Why do we need to be able to talk precisely about color?
Munsell book of color

Swatch identified by three numbers: hue, value (lightness), and chroma (color purity)
What is light?
Light is electromagnetic radiation (oscillating electromagnetic field)
Perceived color is *related to* frequency of oscillation

Most light is not visible to human eye!
Q: Why does your stove turn red when it heats up?

Heat generates light
Spectral power distributions

Describes distribution of power (energy/time) by wavelength

Below: spectrum of various common light sources:

- Daylight
- Incandescent
- Fluorescent
- Halogen
- Cool White LED
- Warm White LED

Figure credit: admesy

Stanford CS248, Spring 2018
Superposition (linearity) of spectral power distributions

Figure credit: Brian Wandell
Additive vs. subtractive models of light

- Spectrum we just saw for different light sources “emission spectrum”
  - How much light is produced (by heat, fusion, etc.)
  - Useful for, e.g., characterizing color of a lightbulb

- Another useful description: “absorption spectrum”
  - How much light is absorbed (e.g., turned into heat)
  - Useful for, e.g., characterizing color of paint, ink, etc.
Absorption spectrum

- Emission spectrum is *intensity* as a function of frequency
- Absorption spectrum is *fraction absorbed* as function of frequency

Q: What color is an object with this absorption spectrum?
Interaction of emission and reflection

- Consider what happens when light gets reflected from a surface
  - $\nu$—frequency (Greek “nu”)
  - Light source has emission spectrum $f(\nu)$
  - Surface has reflection spectrum $g(\nu)$
  - Resulting intensity is the product $f(\nu)g(\nu)$
But what does “visible to the eye” mean?

How does electromagnetic radiation (with a given power distribution) end up being perceived by a human as a certain color?

In other words:
Color is something we perceive. It is not technically correct to say different wavelengths of light are “colors”.
Biological Basis of Color
The eye

Image credit: Georgia Retina (http://www.garetina.com/about-the-eye)
The eye's photoreceptor cells: rods and cones

Image credit: http://colorvisionawareness.com/images/pages/eyeDiagram_rodsCones.png
The eye’s photoreceptor cells: rods and cones

- Rods are primary receptors under dark viewing conditions (scotopic conditions)
  - Approx. 120 million rods in human eye
  - Sense light intensity (shades of gray, not color)

- Cones are primary receptors under high-light viewing conditions (photopic conditions, e.g., daylight)
  - Approx. 6-7 million cones in the human eye
  - Each of the three types of cone feature a different “spectral response”. This will be critical to color vision (much more on this in the coming slides)

Figure credit: http://ebooks.bfwpub.com/life.php Figure 45.18
Density of rods and cones in the retina

- Highest density of cones is in fovea
  (best color vision at center of where human is looking)
- Note “blind spot” due to optic disk

[Roorda 1999]
Tired of this lecture? Make Kayvon disappear

- Cover left eye
- Focus on Wall-E with right eye
- Move head slowly away from display…

(Kayvon will disappear when your eye is about 3x farther from screen than distance between Kayvon and Wall-E. This is when projected image of Kayvon falls on the blind spot of your retina)
Simple model of a light detector

\[ R = \int_{\lambda} \Phi(\lambda) r(\lambda) d\lambda \]

incoming spectrum \( \Phi(\lambda) \)

spectral response function \( r(\lambda) \)

Figure credit: Steve Marschner
Response of human cone cells

- S, M, L cones have different spectral response: (peaked at short (S), medium (M), and long (L) wavelengths)
Spectral response of cones

Three types of cones: S, M, and L cones (corresponding to peak response at short, medium, and long wavelengths)

\[ S = \int_{\lambda} \Phi(\lambda) r_S(\lambda) d\lambda \]
\[ M = \int_{\lambda} \Phi(\lambda) r_M(\lambda) d\lambda \]
\[ L = \int_{\lambda} \Phi(\lambda) r_L(\lambda) d\lambda \]
Spectral response of cones (discrete form)

Three types of cones: S, M, and L cones (corresponding to peak response at short, medium, and long wavelengths)

Discrete form: measured signal computed via dot product

\[
S = \begin{bmatrix} \Phi \\ \Phi \end{bmatrix} \begin{bmatrix} r_S \end{bmatrix}
\]

\[
M = \begin{bmatrix} \Phi \\ \Phi \end{bmatrix} \begin{bmatrix} r_M \end{bmatrix}
\]

\[
L = \begin{bmatrix} \Phi \\ \Phi \end{bmatrix} \begin{bmatrix} r_L \end{bmatrix}
\]
Response of S,M,L cones to monochromatic light

Figure visualizes cone’s response to monochromatic light (light with energy in a single wavelength) as points in 3D space

(plots value of S, M, L response functions as a point in 3D space)
Notice what is happening

- Human eye reduces a $\infty$-dimensional signal (spectrum) to three scalar values (3D)
The human visual system

- Human eye does not directly measure the spectrum of incoming light - a.k.a. the brain does not receive “a spectrum” from the eye
- The eye measures three response values = (S, M, L). The result of integrating the incoming spectrum against response functions of S, M, L-cones

\[ \Phi(\lambda) \]

Focuses light on retina
Cones measure light (photopic case)

cone responses (S, M, L) carried along optic nerve

Brain
Implication: metamerism
Metamers

- Metameters = two different spectrum that integrate to the same (S,M,L) response!

- The fact that metameters exist is critical to color reproduction: we don’t have to reproduce the exact same spectrum as was present in a real world scene to be able to reproduce the perceived appearance of the scene on a monitor (or piece of paper, or paint on a wall)!
Metamerism is a big effect

Example: different spectrum that integrate to the same S, M, L response
Displays producing color: (additive color)

- Given a set of primary lights, each with its own spectral distribution (e.g. R,G,B display pixels):
  \[ s_R(\lambda), \ s_G(\lambda), \ s_B(\lambda) \]

- We can adjust the brightness of these lights and add them together to produce a linear subspace of spectral distribution:
  \[ R \ s_R(\lambda) + G \ s_G(\lambda) + B \ s_B(\lambda) \]

- The color is now described by the scalar values:
  \[ R, \ G, \ B \]
Recall: real LCD screen pixels (Closeup)

Notice R, G, B sub-pixel geometry. Effectively three lights at each (x,y) location.
Example primaries: LCD display

- Curves determined by backlight and color filters
Color reproduction problem

- Goal: at each pixel, choose R, G, B values for display so that the output color matches the appearance of the target color in the real world.

\[ \Phi(\lambda) \]

- Target real spectrum

- Display outputs spectrum

\[ R s_R(\lambda) + G s_G(\lambda) + B s_B(\lambda) \]
Color reproduction as linear algebra

Spectrum produced by display given values R,G,B:

\[ s_{\text{disp}}(\lambda) = R s_R(\lambda) + G s_G(\lambda) + B s_B(\lambda) \]

\[ \begin{bmatrix} s_{\text{disp}} \\ \end{bmatrix} = \begin{bmatrix} s_R & s_G & s_B \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix} \]
Color reproduction as linear algebra

- What color do we perceive when we look at the display?

\[
\begin{bmatrix}
S \\
M \\
L
\end{bmatrix}_{\text{disp}} = \begin{bmatrix}
  r_S & - \\
  r_M & - \\
  r_L & - \\
\end{bmatrix}
\begin{bmatrix}
S_{\text{disp}} \\
\end{bmatrix}
\]

\[
= \begin{bmatrix}
  r_S & - \\
  r_M & - \\
  r_L & - \\
\end{bmatrix}
\begin{bmatrix}
  s_R & s_G & s_B \\
\end{bmatrix}
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix}
\]

We want this displayed spectrum to be a metamer for the real-world target spectrum.
Color reproduction as linear algebra

Color perceived for display spectra with values R,G,B

\[
\begin{bmatrix}
S \\
M \\
L \\
\end{bmatrix}_{\text{disp}} = \begin{bmatrix}
 & r_S & \\
 & r_M & \\
 & r_L & \\
\end{bmatrix} \begin{bmatrix}
s_R \\
s_G \\
s_B \\
\end{bmatrix} = \begin{bmatrix}
R \\
G \\
B \\
\end{bmatrix}
\]

Color perceived for real scene spectra, s

\[
\begin{bmatrix}
S \\
M \\
L \\
\end{bmatrix}_{\text{real}} = \begin{bmatrix}
 & r_S & \\
 & r_M & \\
 & r_L & \\
\end{bmatrix} \begin{bmatrix}
s \\
\end{bmatrix}
\]

How do we reproduce the color of s? Set these lines equal and solve for R,G,B as a function of s!
Color reproduction as linear algebra

Solution:

\[
\begin{bmatrix}
  r_S \\
r_M \\
r_L \\
\end{bmatrix}
\begin{bmatrix}
  s_R & s_G & s_B \\
\end{bmatrix}
\begin{bmatrix}
  R \\
  G \\
  B \\
\end{bmatrix}
= 
\begin{bmatrix}
  r_S \\
r_M \\
r_L \\
\end{bmatrix}
\begin{bmatrix}
  s \\
\end{bmatrix}
\]

\[\rightarrow \begin{bmatrix}
  R \\
  G \\
  B \\
\end{bmatrix}
= \left( \begin{bmatrix}
  r_S \\
r_M \\
r_L \\
\end{bmatrix}
\begin{bmatrix}
  s_R & s_G & s_B \\
\end{bmatrix} \right)^{-1}
\begin{bmatrix}
  r_S \\
r_M \\
r_L \\
\end{bmatrix}
\begin{bmatrix}
  s \\
\end{bmatrix}\]
Color reproduction as linear algebra

Solution (form #1):

\[
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix} = \left( \begin{bmatrix}
1 & r_S & 0 \\
0 & r_M & 0 \\
0 & r_L & 0
\end{bmatrix} \begin{bmatrix}
S_R \\
S_G \\
S_B
\end{bmatrix} \right)^{-1} \begin{bmatrix}
1 & r_S & 0 \\
0 & r_M & 0 \\
0 & r_L & 0
\end{bmatrix} \begin{bmatrix}
R \\
G \\
B
\end{bmatrix}
\]

Solution (form #2):

\[
RGB = \left( M_{SML} M_{RGB} \right)^{-1} M_{SML} s
\]
Color reproduction as linear algebra

Solution (form #3):

\[
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix} = 
\begin{bmatrix}
\frac{r_S}{r_R} & \frac{r_S}{s_G} & \frac{r_S}{s_B} \\
\frac{r_M}{r_R} & \frac{r_M}{s_G} & \frac{r_M}{s_B} \\
\frac{r_L}{r_R} & \frac{r_L}{s_G} & \frac{r_L}{s_B}
\end{bmatrix}^{-1}
\begin{bmatrix}
\frac{r_S}{s} \\
\frac{r_M}{s} \\
\frac{r_L}{s}
\end{bmatrix}
\]
Color matching
Additive color matching experiment
Example Experiment

Slide from Durand and Freeman 06
Example Experiment

Slide from Durand and Freeman 06
Example Experiment

Slide from Durand and Freeman 06
The primary color amounts needed for a match.

Slide from Durand and Freeman 06
Experiment 2: Out of Gamut

Slide from Durand and Freeman 06
Experiment 2: Out of Gamut

Slide from Durand and Freeman 06
Experiment 2: Out of Gamut

Slide from Durand and Freeman 06
We say a “negative” amount of $p_2$ was needed to make the match, because we added it to the test color’s side.

The primary color amounts needed for a match:

Slide from Durand and Freeman 06
CIE RGB color matching experiment

You are given three lasers pointed at the same spot, each producing light of a single wavelength ("monochromatic" primaries)

- 700 nm
- 546.1 nm
- 435.8 nm

You are asked to change the magnitude of the lasers to match the color of light from a fourth laser that has unknown wavelength (find a combination of the primaries that is a metamer to the spectrum emitted by the unknown laser.)

?? nm
Color matching experiment

Each point on the 3D curve describes the amount of each primary used in a combination that appears to be the same color as monochromatic light of a given wavelength.

Note, for visual clarity, this illustration uses primaries of slightly different wavelengths than described on the last slide.
Clarification

- The previous slide plots the results of the color matching experiment in a 3D space
  - That is: the figure plots how much of each primary light source is needed to create a spectrum that appears as the same color as a monochromatic reference (repeating the experiment over many monochromatic references yields a curve in 3D)
  - Repeating the experiment for many different monochromatic light sources yields the curve in the figure
  - The color of monochromatic light is represented as a 3-vector, which can be interpreted as how much of each primary is used to make the color

- Do not confuse this plot with the responses of S,M,L cones to monochromatic light... which was also plotted in 3D space earlier in the lecture
  - The color matching experiment works because the output of the visual system has dimensionality 3 (a.k.a. metameters that are the combination of three primaries exist), but the color matching experiment is not directly measuring the response of S,M,L cones.
CIE tristimulus matching values

- Graph plots “how much” of each laser (primary light) \((\vec{r}(\lambda), \vec{g}(\lambda), \vec{b}(\lambda))\) must be combined to create a spectrum that is perceived to be same color as monochromatic light of given wavelength (x axis)

And for any spectrum \(\Phi(\lambda)\), can express spectrum has weighted combination of primaries

\[
\begin{align*}
c_r &= \int_{\lambda} \Phi(\lambda) \vec{r}(\lambda) d\lambda \\
c_g &= \int_{\lambda} \Phi(\lambda) \vec{g}(\lambda) d\lambda \\
c_b &= \int_{\lambda} \Phi(\lambda) \vec{b}(\lambda) d\lambda
\end{align*}
\]

Wait a minute: negative red?

[Figure credit: Glassner]
Negative red primary?

- There is no positive combination of red, blue, green lasers that yields color that appears the same to a human as monochromatic light of 500 nm (“blue-greenish” light).

- But adding red primary to 500 nm target light yields light whose color can be matched by a combination of blue and green primaries.

[Figure credit: Glassner]
Color matching functions

Recall our analysis of color reproduction as linear algebra

\[
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix} = \left( \begin{bmatrix}
- & r_S & - \\
- & r_M & - \\
- & r_L & -
\end{bmatrix} \begin{bmatrix}
s_R & s_G & s_B
\end{bmatrix} \right)^{-1} \begin{bmatrix}
r_S & - & - \\
r_M & - & - \\
r_L & - & -
\end{bmatrix} \begin{bmatrix}
s
\end{bmatrix}
\]

\[
= \begin{bmatrix}
r_S \cdot s_R & r_S \cdot s_G & r_S \cdot s_B \\
r_M \cdot s_R & r_M \cdot s_G & r_M \cdot s_B \\
r_L \cdot s_R & r_L \cdot s_G & r_L \cdot s_B
\end{bmatrix}^{-1} \begin{bmatrix}
r_S & - & - \\
r_M & - & - \\
r_L & - & -
\end{bmatrix} \begin{bmatrix}
s
\end{bmatrix}
\]

This Nx3 matrix contains, as row vectors, “color matching functions” associated with the primary lights \( s_R, s_G, s_B \).
CIE 1931 XYZ primaries

- Primaries chosen so that convex combination of primaries includes all observable colors (color matching functions for these primaries are non-negative)

For any spectrum $\Phi(\lambda)$, can express spectrum as weighted combination of primaries:

$$X = k \int_\lambda \Phi(\lambda)x(\lambda) d\lambda$$

$$Y = k \int_\lambda \Phi(\lambda)y(\lambda) d\lambda$$

$$Z = k \int_\lambda \Phi(\lambda)z(\lambda) d\lambda$$

- What do the spectrum of the XYZ primaries look like? Unlike previous example where primaries were monochromatic R, G, B monochromatic lights, XYZ primaries do not correspond to any physically realizable light sources (they have negative spectra!)

[Image credit: Wikipedia]
Brightness

- The color matching experiments measure how a human observer perceives color. The goal was to match the perceived color of one spectrum with a new spectrum (a metamer) formed via the combination of three primaries.

- We can also ask the question, given lights with two different colors but equal power, how bright do the lights look?
Luminance (brightness)

- Product of radiance and the eye’s luminous efficiency

\[ Y = \int \Phi(\lambda) V(\lambda) \, d\lambda \]

- Luminous efficiency is measure of how bright light at a given wavelength is perceived by a human (due to the eye’s response to light at that wavelength)

- How to measure the eye’s response curve \( V(\lambda) \)?
  - Adjust power of monochromatic light source of wavelength \( \lambda \) until it matches the brightness of reference 555 nm source (photopic case)
  - Notice: the sensitivity of photopic eye is maximized at \( \sim 555 \) nm

https://upload.wikimedia.org/wikipedia/commons/a/a0/Luminosity.png
Properties of CIE 1931 XYZ primaries

- Color matching function for $Y$ primary is the eye’s luminous efficiency curve (so $Y$ component in XYZ representation of light’s color is the light’s luminance)

- By definition, all observable monochromatic spectra are positive points in XYZ space, so conversion from color representation from space defined by any realizable primaries to XYZ is possible via a linear transform:
  - Consider display with 3 primaries (primaries need not be monochromatic light)
  - Compute XYZ coords of light emitted by display when providing it $(1,0,0), (0,1,0), (0,0,1)$
  - Light generated by display is linear combination of these vectors (non-negative weights)

  \[
  \begin{align*}
  \text{color of R primary ([1,0,0] on display)} & = R_x X + R_y Y + R_z Z \\
  \text{color of G primary ([0,1,0] on display)} & = G_x X + G_y Y + G_z Z \\
  \text{color of B primary ([0,1,0] on display)} & = B_x X + B_y Y + B_z Z
  \end{align*}
  \]

- Example: Converting from CIE RGB to CIE XYZ:

  \[
  \begin{bmatrix}
  X \\ Y \\ Z
  \end{bmatrix} = \frac{1}{0.17697} \begin{bmatrix}
  0.49 & 0.31 & 0.20 \\
  0.17687 & 0.81240 & 0.01063 \\
  0.00 & 0.01 & 0.99
  \end{bmatrix} \begin{bmatrix}
  R \\ G \\ B
  \end{bmatrix}
  \]

  $X$, $Y$, $Z$ components of the color in space of display primaries
  $R$, $G$, $B$ components of the XYZ representation
Intensity-independent representation

- Chromaticity is the intensity-independent component of a color
- Project \((X,Y,Z)\) to \(X+Y+Z=1\) plane in XYZ space

\[
x = \frac{X}{X + Y + Z}
\]
\[
y = \frac{Y}{X + Y + Z}
\]
\[
z = \frac{Z}{X + Y + Z} = 1 - x - y
\]

Can recover \((X, Y, Z)\) from \((x, y, Y)\)

\[
X = \frac{x}{y}
\]
\[
Y = Y
\]
\[
Z = \frac{1 - (x + y)}{y} Y
\]

Plot of Chromaticity \((x, y)\)

Spectrum loci (all monochromatic spectra)

All additive combinations of monochromatic sources
Uses of chromaticity diagram

Complementary colors are colors that can be mixed to form a designated white. A and B and C and D are complementary with respect to reference white W.

Demonstrate colors that fall out-of-gamut for a given choice of primaries.

A display with primaries with chromacities $P_1$, $P_2$, $P_3$ can create colors that are combinations of these primaries (colors that fall within the triangle).
What is white?

“White point” of display is the X,Y,Z value of (1,1,1) in space of display’s primaries

“Warmth” of white light is often described by how chromaticity coordinates of (1,1,1) on display relate to that of spectrum emitted by black-body radiator of given temperature.
Gamut

sRGB is a common color space used throughout the internet

sRGB uses ITU-R BT.709 primaries
Red  Green  Blue  White
x  0.64  0.30  0.15  0.3127
y  0.33  0.60  0.06  0.3290
AdobeRGB(98) uses Red and Blue like sRGB and Green like NTSC
CIE-RGB are the primaries for color matching tests: 700/546.1/435.8nm
Perceptually Organized Color Spaces
HSV (hue-saturation-value)

- Axes of space correspond to natural notions of “characteristics” of color.
Perceptual dimensions of color

- **Hue**
  - the “kind” of color, regardless of attributes
  - colorimetric correlate: dominant wavelength
  - artist’s correlate: the chosen pigment color

- **Saturation**
  - the “colorfulness”
  - colorimetric correlate: purity
  - artist’s correlate: fraction of paint from the colored tube

- **Lightness (or value)**
  - the overall amount of light
  - colorimetric correlate: luminance
  - artist’s correlate: tints are lighter, shades are darker
**Y’CbCr color space**

Common for modern digital video

Y’ = luma: perceived luminance (same as L* in CIELAB)

Cb = blue-yellow deviation from gray

Cr = red-cyan deviation from gray

Conversion from R’G’B’ to Y’CbCr:

\[ Y’ = 16 + \frac{65.738 \cdot R’_D}{256} + \frac{129.057 \cdot G’_D}{256} + \frac{25.064 \cdot B’_D}{256} \]  

\[ C_B = 128 + \frac{-37.945 \cdot R’_D}{256} + \frac{74.494 \cdot G’_D}{256} + \frac{112.439 \cdot B’_D}{256} \]  

\[ C_R = 128 + \frac{112.439 \cdot R’_D}{256} + \frac{94.154 \cdot G’_D}{256} + \frac{18.285 \cdot B’_D}{256} \]  

(Primed notation indicates perceptual (non-linear) space)

Example: compression in Y’CbCr

Original picture of Kayvon
Example: compression in Y’CbCr

Contents of CbCr color channels downsampled by a factor of 20 in each dimension
(400x reduction in number of samples)
Example: compression in Y’CbCr

Full resolution sampling of luma (Y’)

Stanford CS248, Spring 2018
Example: compression in Y’CbCr

Reconstructed result
(looks pretty good)
Perceptual non-uniformity

- In the xy chromaticity diagram at left, MacAdam ellipses show regions of perceptually equivalent color (ellipses enlarged 10x).

- Must non-linearly warp the diagram to achieve uniform perceptual distances.
Perceptual color spaces

Problem: Euclidean distance in color space is not proportional to perceived differences in color

- Idea of CIELAB: non-linearly transform XYZ coordinates into a space where Euclidean distances in more closely match perceived color distances
- \( L^* \) (perceived brightness relative to a reference white) is one basis
- Chromaticity described by two opponent color axes:
  - \( a^* \): blue/green vs. magenta
  - \( b^* \): blue vs. yellow
Opponent color theory

- There’s a good neurological basis for the color space dimensions in CIE LAB
  - the brain seems to encode color early on using three axes:
    - white — black, red — green, yellow — blue
  - the white — black axis is lightness; the others determine hue and saturation
  - one piece of evidence: you can have a light green, a dark green, a yellow-green, or a blue-green, but you can’t have a reddish green (just doesn’t make sense)
    - thus red is the opponent to green
  - another piece of evidence: afterimages (following slides)
Even simple judgments – such as lightness - depend on brain processing (Anderson and Winawer, Nature, 2005)
Everything is relative

Edward H. Adelson
Everything is relative
Everything is relative
Everything is Relative
Everything is relative
Everything is relative
Everything is relative
Everything is relative
Things to Remember

- Physics of Light
  - Spectral power distribution (SPD)
  - Superposition (linearity)

- Tristimulus theory of color
  - Spectral response of human cone cells (S, M, L)
  - Metamers - different SPDs with the same perceived color
  - Color reproduction mathematics
  - Color matching experiment, per-wavelength matching functions

- Color spaces
  - CIE RGB, XYZ, xy chromaticity, LAB, HSV
  - Gamut
Extras
Subtractive Color

- Produce desired spectrum by subtracting from white light (usually via absorption by pigments)
- Photographic media (slides, prints) work this way
- Leads to C, M, Y (cyan, magenta, yellow) as primaries
- Approximately, 1 – R, 1 – G, 1 – B
Subtractive color spaces

- Up to this point, we’ve described color in terms of superposition of primaries. (addition of primaries)
  - Good description of color formed by mixing light from multiple light sources (e.g., displays)

- When describing color of reflected light, each mixed primary contributes to absorption of light (requires a subtractive color space)
  - Describes how to form colors by mixing inks (e.g., printing)

CMYK Representation

- CMYK adds 4th component (K=black) as using a single black ink gives better black (and is more ink efficient) than mixing CMY primaries

(0,0,0) is white
(1,1,1,) is black
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