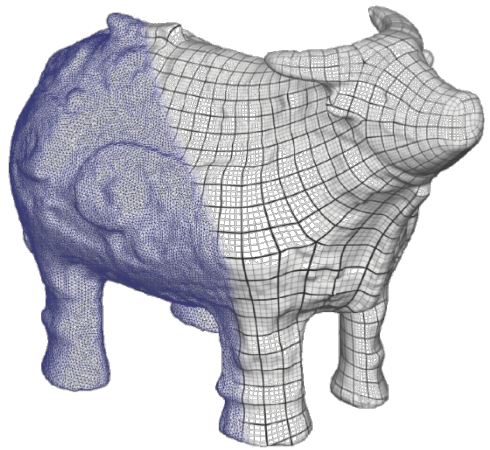
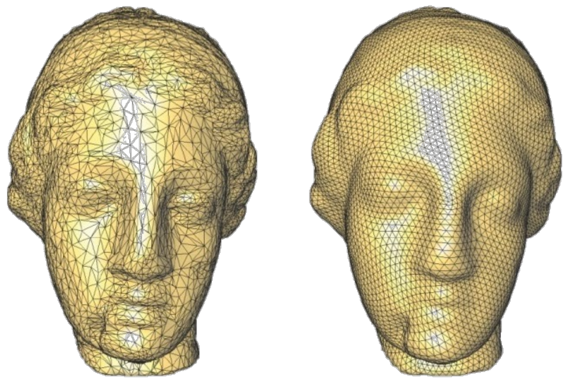
CS348a: Computer Graphics -- Geometric Modeling and Processing





Leonidas Guibas
Computer Science Department
Stanford University

Leonidas Guibas Laboratory
Geometric Computing

Slides adapted from Peter Schröder (Caltech) and Denis Zorin (NYU)

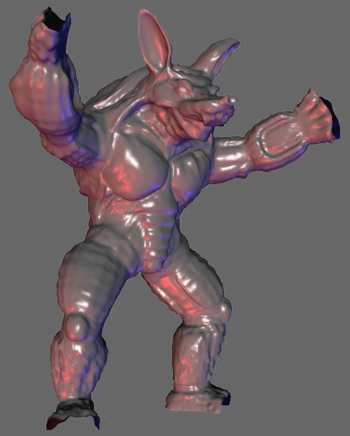
Subdivision Surfaces

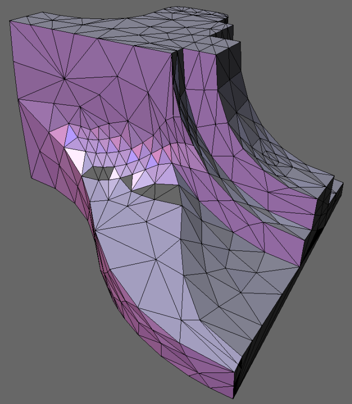
Peter Schröder, Caltech
Denis Zorin, NYU

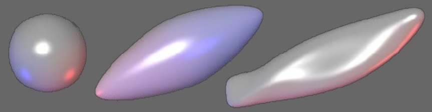
Geometric Modeling

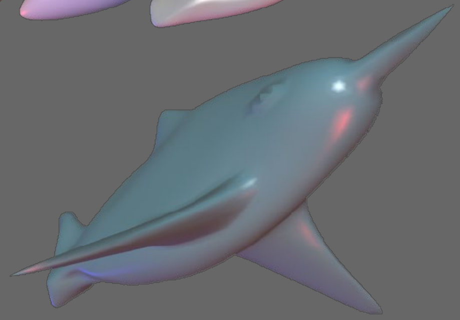
Surface representations

- large class of surfaces
- interactive manipulation/display
- numerical modeling tasks









Representations

Desirable properties

- compatible with hardware and existing representations
- scalable: large data on small machines
- solid mathematical foundation
- suitable for animation and simulation

Representations

General philosophy

- same core representation for multiple purposes
  - transmission
  - rendering
  - simulation
  - editing

Multiresolution

What does it offer?

- good editing semantics
- deep connection with wavelets
  - compression, solvers, speed/accuracy tradeoff, approximation properties
- builtin support for LOD display
- very efficient

Splined Surfaces

Advantages

- high level control (control points)
- compact representation
- multiresolution structure

Disadvantages

- difficult to maintain and manage
- naïve rendering of large models slow

Polygonal Meshes

Advantages

- very general
- direct hardware implementation

Disadvantages

- heavy weight representation
- good editing semantics difficult
- limited multiresolution structure

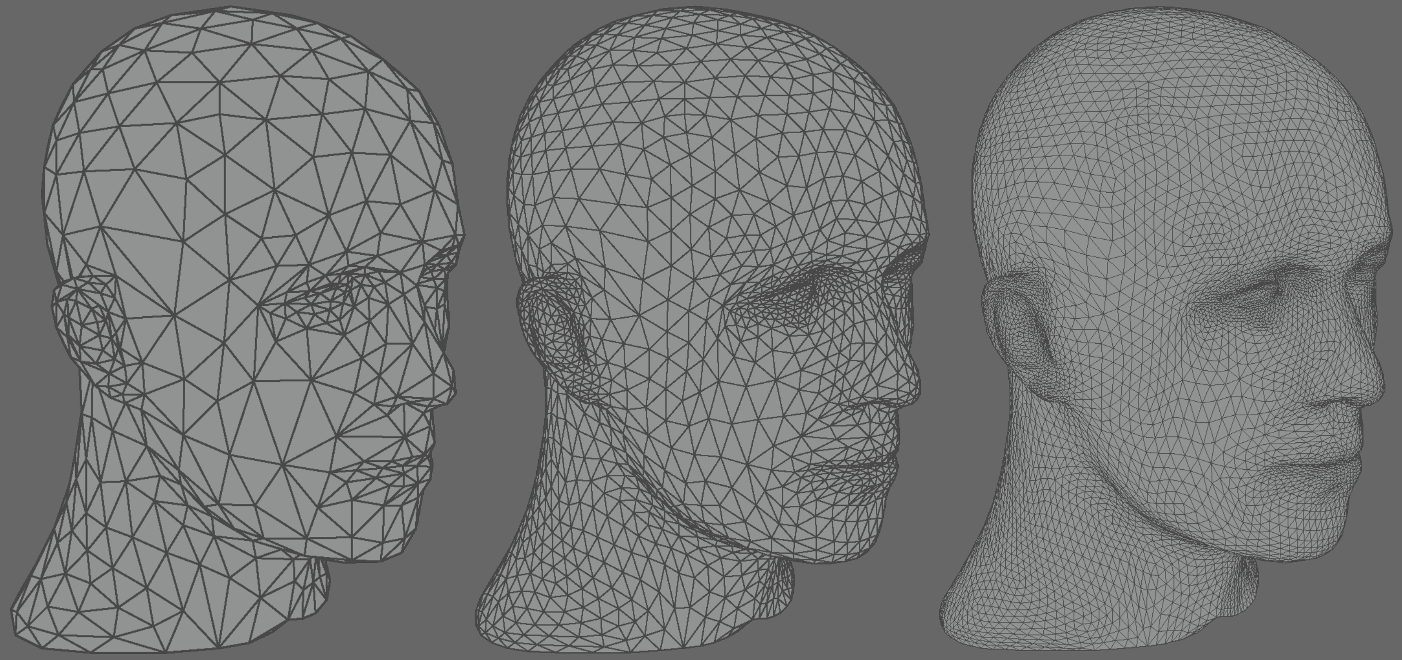
Subdivision Surfaces

Important modeling primitive

- smooth, arbitrary topology surface modeling
- generalizes spline patches
- covers range of representations from “pure” spline patches to “pure” meshes
  - BUT: special connectivity (more on that later)

Subdivision

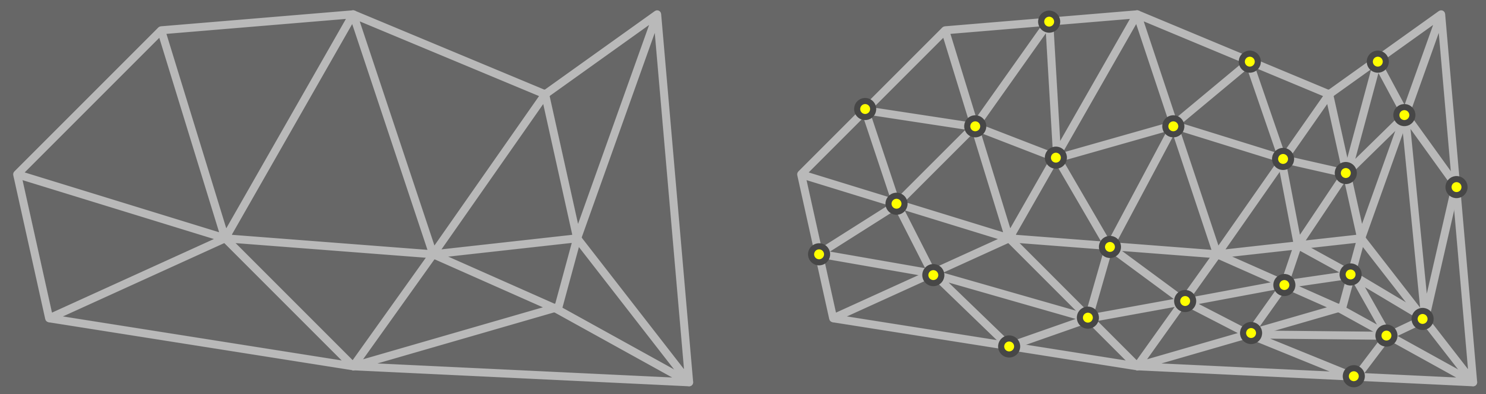
Smooth surfaces as the limit of a sequence of successive refinements



Subdivision

Topological rule

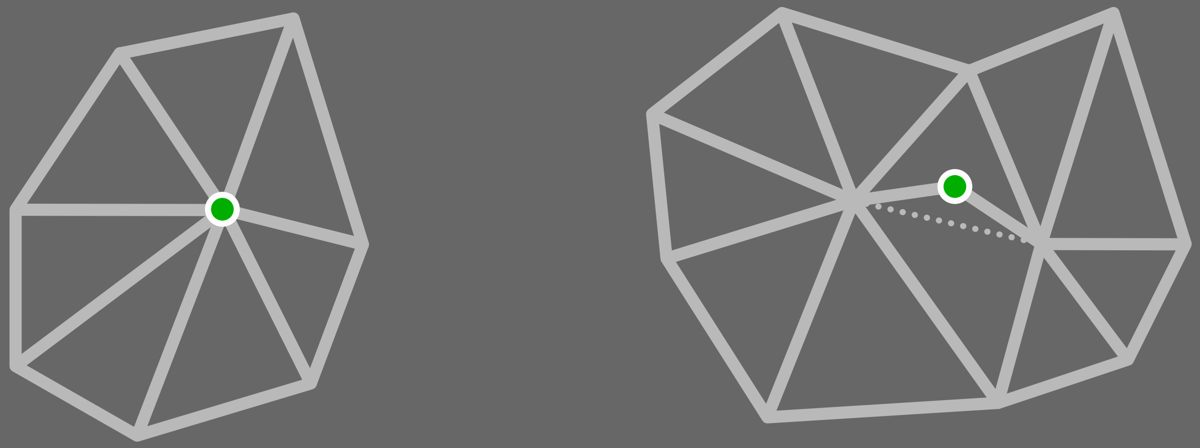
- refinement of abstract simplicial complex
- refine a triangular graph



Subdivision

Geometric rule

- geometry is a map defined on graph
- extension rule



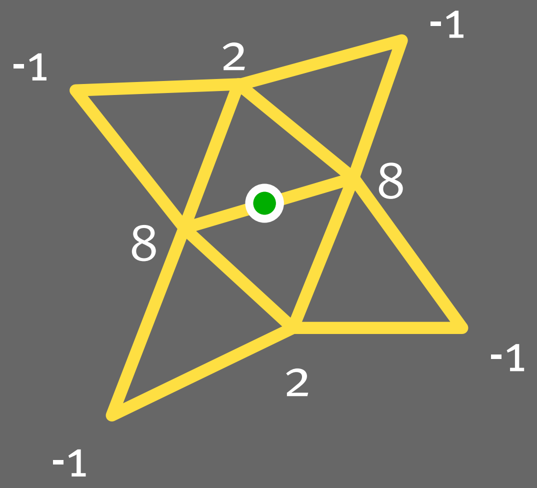
even at level i

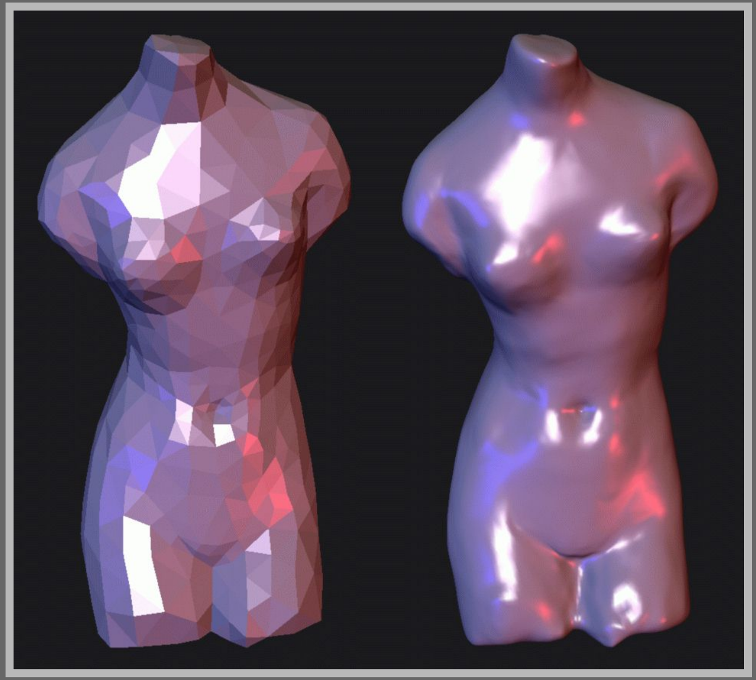
odd at level i

Interpolating

Keep old points, insert new ones

- affine combination of nearby points

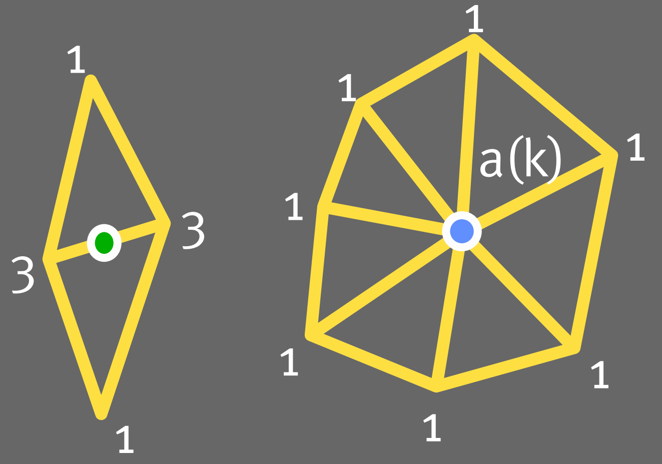


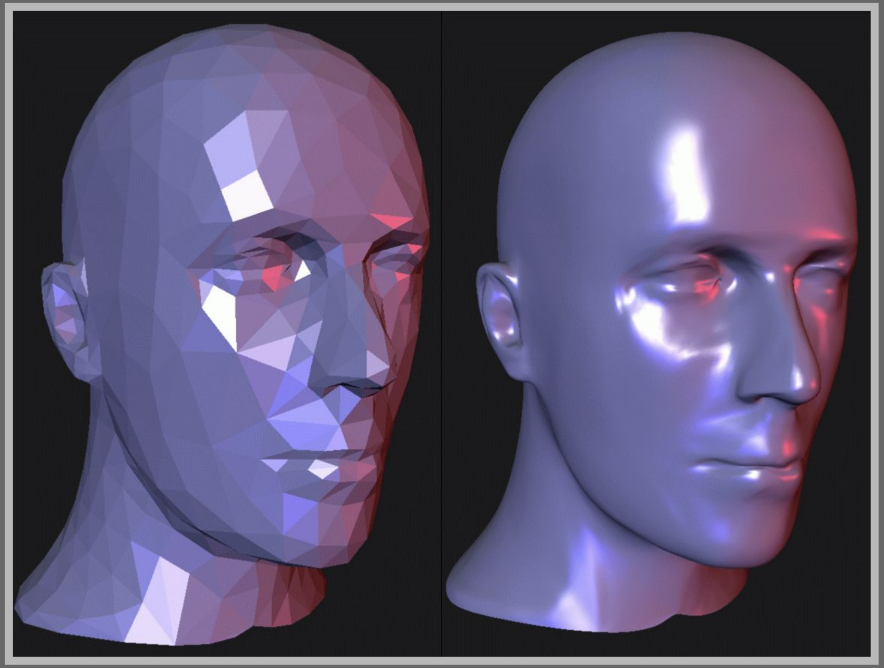


Approximating

Insert new, smooth new and old

- generalizes spline patches





Subdivision

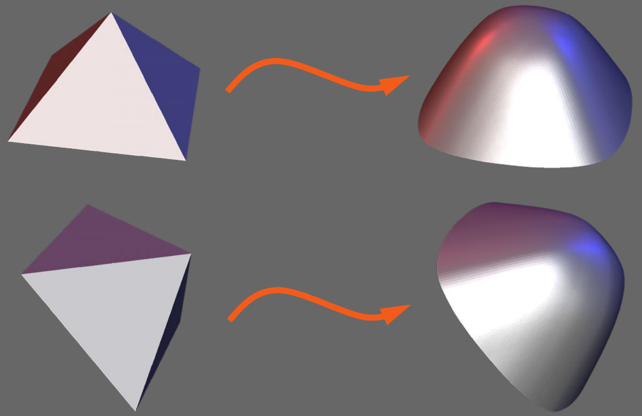
Established schemes

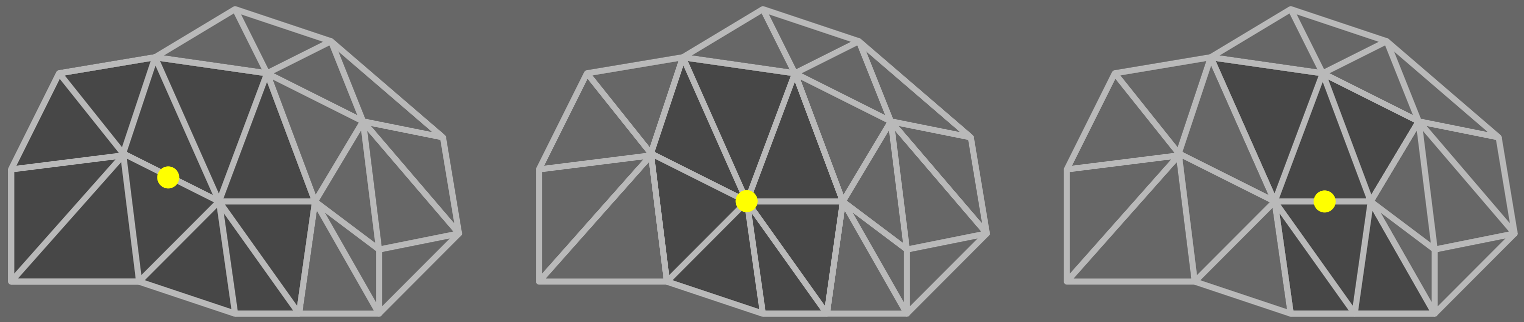
- Catmull-Clark
  - generalizes bi-cubic patches
- Loop
  - generalizes quartic box splines
- many others:
  - Doo-Sabin, Butterfly, Kobbelt, Peters/Reif

Subdivision

Properties

- affine invariance
- local definition
- compact support





Why Subdivision?

Advantages

- arbitrary topology, smooth surfaces
- support for compression and LOD
- suitable for wavelet-based numerical solvers

Scalability

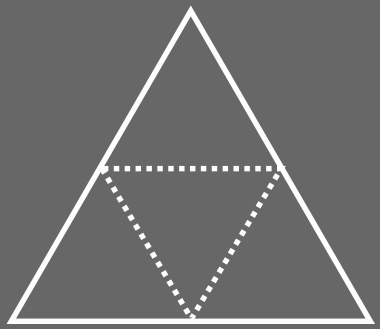
- large datasets on small machines

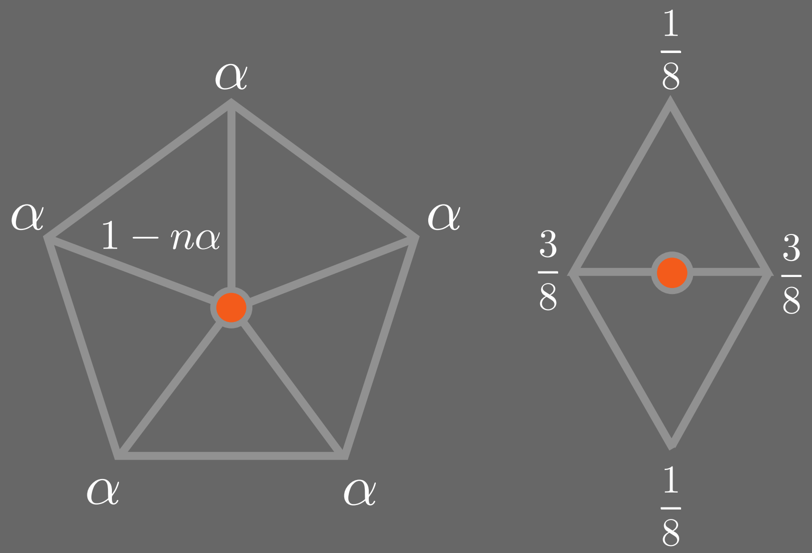
Algorithms

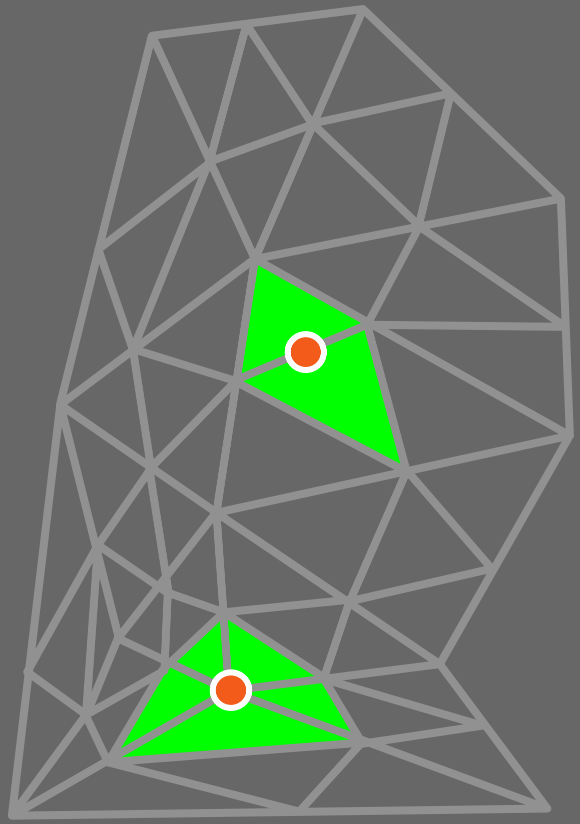
Properties

- exact evaluation
  - value, tangent planes, derivatives
  - moments
- little computation
- simple data structures
- integrates well with spline methods

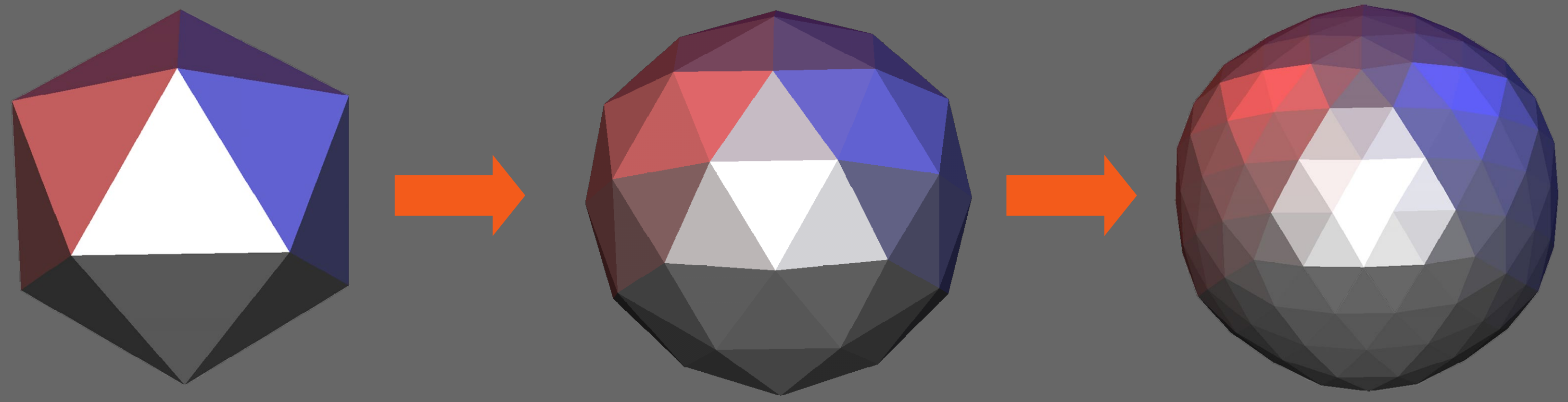
Example: Loop scheme







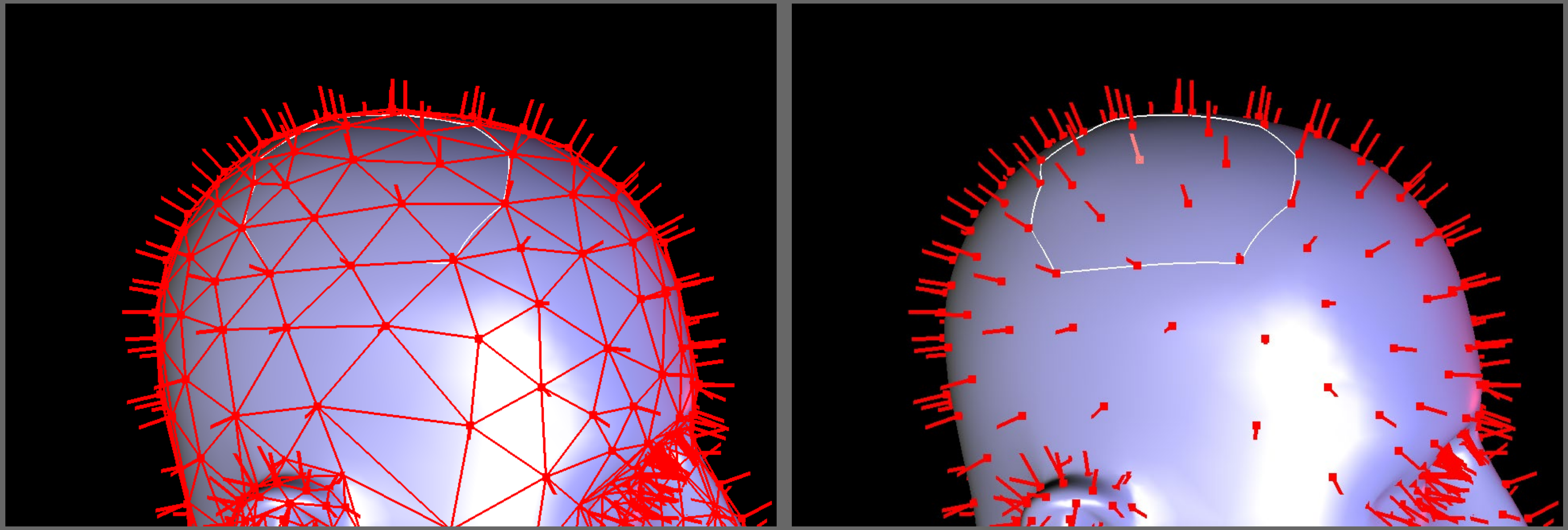
Example: Loop scheme



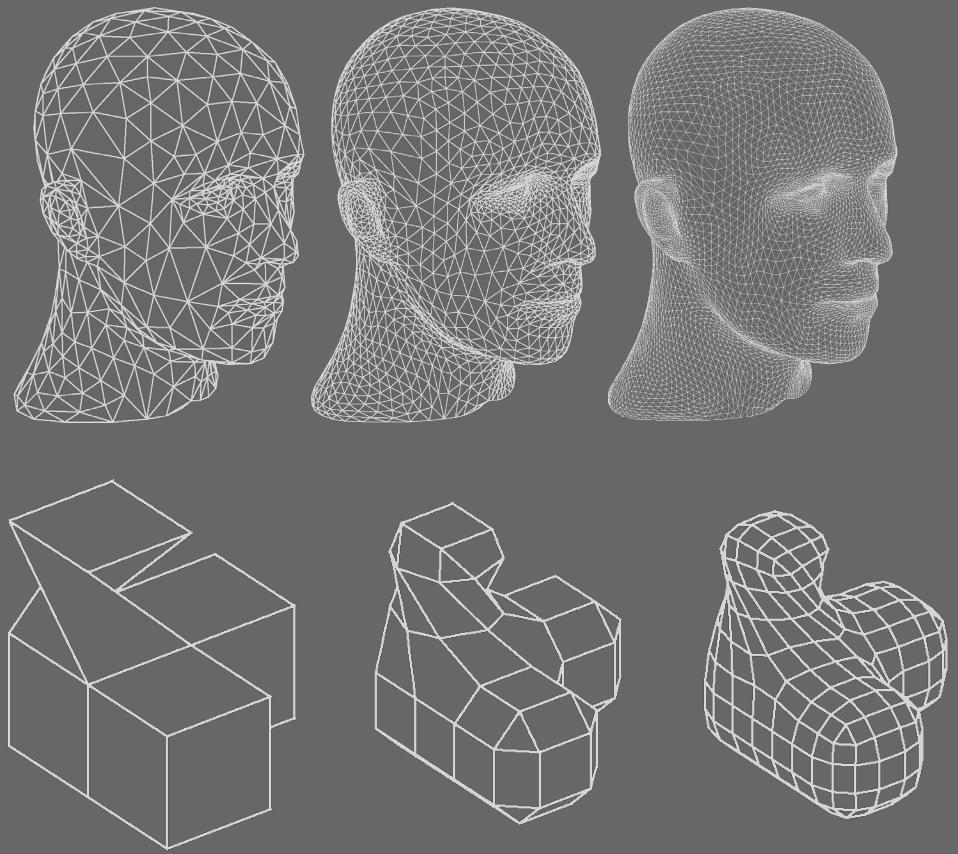
- For a “good” scheme, recursive application approximates a smooth surface

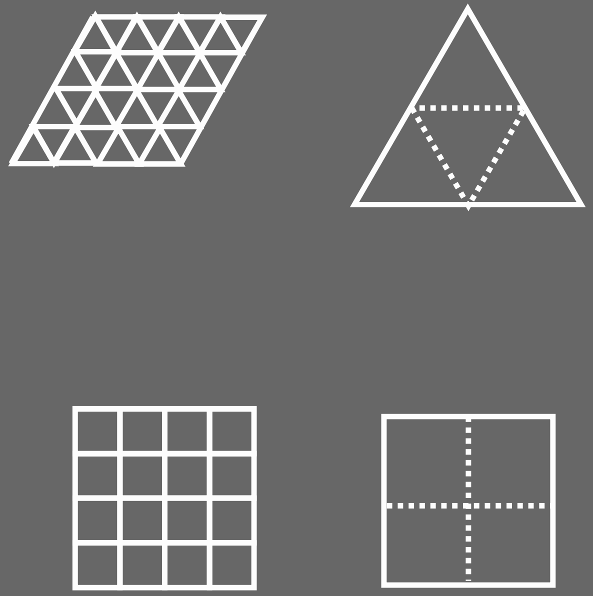
Control points

- vertices of the initial mesh define the surface
- each vertex influences a finite part of the surface



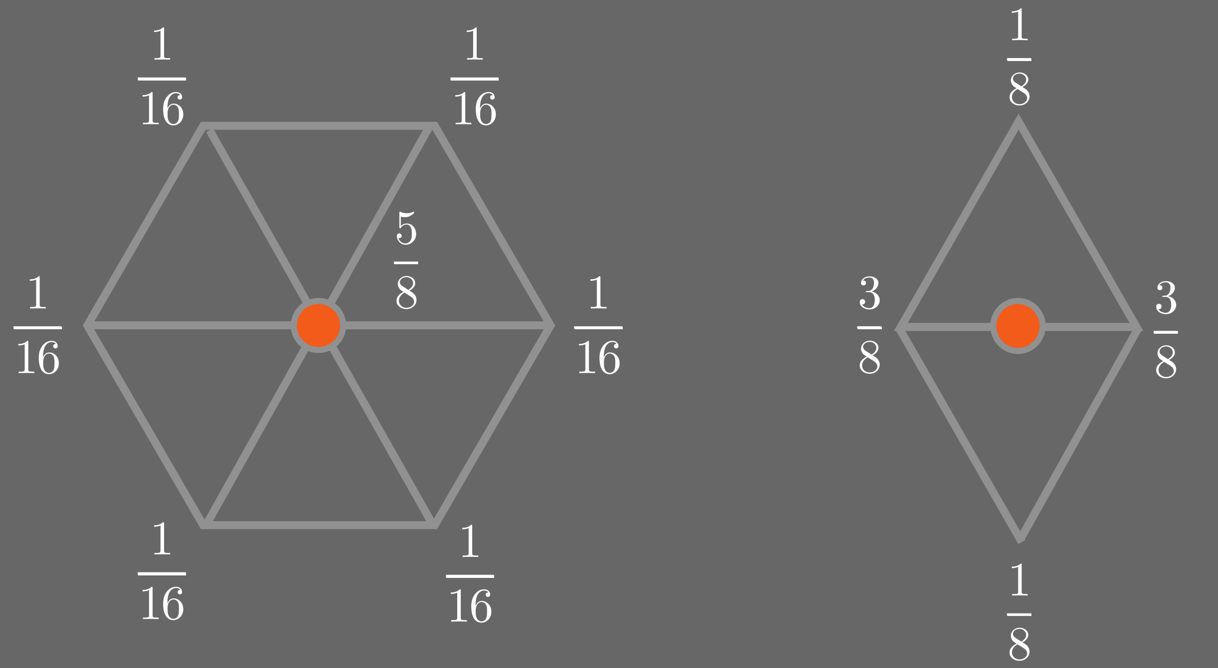
Triangular and quadrilateral subdivision





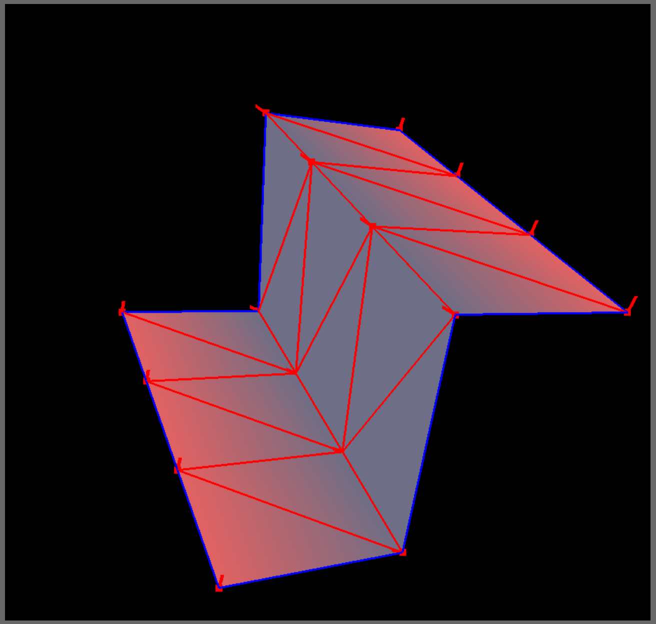
Subdivision and splines

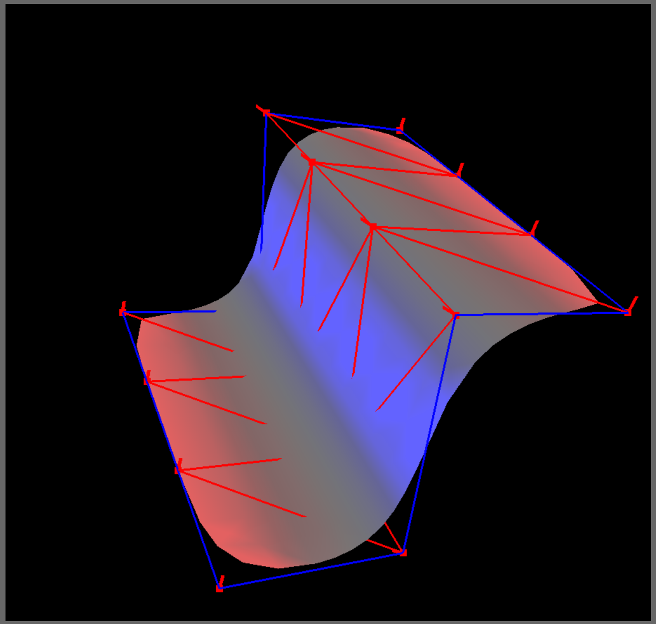
- uniform splines can be computed using subdivision rules
- triangular spline



Subdivision and splines

- For splines, the control mesh is regular

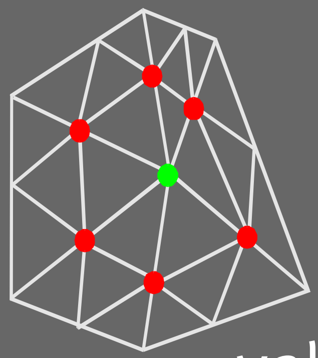




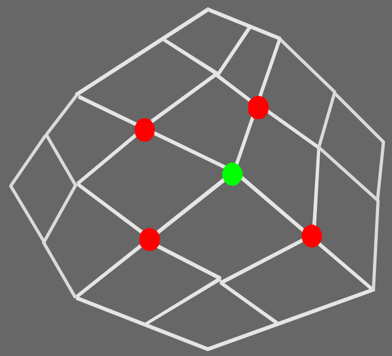
Extraordinary Vertices

Triangular meshes

Quad meshes

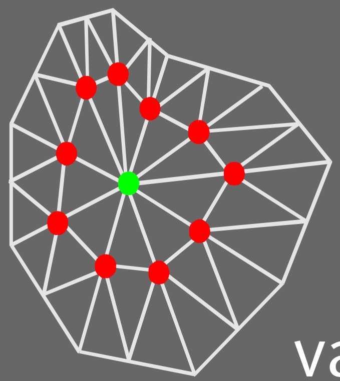


regular

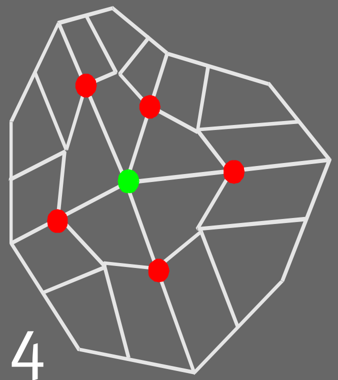


valence 6

valence 4



extraordinary



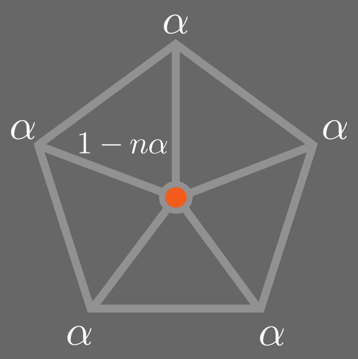
valence $\neq 6$

valence $\neq 4$

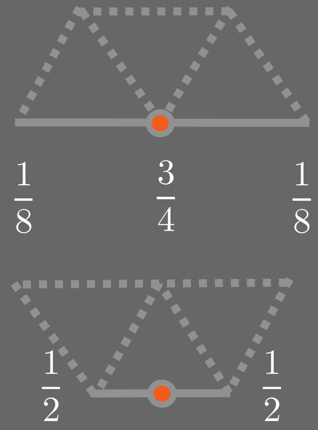
Constructing the rules

- Start with spline rules (or other smooth rules)
- Define rules for:

Extraordinary vertices



Boundaries

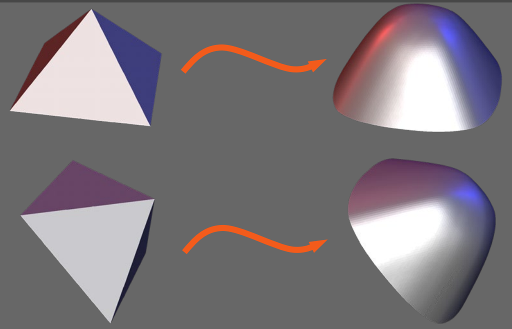


Creases etc.

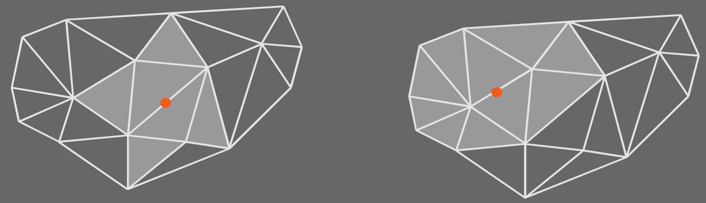
• • •

Constructing the Rules

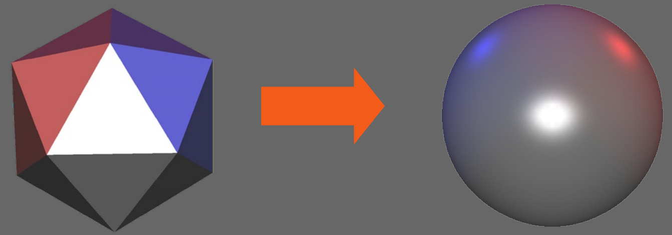
- Invariance under rotations and translations



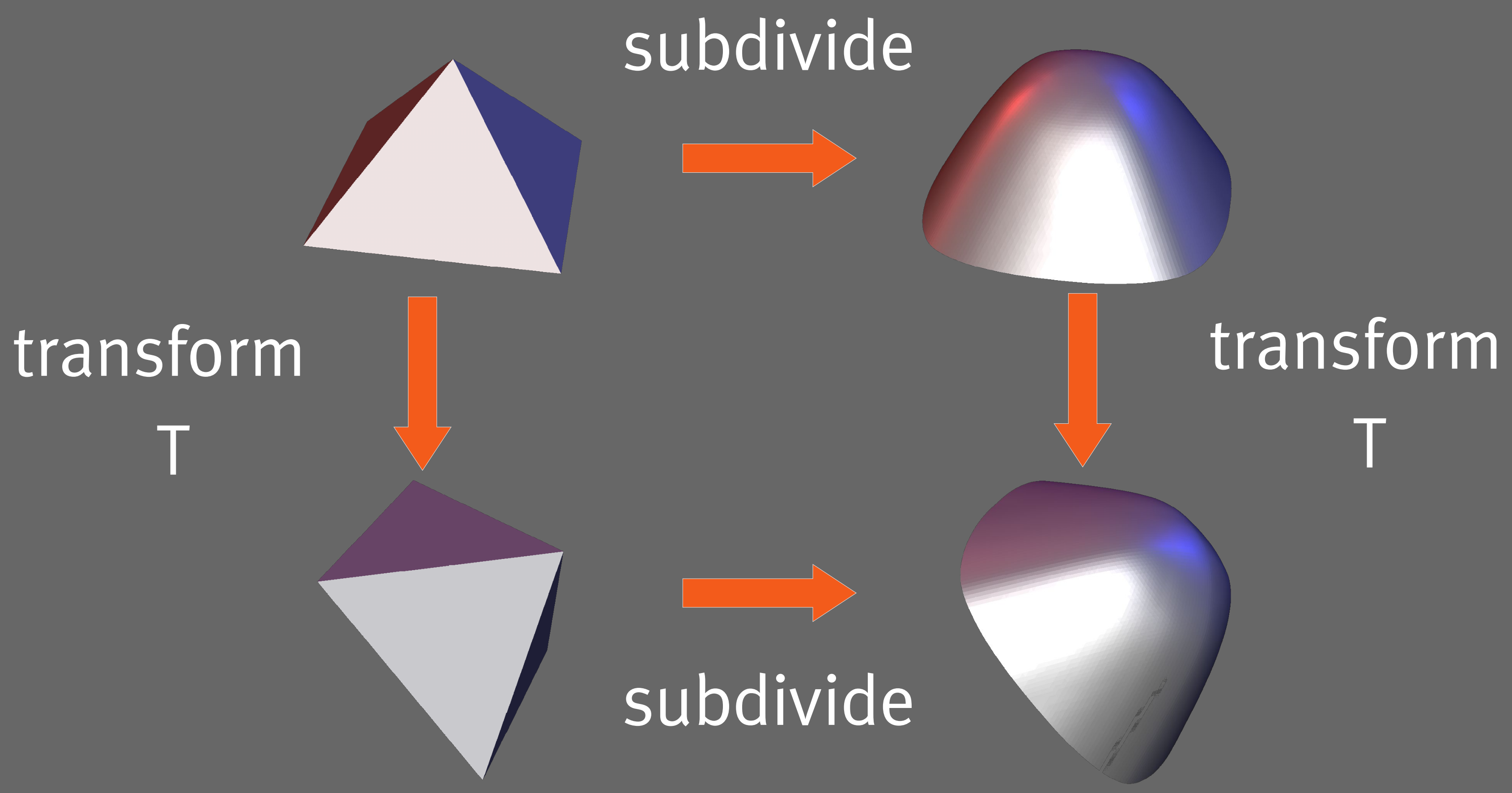
- Small support



- Smoothness and Fairness

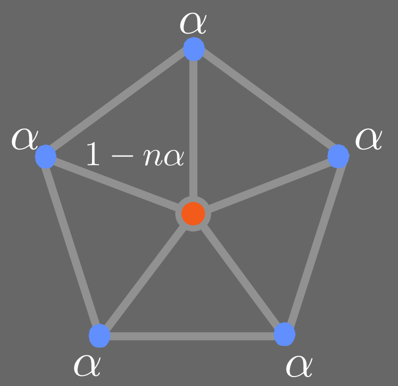


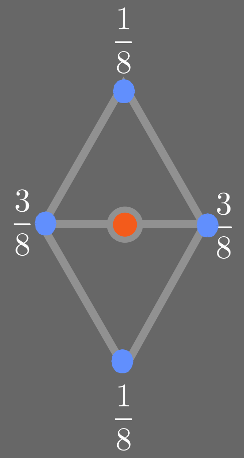
Affine invariance



Affine invariance

- the coeficients of any mask should sum up to 1





$$p = \sum a_i p_i$$

displacement

$$\sum a_i (p_i + t) = \underbrace{\left(\sum a_i\right)}_{1} t + p$$

SUBDIVISION ZOO

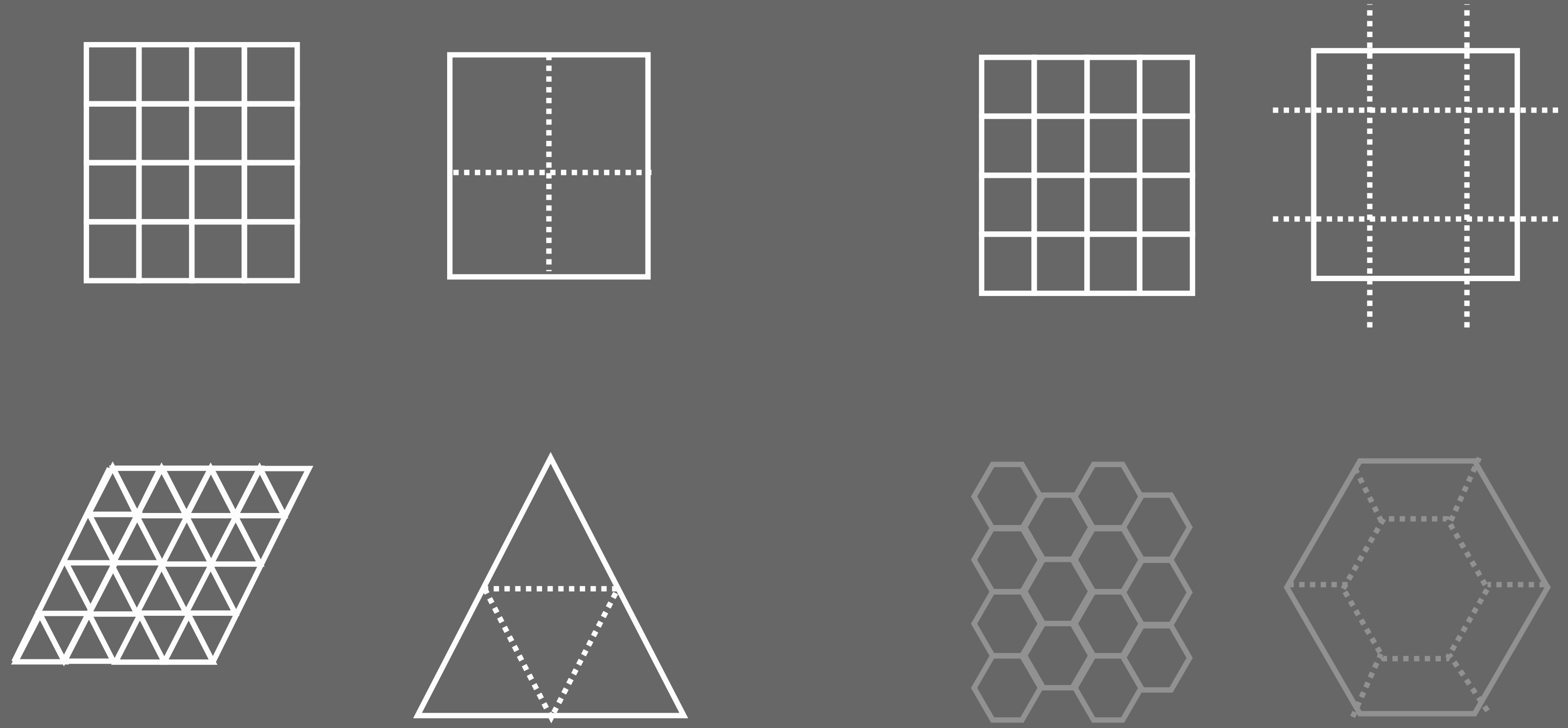
Classification of schemes

Classification criteria

- type of refinement rule (primal or dual)
- type of mesh (triangular or quad or…)
- approximating or interpolating

Refinement rules

- Primal (vertex insertion)
- Dual (corner cutting)



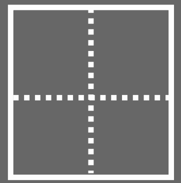
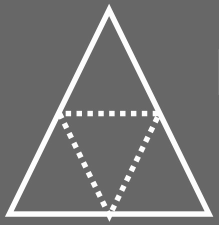
APPROXIMATION AND INTERPOLATION

Advantages

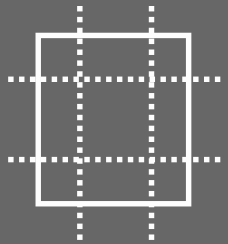
- approximating schemes
  - based on splines, small support
- interpolating schemes
  - control points on surface
  - in-place implementation

Subdivision schemes

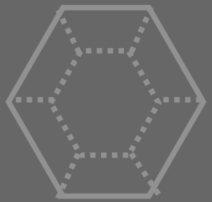
- Primal (vertex insertion)

	Approximating	Interpolating
	Catmull-Clark	Kobbelt
	Loop	Butterfly

- Dual (corner cutting)



Doo-Sabin,
Midedge



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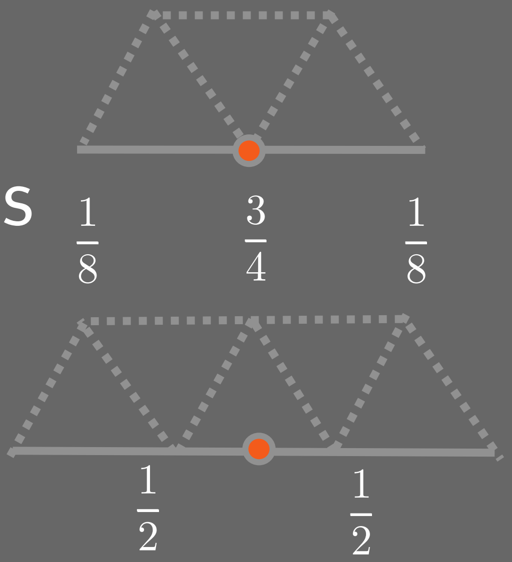
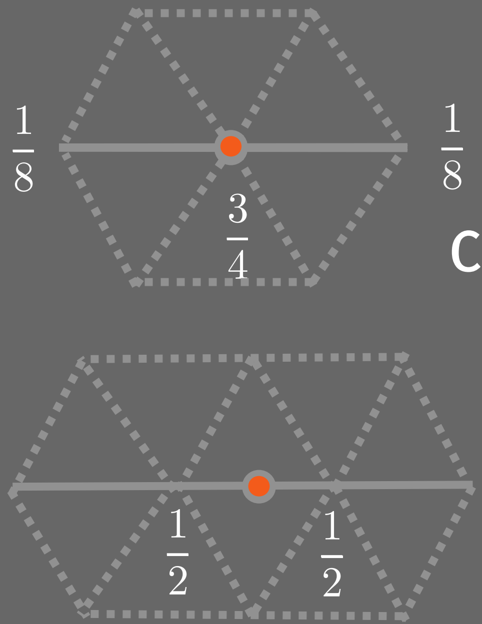
Boundaries and creases

- special rules on and near the boundary
- boundary independent of the interior

boundaries



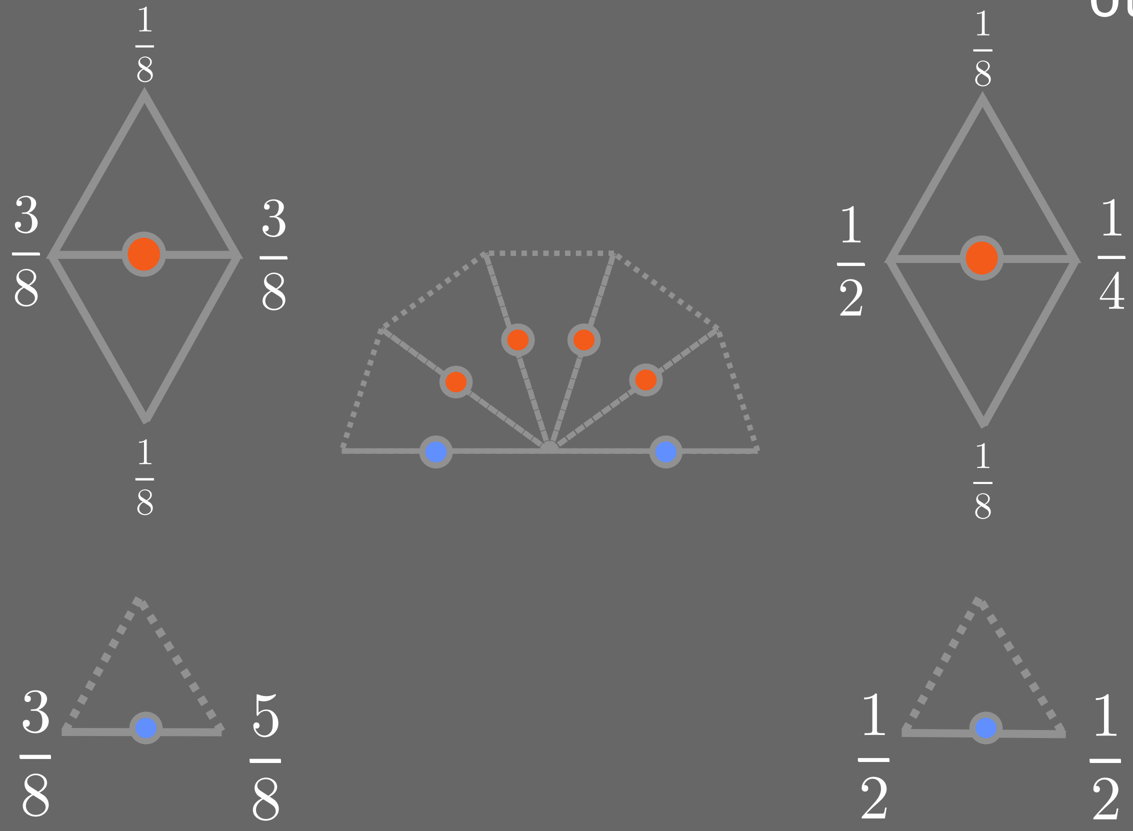


creases

Loop scheme, boundaries and creases

Hoppe et al

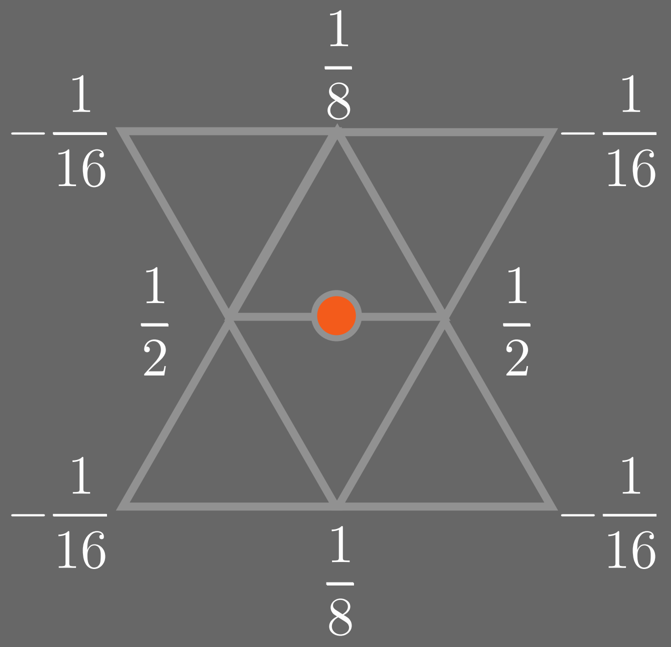
our rules



Modified Butterfly scheme

- triangular meshes, interpolating: only one rule
- needs larger support

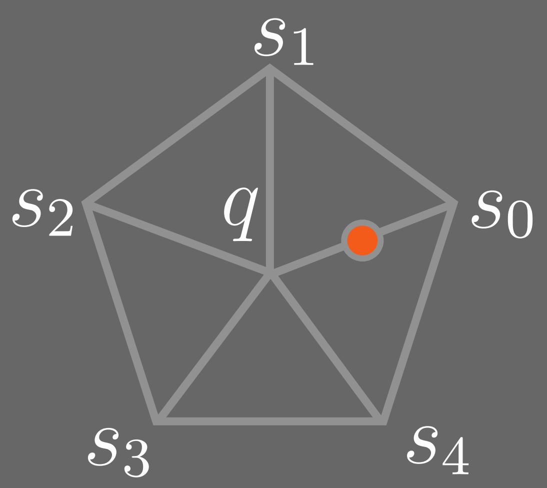
regular mask



Modified Butterfly scheme

exraordinary vertices

- coefficients derived to ensure good eigenvalues and eigenvectors

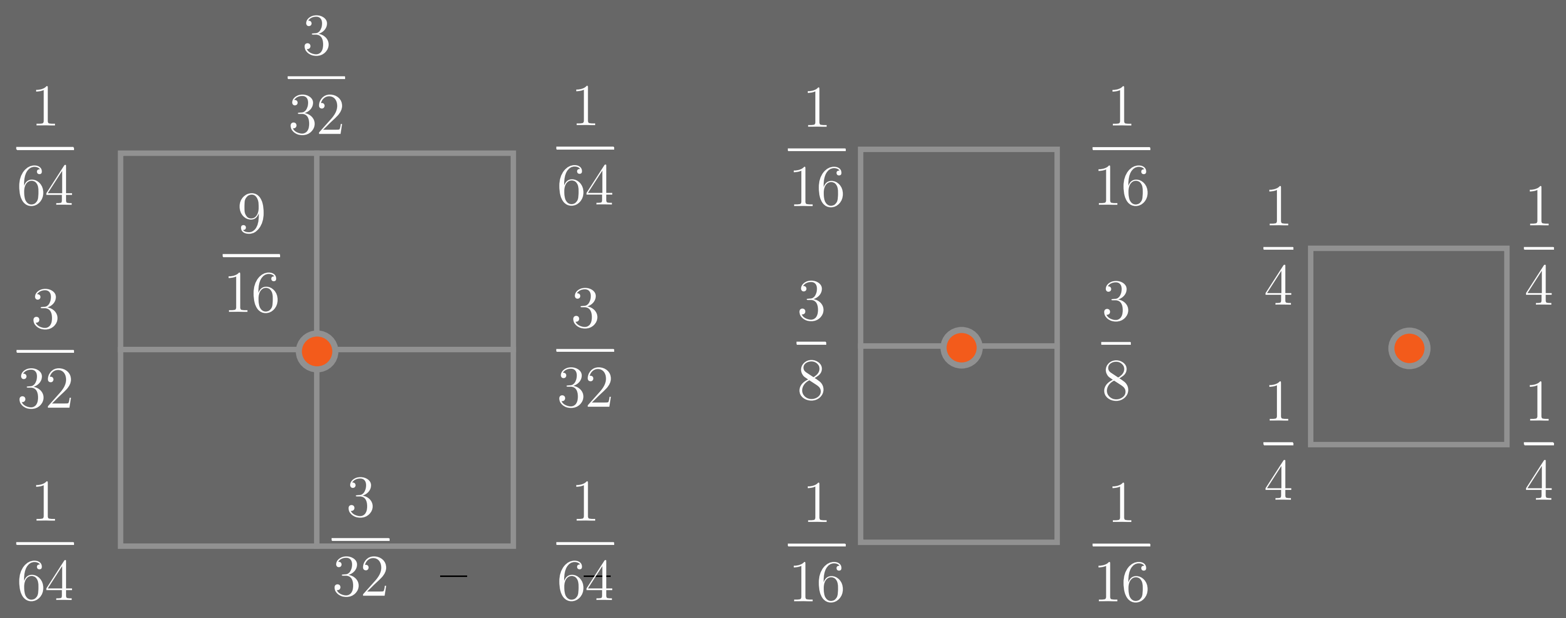


$$s_j = \frac{1}{K}\left(\frac{1}{4} + \cos\frac{2j\pi}{K} + \frac{1}{2}\cos\frac{4j\pi}{K}\right)$$

$$K > 4$$

Catmull-Clark scheme

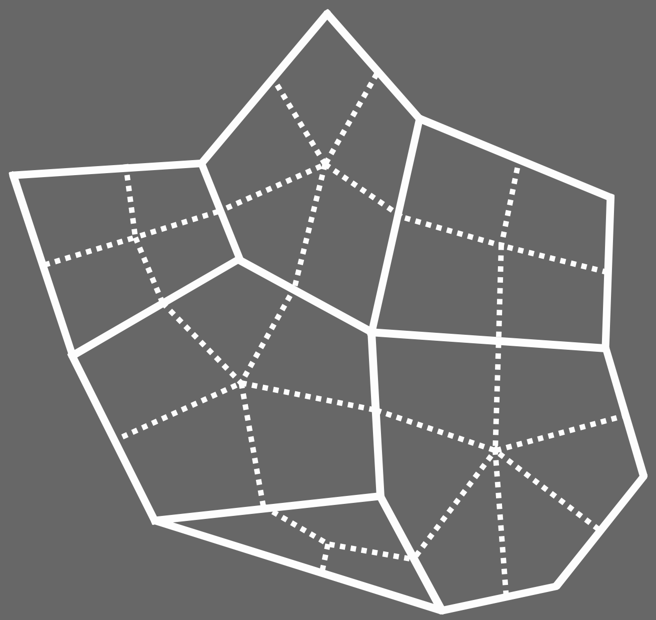
- quadrilateral, approximating
- tensor-product bicubic splines



Catmull-Clark scheme

Reduction to a quadrilateral mesh

- do one step of subdivision with special rules; all polygons become quads

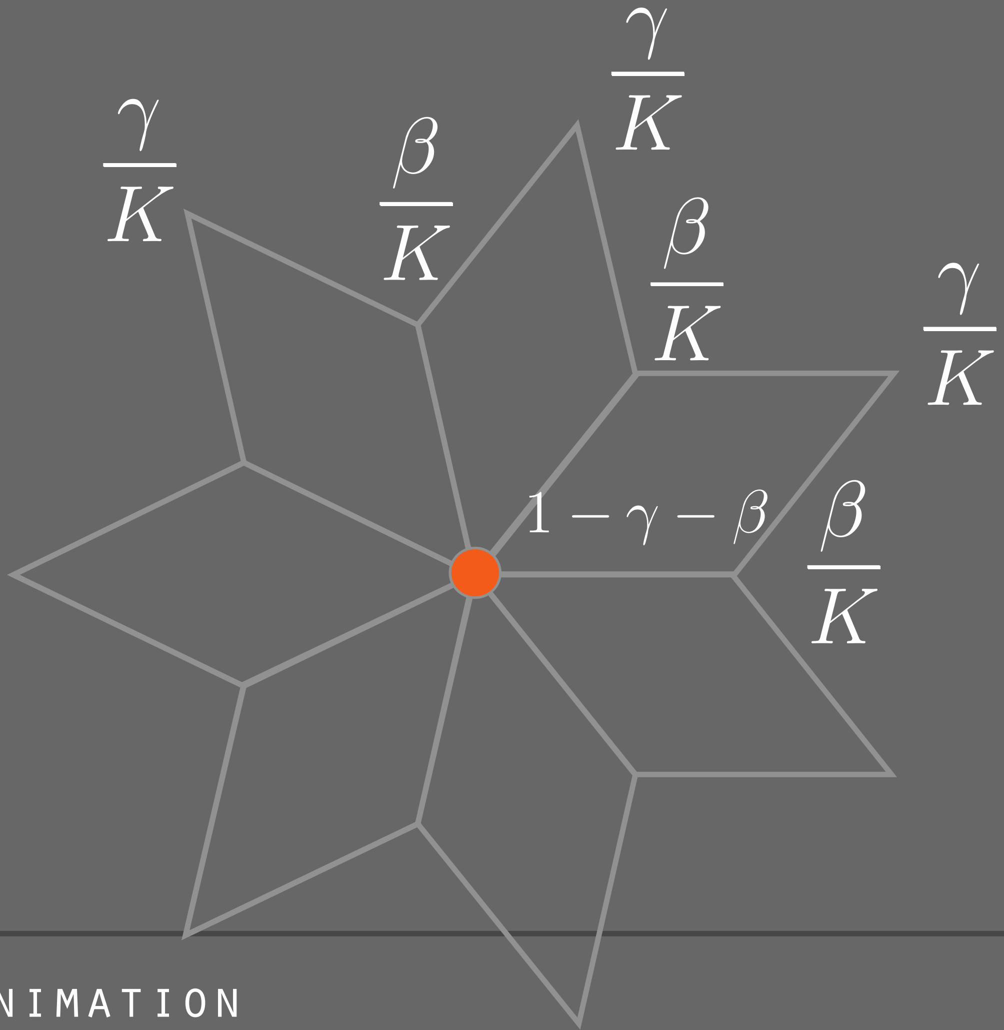


Catmull-Clark scheme

Extraordinary vertices

$$\gamma = \frac{1}{4K}$$

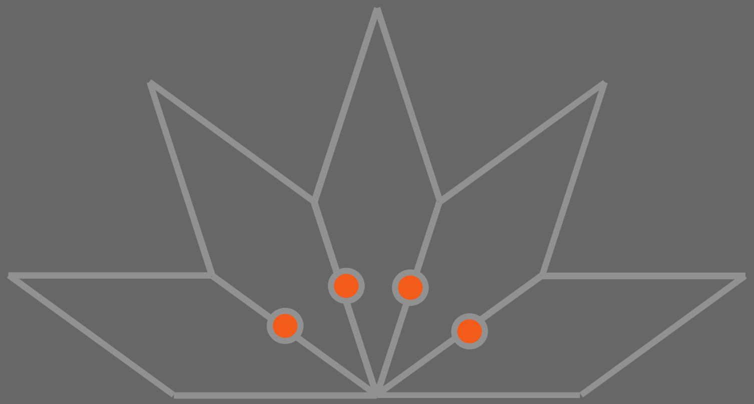
$$\beta = \frac{3}{2K}$$

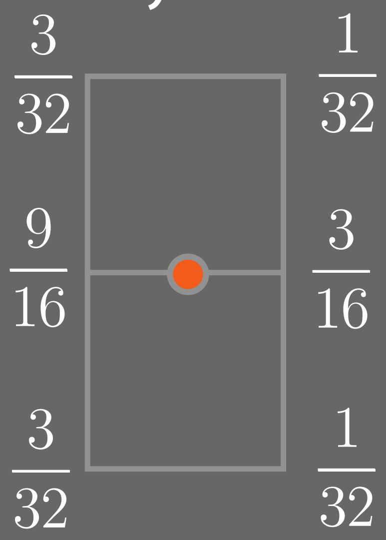


Catmull-Clark scheme

boundaries and creases

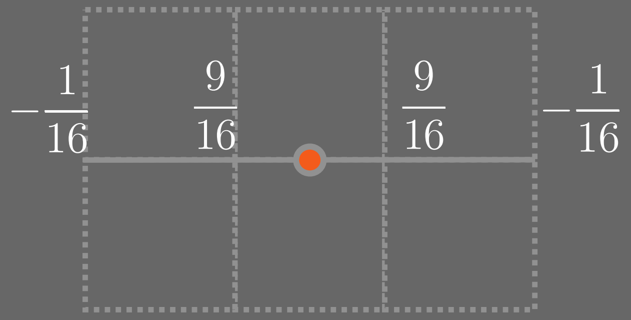
- cubic spline (same as Loop!)
- need to fix rules for C1-continuity

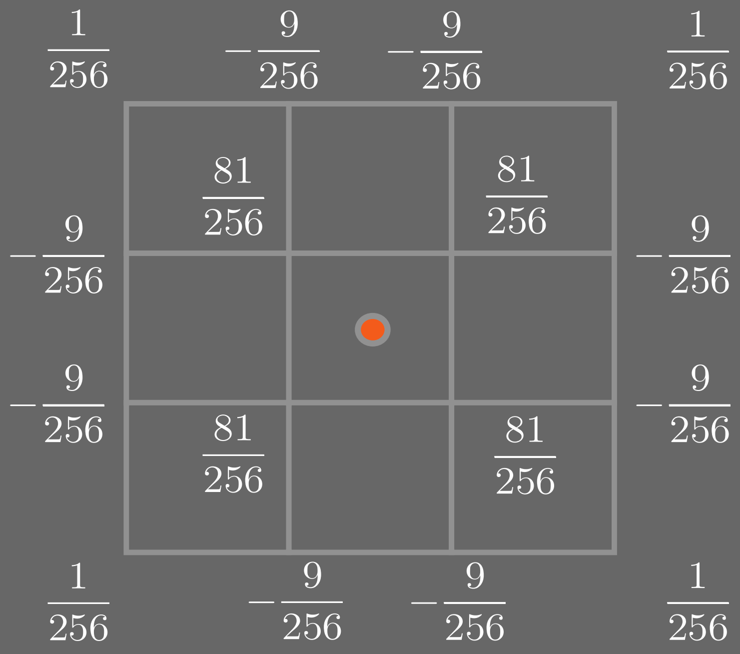




Kobbelt scheme

- quadrilateral, interpolating
- tensor product 4-point scheme

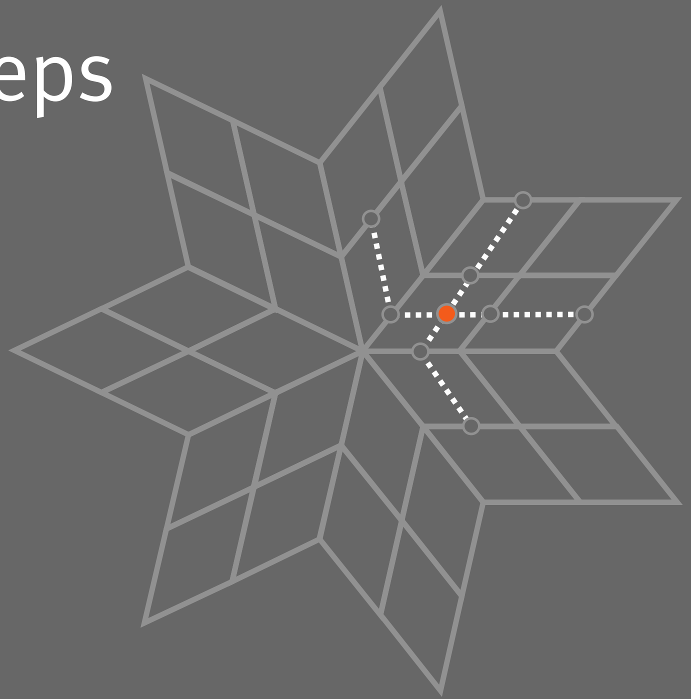




Kobbelt scheme

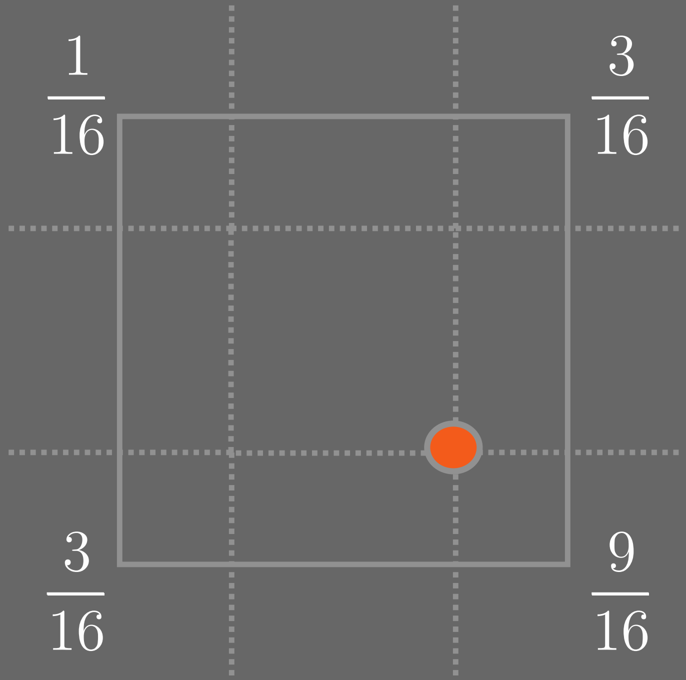
extraordinary vertices

- computation in two steps
  - edge vertices
  - face vertices
- two ways to compute
- results are the same



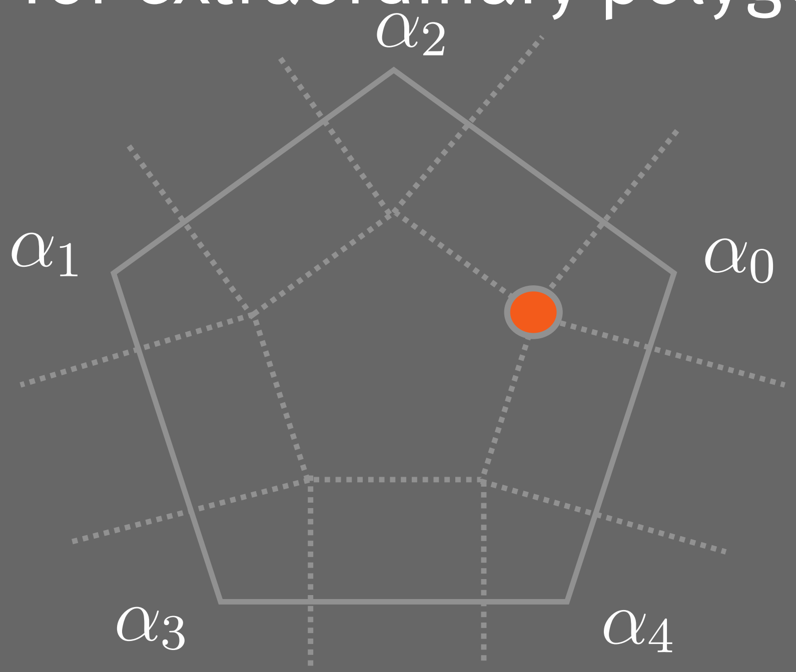
Doo-Sabin scheme

- dual scheme, quadrilateral
- extends tensor-product biquadratic splines



Doo-Sabin scheme

- after one step, all valences = 4
- rule for extraordinary polygons:



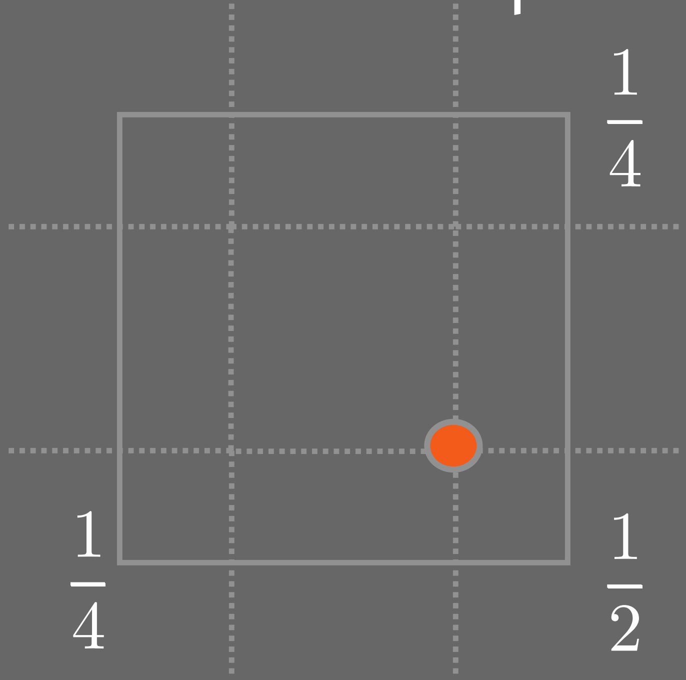
$$\alpha_0 = \frac{1+5K}{4}$$

for $i = 1 \dots K-1$

$$\alpha_i = \frac{1}{K}\left(3 + 2\cos\frac{2i\pi}{K}\right)$$

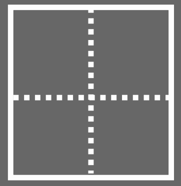
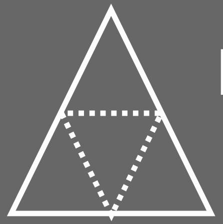
Midedge scheme

- dual scheme, quadrilateral
- extends 4-directional box spline

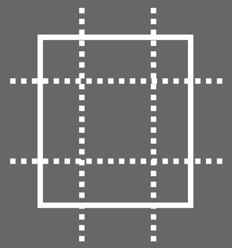


Summary

- Primal (vertex insertion)

	Approximating	Interpolating
	Catmull-Clark	Kobbelt
	Loop	Butterfly

- Dual (corner cutting)



Doo-Sabin,
Midedge

Limitations of subdivision

- no C2 with small support
- decrease of smoothness with valence
- ripples
- no direct control of fairness

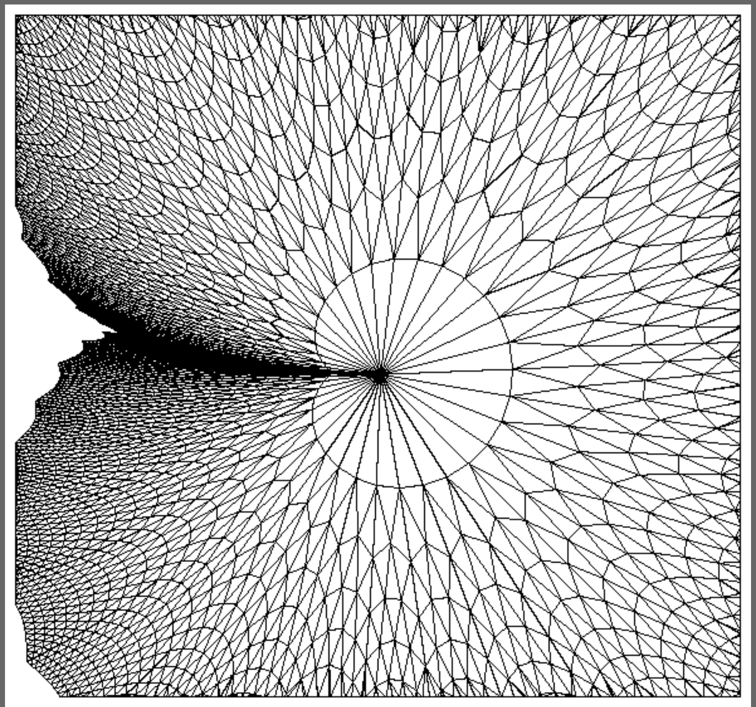
Limitations of stationary subdivision

Problems with curvature continuity

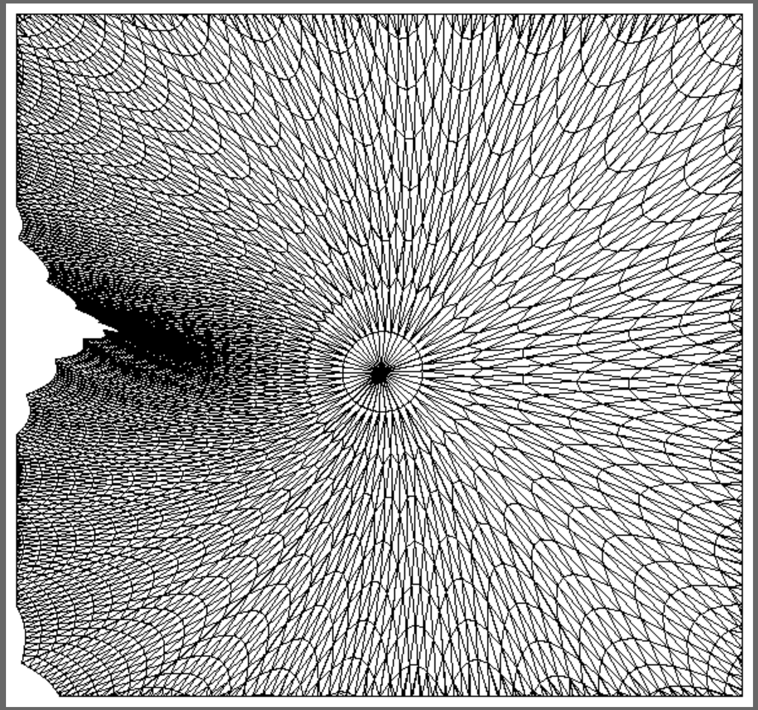
- the only practical C2 schemes (Umlauf) have “flat spots” at extraordinary vertices
- “true” C2 has to have very large support
- lack of C2-continuity = nonsmooth normal changes
- visible for high valences

Limitations of subdivision

- decrease of smoothness with valence; possible to fix



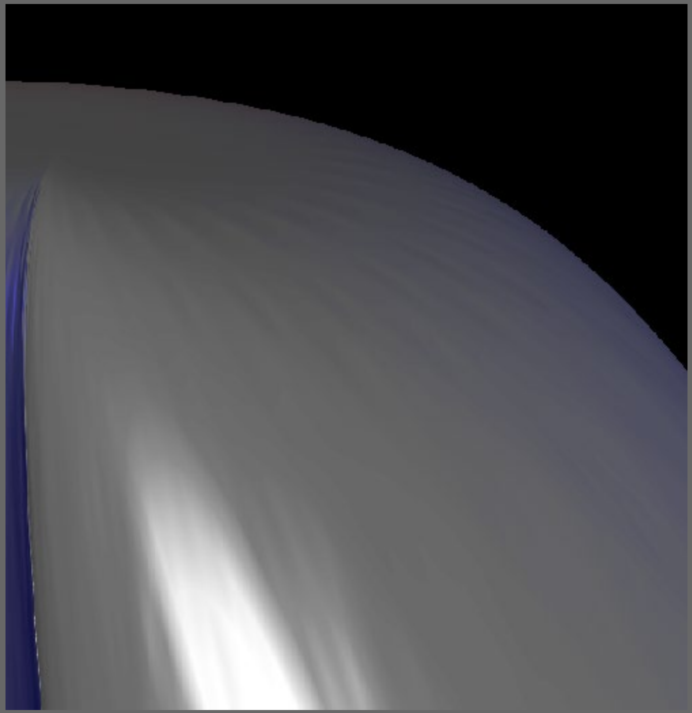
Loop



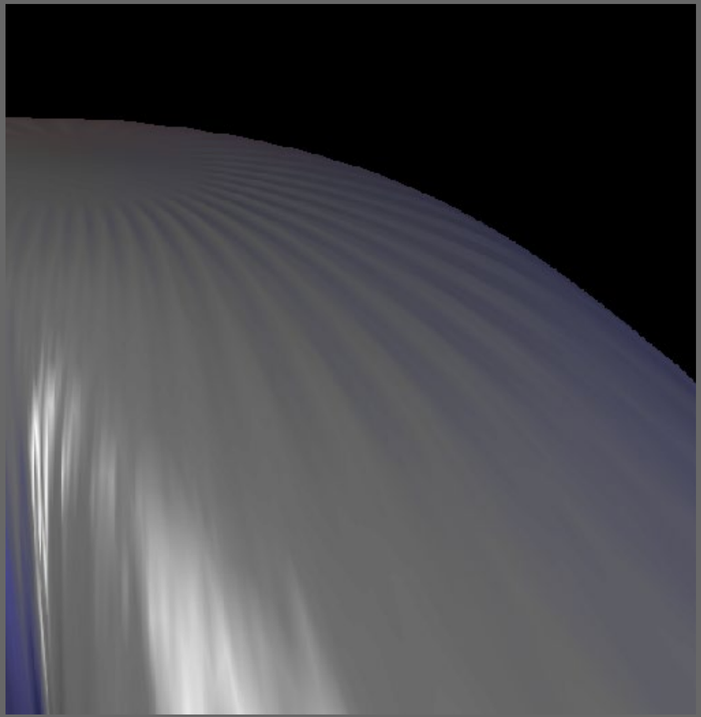
Modified Loop

Limitations of subdivision

- Ripples



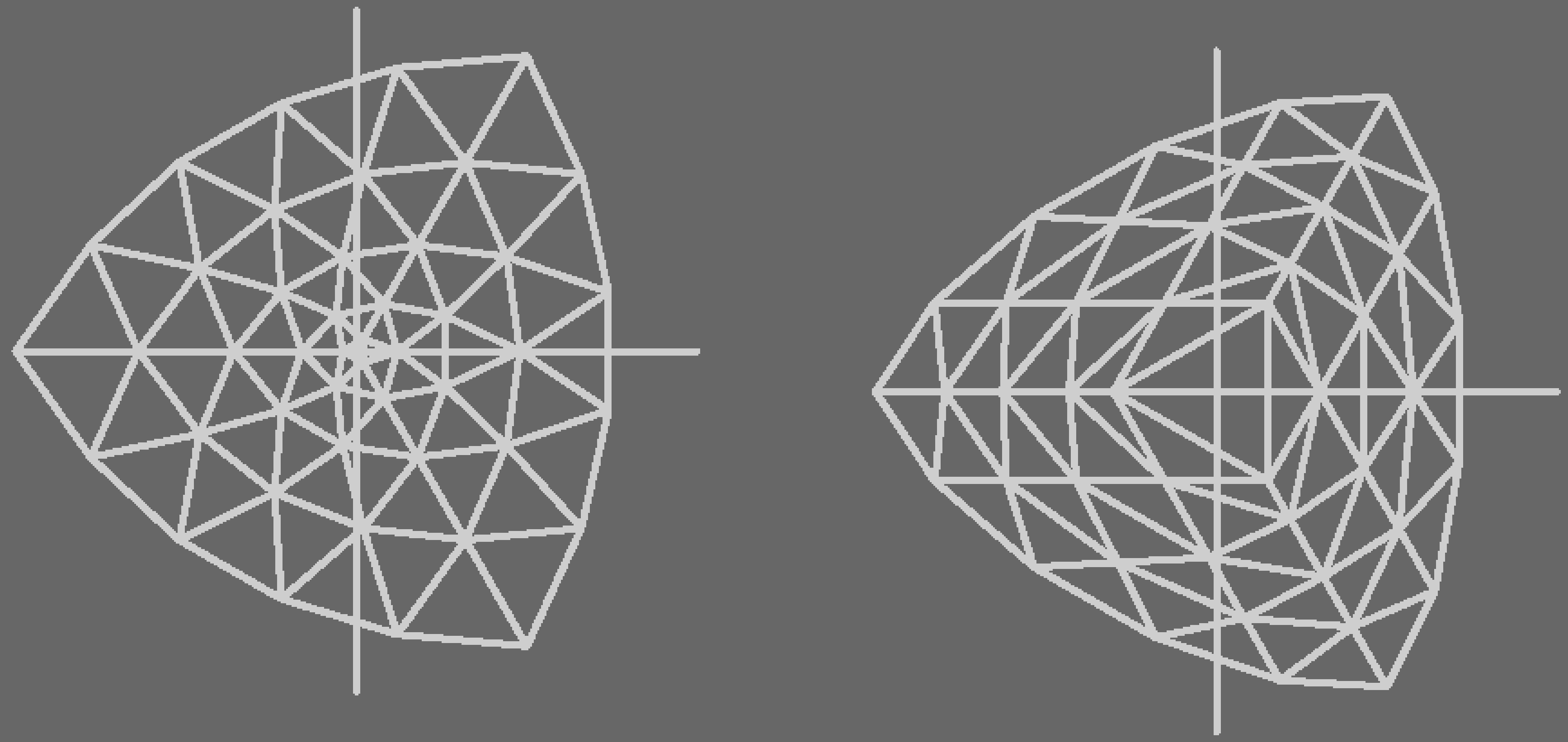
Loop



Modified Loop

Limitations of subdivision

- Uneven structure of the mesh



Extensions

Umlauf

- nonstatonary; C2 surfaces; small size of flat spot

Sederberg et al

- allows one to integrate NURBS with subdivision

Variational subdivision: Kobbelt, Warren

- slower, more complex; higher quality surfaces