Desynchronization Metrics

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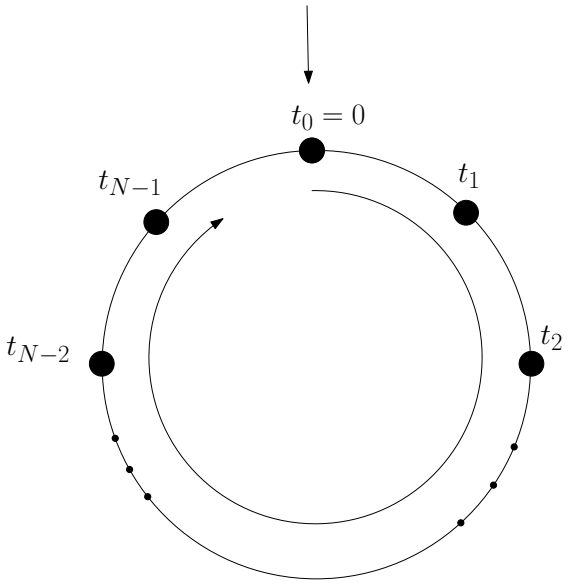
1 One-hop

We begin with a description of the metric optimized. In the one hop case, we have a complete graph with N motes. If we consider the firing time of each mote globally, we denote them

$$t_0 < t_1 < \ldots < t_{N-1}$$

where the starting point is arbitrary and $t_{N-1} < T$, where T is the period. As an example:



The starting point is chosen as t_0 and the points named according to their order on the circle until we reach t_0 again.

We would like the points to be spread out as evenly as possible, so the metric to minimize is

$$\varepsilon = \frac{1}{N} \sum_{i=0}^{N-1} \left| \left(t_{(i+1)\%N} - t_i\right) \%T - \frac{1}{N} \right|$$

The modulo operator just makes the wrap around so that we compute the difference between the last and the first mote.What the above equation denotes is the average error of each interval from the optimum. That is under ideal conditions, the N motes have a spacing of $\frac{1}{N}$.

Now we extend that to arbitrary topologies.

2 General metric

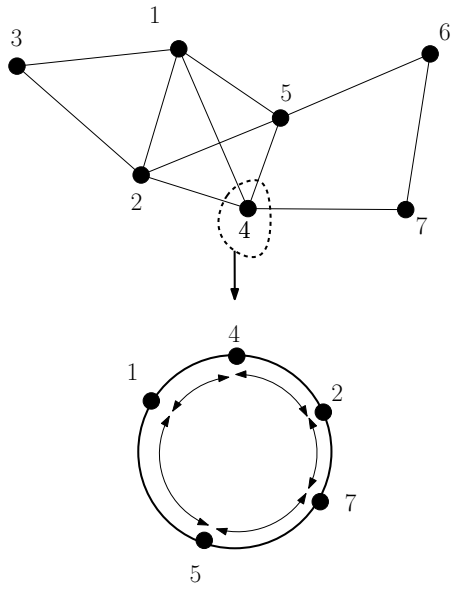
While there is still a global ordering on the motes, it is not clear what the "correct" metric is. The metric we propose is an average of the above metric taken at each mote. To be more precise for each mote we create an ordering. For example for mote j we form an ordering on node j and its neighbors $\mathcal{N}_j$. The cardinality is defined as $n_j = |\mathcal{N}_j| + 1$

$$\varepsilon_j = \sum_{i=0}^{n_j-1} \left| \left(t_{(i+1)\%n_j} - t_i\right) \%T - \frac{1}{n_j} \right|$$

where the total error is

$$\varepsilon = \frac{1}{N} \sum_{j=1}^{N} \varepsilon_j.$$

Graphically depicted



Each node looks at its immediate neighbors and orders them beginning with itself. The differences in this ordering should be one over the neighborhood size of that node (including the node itself). The total error is simply the sum of the errors at all the nodes.

Notice there are some problems with this metric. The largest problem is that if we have two neighboring motes with different number of neighbors, there may be no perfect solution (one with zero error).