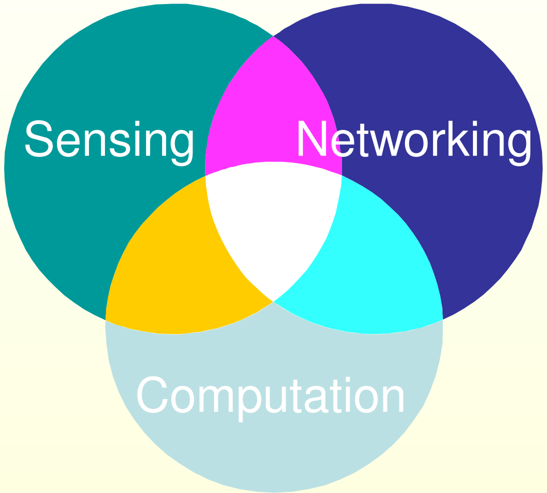
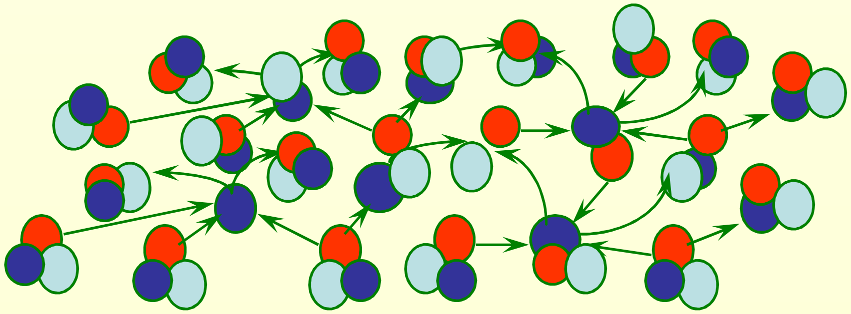
CS321: Wireless Links and Communication Models

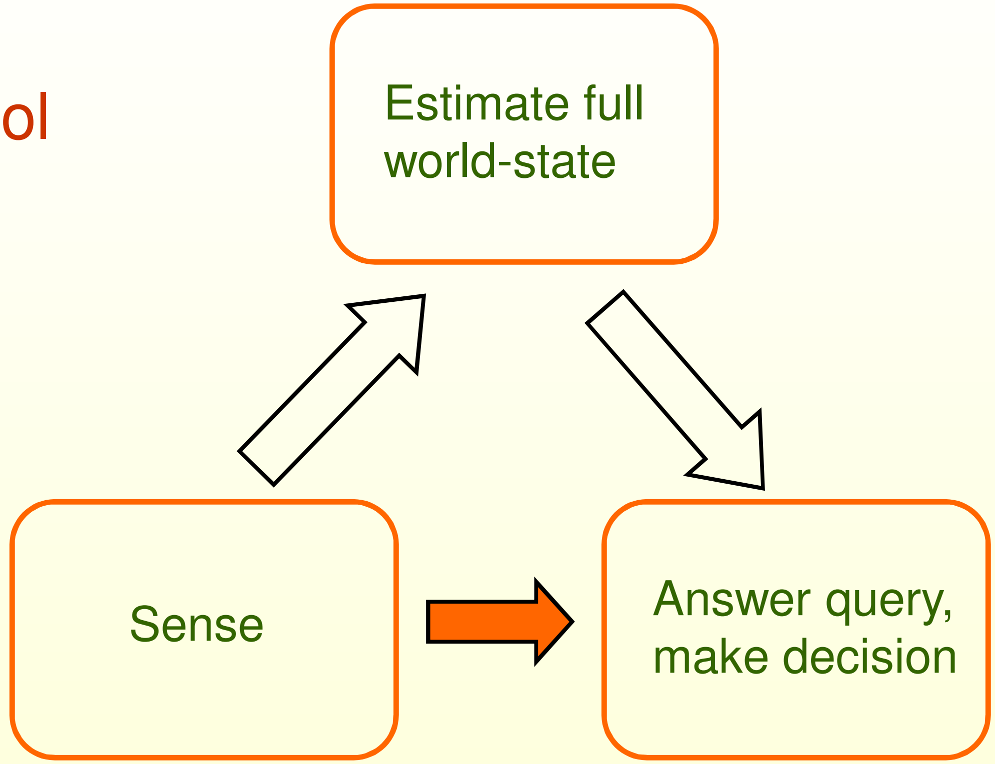


Leonidas Guibas
Computer Science Dept.
Stanford University



Sensor Network Research

- power awareness
- sensor tasking and control
- formation of sensor collaboration groups
- in-network, distributed processing
- node management, service establishment, software layers
- coping with noise and uncertainty in the environment



Estimate full world-state

Sense

Answer query, make decision

A key algorithmic problem is how to sense and aggregate only the portions of the world-state relevant to the task at hand, in a lightweight, energy-efficient manner.

Wireless Links

Radio Signal Propagation

- Ideal (free space, isotropic)

$$P_r = \frac{P_t A_r}{4\pi r^2}$$

- In reality, mutlipath, fading, and shadowing effects result in a law more like

$$P_r \propto \frac{P_t}{r^\alpha}, \text{ where } 2 \leq \alpha \leq 6$$

- Many multipath and shadowing models exist (Raleigh, Rice, lognormal, ...) for different settings and materials

Power Attenuation

- In log (decibel) form, say for an office environment, power loss (PL) as a function of distance is:

$$\mathrm{PL}(d) = \mathrm{PL}(d_0) + 10\alpha \log(d/d_0) + \mathrm{FAF} + \sum \mathrm{PAF}$$

- So, say for *d* = 10 m on same floor, w. 3 partitions, d_0 = 1 m, PL(d_0) = 3, α = 3, FAF = 12, PAF = 1, we get:

$$\mathrm{PL}(10) = 3 + 10 \times 3 \log(10) + 0 + 3 \times 1 = 36\,\mathrm{db}$$

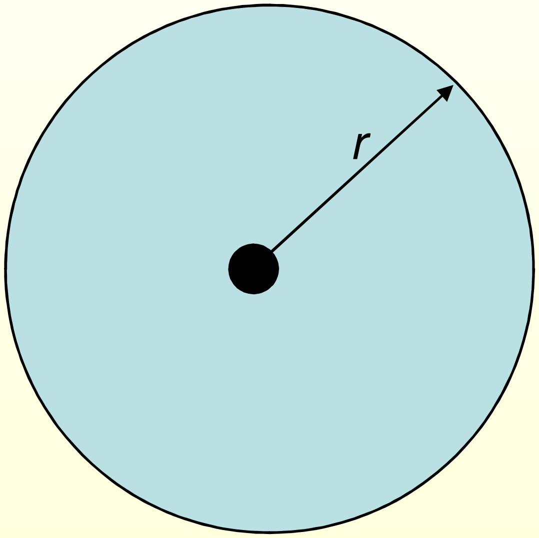
[From Pottie and Kaiser ‘05]

Simplistic Signal Reception

- Received signal strength (RSS) has to be high enough
- Assume no interference

$$P_r \geq \beta, \text{ or equivalently } P_t \geq \beta d^{\alpha}$$

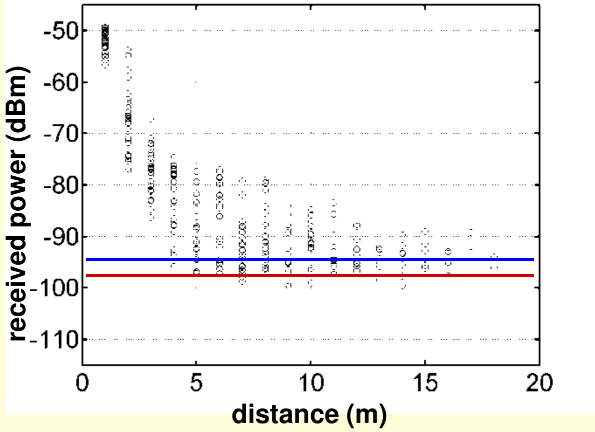
$$\text{SNR actually, } P_r/N_r \geq \beta$$



`Unit-disk graph' (UDG) model

Circular radio range with perfect reception within & zero reception outside

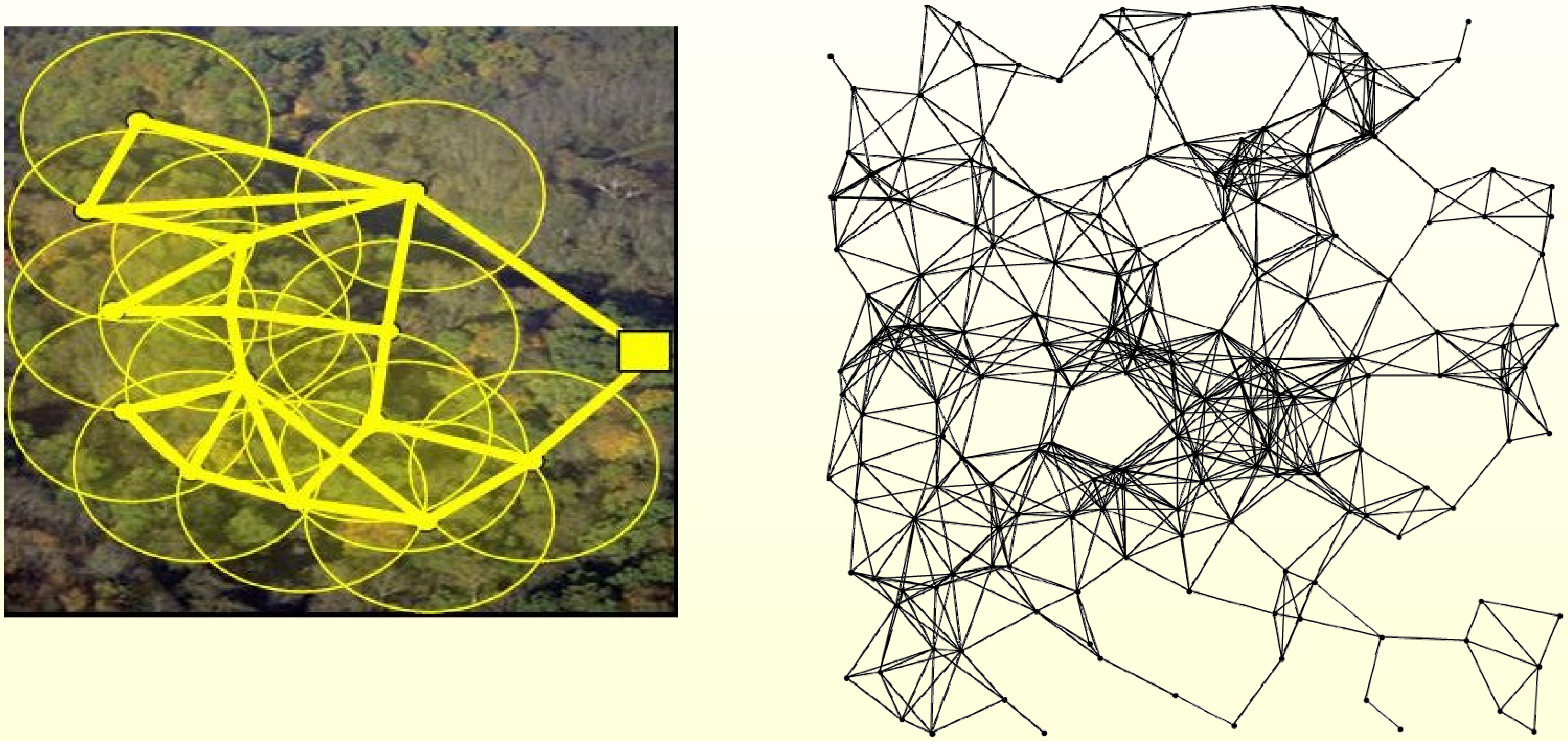
Empirical: channel behavior



RSS

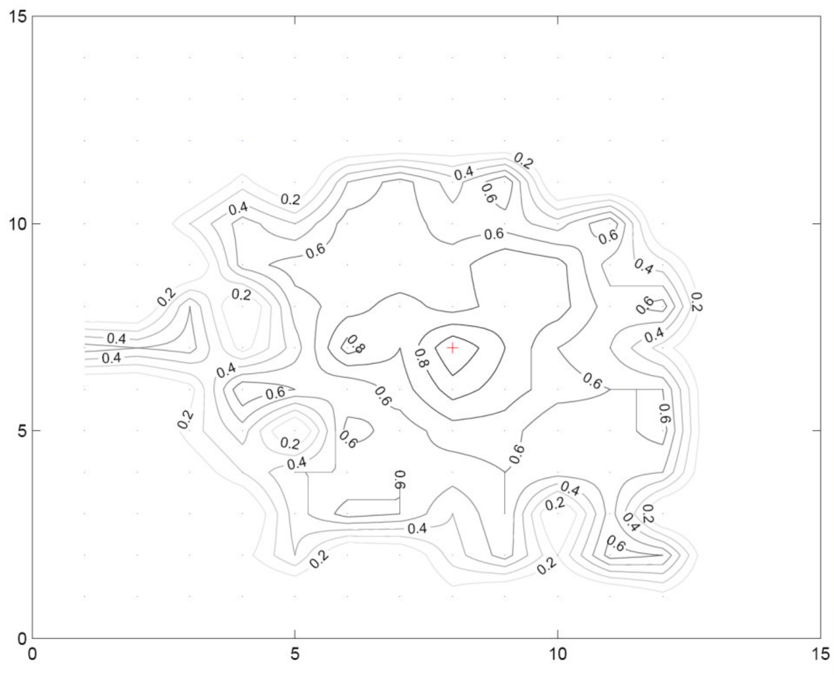
[From Zuniga et. al. '07]

Network Connectivity Graph



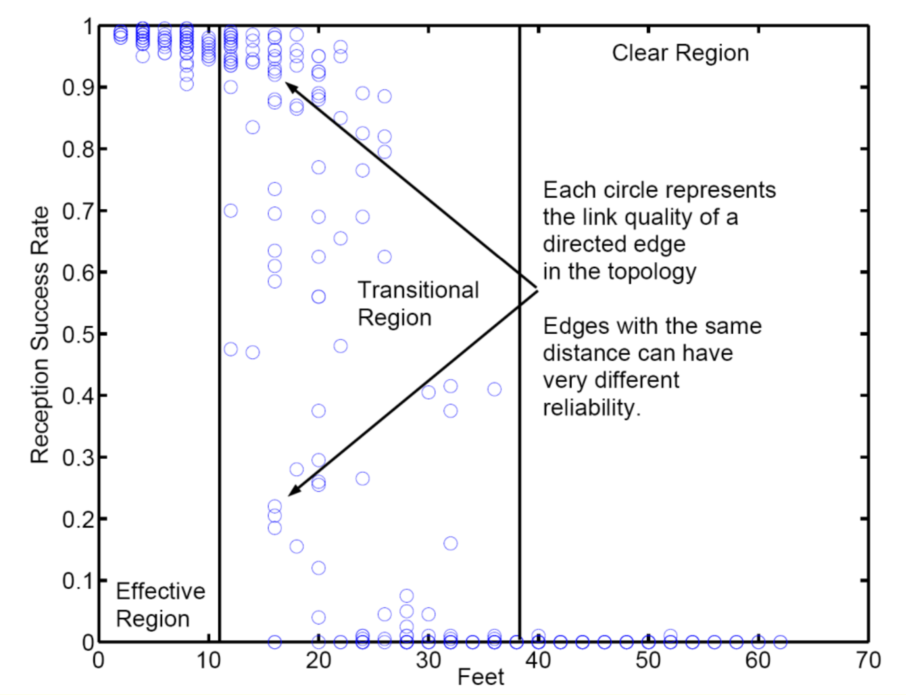
But In Reality

Propagation is not isotropic



[From Ganesan et.al. '02]

Links are not just there or not

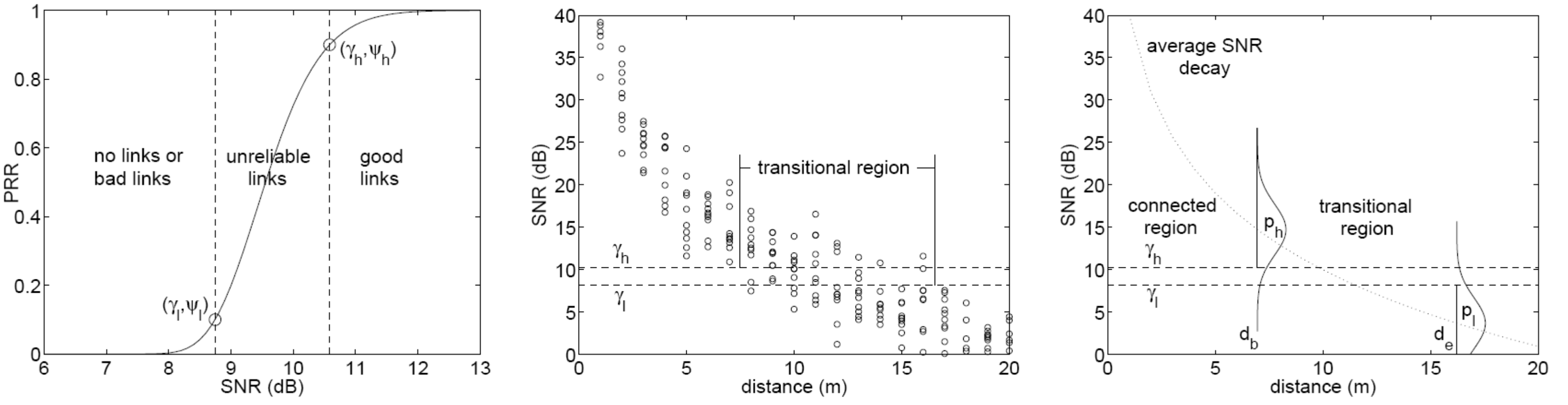


Each circle represents the link quality of a directed edge in the topology

Edges with the same distance can have very different reliability.

[From Woo et.al. '03]

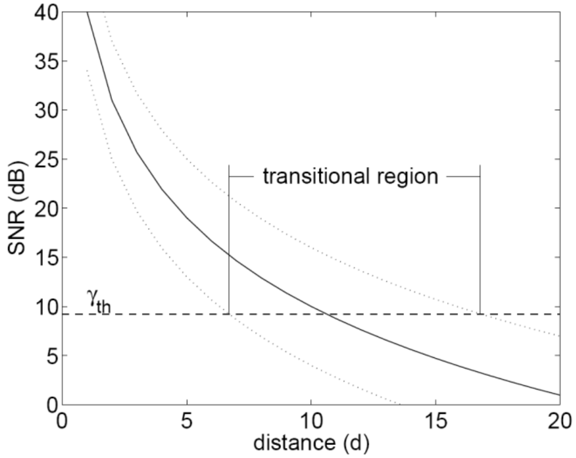
There is a Transitional Region



(a) (b) (c)

- SNR = signal to noise ratio
- PRR = packet reception rate

[From Zuniga et. al. '07]

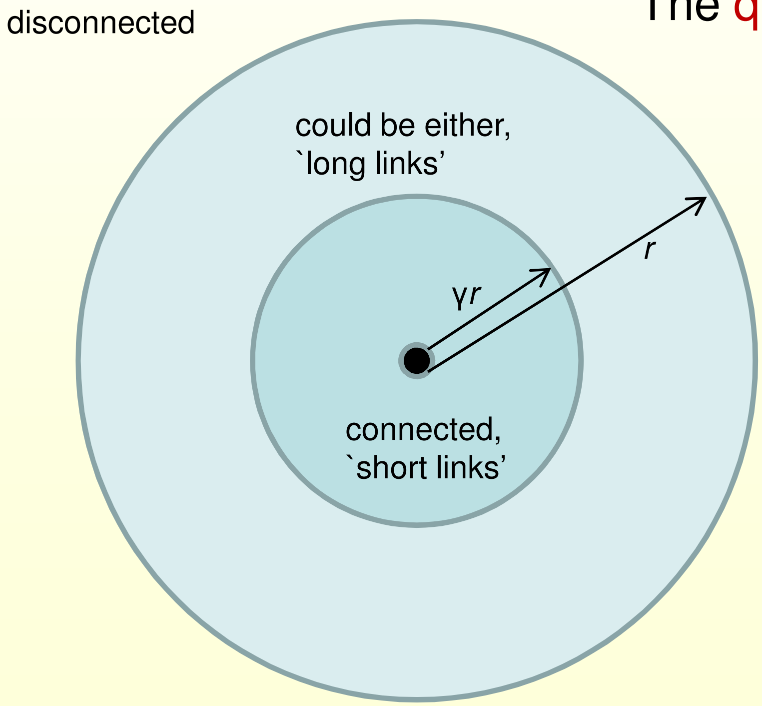


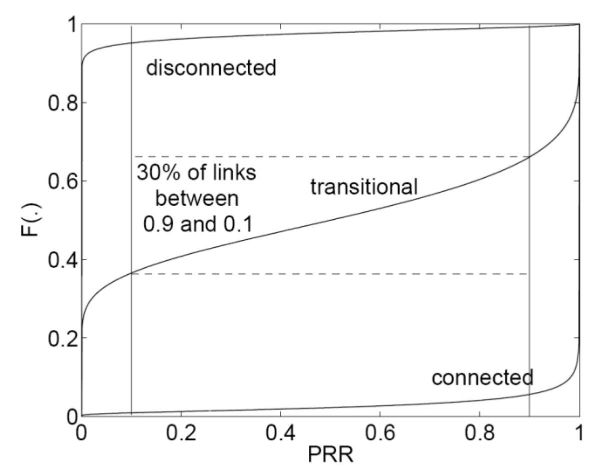
$$P_r(d) = P_t - \mathrm{PL}(d_0) - 10\,\alpha\,\log(d/d_0) + \mathcal{N}(0,\sigma)$$

Quasi-Unit Disk Graph Models

A majority of the links are either good (PRR above 90%) or bad (PRR below 10%), matching empirical findings (e.g., Cerpa *et al.* ’05)

The q-UDG model



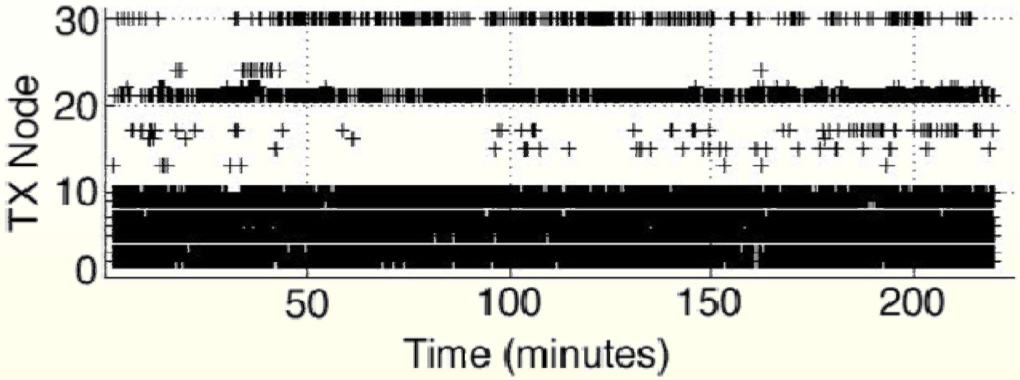


[From Zuniga et. al. ‘07]

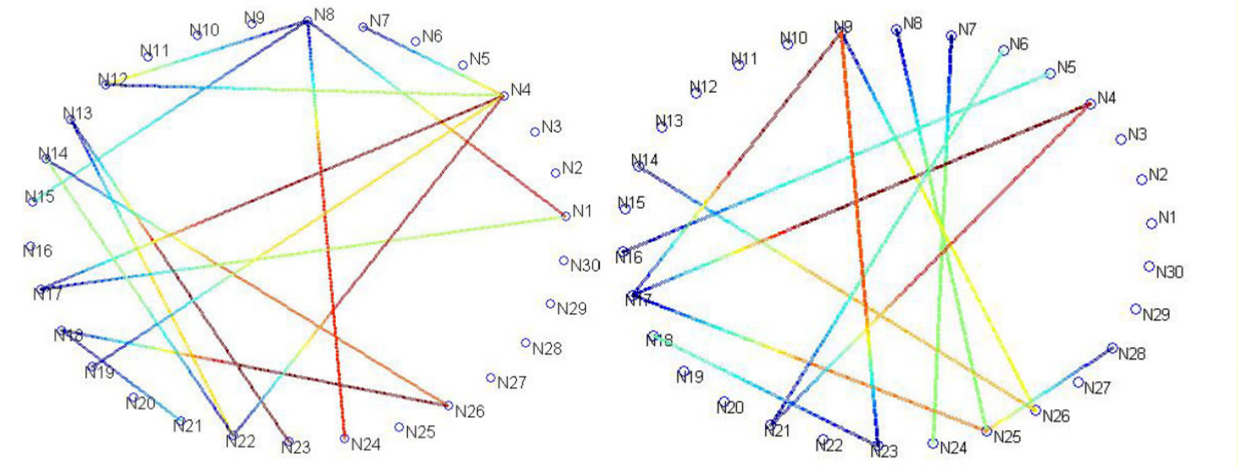
Link Modeling Headaches

- Should we try to exploit the long transitional links? Studies have shown that opportunistic exploitation of long links can be beneficial...
- On the other hand, we cannot rely on these links – but hop-minimizing protocols will always want to use them ...
- Asymmetries often arise in the transition region (*A* can talk to *B*, but not *B* to *A*) – there are two types.
- There are also significant temporal variations in link quality – especially in the transitional regions. Bursty behavior is common.

Temporal Link Variability



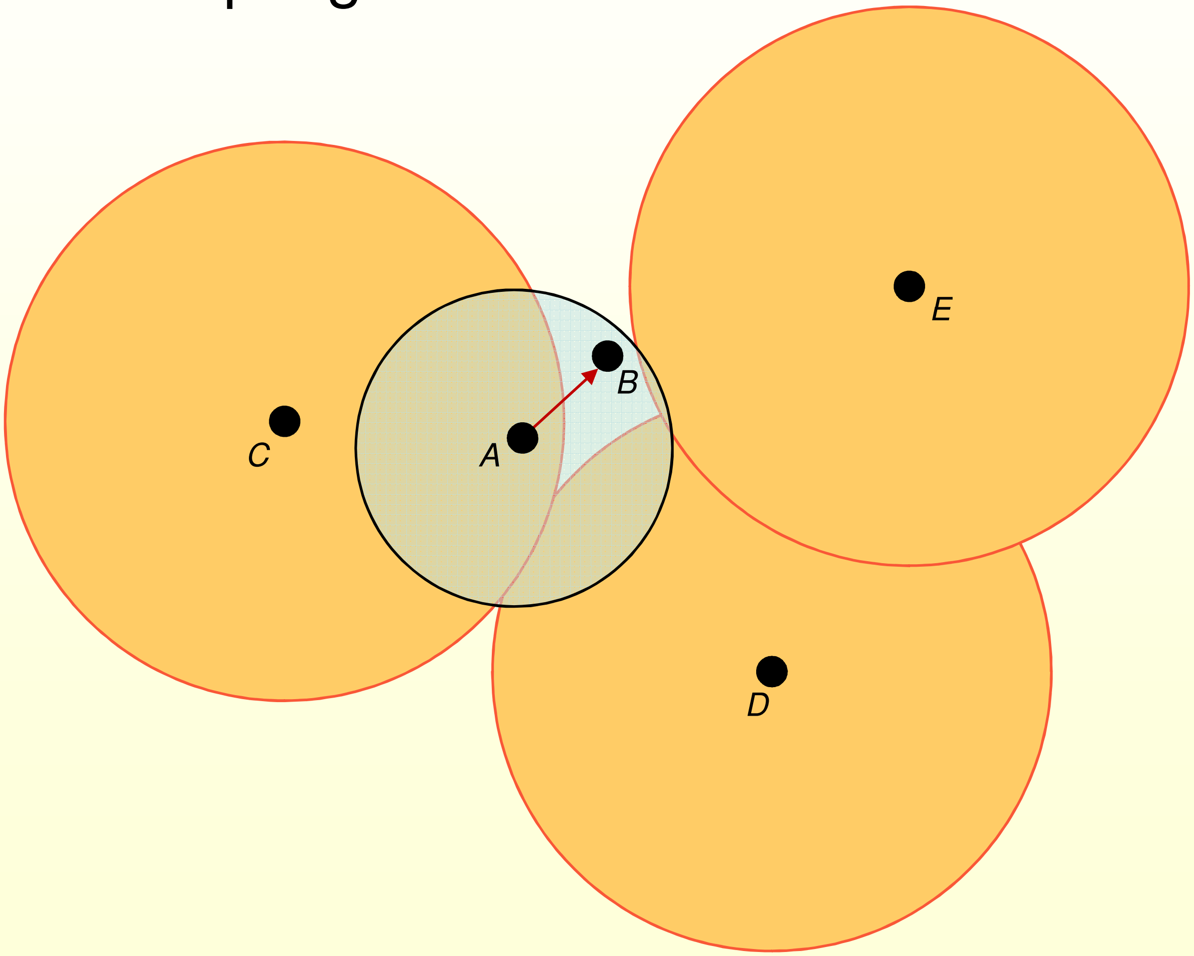
[From Srinivasan et. al. ‘06]



(a) Wednesday (b) Saturday

Modeling Interference

- A simple geometric model

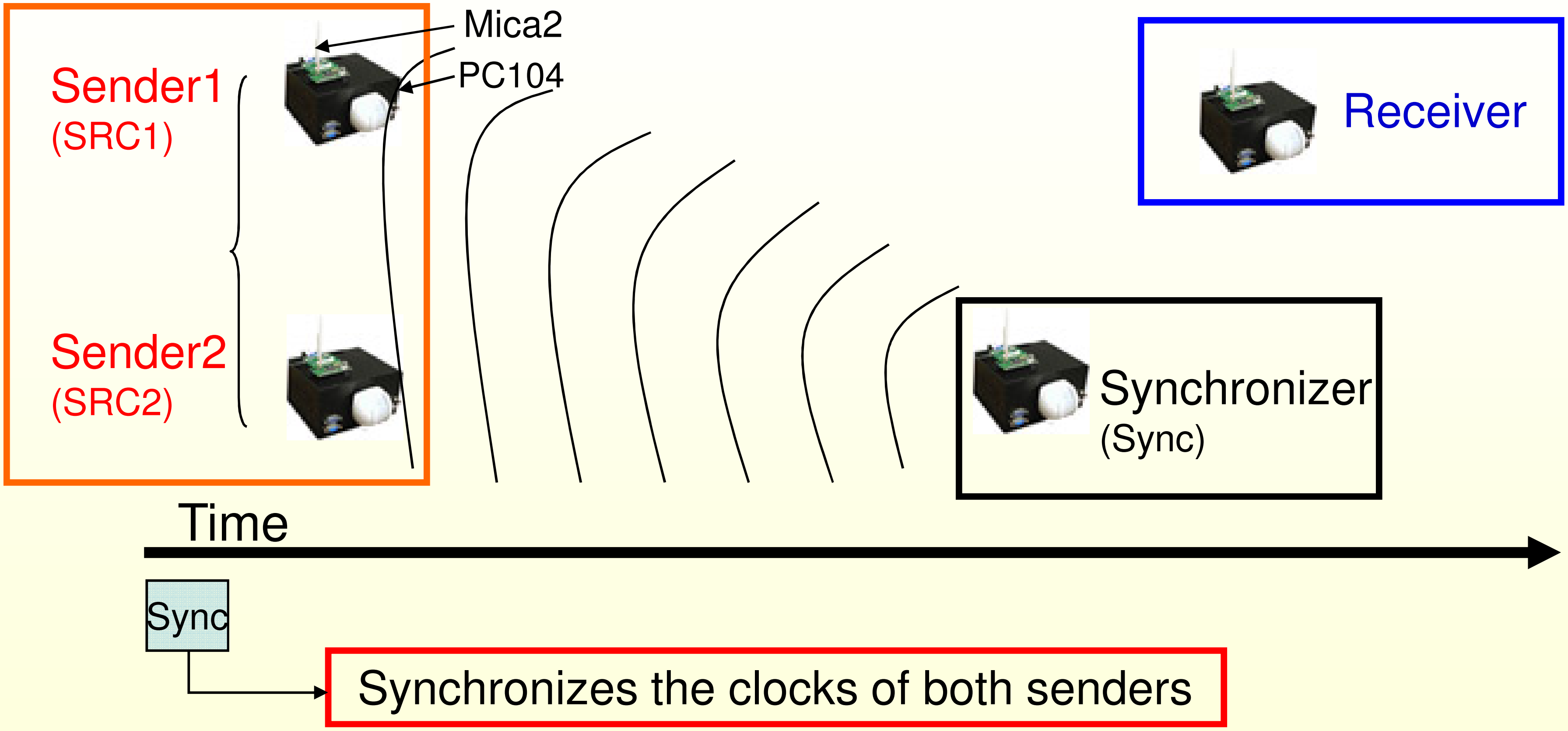


Each node has both
a communication range
and a (usually larger)
interference range

Collision Avoidance (CSMA/CA)

- MACA [Karn ‘90]: A way of virtually sensing the channel at receiver, with “request-to-send”/“clear-to-send” messages:
    - If a node has packet to send, it first transmits a RTS packet.
    - If available, the receiver responds with CTS.
    - After sender receives CTS, it transmits data.
    - Nodes that hear RTS wait long enough for transmitter to receive CTS.
    - Nodes that hear CTS back off long enough to allow receiver to receive data.

Single Interferer



Sender1
(SRC1)

Sender2
(SRC2)

Mica2
PC104

Receiver

Synchronizer
(Sync)

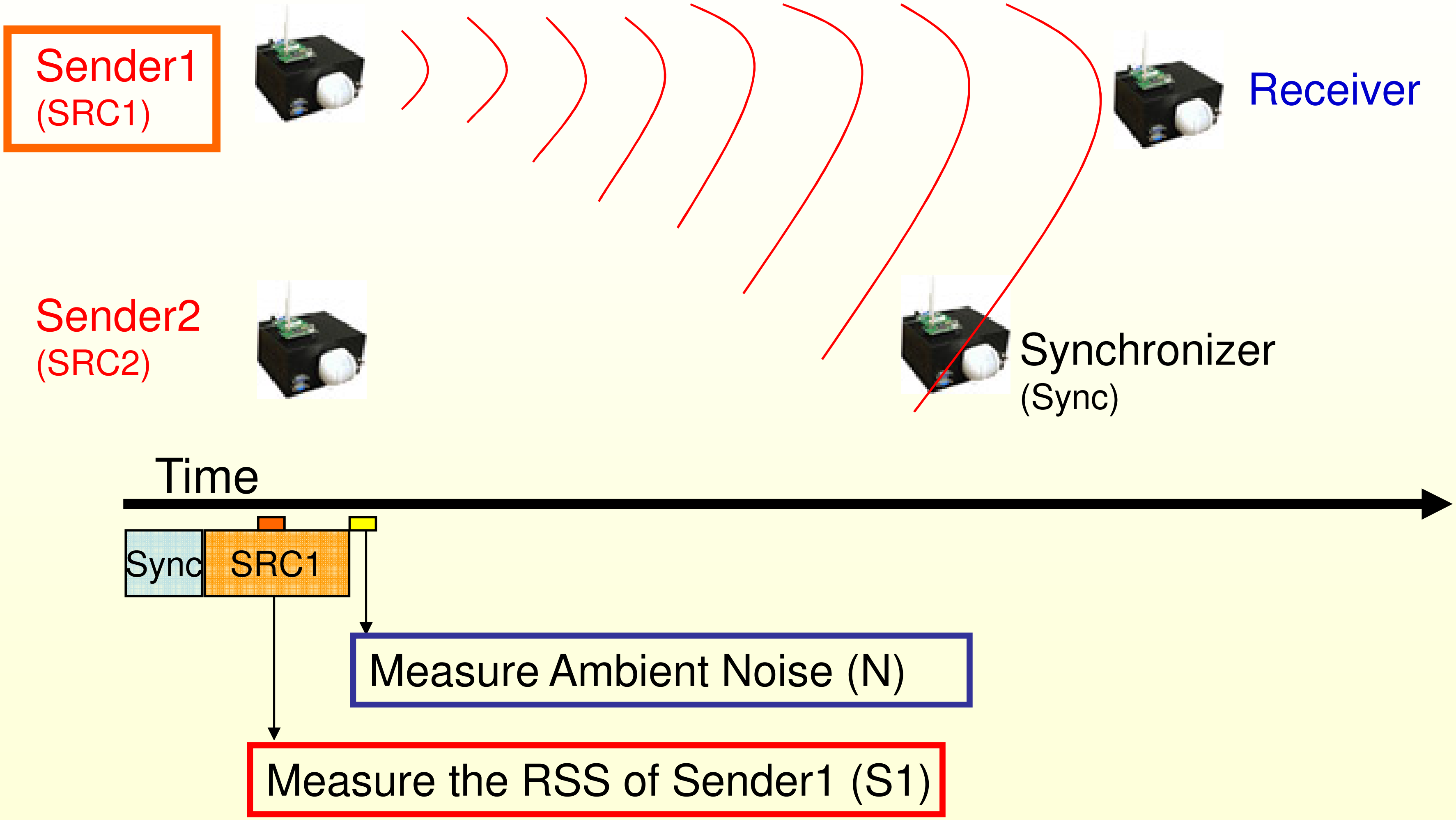
Time

Sync

Synchronizes the clocks of both senders

[From Son et. al. 06]

Single Interferer



Sender1
(SRC1)

Receiver

Sender2
(SRC2)

Synchronizer
(Sync)

Time

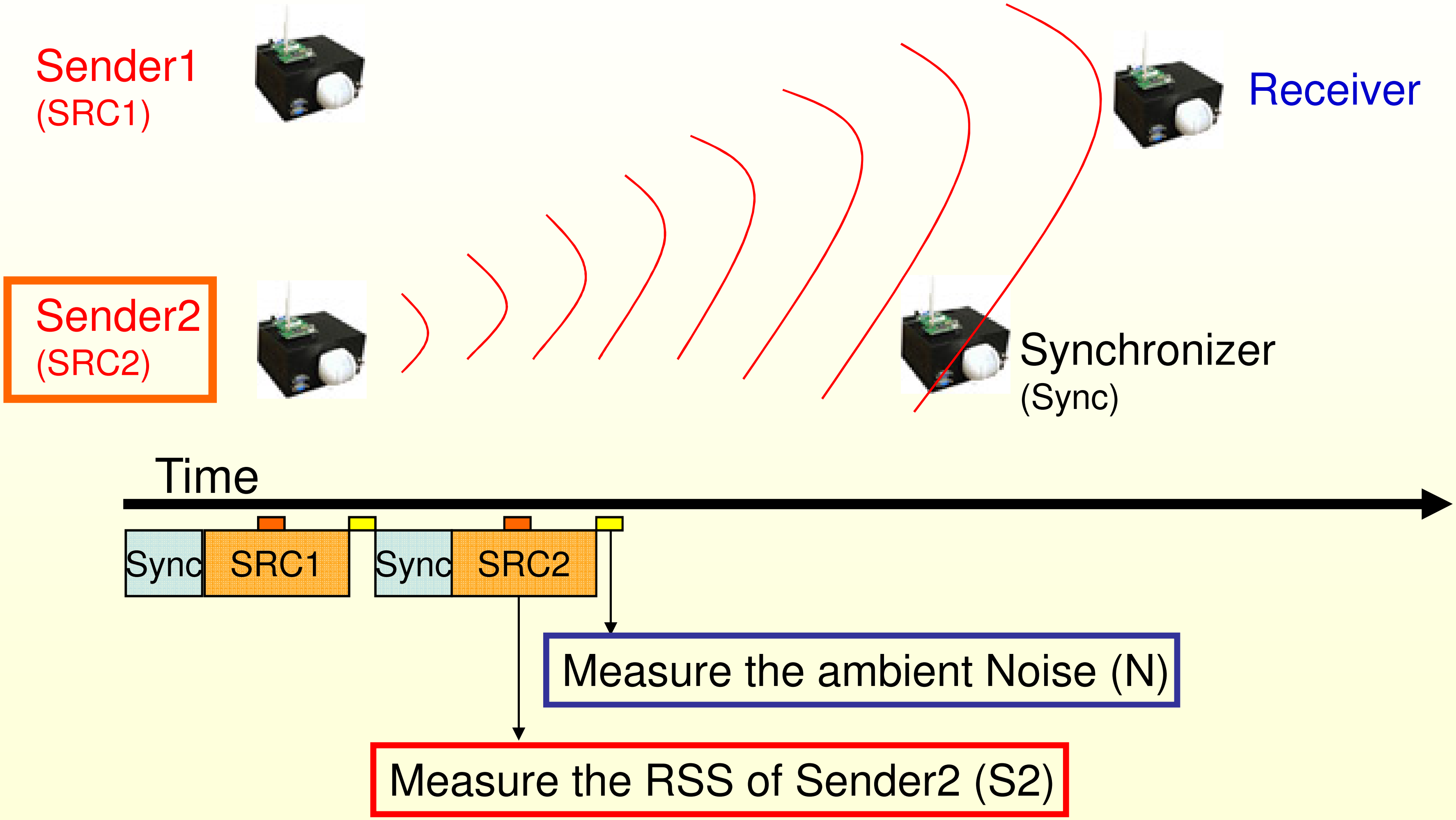
Sync | SRC1

Measure Ambient Noise (N)

Measure the RSS of Sender1 (S1)

[From Son et. al. 06]

Single Interferer



Sender1 (SRC1)

Sender2 (SRC2)

Receiver

Synchronizer (Sync)

Time

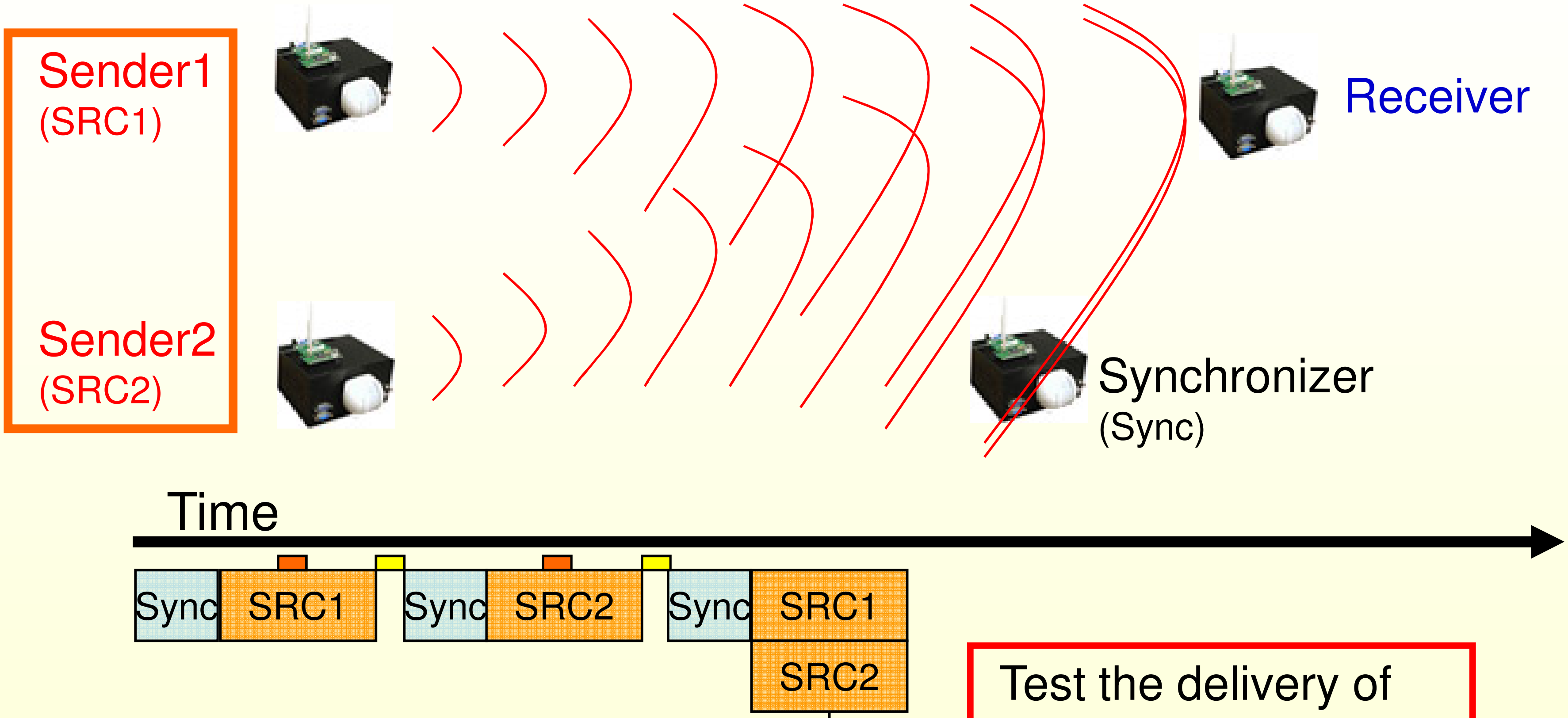
Sync SRC1 Sync SRC2

Measure the ambient Noise (N)

Measure the RSS of Sender2 (S2)

[From Son et. al. 06]

Single Interferer



Sender1
(SRC1)

Sender2
(SRC2)

Receiver

Synchronizer
(Sync)

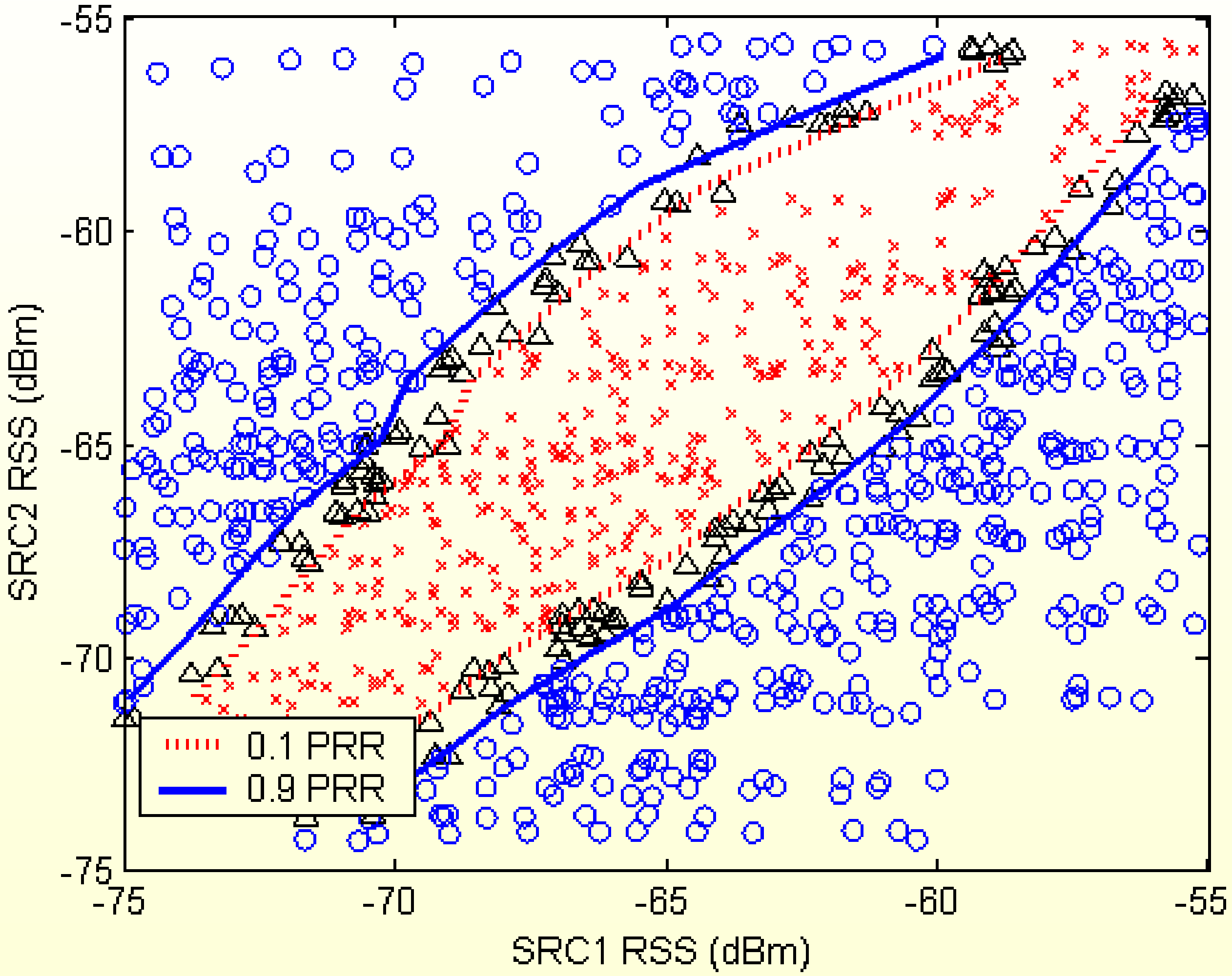
Time

Sync SRC1 Sync SRC2 Sync SRC1
SRC2

Test the delivery of the sender's packet under the CTX

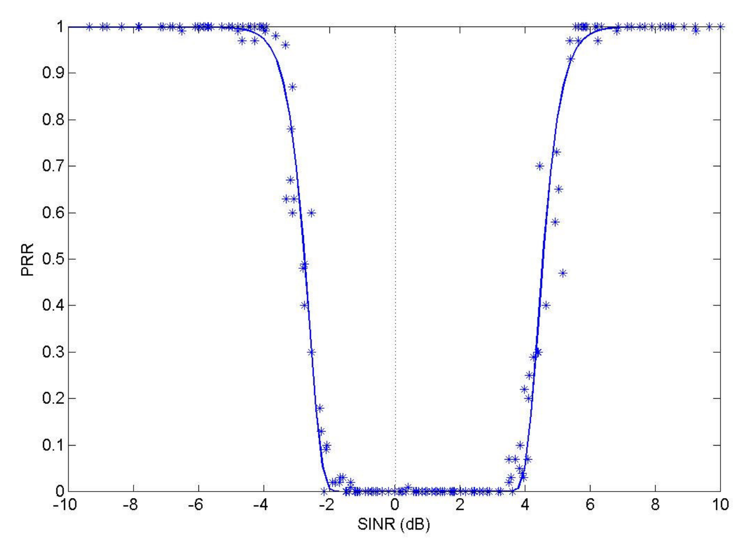
[From Son et. al. 06]

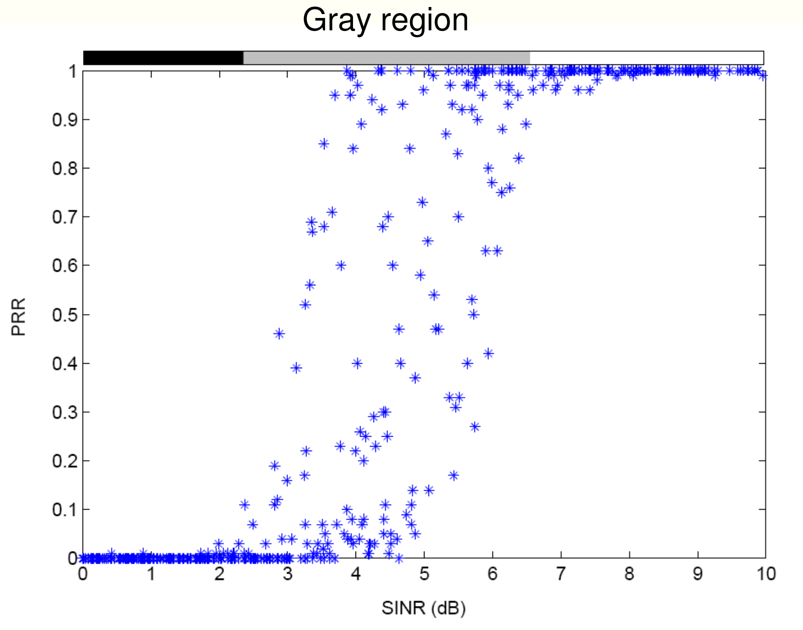
The Capture Effect



[From Son et. al. 06]

Signal to Interference plus Noise Ratio (SINR)



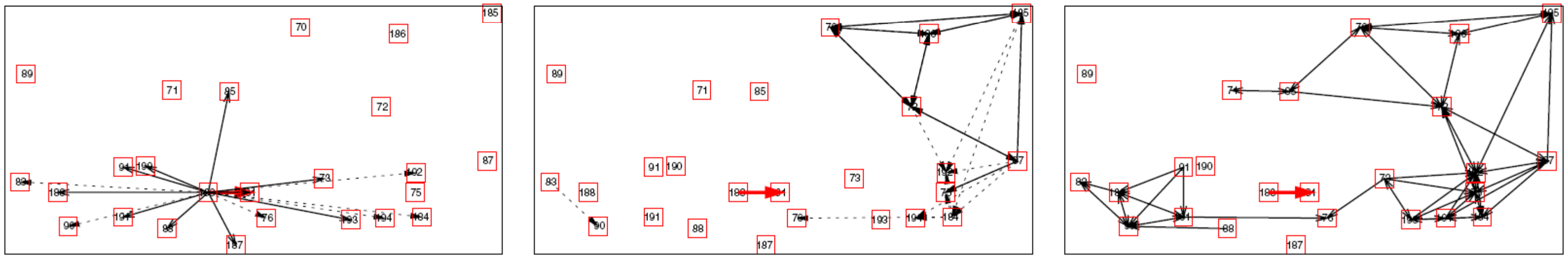


For packet reception, SINR has to be above a certain threshold –
but the threshold can depend on both the received interference strength,
as well as on the particular hardware involved.

[From Son et. al. 06]

Talking Across Interference

- More links may be available than geometric model would allow



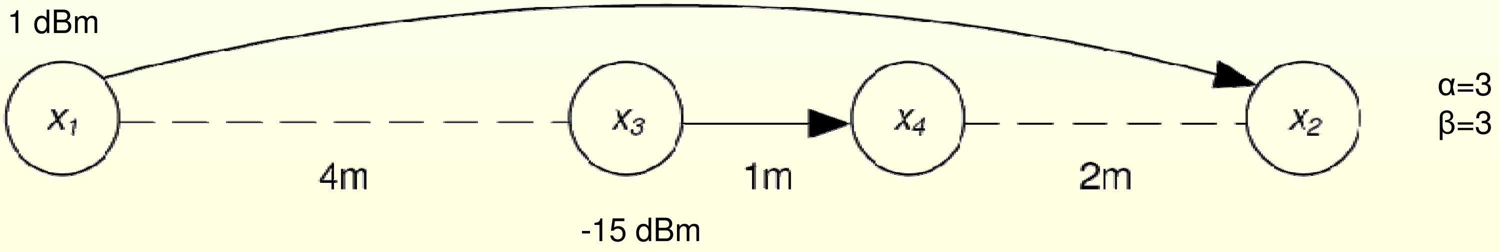
(a) Interference from the sender (b) CTXable links under old model (c) CTXable links with new model

[From Son et. al. 06]

Physical Interference Model

- Packet reception from x_i to x_j happens if SINR is high enough

$$\frac{P_i / d(x_i, x_j)^\alpha}{N + \sum_{x_k \in X \setminus \{x_i\}} P_k / d(x_j, x_k)^\alpha} \geq \beta$$



- Theoretically, the physical interference model allows information transmissions at rates exponentially larger than the geometric model [Moscibroda '07]

Dartmouth Study: Questionable Wireless Network Assumptions

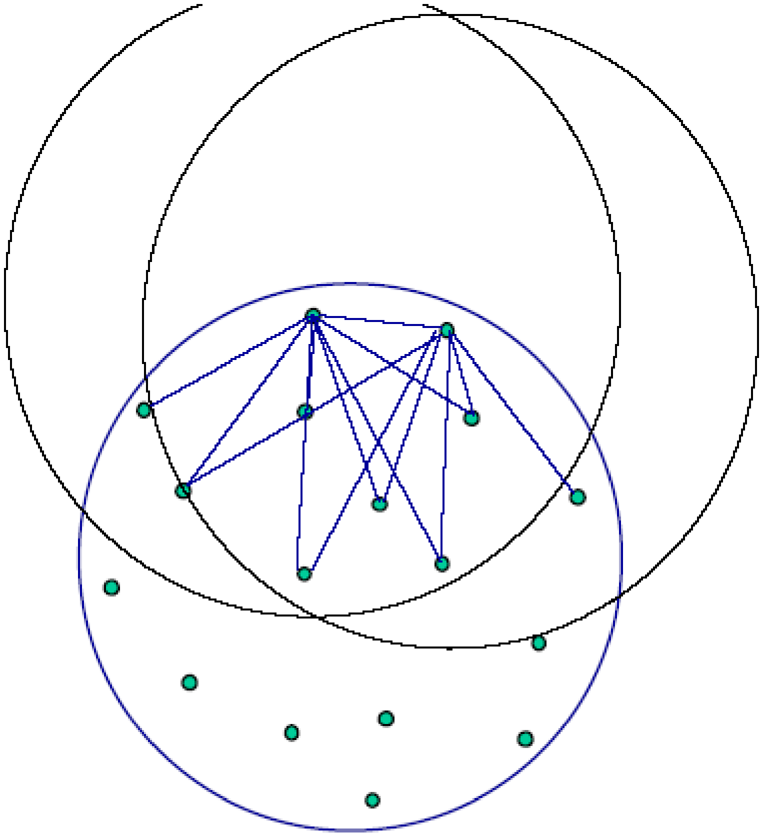
- The world is flat
- A radio’s transmission range is circular
- All radios have equal range
- If I can year you, you can hear me (symmetry)
- If I can hear you at all, I can hear you perfectly
- Signal strength is a simple function of distance

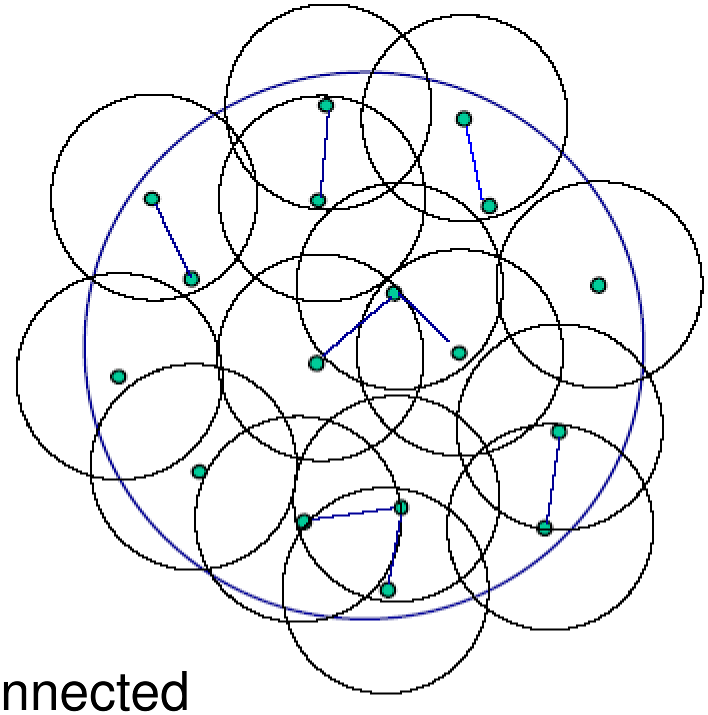
“The Mistaken Axioms of Wireless-Network Research”
(Kotz et. al. ‘03)

Basic Communication Infrastructure Establishment: Topology Discovery and Control

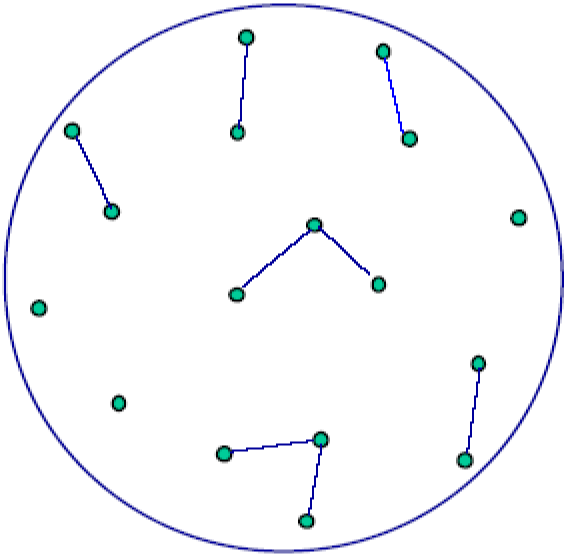
Topology Discovery and Control

- Each node must discover which other nodes it can talk to directly
- This depends on the radio power setting – a node may be able to vary that setting according to local conditions
- These elementary connections establish the topology of the network
- We always want radio power settings so that a network that is connected
- But ranges that are too long waste power and cause interference
- Assume for now that node locations are known

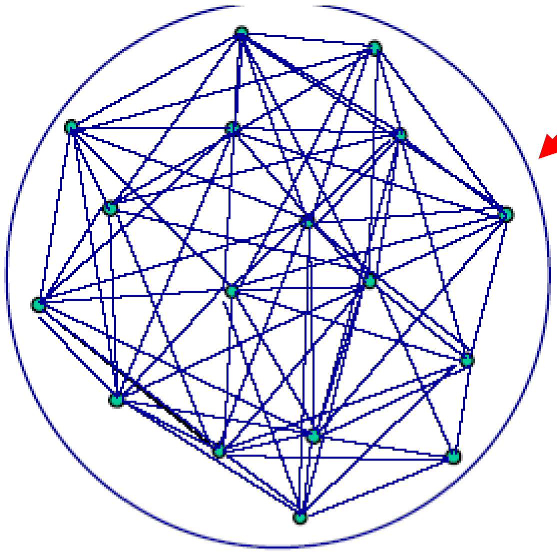




underconnected

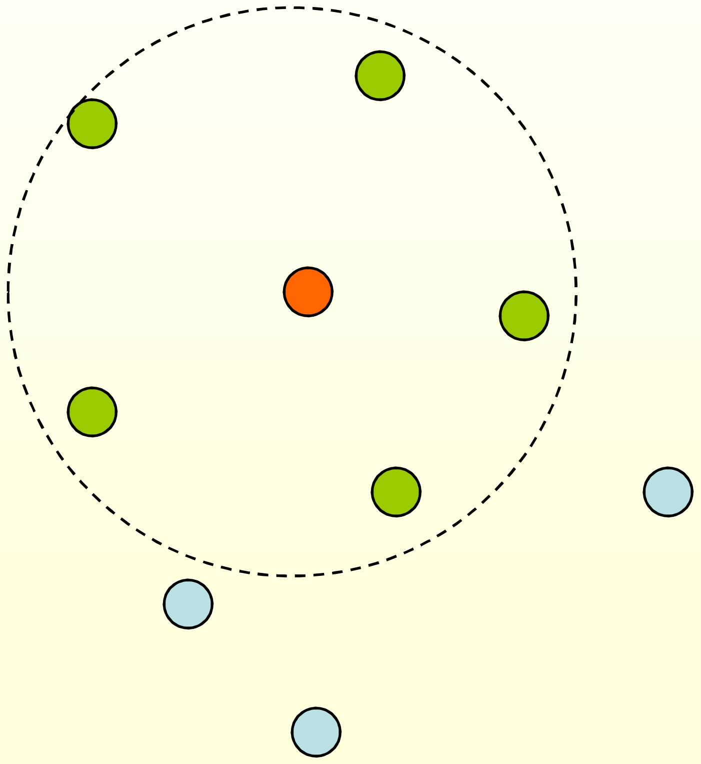


overconnected

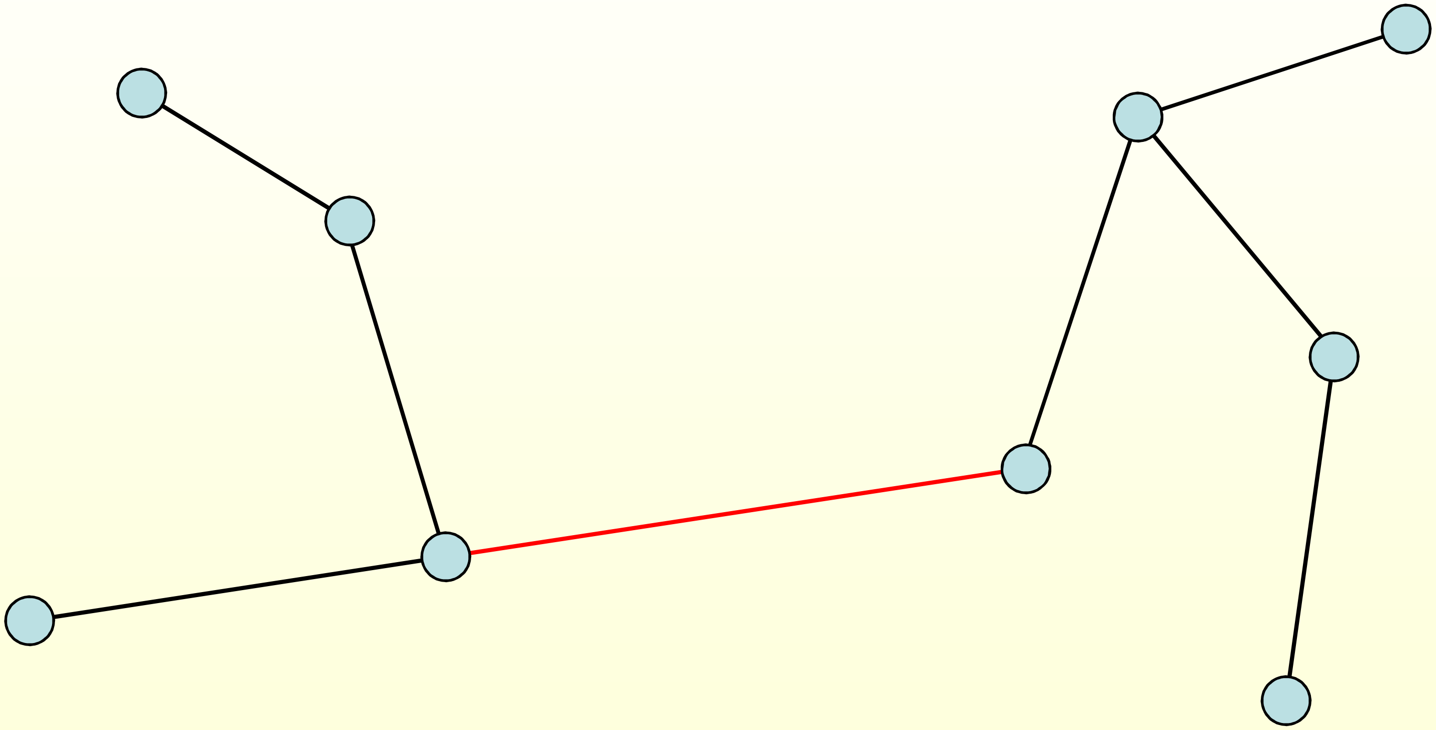


The Critical Transmitting Range (CTR) Problem

- Assume all nodes must use exactly the same radio range
- How can we compute the minimum radio range that is guaranteed to just connect all the nodes?
- Theorem: It is the length of the longest edge of an MST connecting the nodes
- The MST can be computed in a distributed fashion



Why The MST Solves the CTR Problem



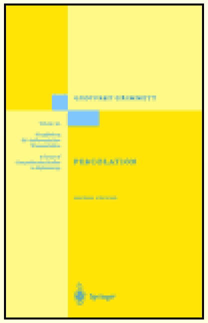
A Probabilistic Variant

- Say *n* points are dropped in the unit square randomly and uniformly. What can we say about the CTR *r* ?
- With high probability it will be:

$$r = c\sqrt{\frac{\log n}{n}}$$

- Result from *Percolation Theory* and *Geometric Random Graph Theory* – there is a critical constant *c* for a threshold effect

A Little Excursion into Percolation Theory

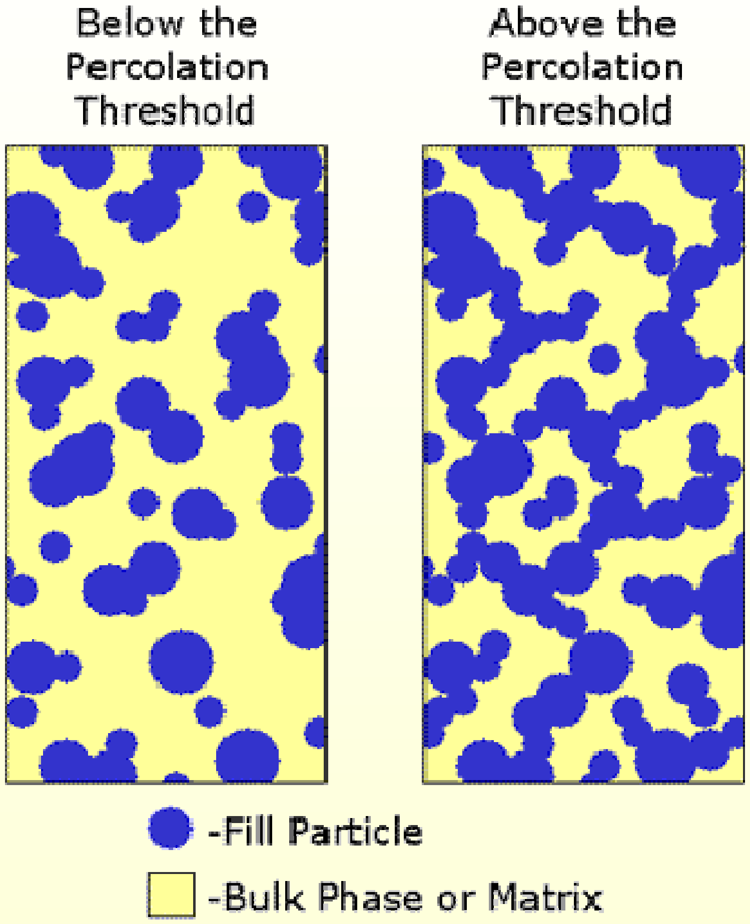


Geoffrey Grimmett, **Percolation**, first chapter, Second edition, Springer, 1999

On A Rainy Day ...

- Observe the raindrops falling on the pavement. Initially the wet regions are isolated and we can find a dry path. Then after some point, the wet regions are connected and we can find a wet path.

- There is a critical density where sudden change happens.



Phase Transitions

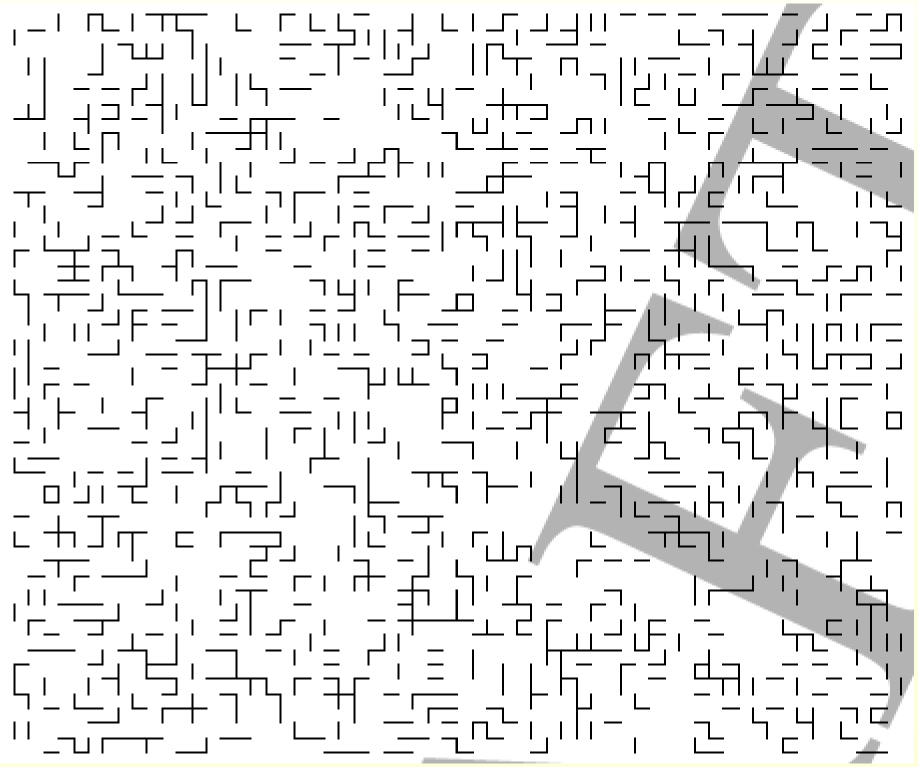
- In physics, a **phase transition** is the transformation of a thermodynamic system from one **phase** to another. The distinguishing characteristic of a **phase transition** is an abrupt sudden change in one or more physical properties, in particular the heat capacity, with a small change in a thermodynamic variable such as the temperature.

- Solid, liquid, and gaseous phases.
- Different magnetic properties.
- Superconductivity of metals.

- This generally stems from the interactions of an extremely large number of particles in a system, and does not appear in systems that are too small.

Bond Percolation

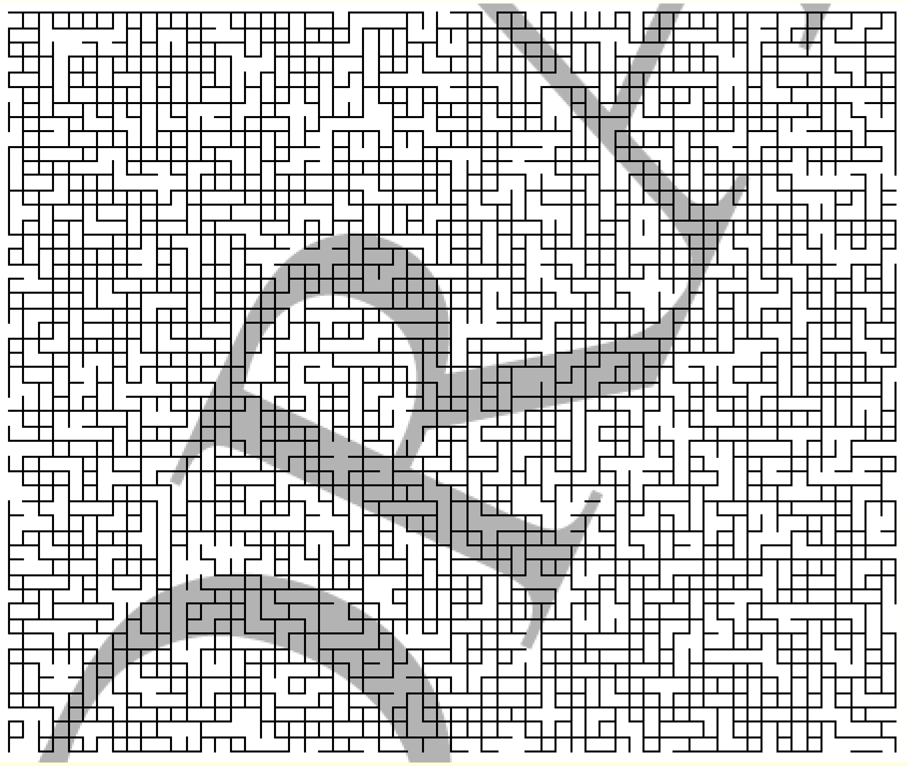
- An infinite grid Z^2, with each link to be “open” (appear) with probability p independently. Now we study the connectivity of this random graph.



p=0.25

Bond Percolation

- An infinite grid Z^2, with each link to be “open” (appear) with probability *p* independently. Now we study the connectivity of this random graph.

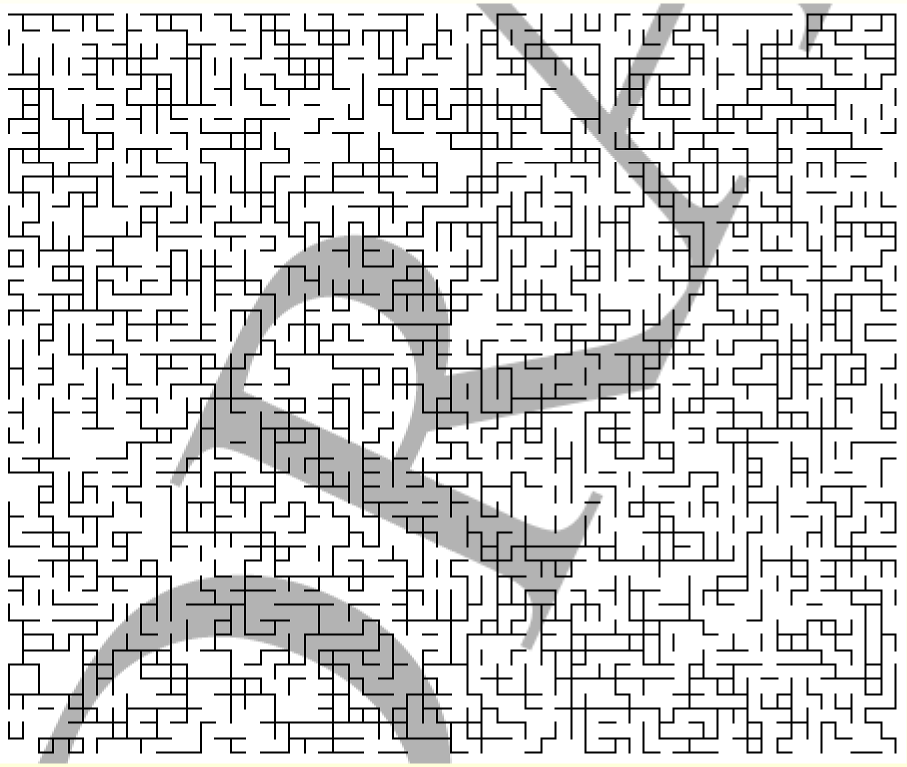


p=0.75

Bond Percolation

- An infinite grid Z^2, with each link to be "open" (appear) with probability *p* independently. Now we study the connectivity of this random graph.

No path from left to right

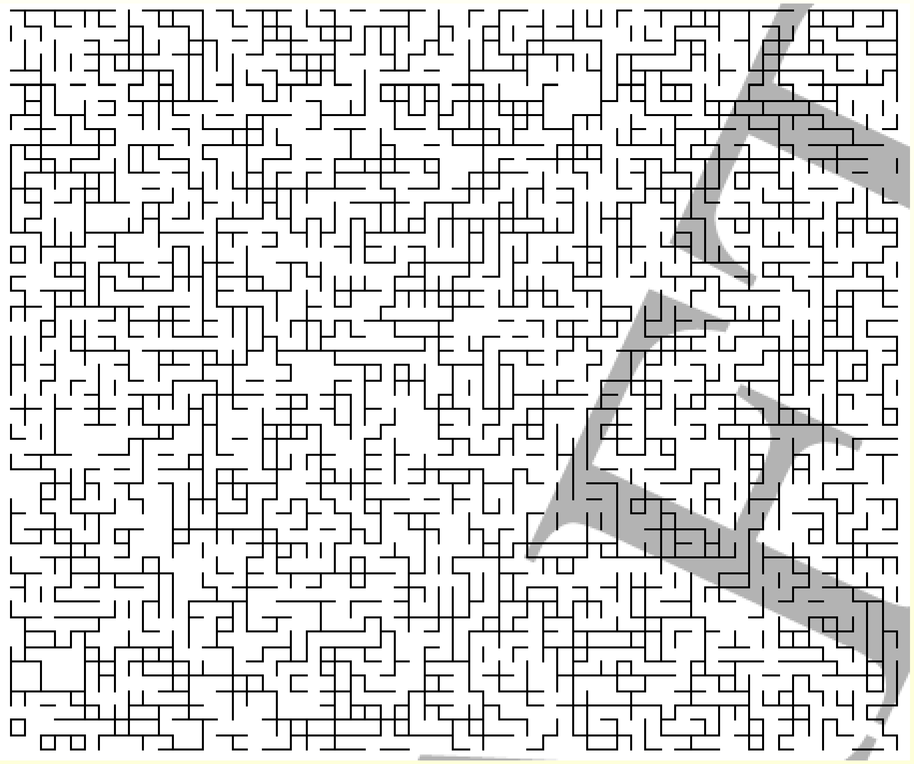


p=0.49

Bond Percolation

- An infinite grid Z^2, with each link to be "open" (appear) with probability *p* independently. Now we study the connectivity of this random graph.

There is a path from left to right!

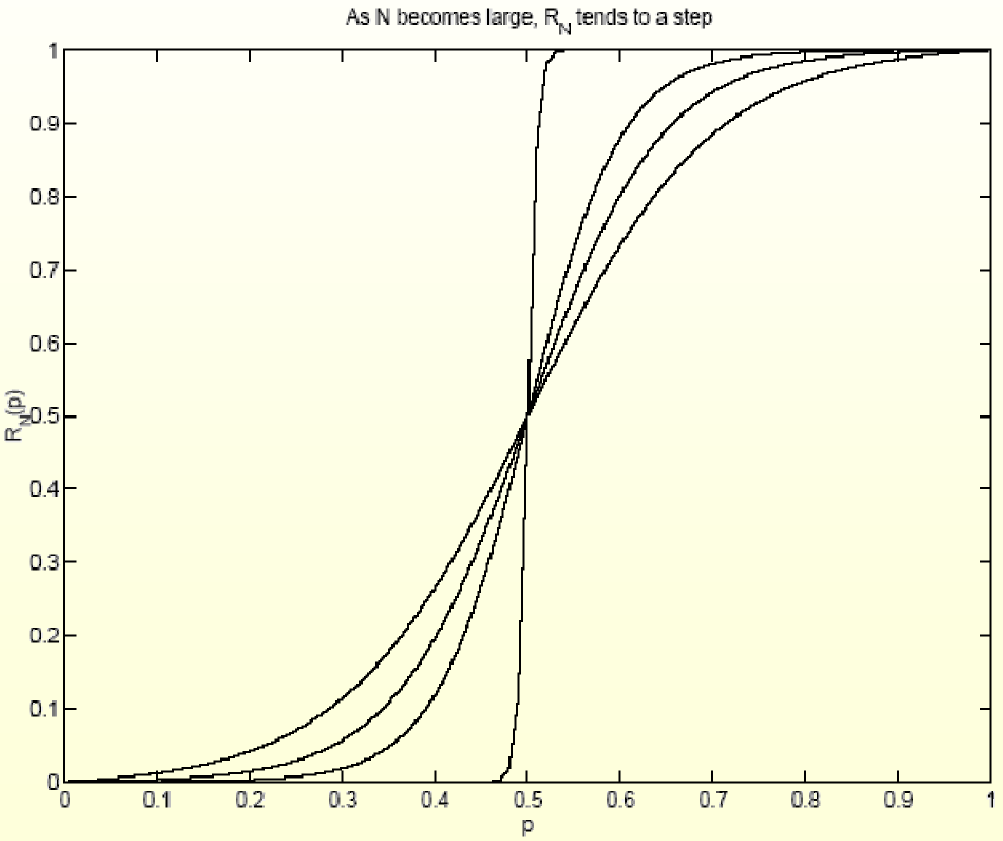


p=0.51

Bond Percolation

- There is a critical threshold *p*=0.5.

The probability that there is a “bridge” cluster that spans from left to right.



Bond Percolation

- There is a critical threshold $p = 0.5$.
- When $p > 0.5$, there is a unique infinite size cluster almost always.
- When $p < 0.5$, there is no infinitely size cluster.
- When $p = 0.5$, the critical value, there is no infinite cluster.

- Percolation theory studies such phase transitions in random structures.

Examples of Percolation

- **Spread of epidemics, virus infection on the Internet.**
  - Each “sick” node has probability p to infect a neighbor node.
  - Denote by *p* the contagious parameter. If *p* is above the percolation threshold, then the disease will spread world wide.
  - The real model is more complicated, taking into account the time variation, healing rate, etc.
- **Gossip-based routing, content distribution in P2P network, software upgrade.**
  - The graph is important in deciding the critical value.
  - An interesting result is about the “scale-free” graphs (also called power-law) that model the topology of the Internet or social networks: in one such model (random attachment with preferential rule), the percolation threshold vanishes.

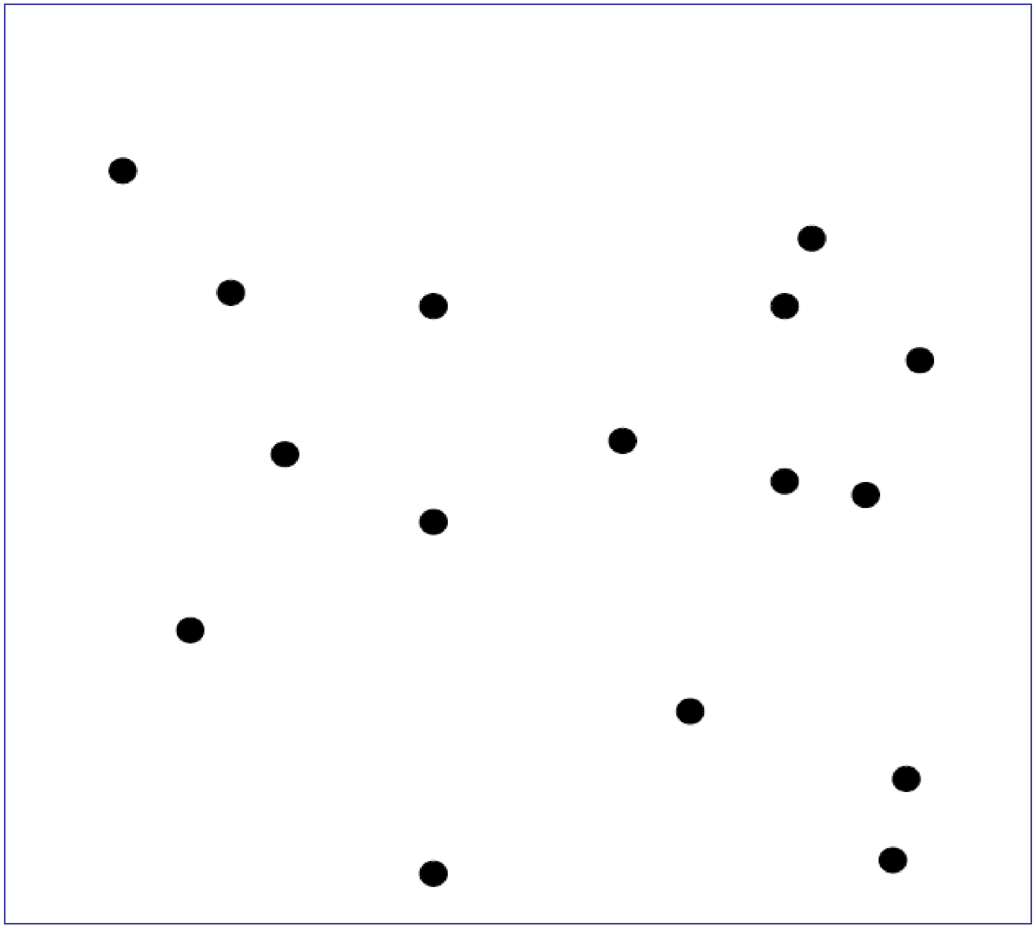
More Examples

- **Connectivity of unreliable networks.**
  - Each edge goes down randomly.
  - Is there a path between any two nodes, with high probability?
  - Resilience or fault tolerance of a network to random failures.
- **Random geometric graph, density of wireless nodes (or, critical communication range).**
  - Wireless nodes with Poisson distribution in the plane.
  - Nodes within distance *r* are connected by an edge.
  - There is a critical threshold on the density (or the communication range) such that the graph has an infinitely large connected component.

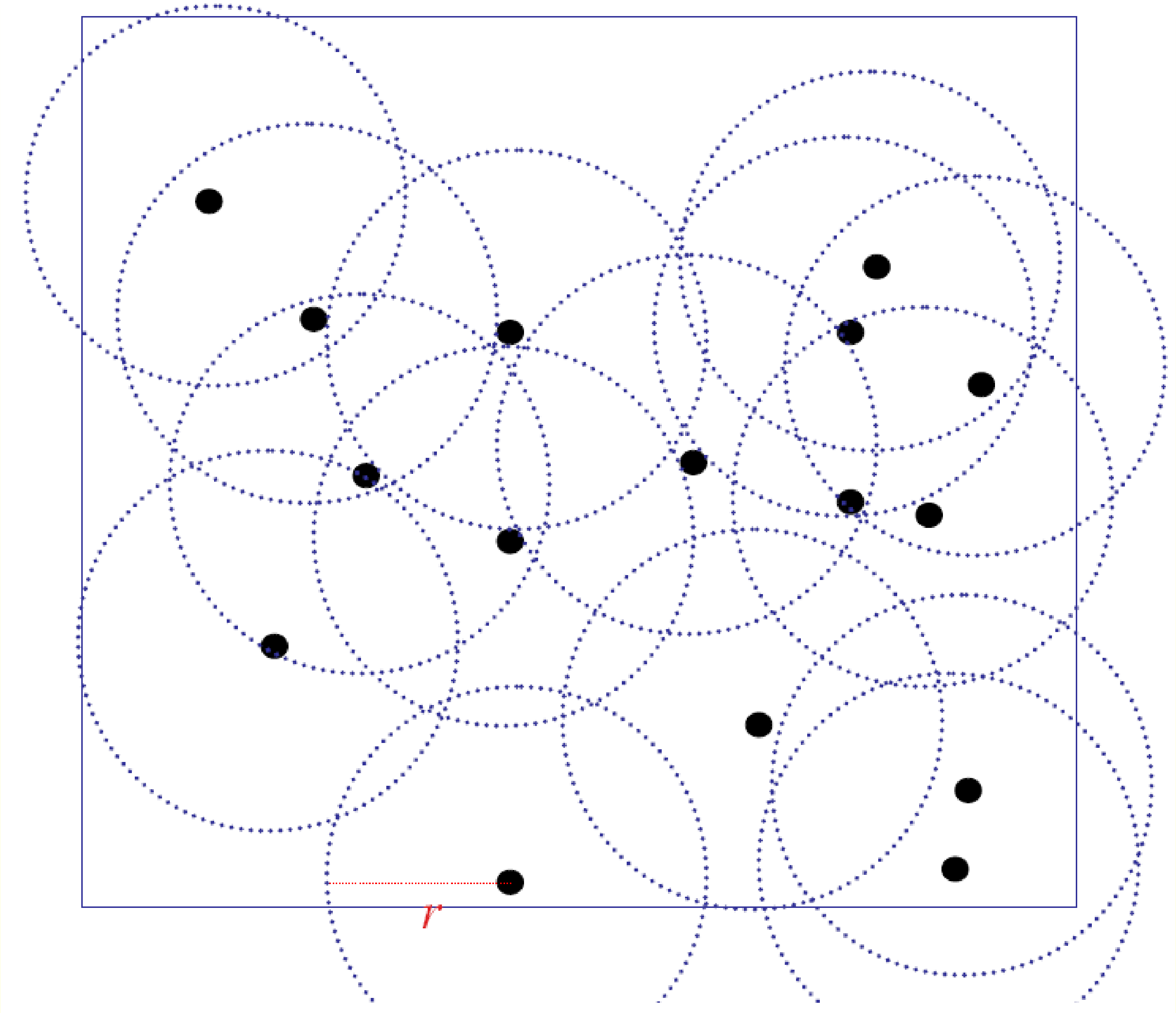
Continuum Percolation

- **Random plane network**, by Gilbert, in 1961.
- Pick points on the plane by a Poisson process with density of λ points per unit area.
- Join each pair of points if they are at distance less than r.
- Equivalently,
- In the unit square [0, 1] by [0, 1], throw *n* points uniformly randomly.
- Connect two nodes with distance less than r.
- This graph is denoted as $G(n, r)$.

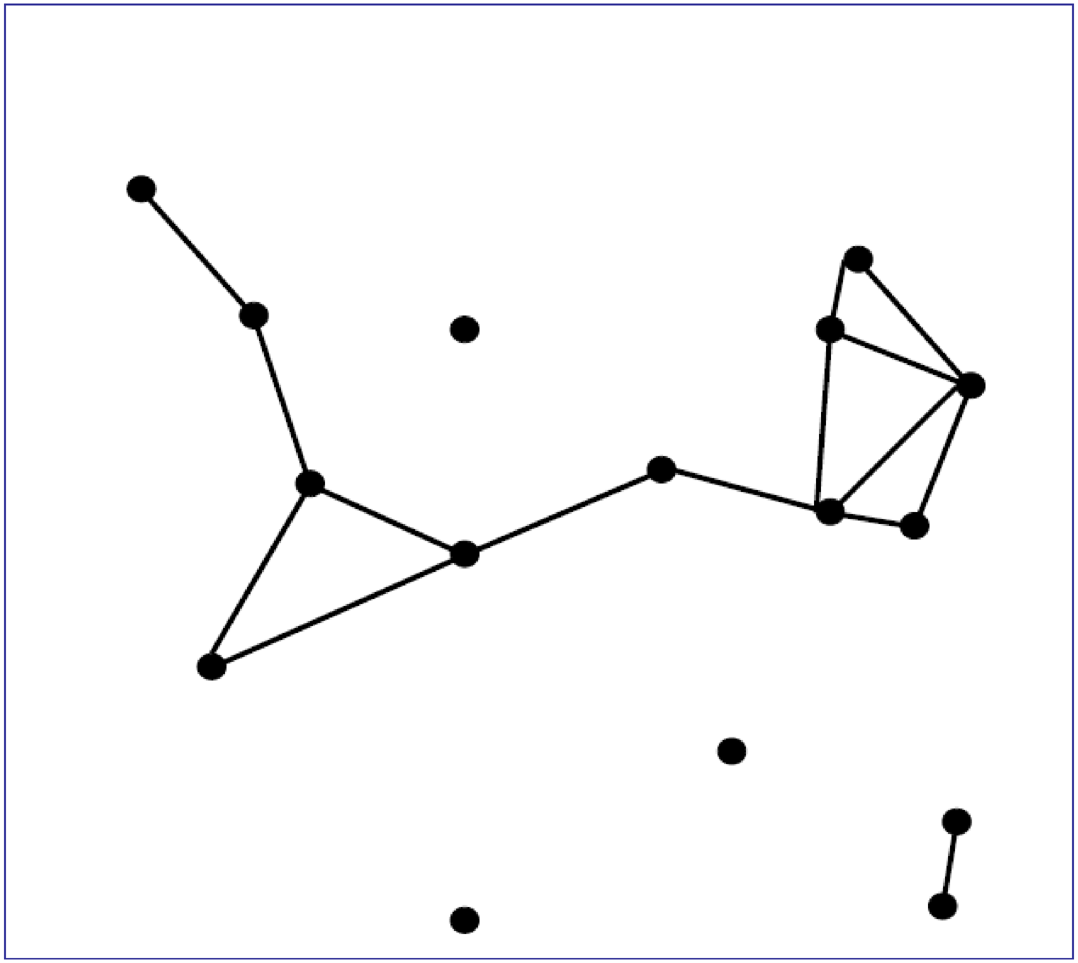
Random Geometric Graph



Random Geometric Graph



Random Geometric Graph



Connectivity in Random Geometric Graphs

- *G*(*n*, *r*): n nodes randomly distributed in a unit square, each node has transmission range *r*.
- Claim: the network is connected with *r* =

$$\Theta(\sqrt{\frac{\log n}{n}})$$

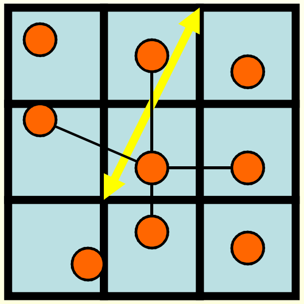
- Proof:
  - Partition the square into small squares of side length

$$\alpha(n) = \sqrt{2\frac{\log n}{n}}$$

  - # squares = $n/(2 \log n)$.

Connectivity in Random Geometric Graphs

- Proof cont.
  - We have $m = n/(2 \log n)$ squares. Throw $n = 2m \log n$ nodes randomly in the squares.
  - With high probability each square gets at least one node --- coupon collector problem.
  - Set $$r(n) = \sqrt{10\frac{\log n}{n}}$$
  - $r(n) = \sqrt{5}\,\alpha(n)$
  - Each node can connect to its neighboring 4 squares.
  - Thus the graph is connected.



Coupon Collector Problem

- You want to gather all n distinct coupons. Each time you receive a random one. How many coupons do you get until you get all the different coupon types?
- Answer: $n \log n$.
  - N_i = time until one gets i different coupons.
  - A random draw gives a new coupon with prob=$(n-i)/n$
  - $E(N_{i+1}-N_i)$ = Expected time to get a new coupon, with i different coupons already at hand = $n/(n-i)$.
  - Thus $E(N_n)=1+n/(n-1)+n/(n-2)+\ldots+n = n \log n$.
  - High probability analysis possible.

Percolation with Noisy Links

- Each pair of nodes is connected according to some (probabilistic) function of their (random) positions.
- A pair of points (i, j) is connected with probability $g(x_i - x_j)$, where g is a general function that depends only on the distance.
- In order to keep the average degree the same, fix the effective area

$$e(g) = \int_{x \in \mathcal{R}^2} g(x) dx$$

- The average degree = $\lambda \, e(g)$.

[From Franceschetti et. al. 05]

Percolation with Noisy Links

- Percolation threshold

$$0 < \lambda_c(g) = \inf\{\lambda : \exists \text{ infinite connected component a.s.}\} < \infty$$

- Question: what is the relationship between the percolation threshold and the function *g*?

[From Franceschetti et. al. 05]

Percolation with Noisy Links

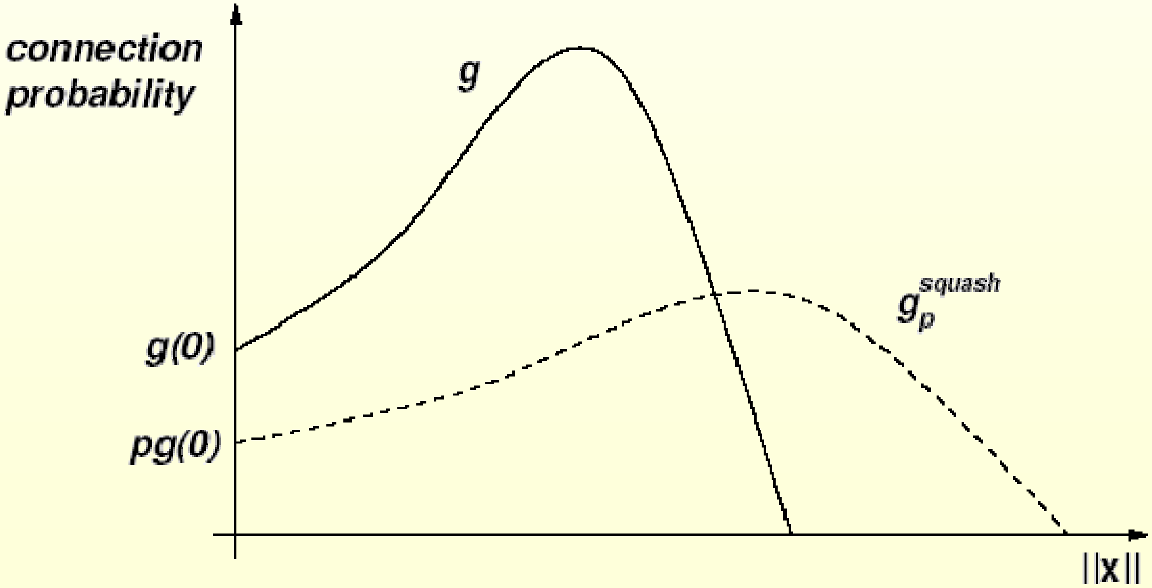
- Question: what is the relationship between the percolation threshold and the function g?
- Each node is connected to the same number of edges on average. So whom should the node be connected to, in order to have a small percolation threshold?
- Which distribution has the best graph connectivity?
- Should I use reliable short links? Or unreliable long links? Or something more complex, say an annulus?

[From Franceschetti et. al. 05]

Squashing

- Probabilities are reduced by a factor of *p*, but the function is spatially stretched to maintain the same effective area (e.g., the same average degree).

$$g_p^{\text{squash}}(x) = p \cdot g(\sqrt{p}x)$$



[From Franceschetti et. al. 05]

Squashing

- Probabilities are reduced by a factor of p, but the function is spatially stretched to maintain the same effective area (e.g., the same average degree).

$$g_p^{\text{squash}}(x) = p \cdot g(\sqrt{p}x)$$

- Theorem: $\lambda_c(g) \geq \lambda_c(g_p^{\text{squash}})$
- It's beneficial for the connectivity to use long unreliable links!
- If the effective area is spread out, then the threshold density goes to 1.
- Question: what makes the difference? The guess is the existence of long links.

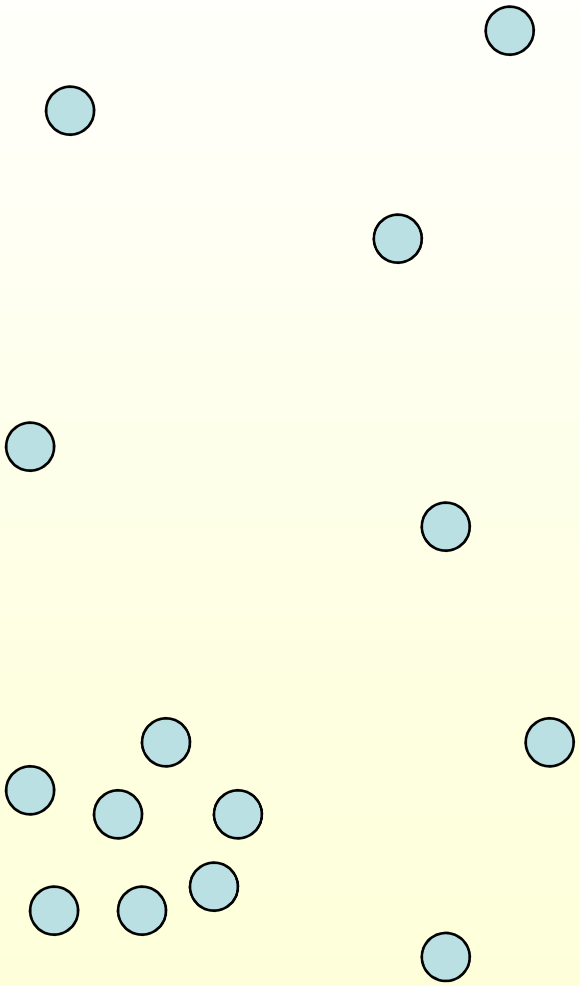
Back to Wireless Sensor Networks

Variable Transmitting Ranges

- If the node density is highly variable, then we should choose short ranges when the density is high, and long when it's low
- The goal is to minimize

$$\sum_{i=1}^{n} r_i^{\alpha},$$

while still connecting the network



The Range Assignment Problem

- The previous minimization problem is known as the range assignment problem
- Unfortunately, it is NP-complete ...
- The MST of the nodes provides a factor 2 approximation
  - define graph weights by $w(i,j) = \delta^{\alpha}(i,j)$
  - Solve the MST problem
  - set the range of each node so as to reach all of its MST neighbors

The COMPOW Protocol

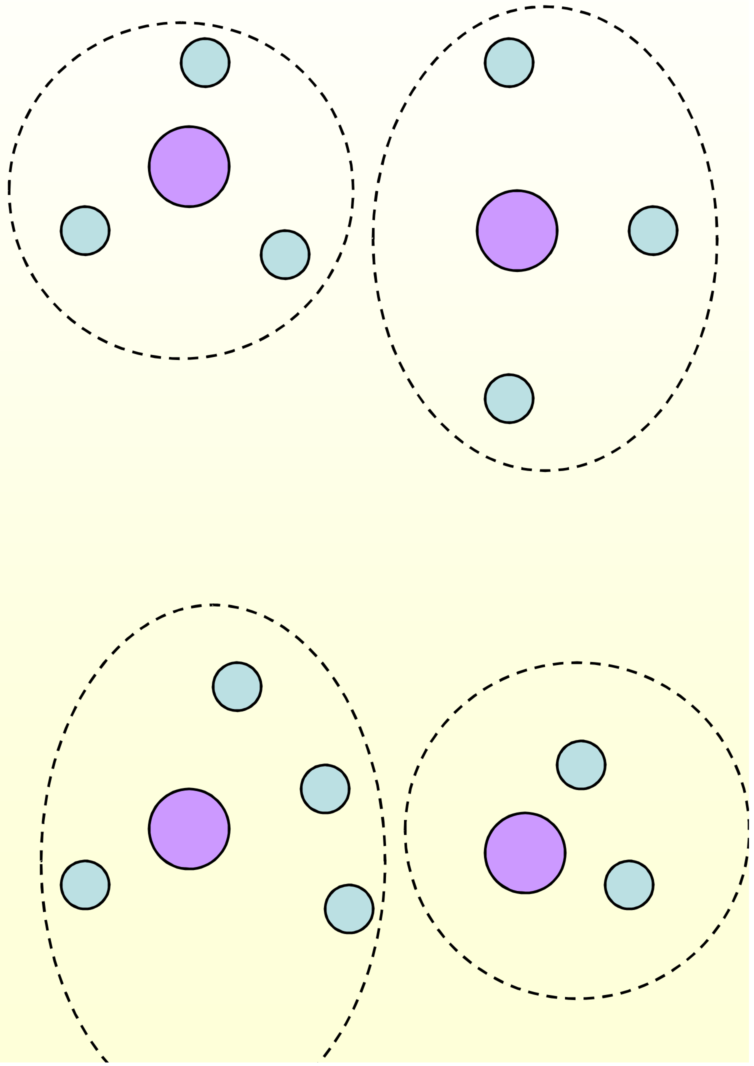
- In practice, we use greedy methods
- The COMPOW protocol computes routing tables for each node at different power levels
- A node selects the minimum transmitting power so that its routing table has paths to all the other nodes

[From Narayanaswamy er.al. ‘02]

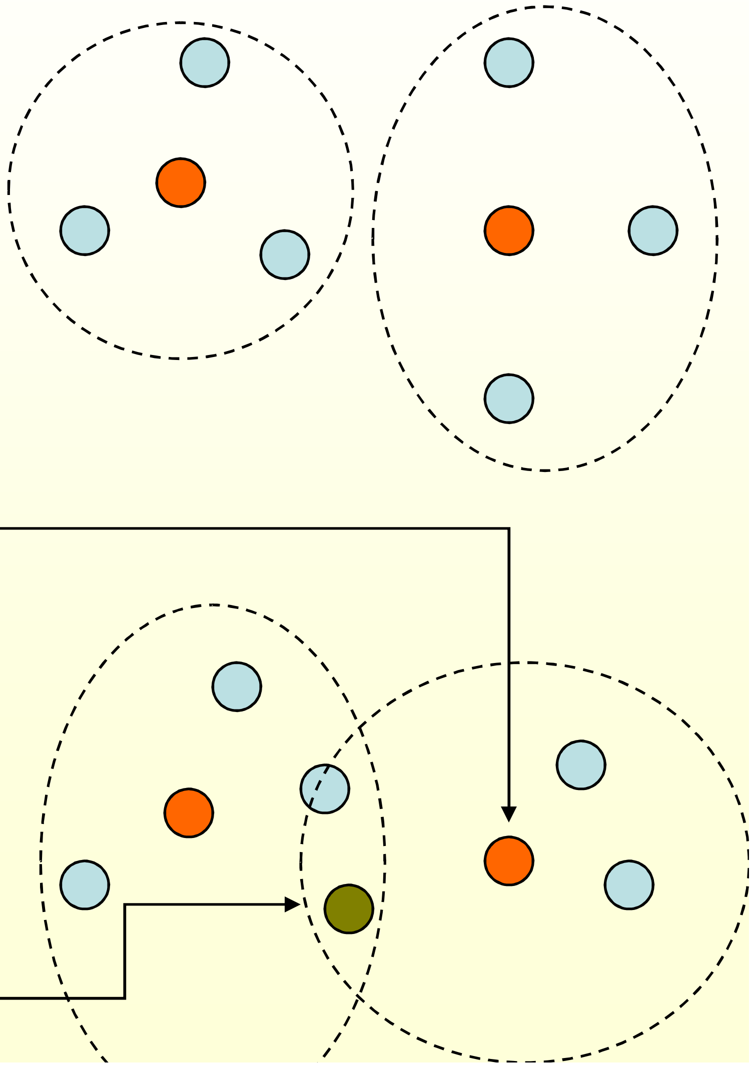
Clustering
Nodes

Clusters and Other Hierarchies

- Node clustering is extremely common in sensor networks
- It is natural in settings where nodes of different capabilities are available



Clustering is Useful Even in Homogeneous Networks



- Clusters are usually of size comparable with the node communication range
- Clusters allow better resource utilization
- Each cluster elects a node as its clusterhead
- Nodes belonging to multiple clusters can function as gateways

A Two-Level Communication Network

- Local traffic: within a cluster, directly or via the clusterhead
- Long-distance traffic: via clusteheads and gateways
- Clustering can even out node density in a network

Clusterhead Election

- Assume each node has a unique ID
- Each node nominates the highest ID node it can hear to become a clusterhead
- All nominated nodes become clusterheads -- and form a cluster with their nominators

The End