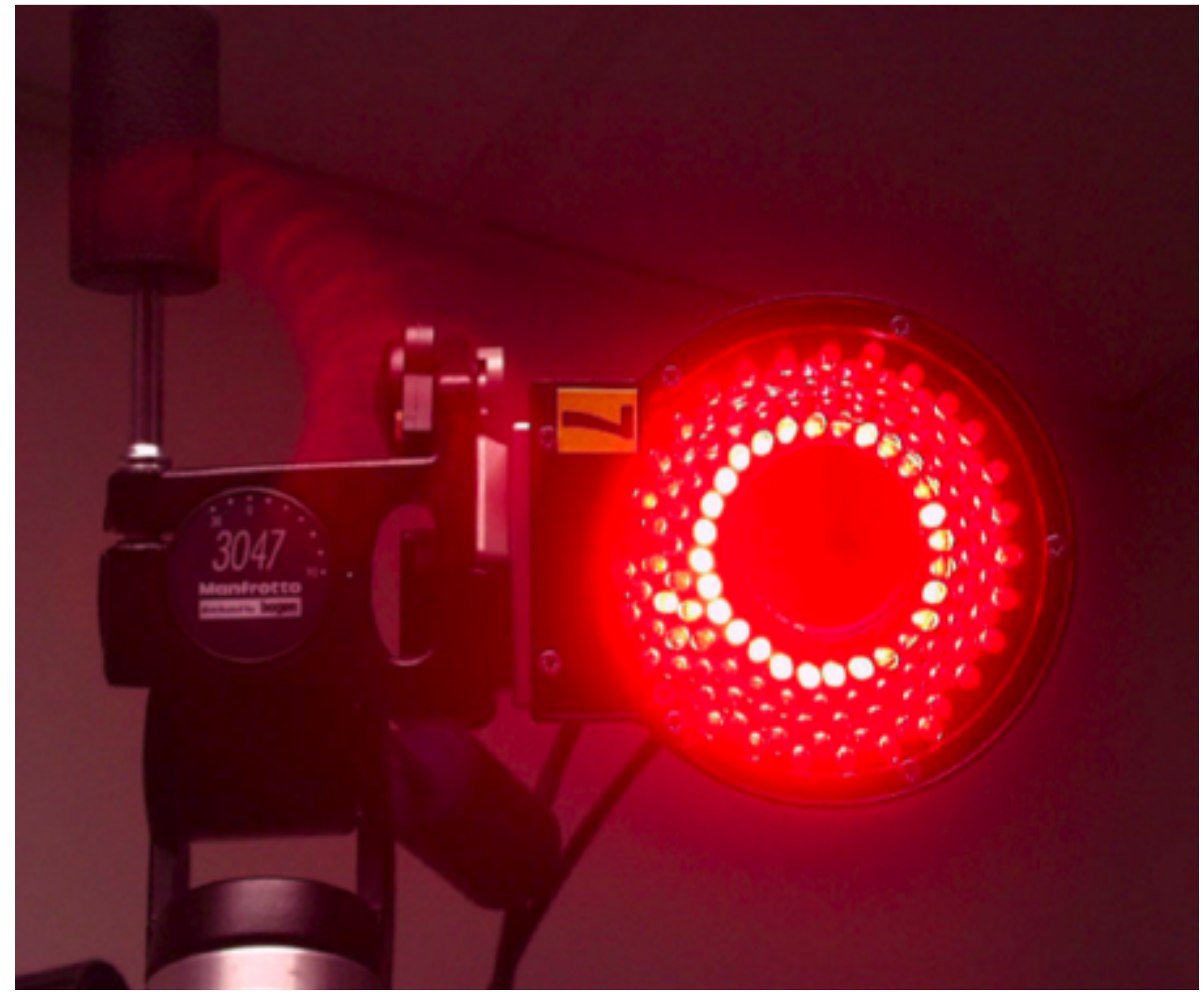
 

Source: http://fightland.vice.com/blog/ronda-rousey-20-the-queen-of-all-media

Ronda Rousey in Electronic Arts' motion capture studio

Optical motion capture

Retroreflective markers attached to subject



IR illumination and cameras

- Affix markers to joints of subject
- Compute 3D positions by triangulation from multiple cameras
- 8+ cameras, 240 Hz, occlusions are difficult

Slide credit: Steve Marschner

Motion capture pros and cons

- **Strengths**
  - Can capture large amounts of real data quickly
  - Realism can be high
- **Weaknesses**
  - Complex and costly set-ups (but progress in computer vision may be changing this)
  - Captured animation may not meet artistic needs, requiring alterations

Challenges of facial animation

- "Uncanny valley"
  - In robotics and graphics
  - As artificial character appearance approaches human realism, our emotional response goes negative, until it achieves a sufficiently convincing level of realism in expression

Cartoon.
Brave, Pixar

Semi-realistic. Polar Express, Warner Bros

Challenges of facial motion capture

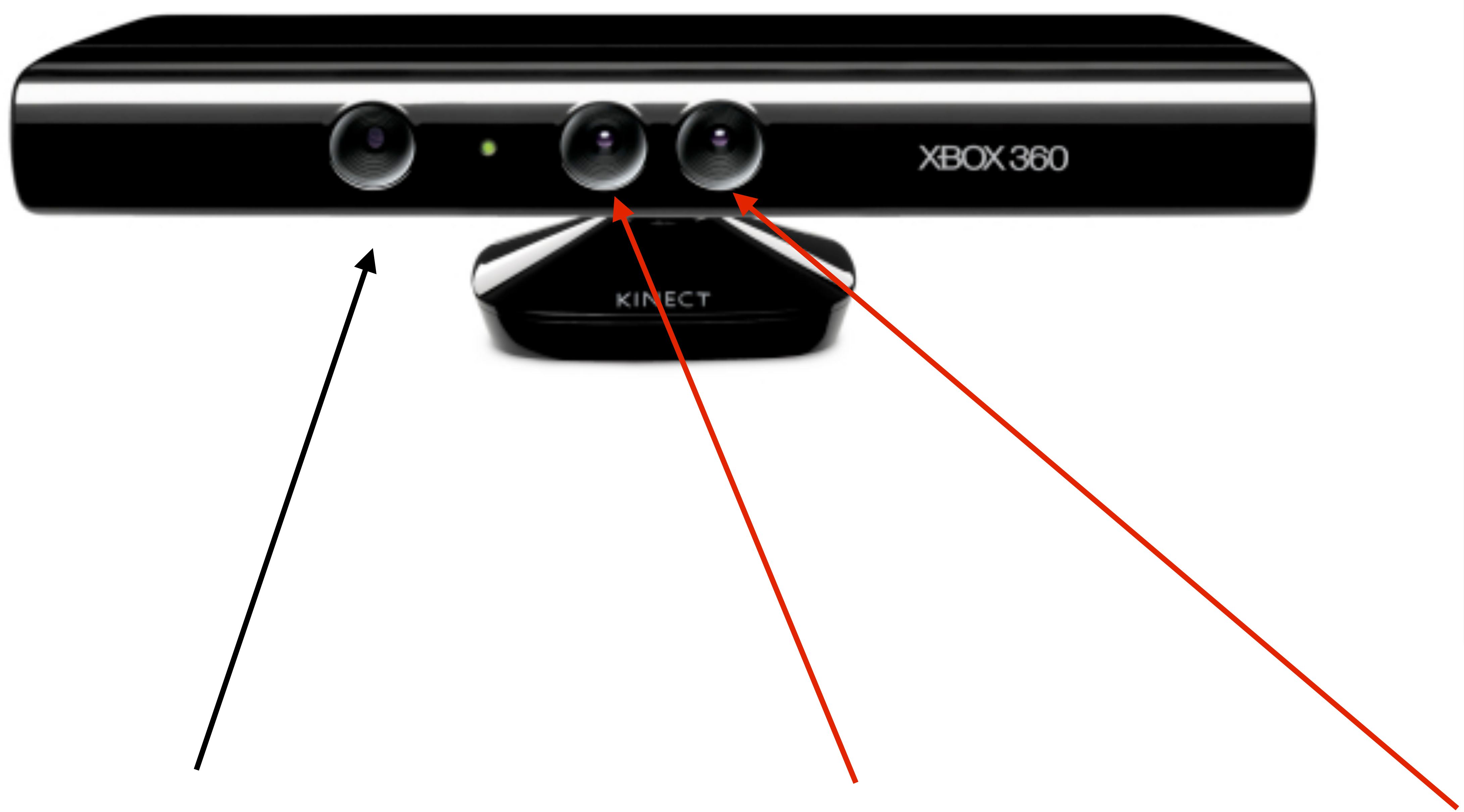
Final Fantasy Spirits Within

Facial motion capture

Discovery, “Avatar: Motion Capture Mirrors Emotions”, https://youtu.be/1wK1Ixr-UmM

Aside: lower-cost forms of capture

Microsoft XBox 360 Kinect



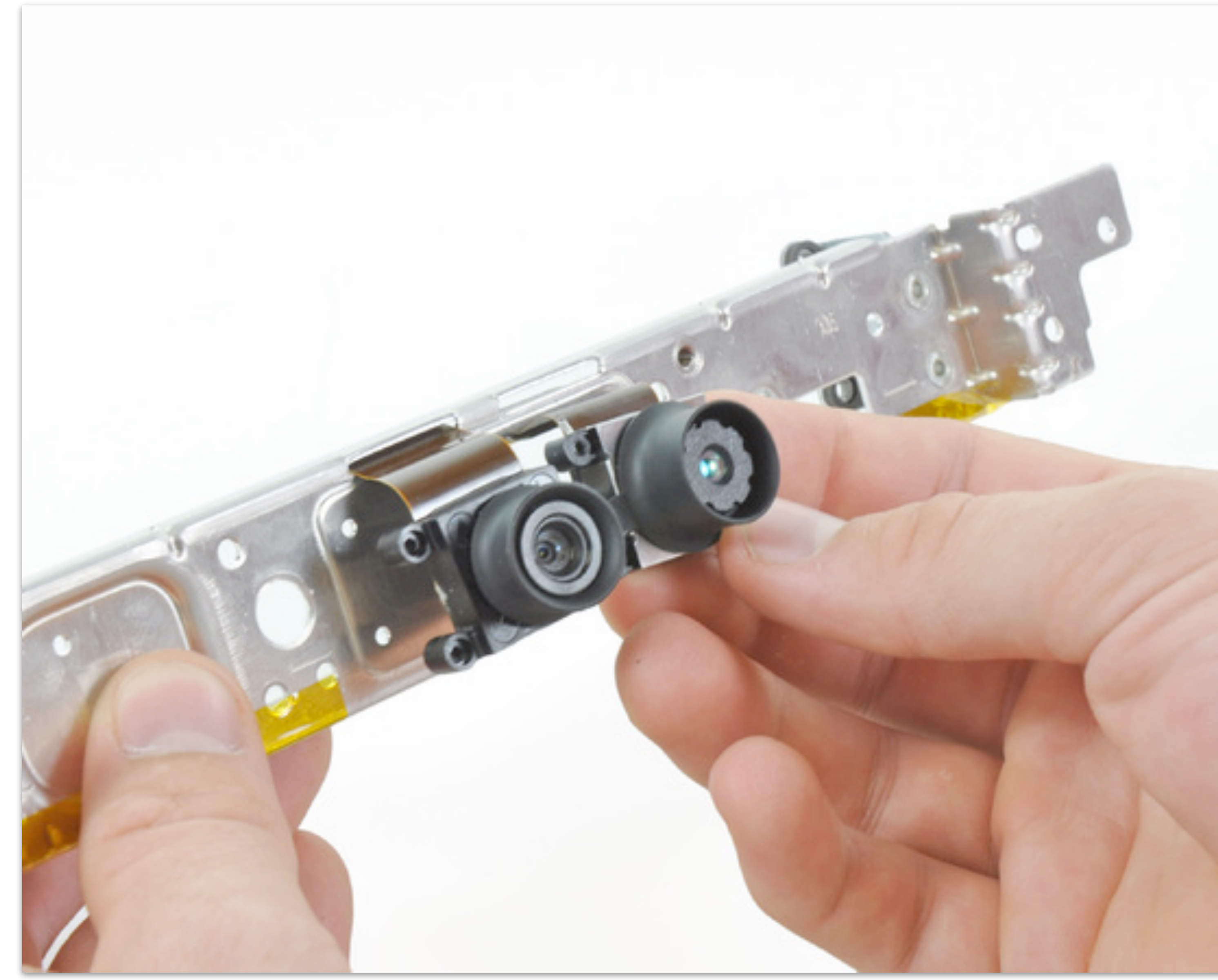


Image credit: iFixIt

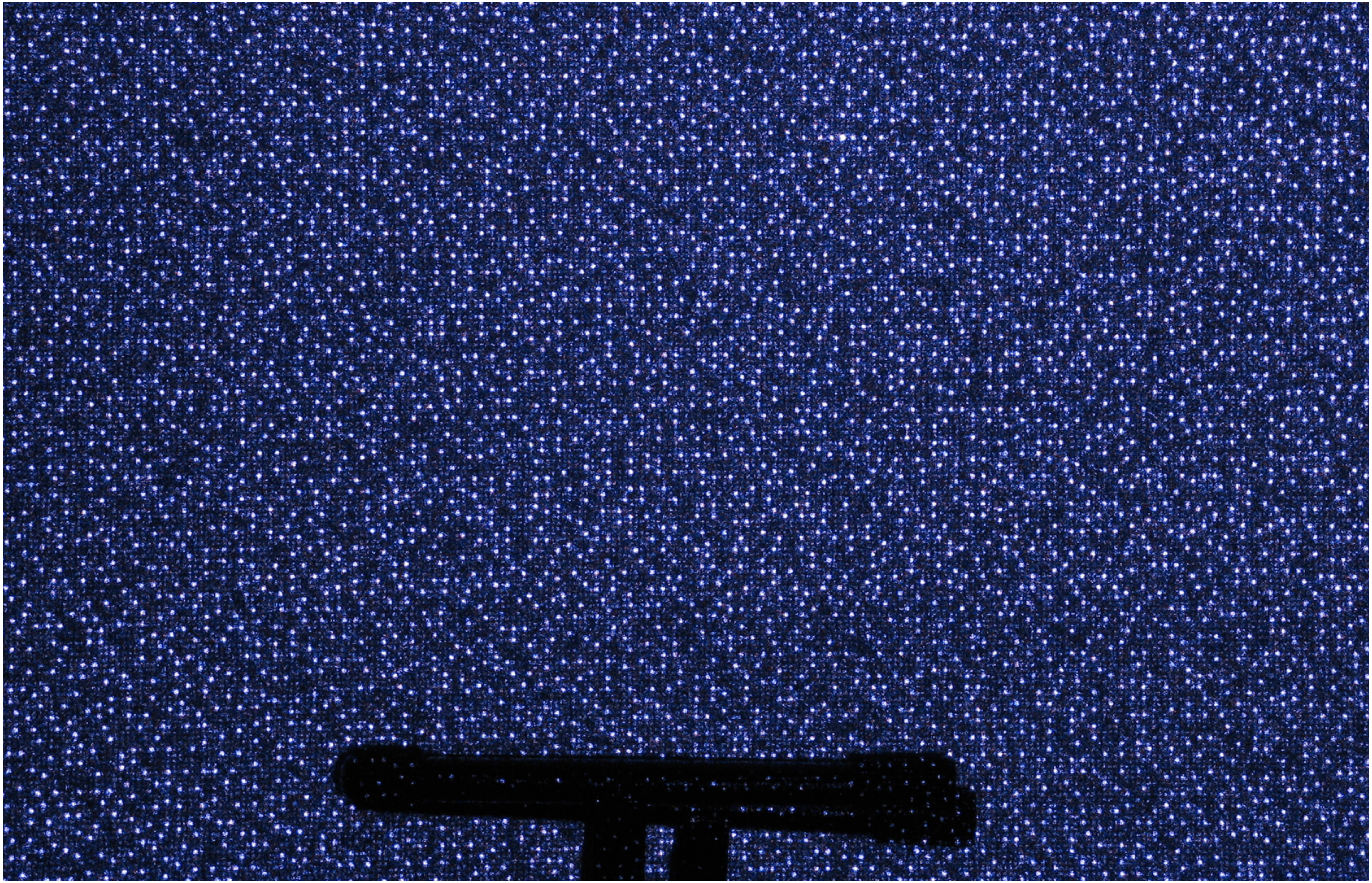
Illuminant
(Infrared Laser + diffuser)

RGB CMOS Sensor
640x480 (w/ Bayer mosaic)

Monochrome Infrared
CMOS Sensor
(Aptina MT9M001)
1280x1024 **

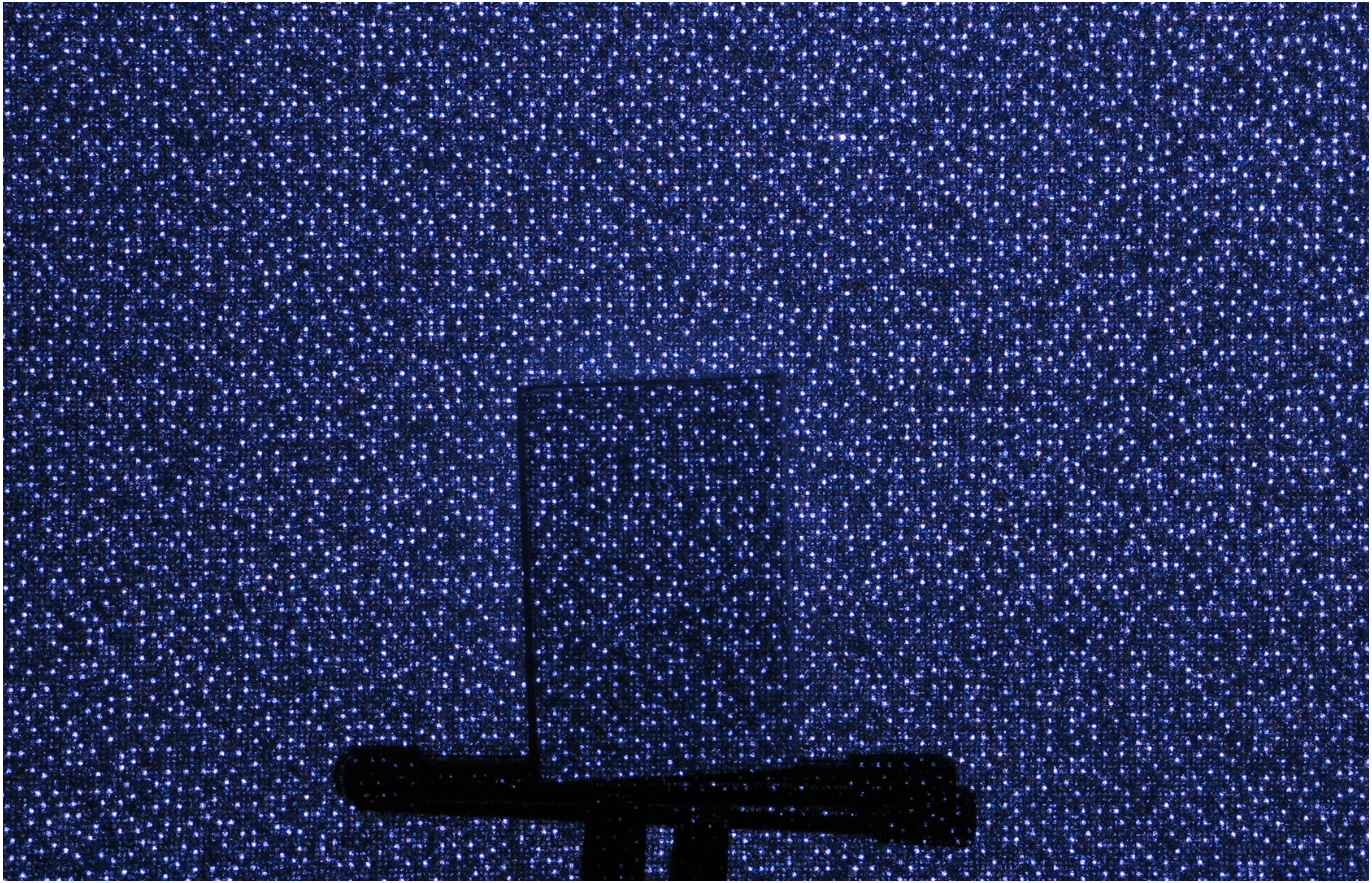
** Kinect returns 640x480 disparity image, suspect sensor is configured for 2x2 pixel binning down to 640x512, then crop

Infrared image of Kinect illuminant output



Credit: www.futurepicture.org

Infrared image of Kinect illuminant output



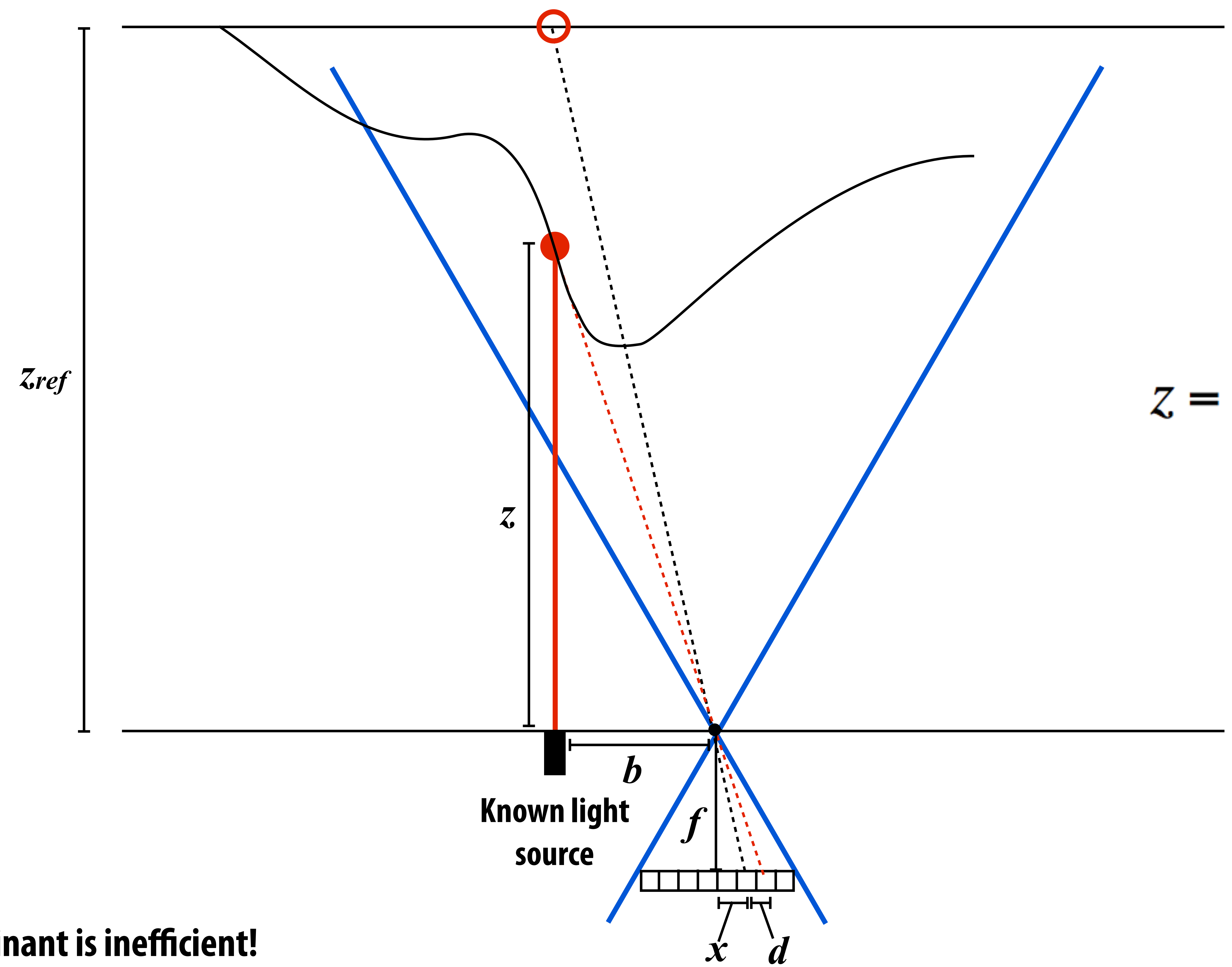
Credit: www.futurepicture.org

Depth from "disparity" using structured light

System: one light source emitting known beam + one camera measuring scene appearance

If the scene is at reference plane, image that will be recorded by camera is known

Movement of observed dot from from reference gives depth.



$$z = \frac{bf}{x+d}$$

Single spot illuminant is inefficient!

(Must "scan" scene to get depth, so high latency to retrieve a single depth image. Hence the dot pattern on the Kinect)

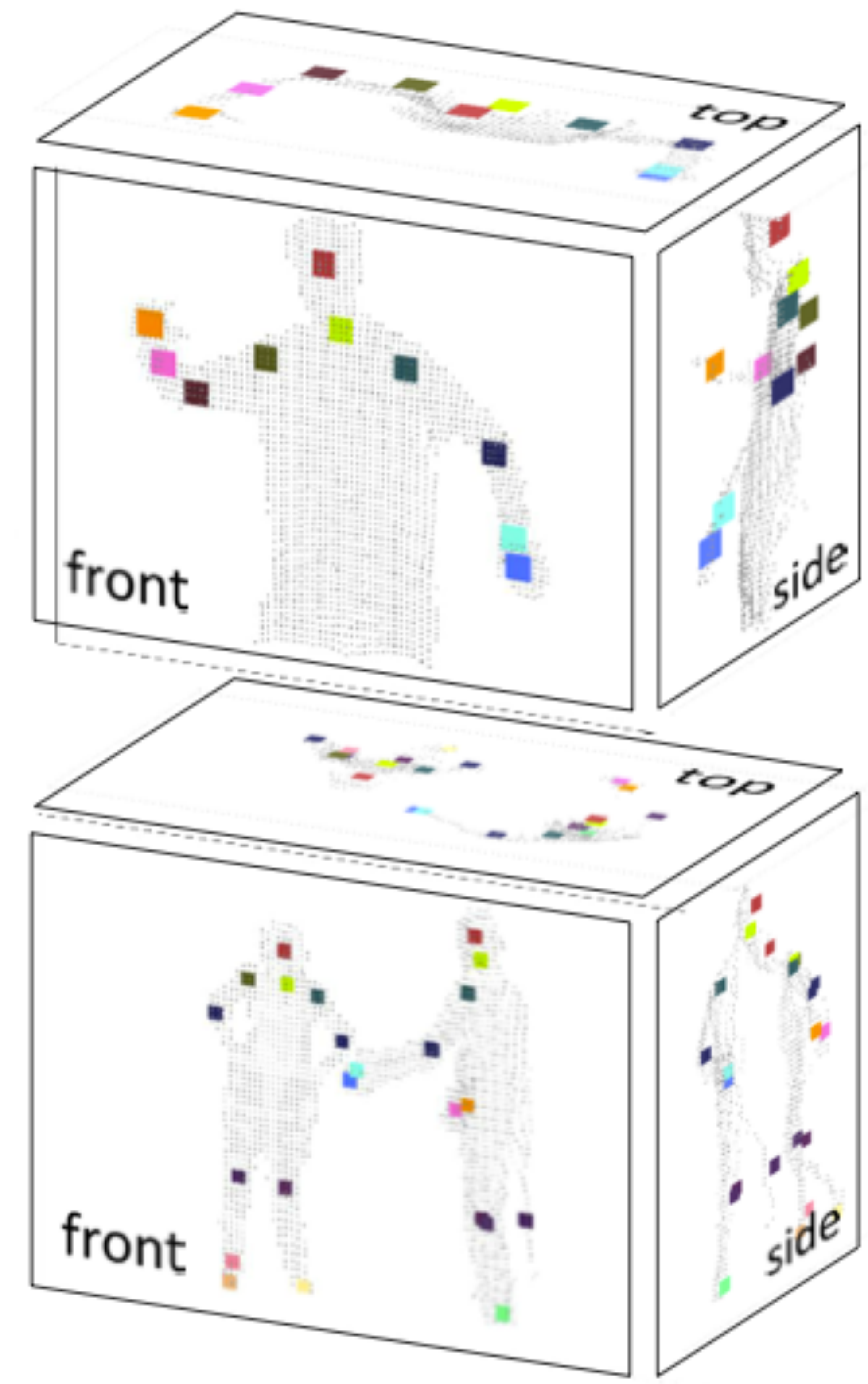
Extracting the player's 2D skeleton

[Shotton et al. 2011]

(enabling full-body game input)

Challenge: how to determine player's position and motion from (noisy) depth images... without consuming a large fraction of the XBox 360's compute capability?

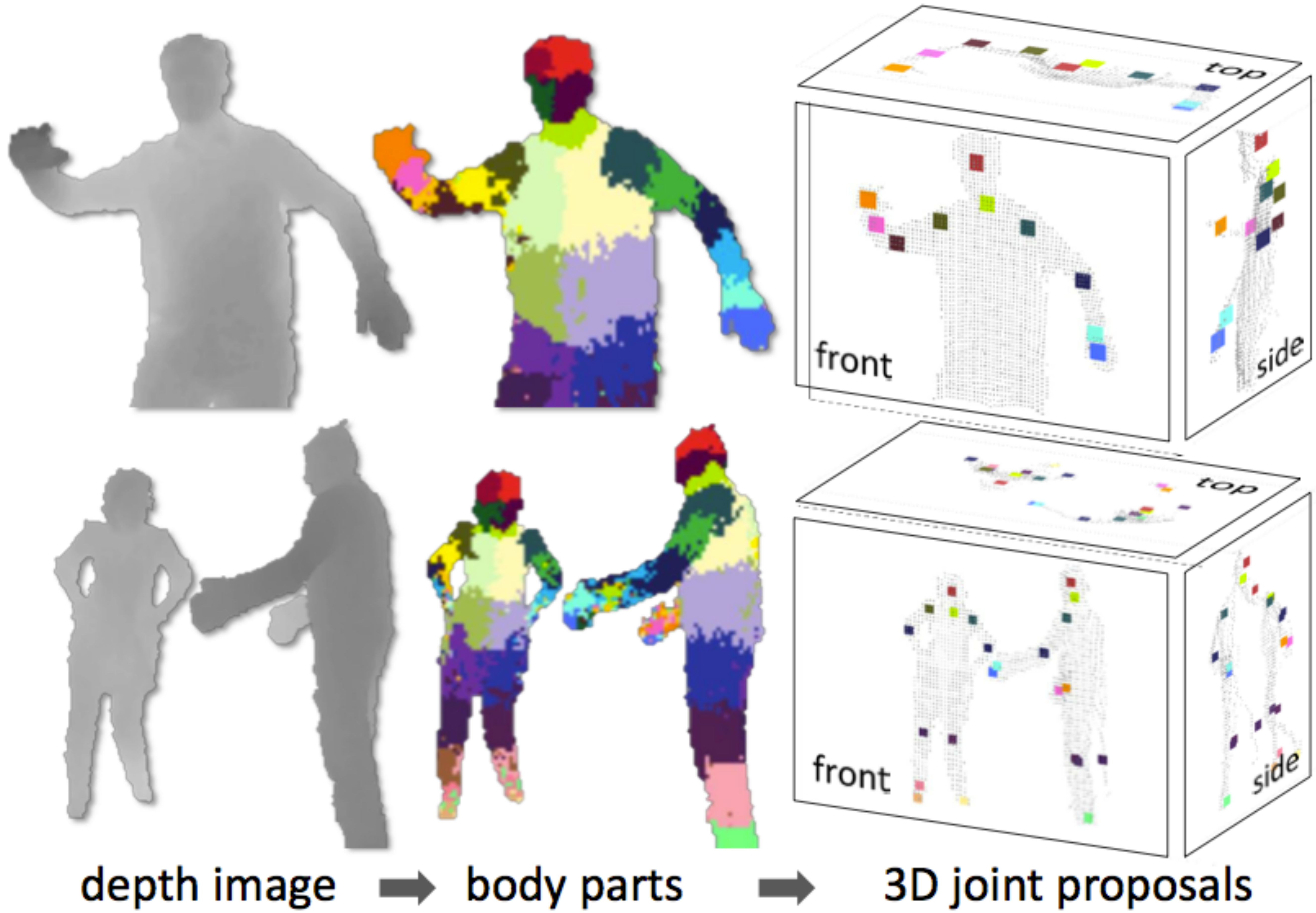
Depth Image



Character Joint Angles

Key idea: classify pixels into body regions

[Shotton et al. 2011]



Shotton et al. represents body with 31 regions

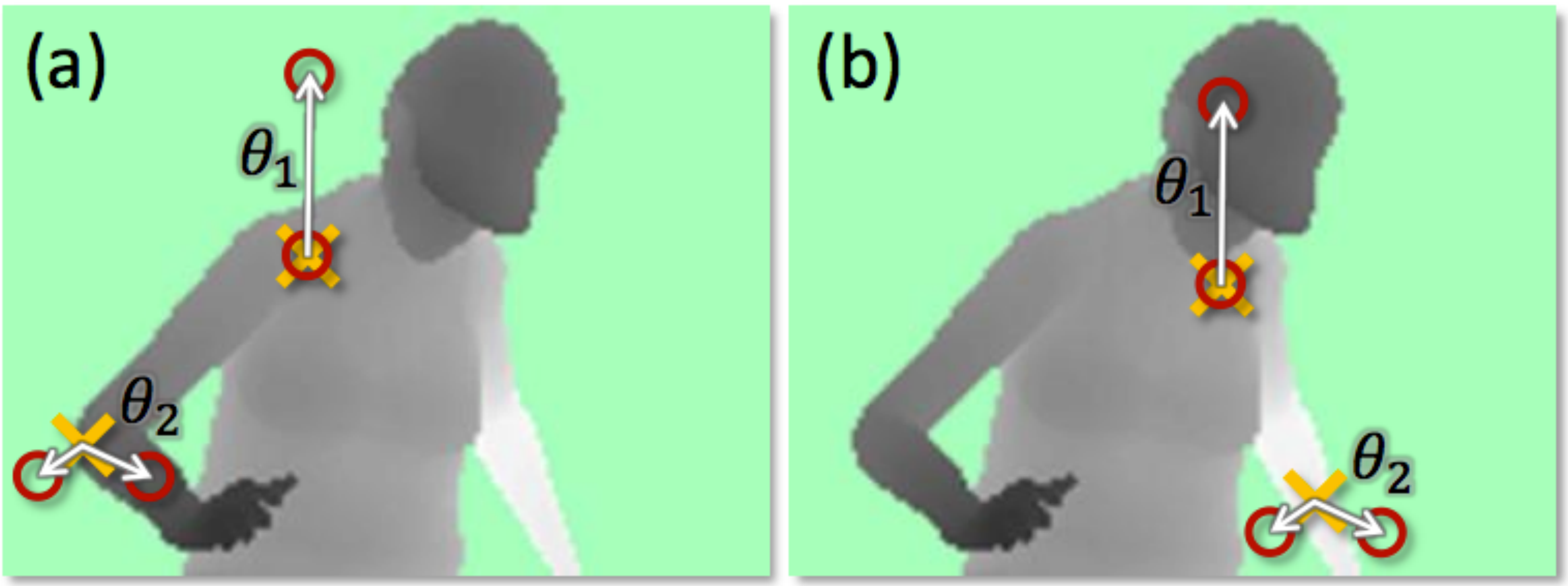
Pixel classification

[Shotton et al. 2011]

For each pixel: compute features from depth image

$$f_\theta(I,\mathrm{X}) = d_I\left(\mathrm{X} + \frac{u}{d_I(\mathrm{X})}\right) + d_I\left(\mathrm{X} + \frac{v}{d_I(\mathrm{X})}\right)$$

Where $\theta = (u,v)$ and $d_I(X)$ is the depth image value at pixel X.



Two example depth features

Features are cheap to compute + can be computed for all pixels in parallel

- Features do not depend on velocities: only information from current frame

Classify pixels into body parts using randomized decision forest classifier

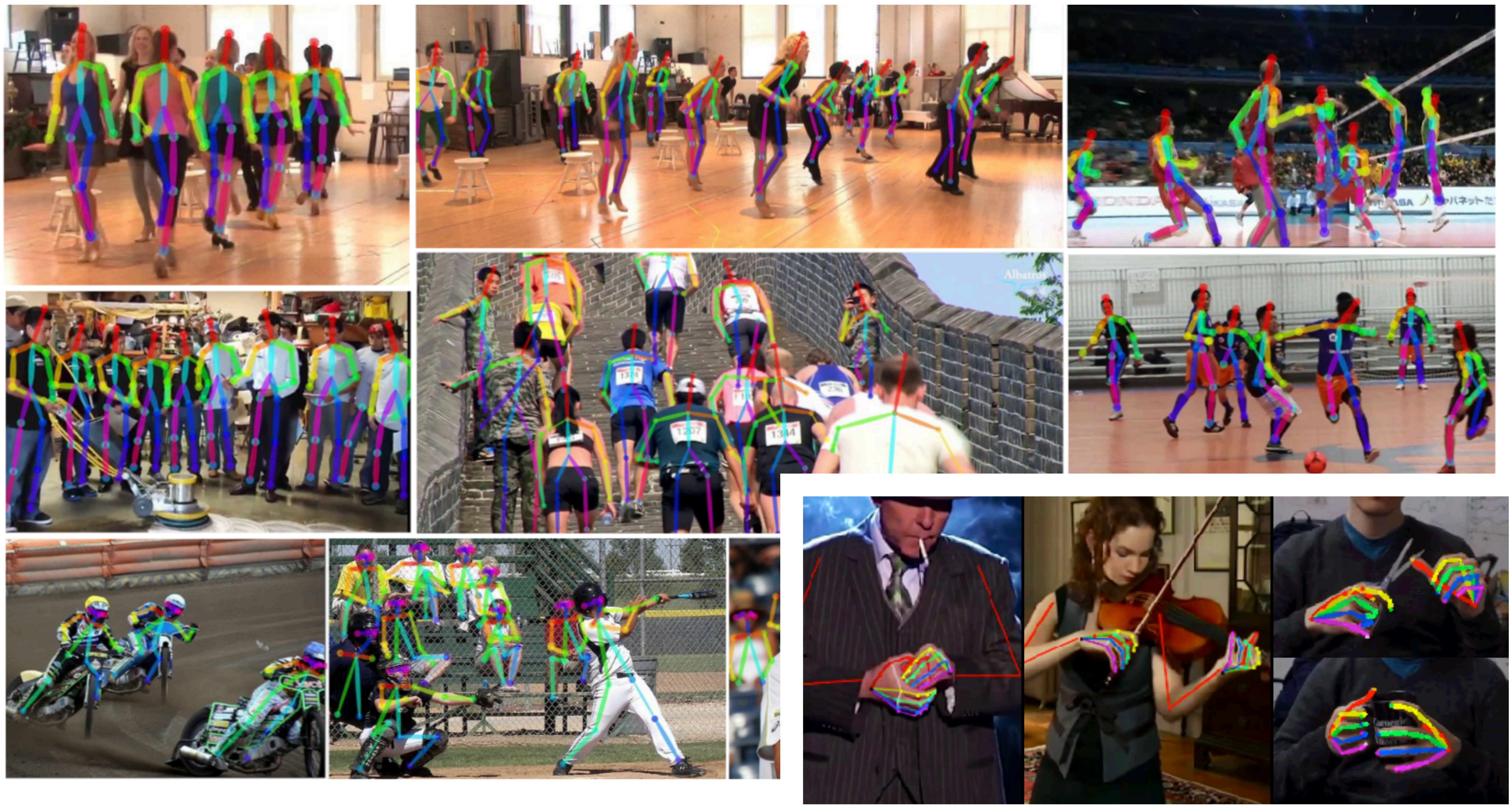
- Trained on 100K motion capture poses + database of rendered images as ground truth

Result of classification: $P(c|I,\mathbf{x})$ (probability pixel x in depth image I is body part c)

Per-pixel probabilities pooled to compute 3D spatial density function for each body part c (joint angles inferred from this density)

Modern computer vision approaches

- "OpenPose": 2D (but not 3D) skeleton from single RGB image



Hands/Fingers

Ongoing research to obtain high-quality 3D poses

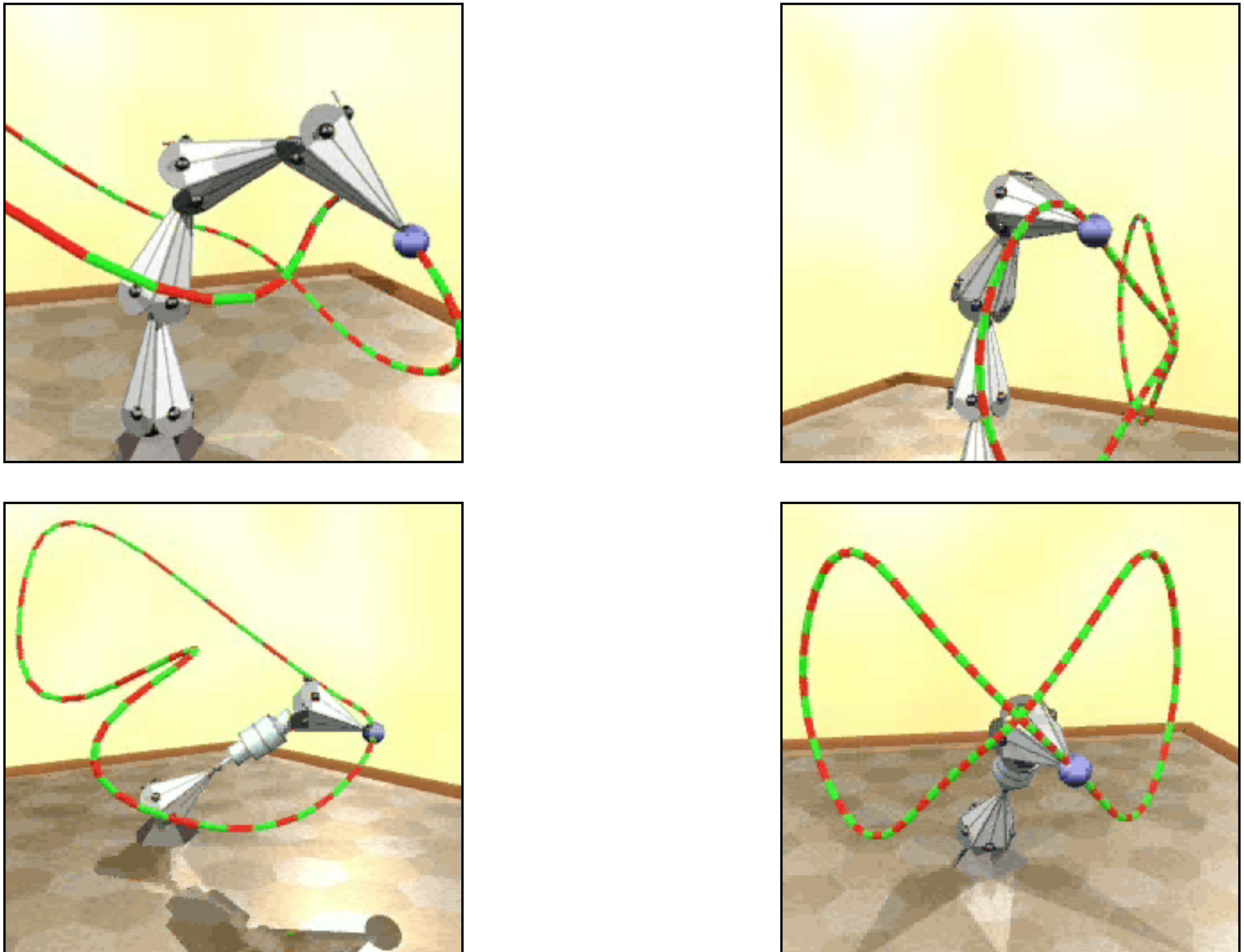
Image credits: Cao et al 2017, Simon et al 2017

So far… hand animate or record joint positions

Inverse Kinematics

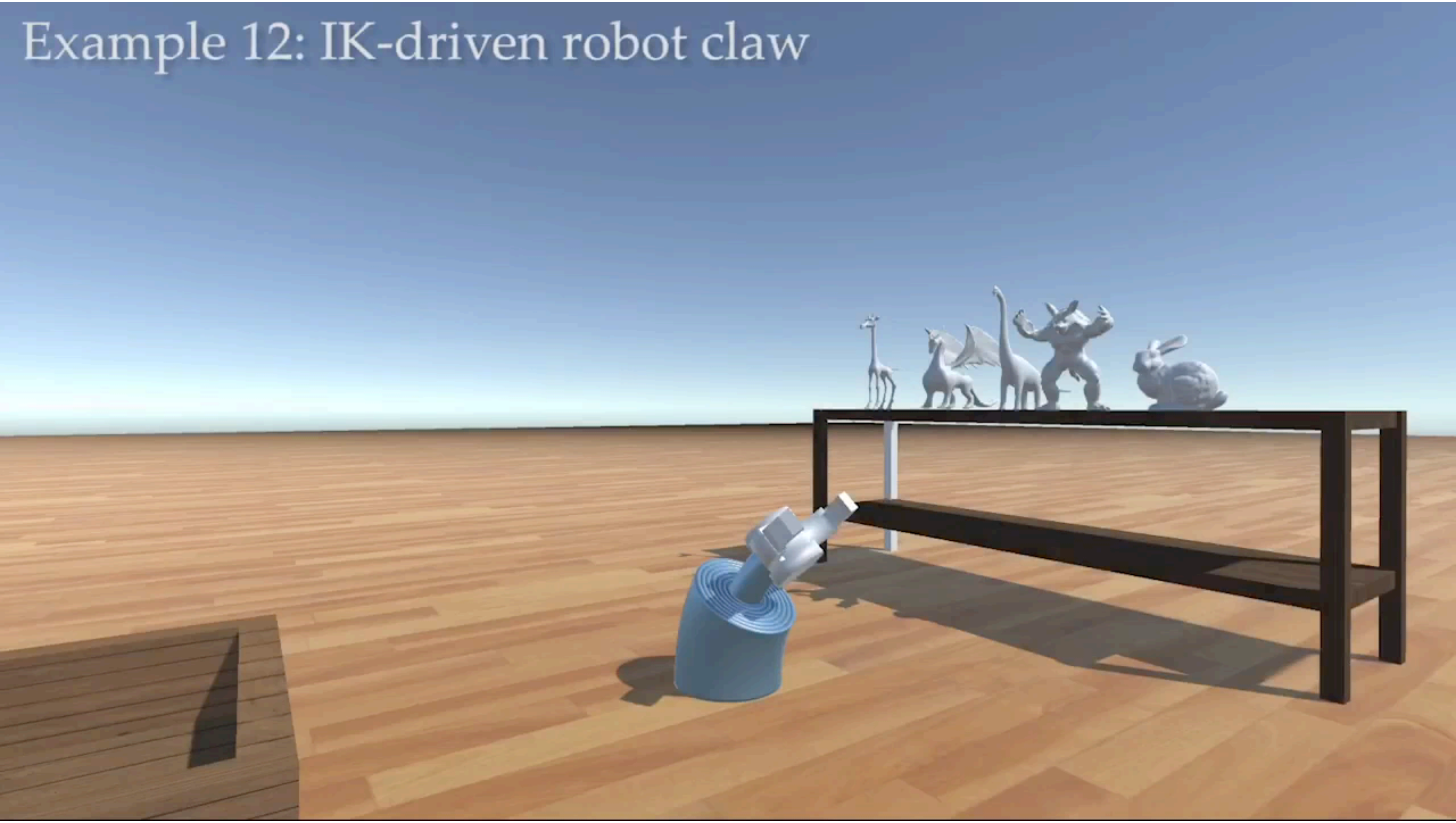
(computer solves for joint angles based on high-level goal)

Example: inverse kinematics



Egon Paszor

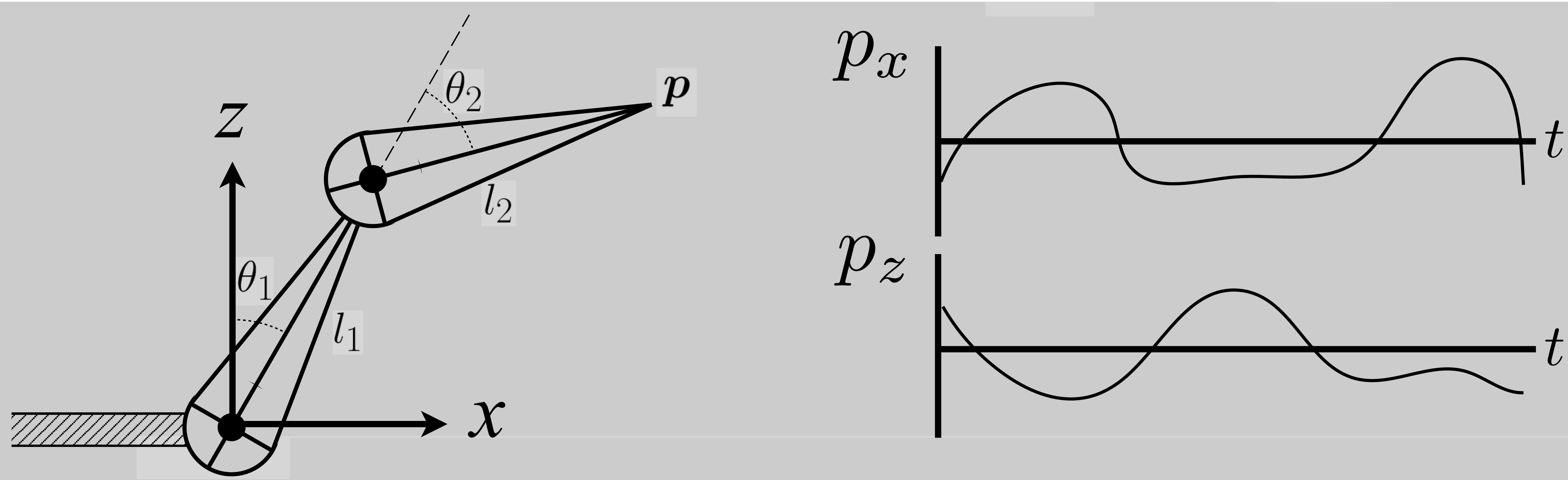
Example: inverse kinematics



Inverse kinematics

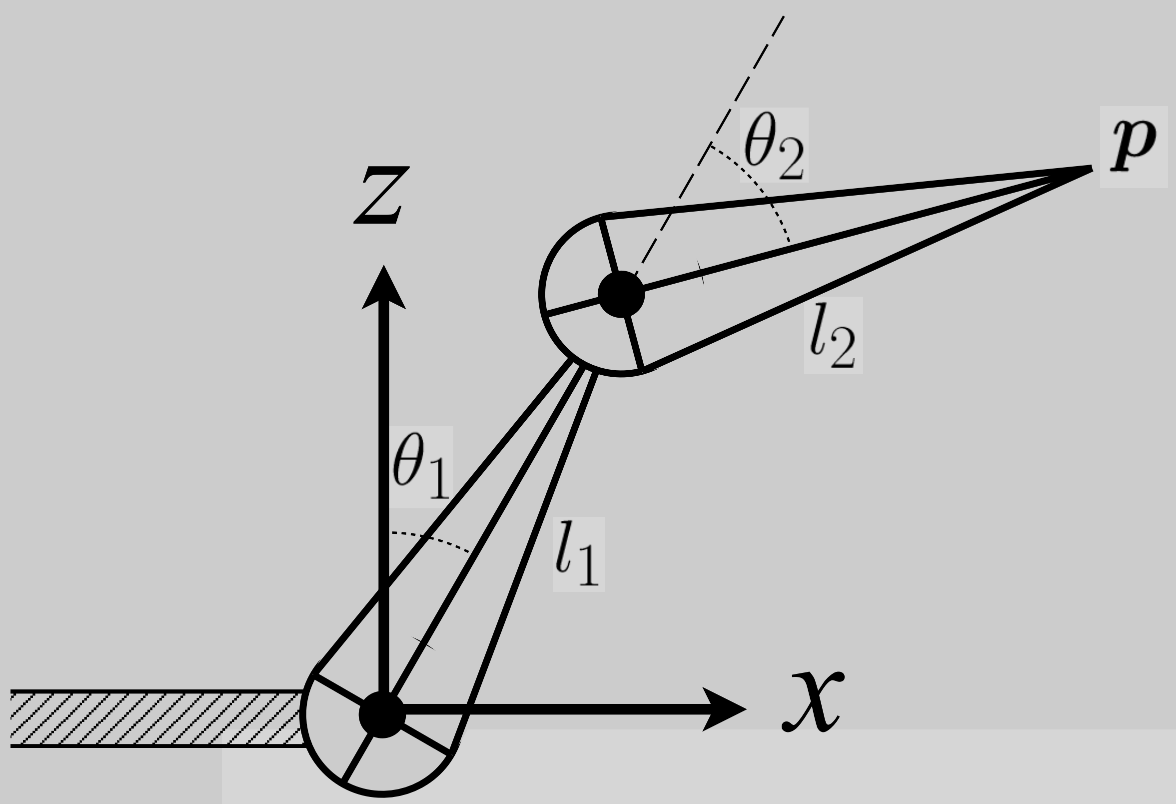
Input: animator provides position of end-effector

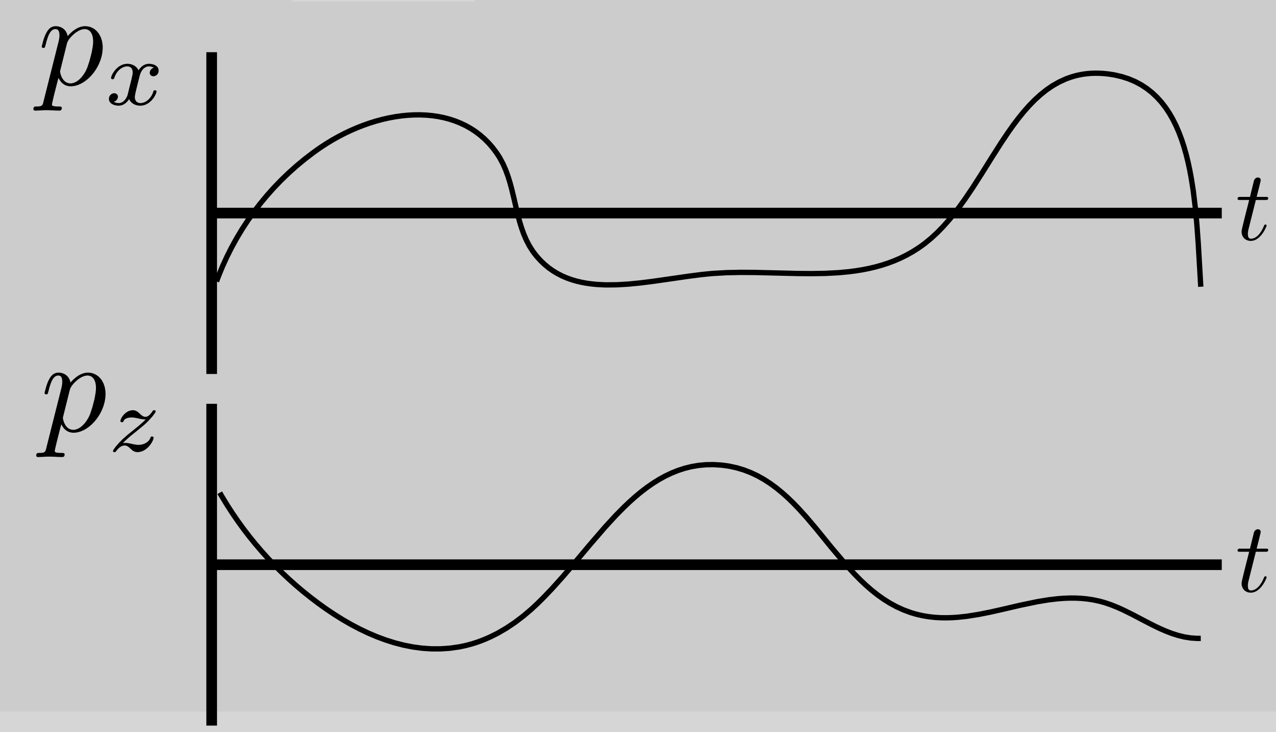
Output: computer must determine joint angles that satisfy constraints



Inverse kinematics

Direct inverse kinematics: for two-segment arm, can solve for parameters analytically (not true for general N-link problem)



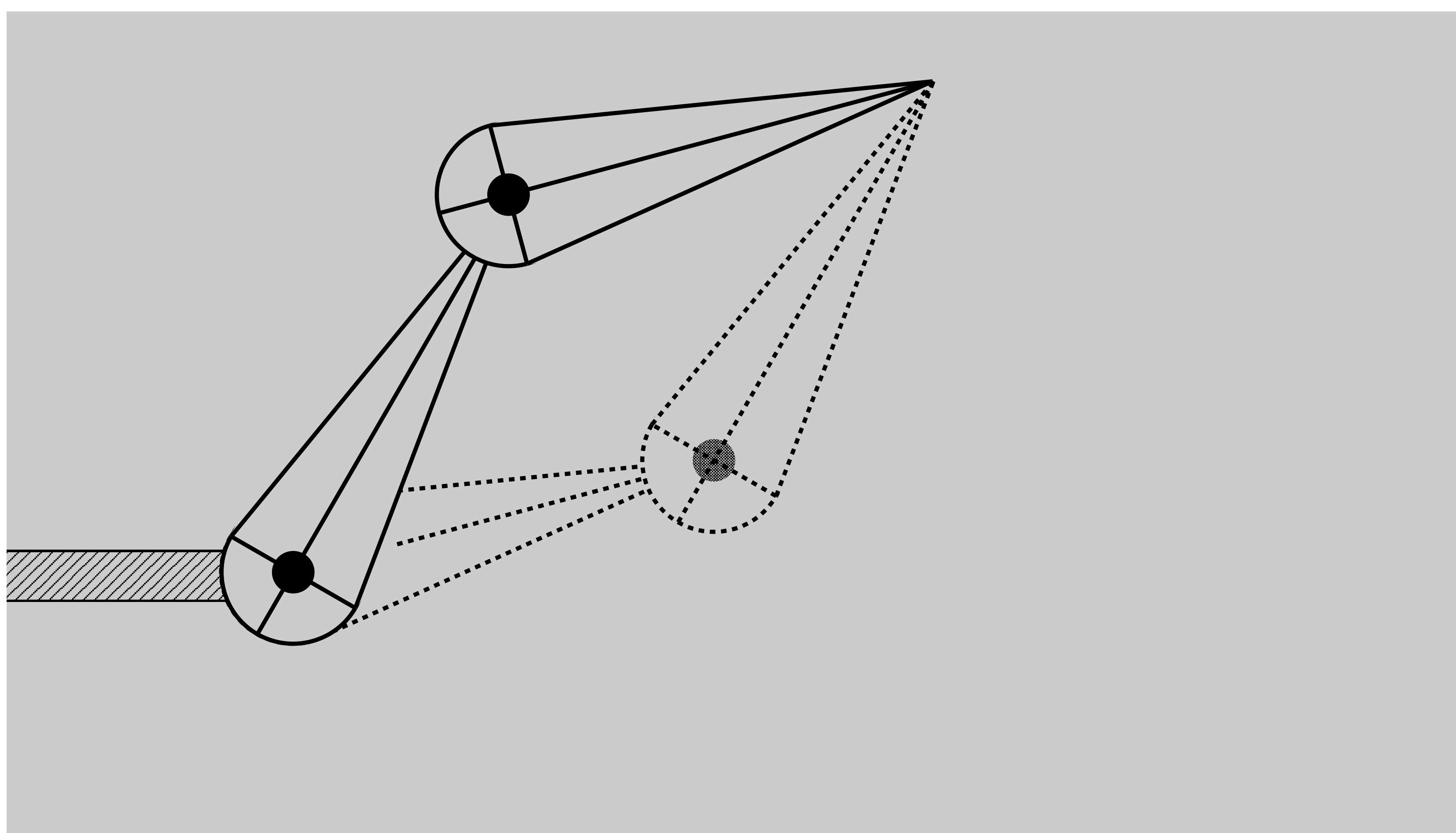


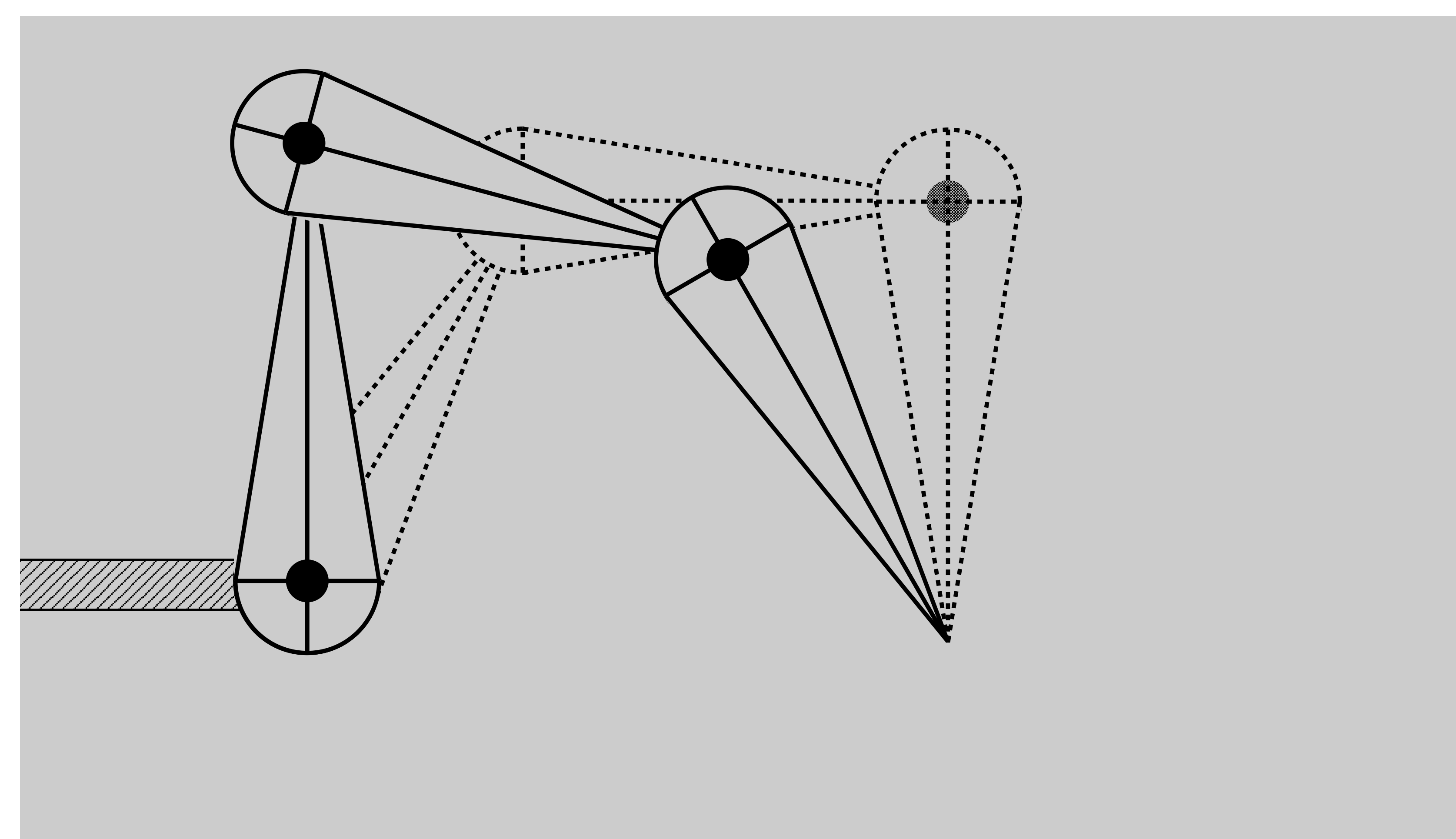
$$\theta_2 = \cos^{-1}\left(\frac{p_z^2 + p_x^2 - l_1^2 - l_2^2}{2l_1l_2}\right)$$

$$\theta_1 = \frac{-p_z l_2 \sin(\theta_2) + p_x(l_1 + l_2\cos(\theta_2))}{p_x l_2 \sin(\theta_2) + p_z(l_1 + l_2\cos(\theta_2))}$$

Inverse kinematics

- Why is the problem hard?
  - Multiple solutions in configuration space (may not be nearby)
  - Solution may not be possible





Inverse kinematics

- **Numerical solution to general N-link IK problem**
  - Choose an initial configuration
  - Define an error metric (e.g. square of distance between goal and current position)
  - Apply *optimization method* to solve for joint angles given the desired (goal) end effector position

A few bits on optimization

(a commonly used tool in graphics)

Optimization problem in standard form

- **Can formulate most continuous optimization problems this way:**

"objective": how much does solution x cost?

$$(f_i : \mathbb{R}^n \to \mathbb{R},\ i = 0, \ldots, m)$$

often (but not always) continuous, differentiable, ...

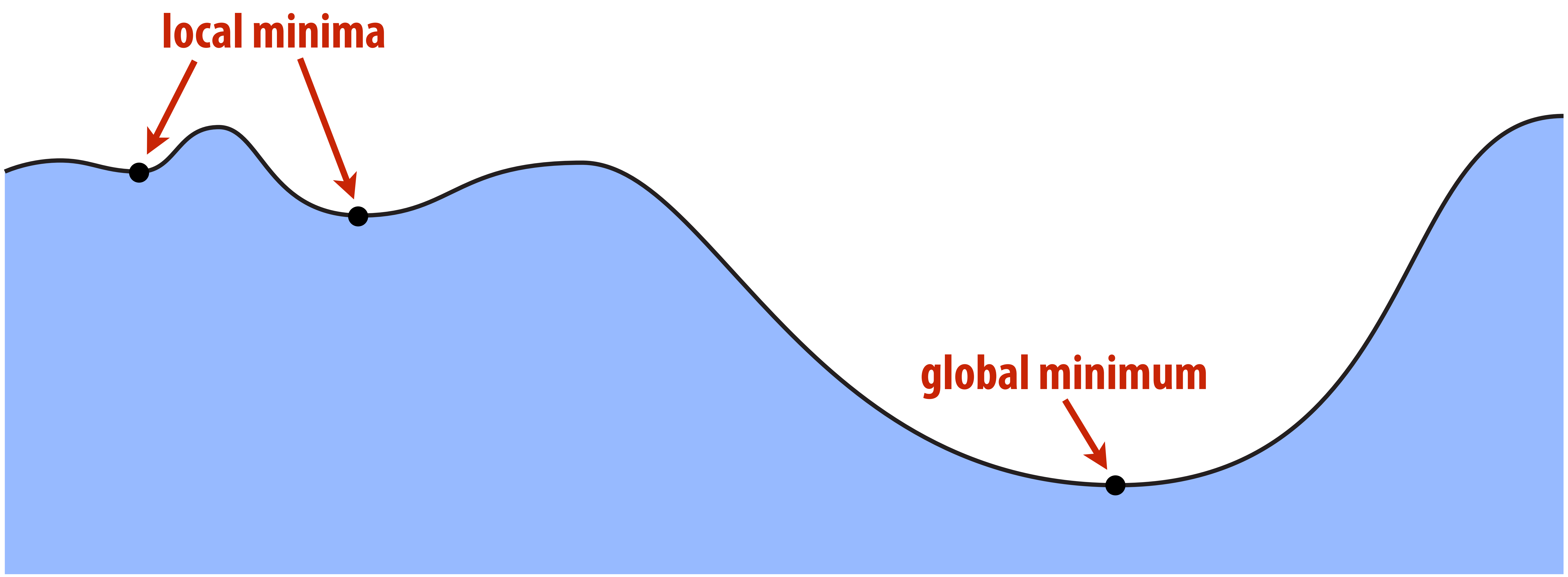
$$\min_{x \in \mathbb{R}^n} \quad f_0(x)$$
$$\text{subject to} \quad f_i(x) \leq b_i,\ i = 1, \ldots, m$$

"constraints": what must be true about x? ("x is *feasible*")

- ***Optimal solution*** **x* has smallest value of f_0 among all feasible x**
- **Q: What if we want to *maximize* something instead?**
- **A: Just flip the sign of the objective!**
- **Q: What if we want *equality* constraints, rather than inequalities?**
- **A: Include two constraints: g(x) ≤ c and g(x) ≤ -c**

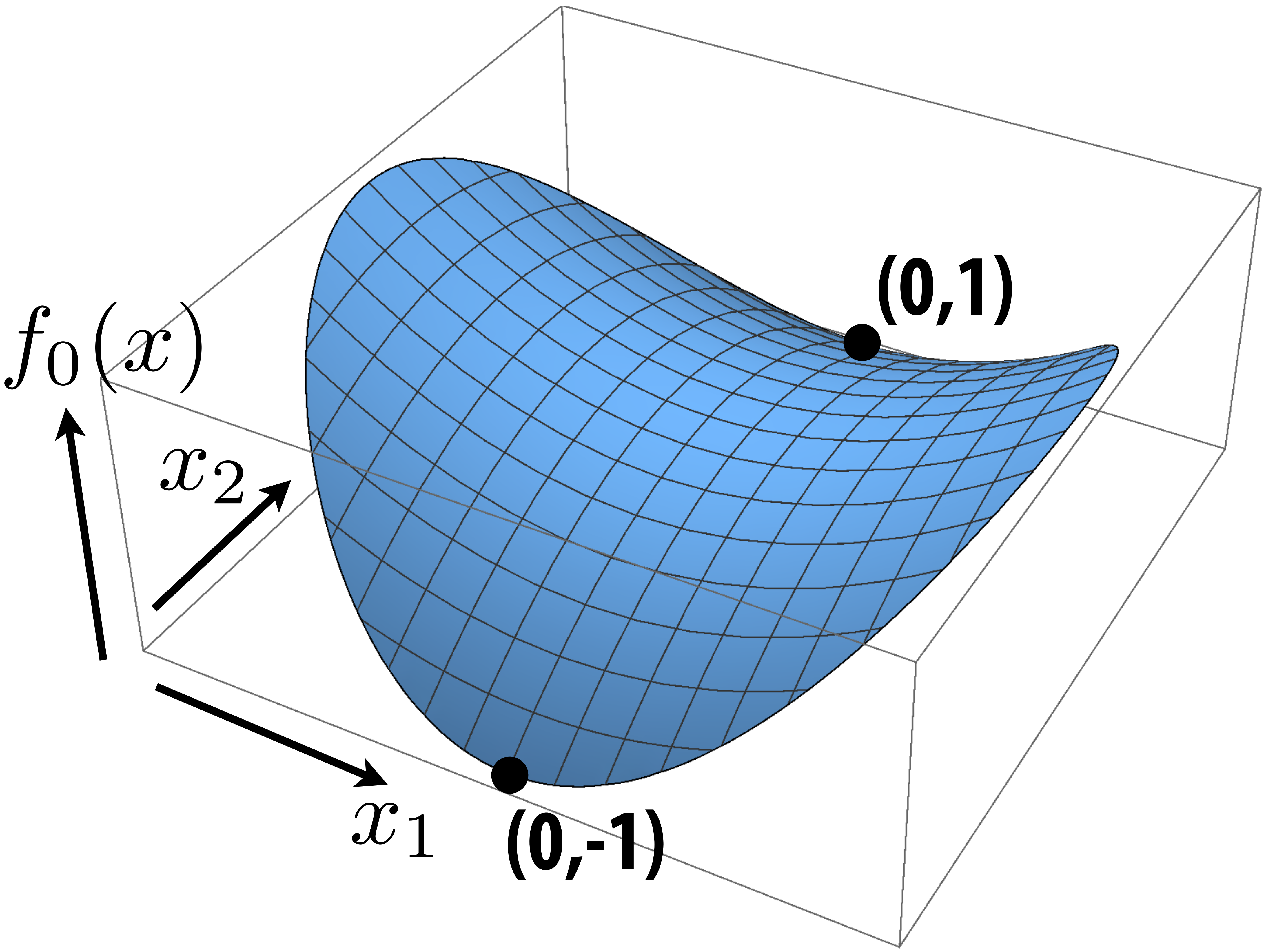
Local vs. global minima

- *Global* minimum is absolute best among all possibilities
- *Local* minimum is best "among immediate neighbors"



Philosophical question: does a local minimum *"solve"* the problem?

Optimization problem, visualized



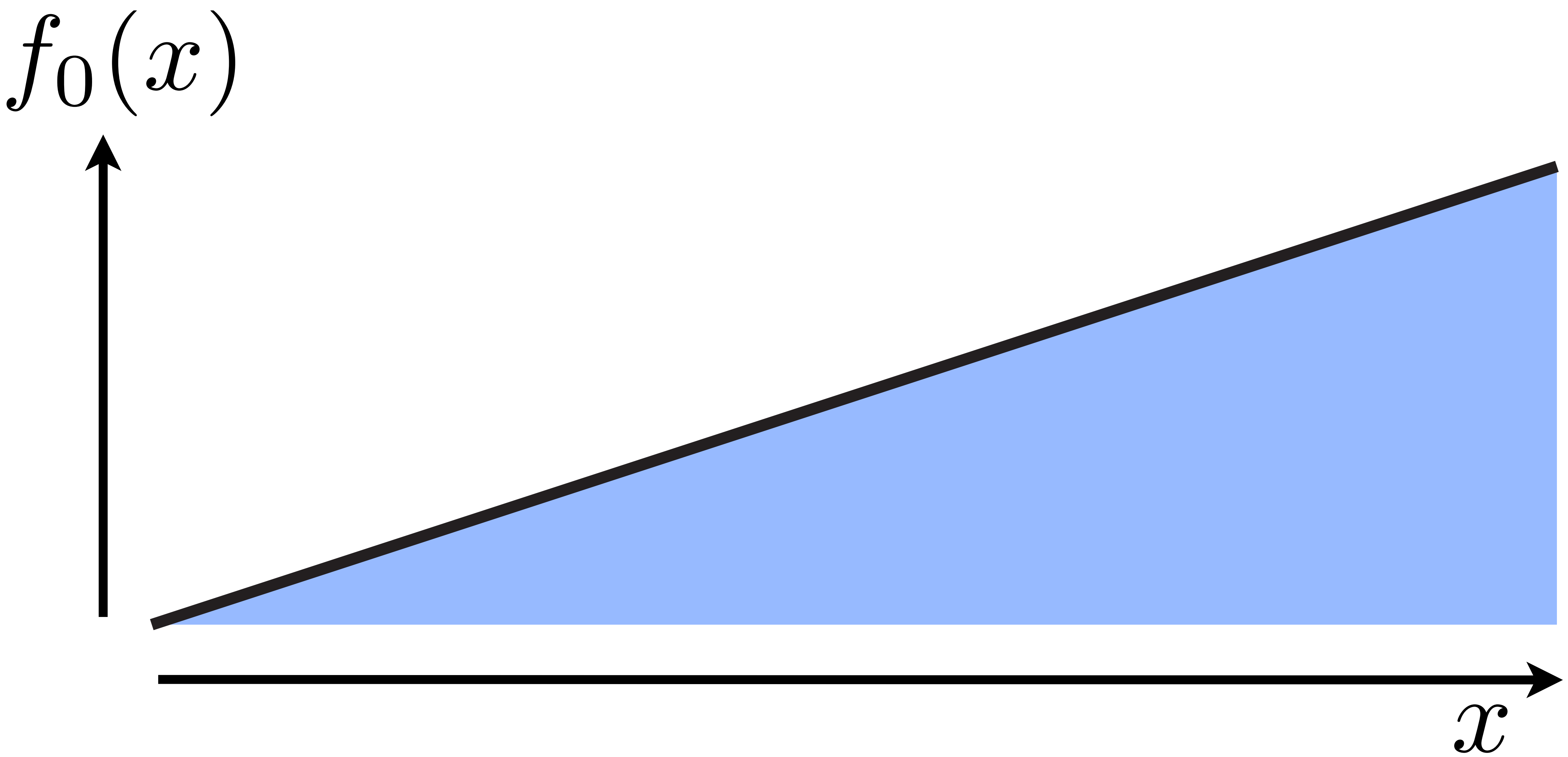
$$\min_{x \in \mathbb{R}^2} \quad x_1^2 - x_2^2$$
$$\text{s.t.} \quad x_1^2 + x_2^2 - 1 \leq 0$$

Q: Is this an optimization problem in standard form? **A: Yes.**

Q: Where is the optimal solution? **A: There are two, (0,1), (0,-1).**

Existence and uniqueness of minimizers

- Already saw that (global) minimizer is not unique
- Does it always exist? Why?
- Just consider all possibilities and take the smallest one, right?



perfectly reasonable optimization *problem*

$$\min_{x \in \mathbb{R}} x$$

clearly has no *solution* (can always pick smaller x)

- WRONG! Not all objectives are bounded from below.
- It's like that old adage: *"no matter how good you are, there will always be someone better than you."*

Feasibility

- **Ok, but suppose the objective is bounded from below.**
- **Then we can just take the best feasible solution, right?**

value of objective doesn't depend on x;
all feasible solutions are equally good

$$\begin{aligned} \min_{x \in \mathbb{R}^n} \quad & 0 \\ \text{subject to} \quad & f_i(x) \le b_i, \;\; i = 1, \ldots, m \end{aligned}$$

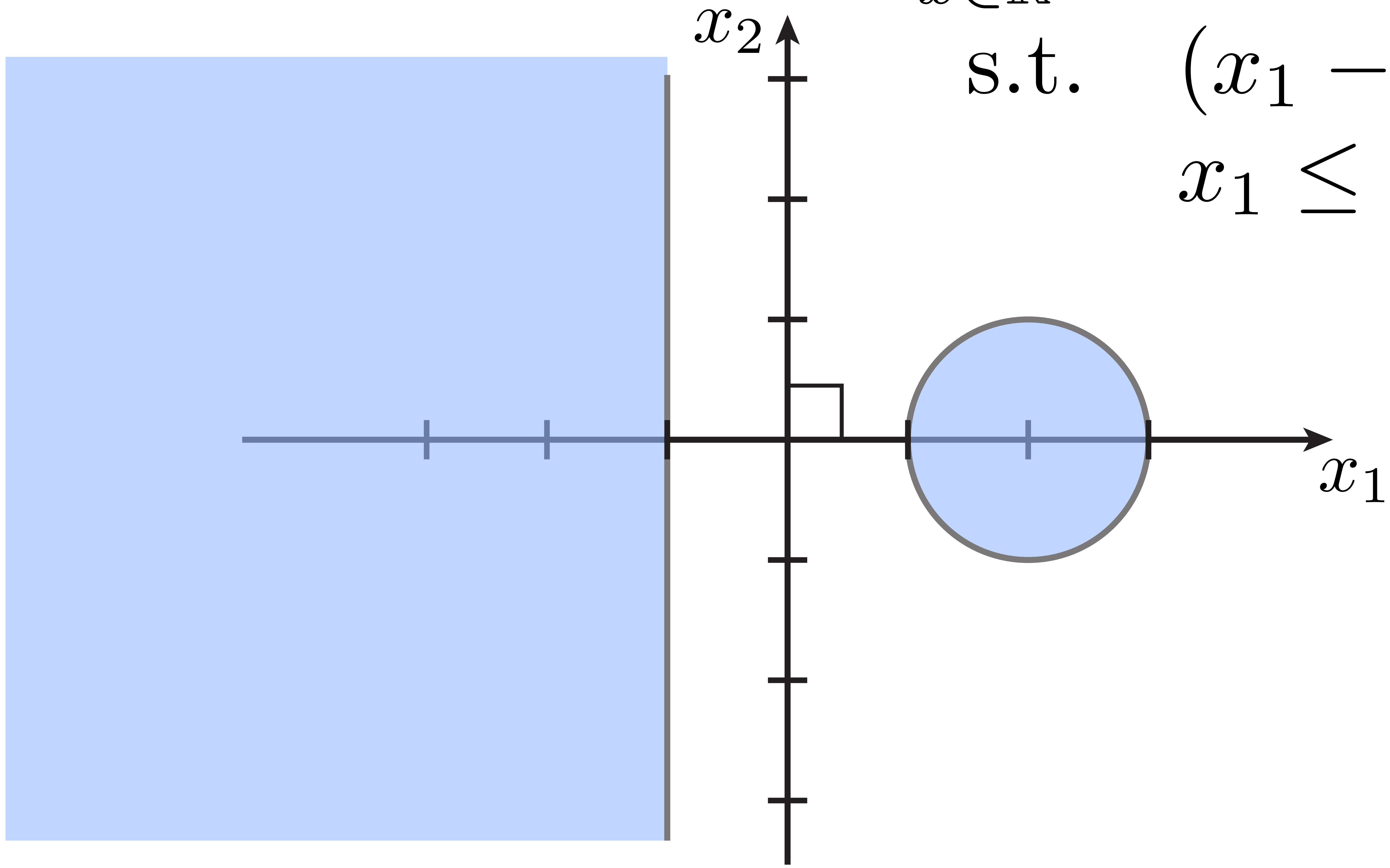
problem now is just finding a feasible solution—
which can be really hard (or impossible!)

- **Not if there aren't any!**
- **Not all problems have solutions!**

Feasibility - example

Q: Is this problem feasible?

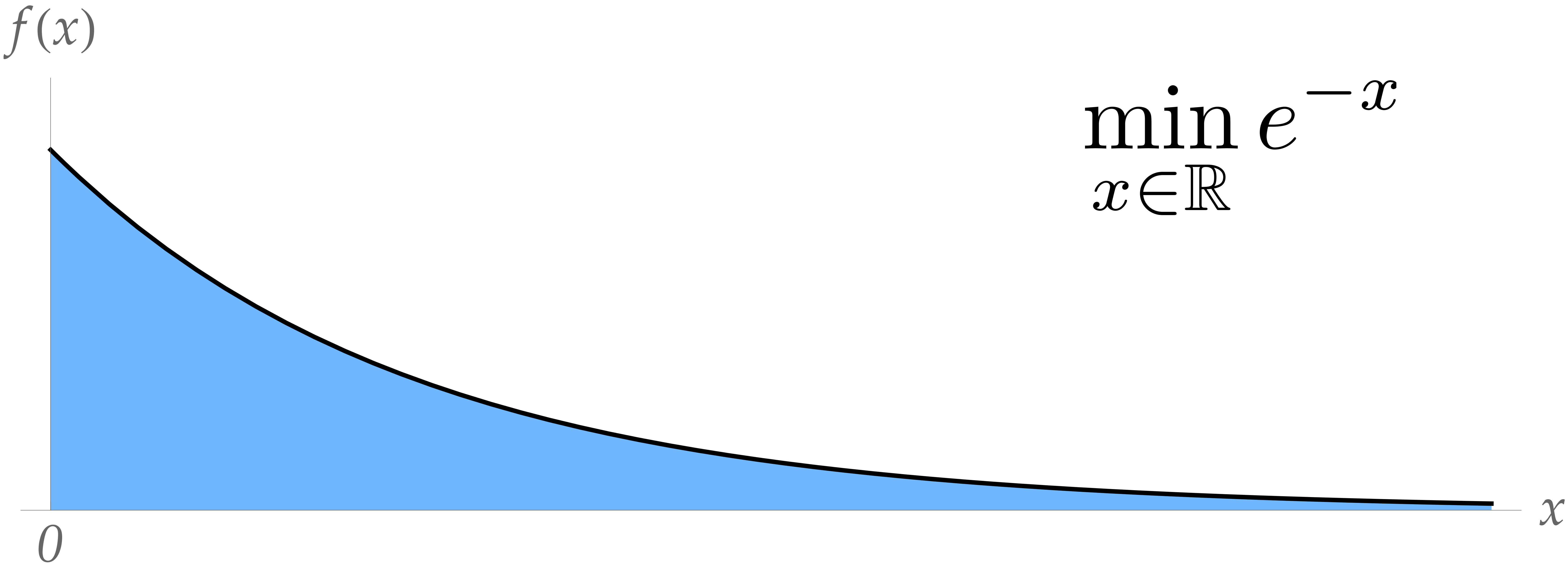
$$\min_{x \in \mathbb{R}^2} \quad \sin(x_1) + x_2^2$$
$$\text{s.t.} \quad (x_1 - 2)^2 + x_2^2 \leq 1,$$
$$x_1 \leq -1$$



A: No—the two sublevel sets (points where f_i(x) ≤ 0) have no common points, i.e., they do not overlap.

Existence and uniqueness of minimizers, cont.

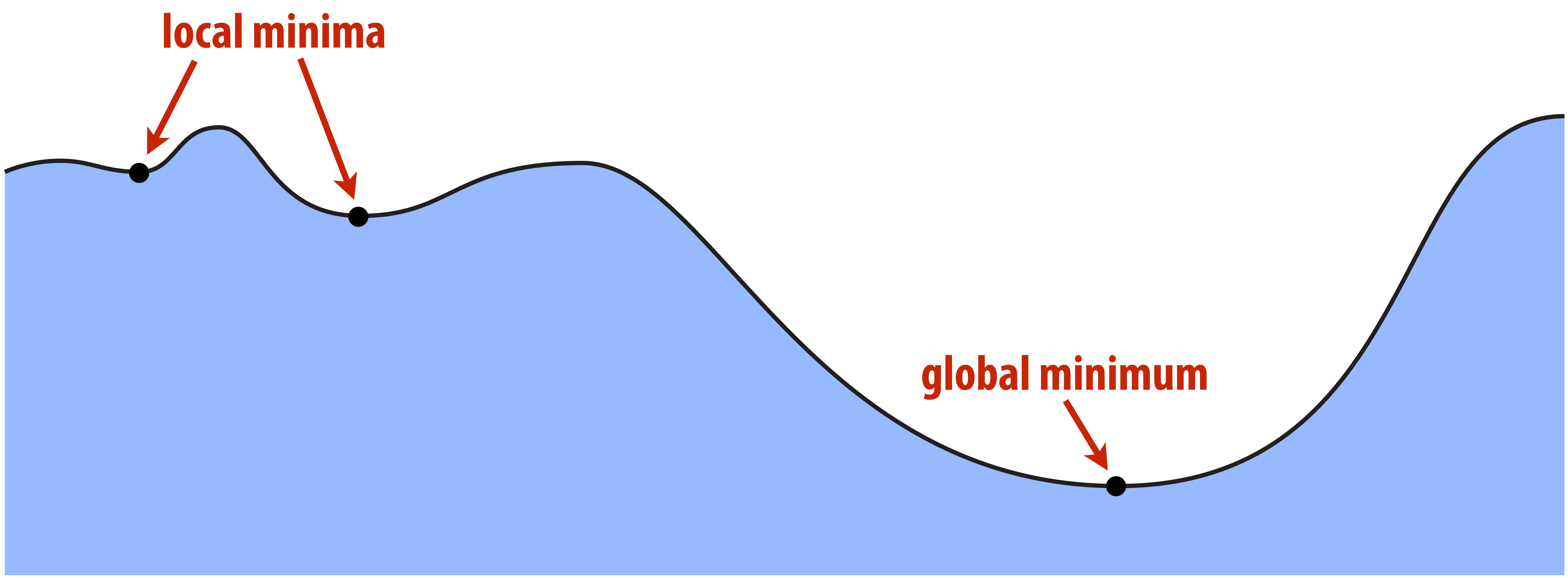
- Even being bounded from below is not enough:



- No matter how big x is, we never achieve the lower bound (0)

Characterization of minimizers

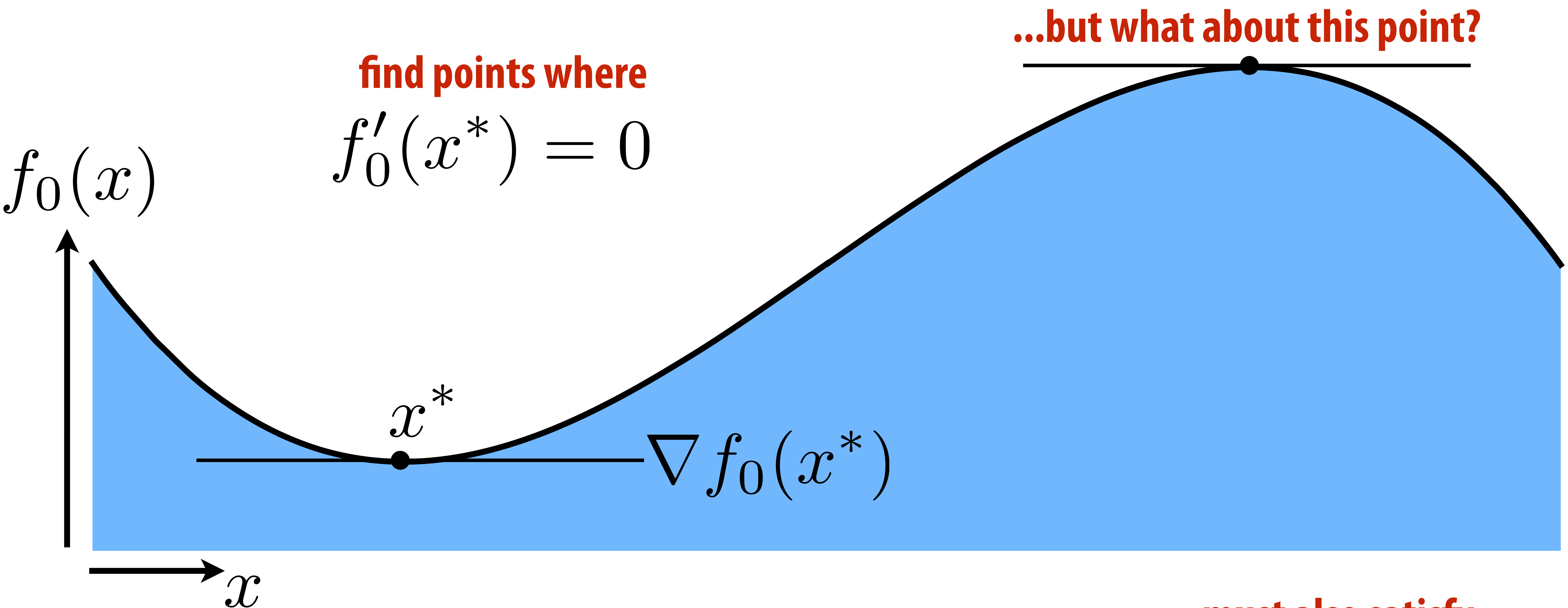
- Ok, so we have some sense of when a minimizer might *exist*
- But how do we know a given point x is a minimizer?



- Checking if a point is a global minimizer is (generally) hard
- But we can certainly test if a point is a local minimum (ideas?)
- (Note: a global minimum is also a local minimum!)

Characterization of local minima

- Consider an objective f_0: R → R. How do you find a minimum?
- (Hint: you may have memorized this formula in high school!)



must also satisfy

$$f_0''(x^*) \geq 0$$

- Also need to check *second* derivative (how?)
- Make sure it's *positive*
- Ok, but what does this all mean for more general functions f_0?

Optimality conditions (unconstrained)

- **In general, our objective is f0: $R^n \rightarrow R$**
- **How do we test for a local minimum?**
- **1st derivative becomes *gradient*; 2nd derivative becomes *Hessian***

$$\nabla f := \begin{bmatrix} \partial f / \partial x_1 \\ \vdots \\ \partial f / \partial x_n \end{bmatrix}$$

GRADIENT
(measures "slope")

$$\nabla^2 f := \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \cdots & \frac{\partial f}{\partial x_1 \partial x_n} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} & \cdots & \frac{\partial f}{\partial x_2 \partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 f}{\partial x_n \partial x_1} & \frac{\partial^2 f}{\partial x_n \partial x_2} & \cdots & \frac{\partial f}{\partial x_n^2} \end{bmatrix}$$

HESSIAN
(measures "curvature")

- **Optimality conditions?**

positive semidefinite (PSD)
($u^T A u \geq 0$ for all u)

$$\nabla f_0(x^*) = 0$$

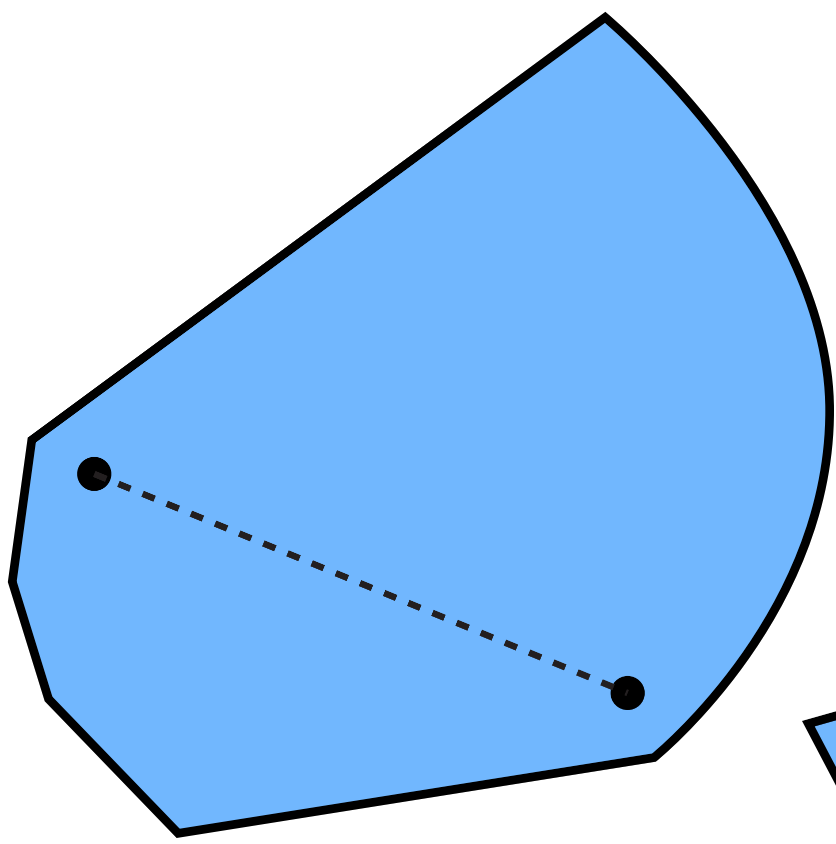
1st order

$$\nabla^2 f_0(x^*) \succeq 0$$

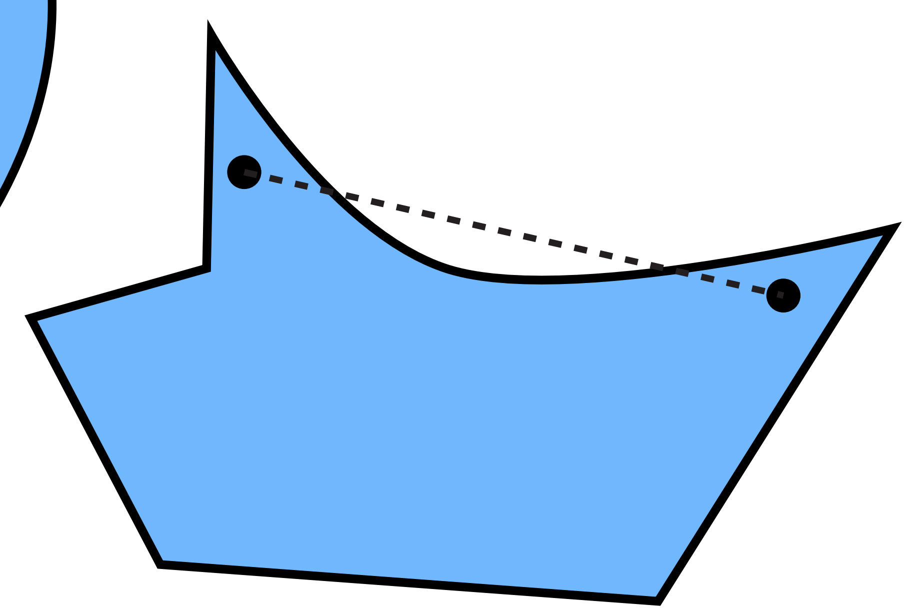
2nd order

Convex optimization

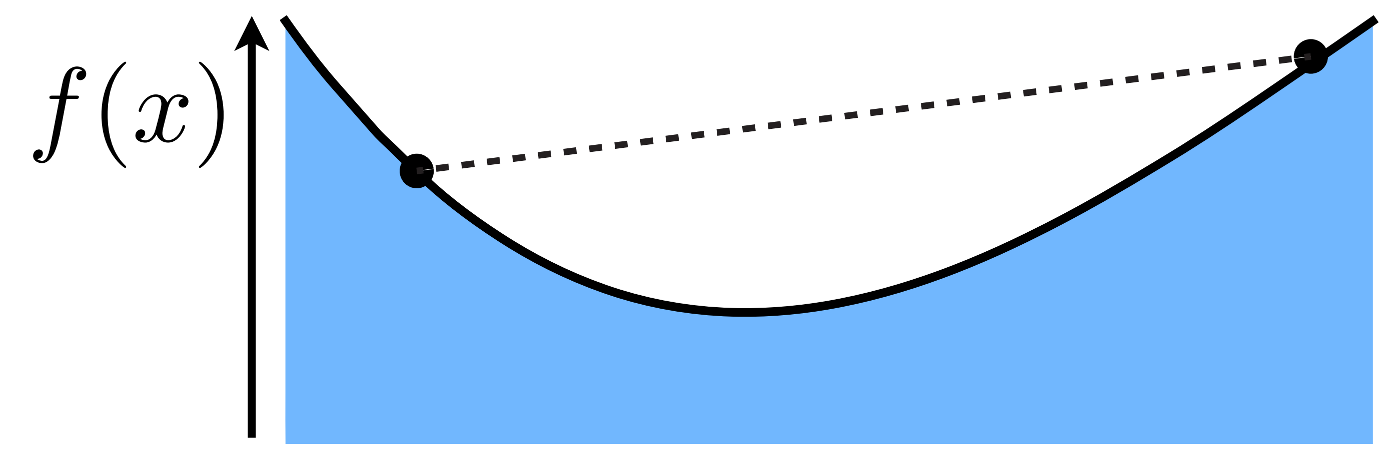
- Special class of problems that are almost always "easy" to solve (polynomial-time!)
- Problem is *convex* if it has a convex domain *and* convex objective



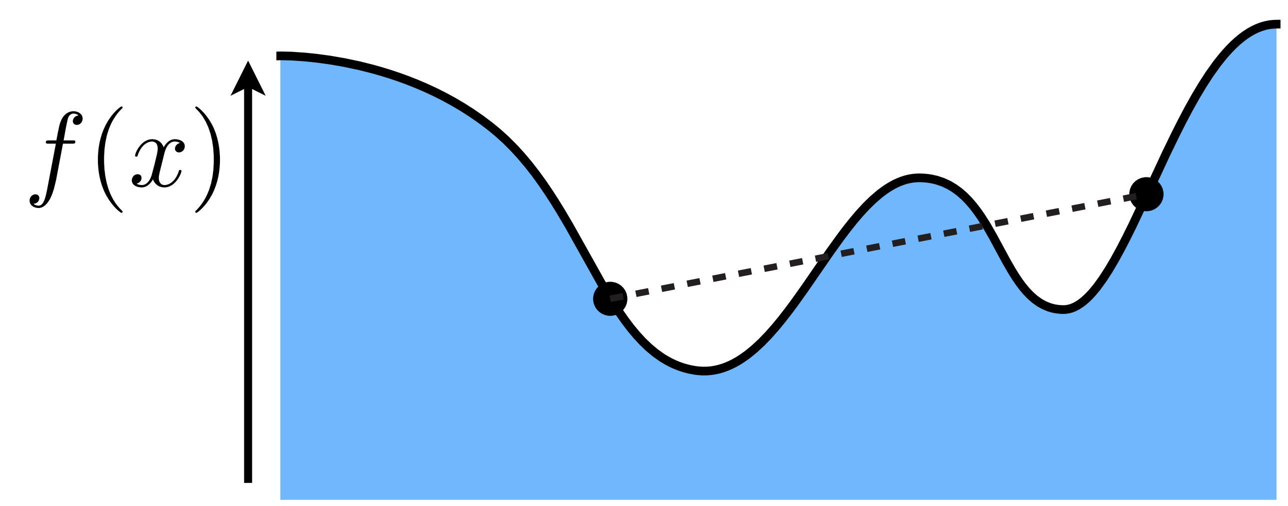
convex domain



noconvex domain



convex objective



nonconvex objective

- Why care about convex problems in graphics?
  - can make guarantees about solution (always the best)
  - doesn't depend on initialization (strong convexity)
  - often quite efficient, but not always

Sadly, life is not usually that easy.
How do we solve optimization problems in general?

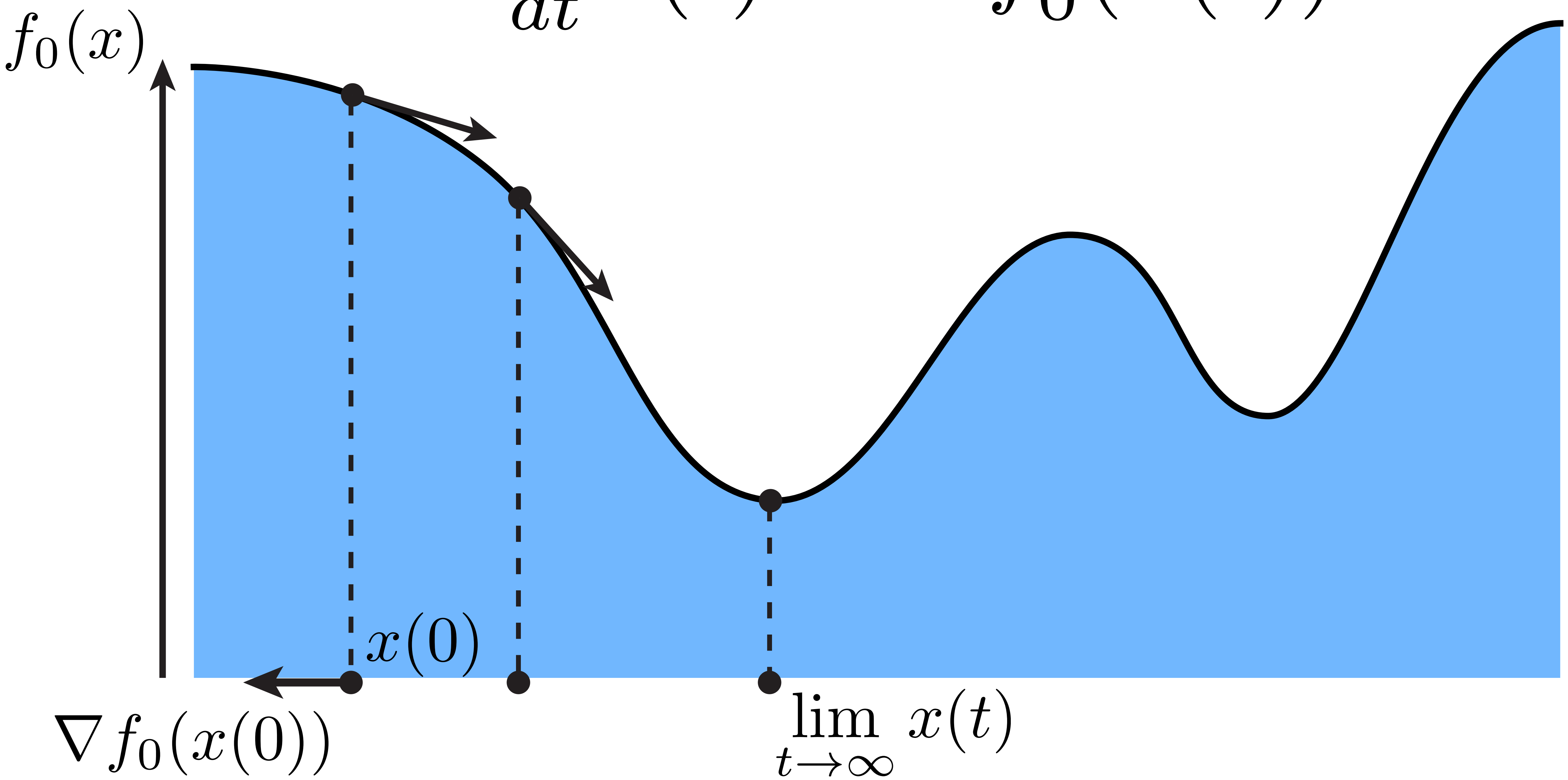
Descent methods

An idea as old as the hills:

Gradient descent (1D)

- **Basic idea: follow the gradient "downhill" until it's zero**
- **(Zero gradient was our 1st-order optimality condition)**

$$\frac{d}{dt}x(t) = -f_0'(x(t))$$



- **Do we always end up at a (global) minimum?**
- **How do we compute gradient descent in practice?**

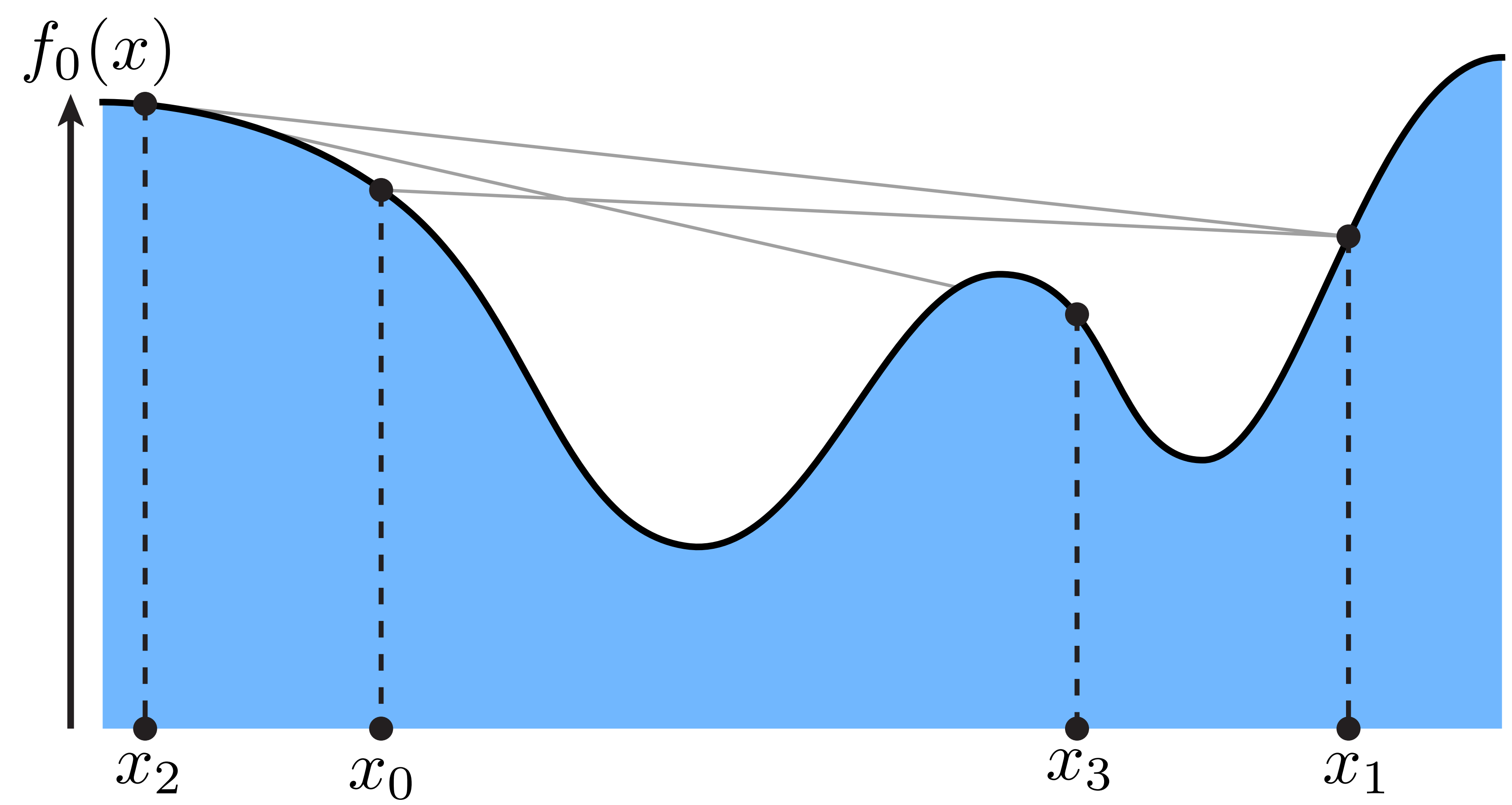
Gradient descent algorithm (1D)

- "Walk downhill"

$$x_{k+1} = x_k - \tau f_0'(x_k)$$

new estimate

step size



- Q: How do we pick the step size?
- If we're not careful, we'll go zipping all over the place; won't make any progress.
- Basic idea: use *"step control"* to determine step size based on value of objective and derivatives
- For now we will do something simple: make τ *small!*

Gradient descent algorithm (n-D)

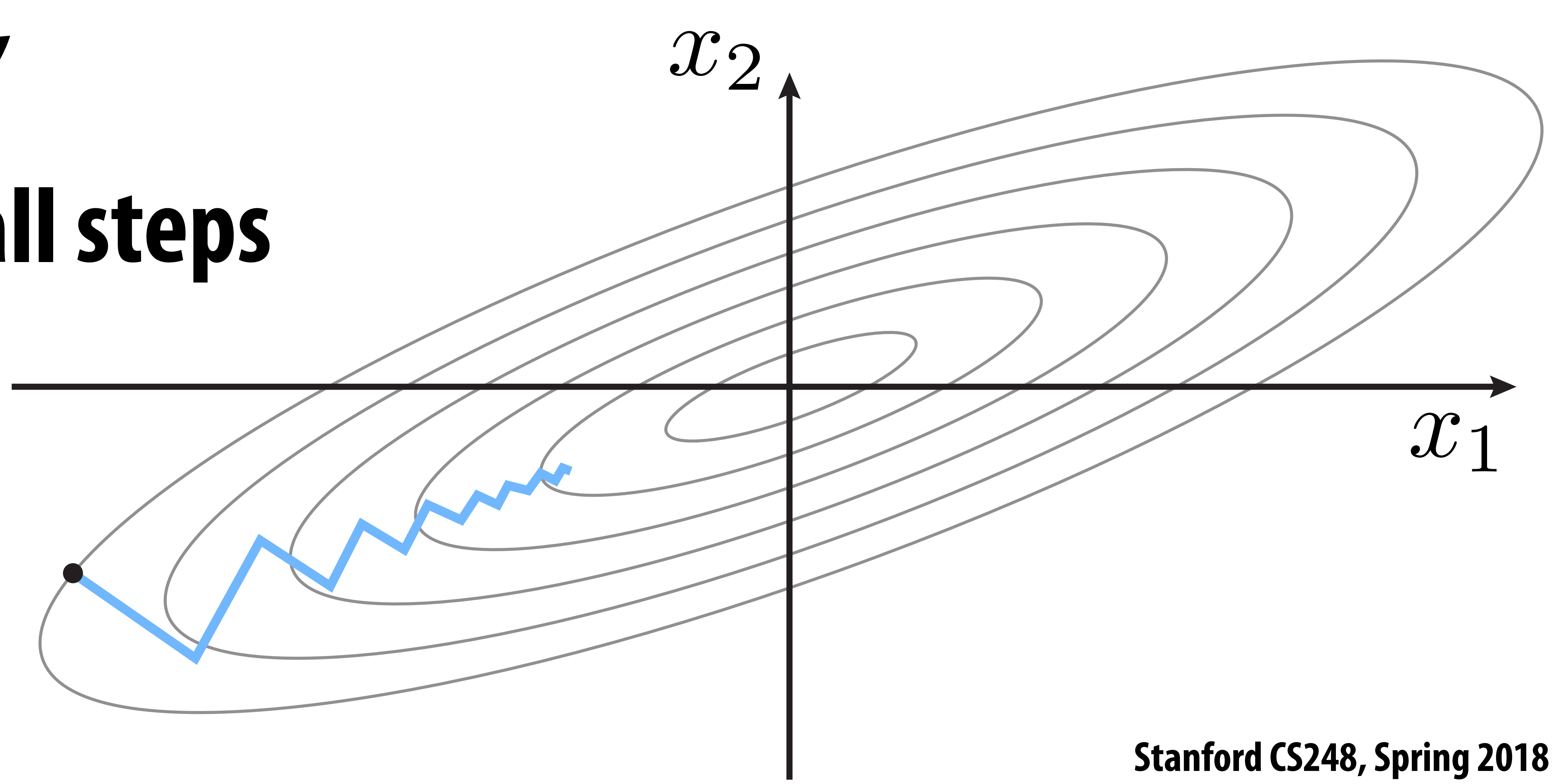
- **Q: How do we write gradient descent equation in general?**

$$\frac{d}{dt}x(t) = -\nabla f_0(x(t))$$

- **Q: What's the corresponding discrete update?**

$$x_{k+1} = x_k - \tau \nabla f_0(x_k)$$

- **Basic challenge in nD:**
  - solution can "oscillate"
  - takes many, many small steps
  - very slow to converge



Simple inverse kinematics algorithm

- **What is the objective?**
  - Distance from end effector position (given current joint parameters) to target position.

position of end effector (given θ)

$$f_0(\theta) = \|p_{\text{current}} - p_{\text{target}}\|^2$$

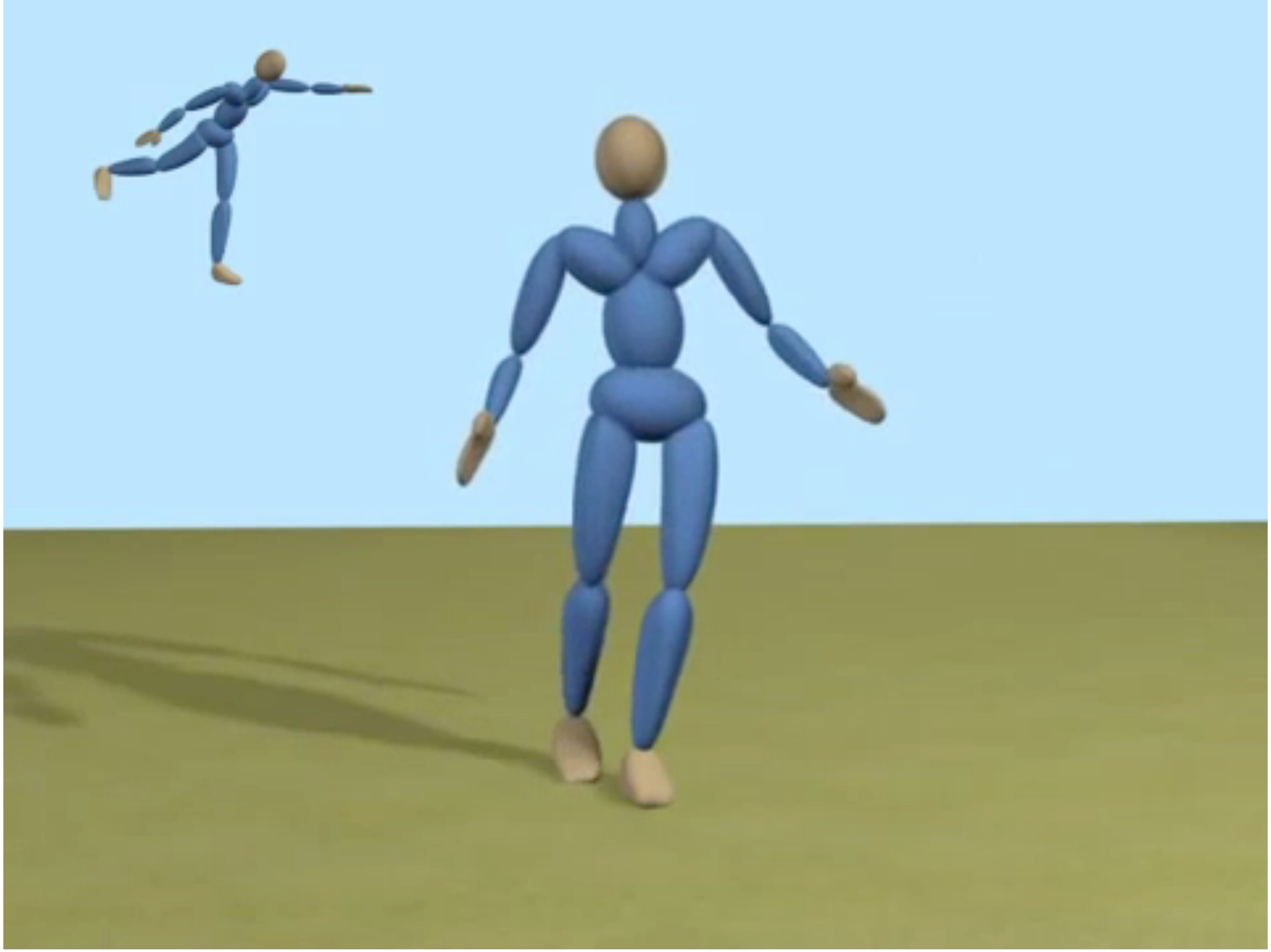
vector of joint angles (to optimize)

desired position

- **Constraints?**
  - Could limit range of motion of a joint

- **How to optimize for joint angles:**
  - Compute gradient of objective with respect to joint angles
  - Apply gradient descent

Many other uses of optimization in animation (and graphics in general)



Sumit Jain, Yuting Ye, and C. Karen Liu, ***"Optimization-based Interactive Motion Synthesis"***

Summary

- **Kinematics: how objects move, without regard to forces that create this movement**

- **Today: multiple ways of obtaining joint motion**
  - **Direct hand authoring of joint angles**
  - **Via measurement (motion capture)**
  - **As a result of solving for angles that yield a particular higher level result (inverse kinematics)**

- ***Acknowledgements: thanks to Keenan Crane, Ren Ng, Mark Pauly, Steve Marschner, Tom Funkhouser, James O'Brien for presentation resources.***