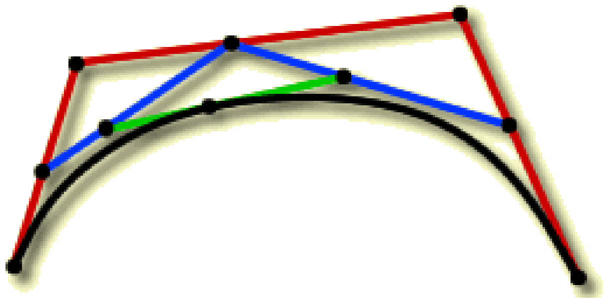
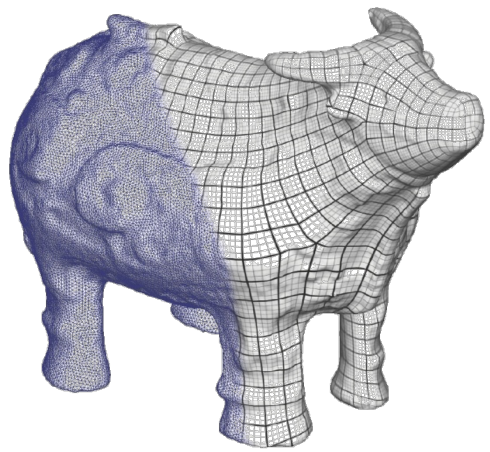
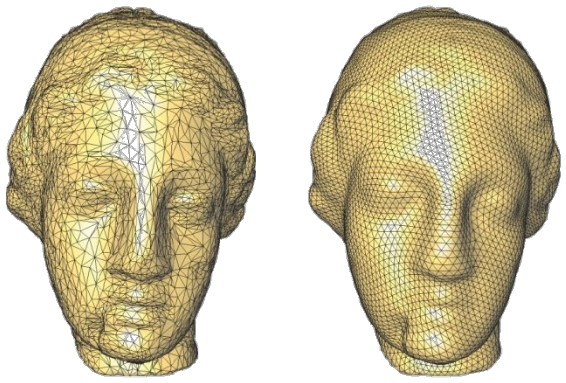
CS348a: Geometric Modeling and Processing







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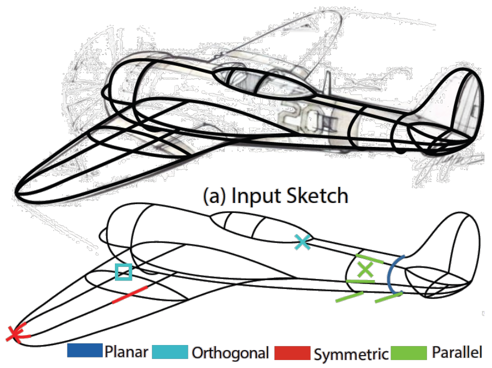
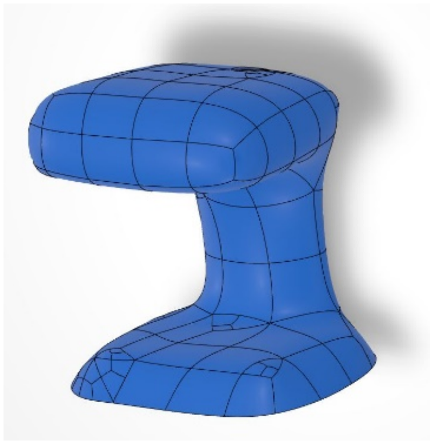
Geometric Computing

Last Time:
Designed Shapes,
Polynomial Curves

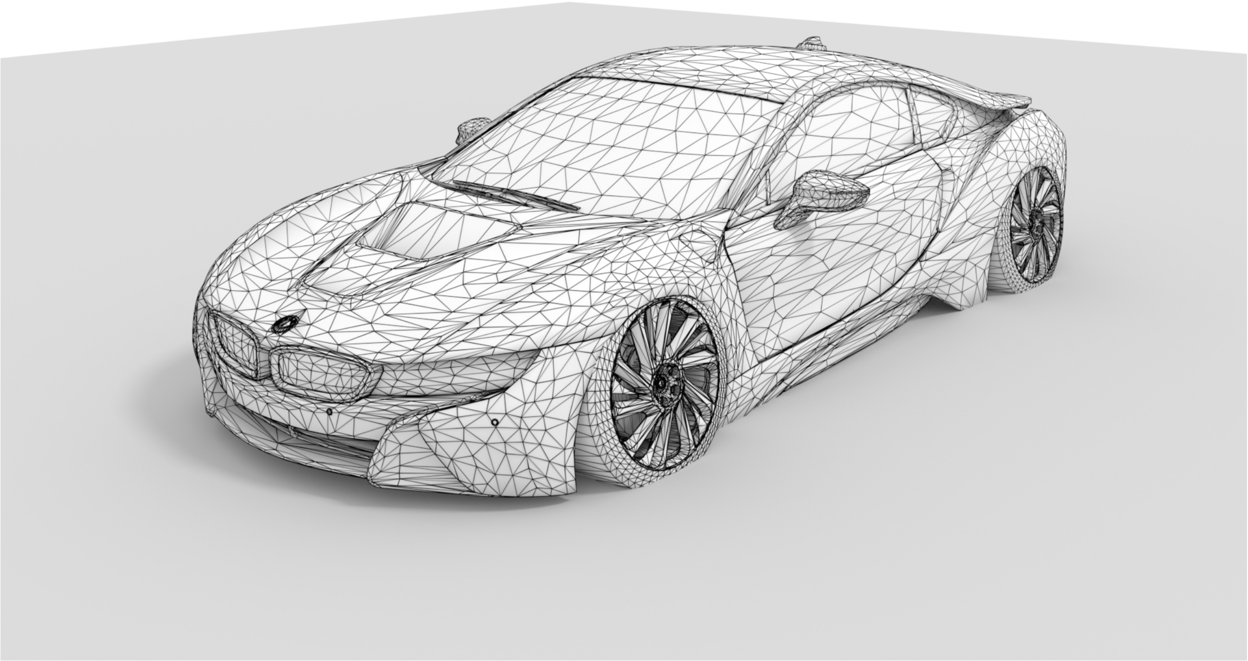
Shapes Representations for Human Design

- Boundary-based or volume-based?
  - Boundary
- One piece or many? – Splines
  - Many – splines
- What class of mathematical functions?
  - polynomial, rational
- Of what degree?
  - 1, 2, 3
- Parametric or implicit?
  - mostly parametric





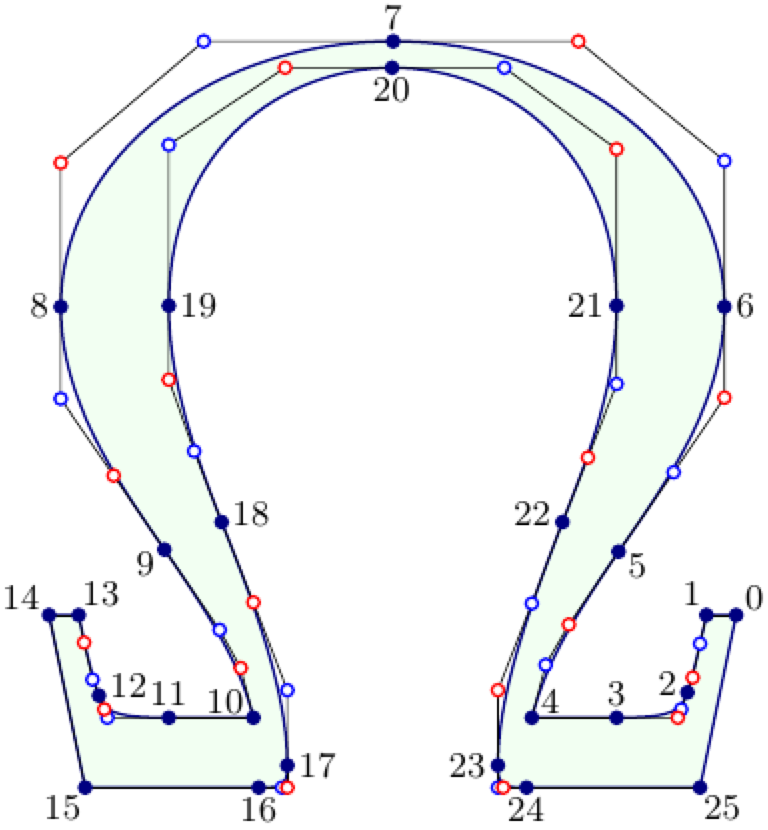
Machinery for Smooth Shapes



Typical manufactured (man-made) objects:
vehicles, airplanes, furniture …

Natural irregular objects:
trees, plants, clouds, coastlines

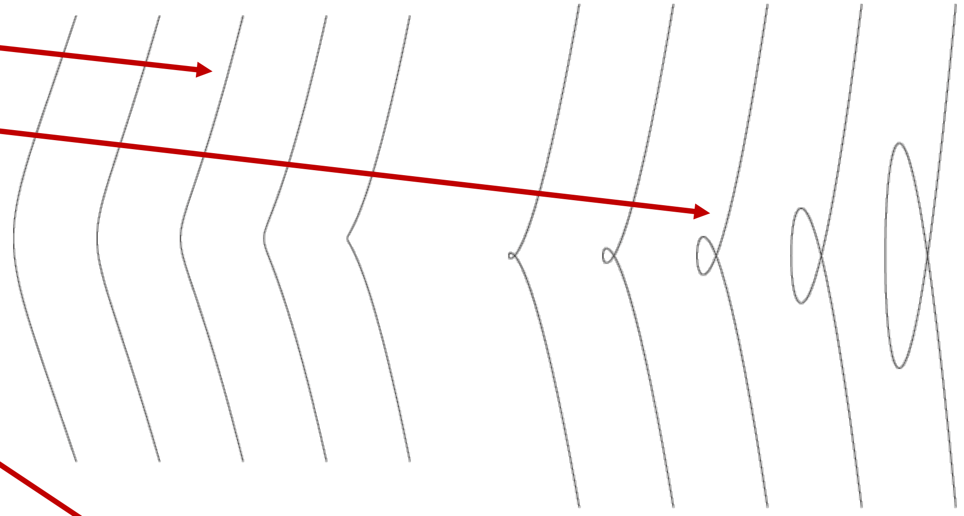
Modeling 2D Shapes with Spline Curves

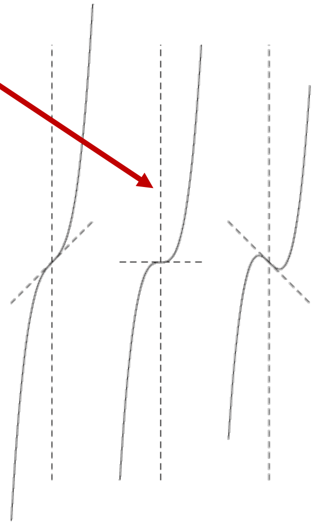


The fonts we use …

Our Basic Shapes: Parametric Polynomial Cubics

- Standard humpy: $H(r) := < r^2, r^3 + r >$
- Standard loopy: $L(r) := < r^2, r^3 - r >$
- Standard pointy: $P(r) := < r^2, r^3 >$
- Standard S-shaped: $S(r) := < r, r^3 >$
- Standard parabola: $Q(r) := < r, r^2 >$
- Standard line: $A(r) := < r, r >$



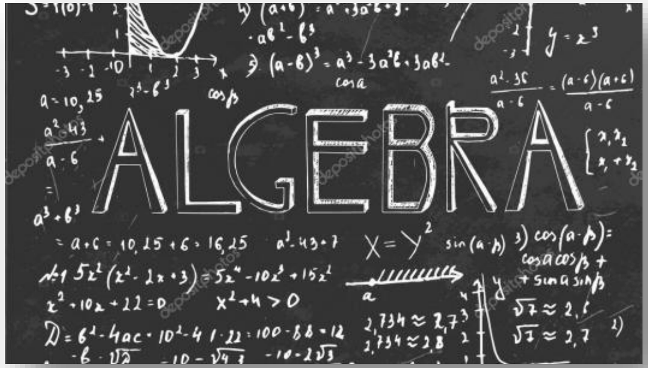


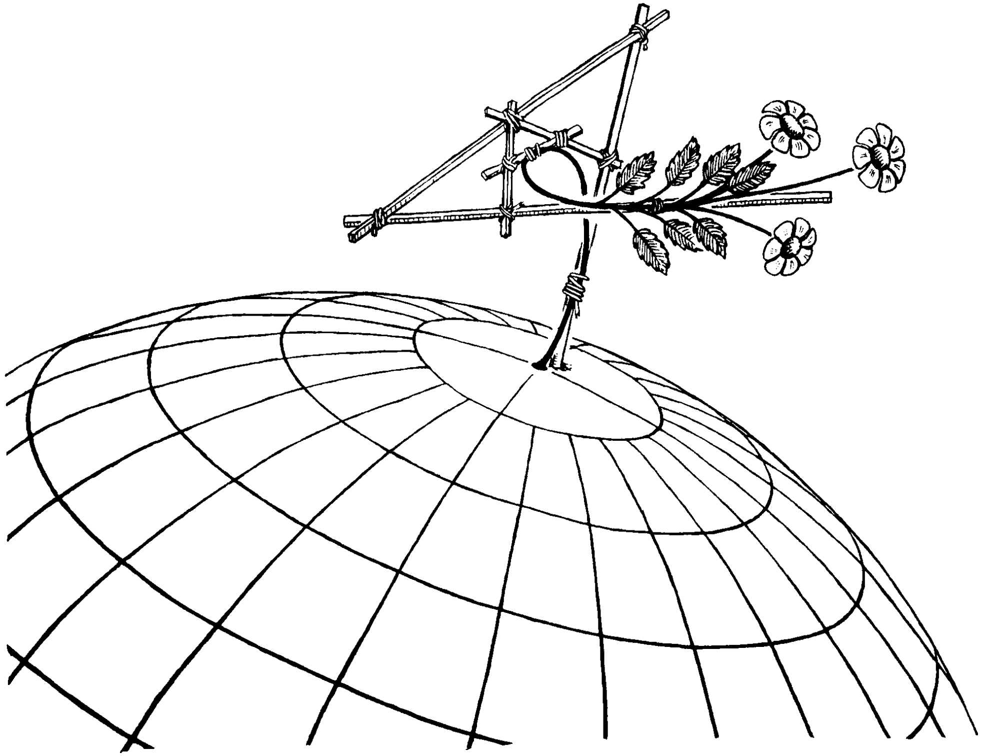
Every planar cubic is affinely equivalent to one of the above

Today:
Polar Forms, Bézier Control Points, de Casteljau Subdivision

Polar Forms and Blossoms

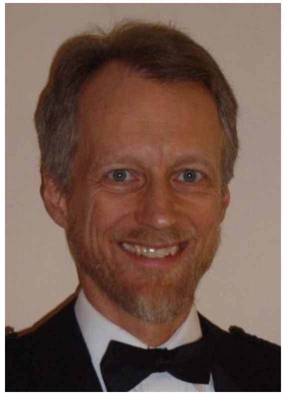
- Homogenization
- Polarization





Pierre
Bézier

Paul
de Casteljau



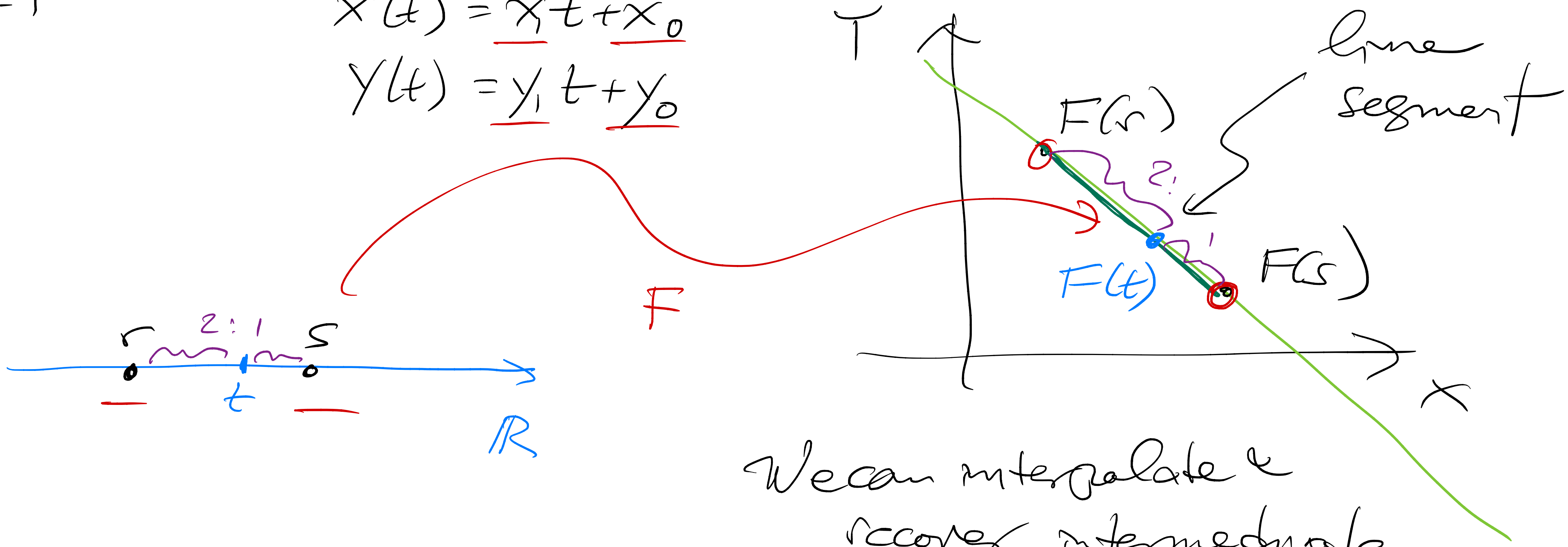
Lyle
Ramshaw

Whiteboard

Arcs of Polynomial Curves

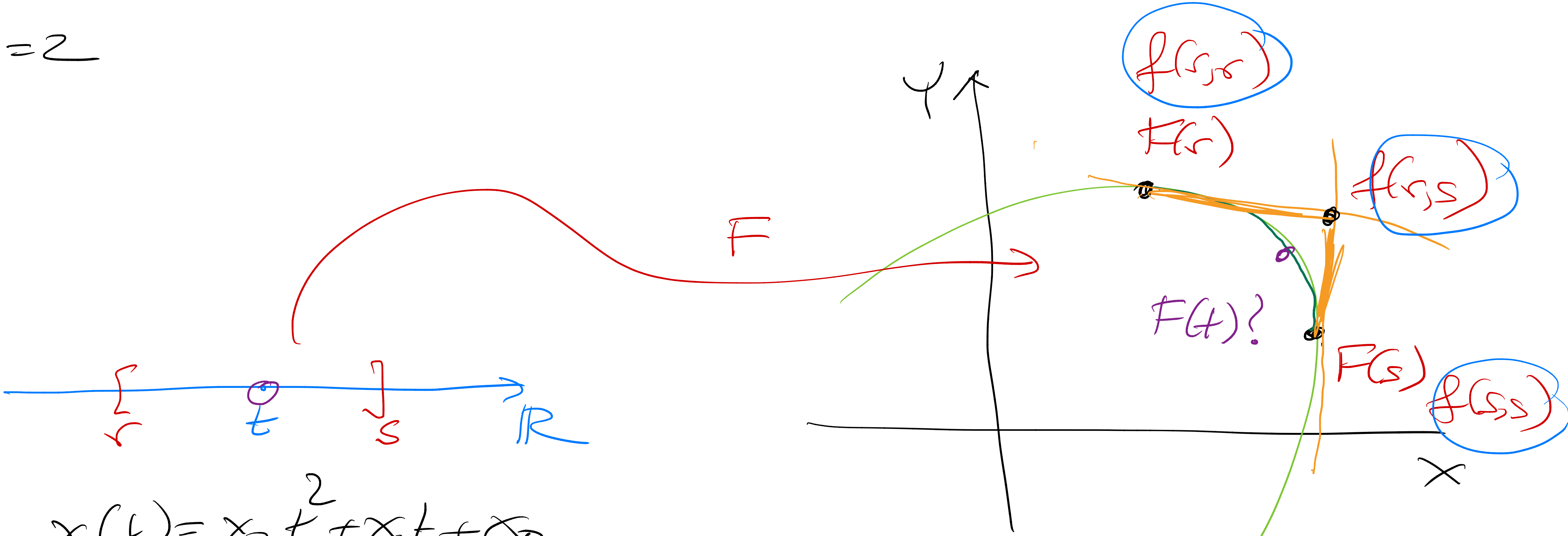
$d = 1$

$$X(t) = x_1 t + x_0$$
$$Y(t) = y_1 t + y_0$$



We can interpolate & recover intermediate points of the arc

$d = 2$



$$x(t) = x_2 t^2 + x_1 t + x_0$$
$$y(t) = y_2 t^2 + y_1 t + y_0$$

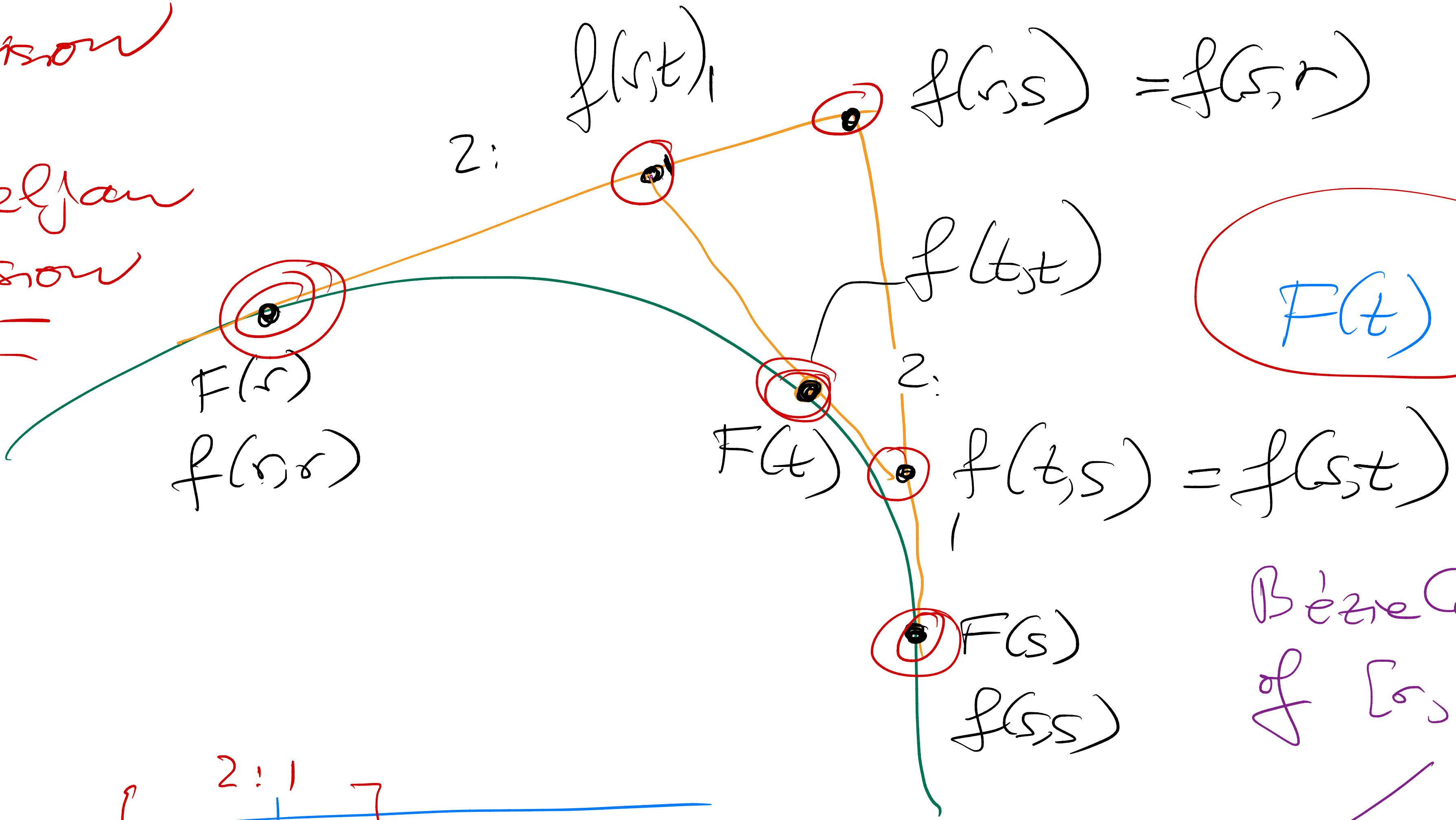
Bézier Control Points of the arc

Bézier Control Polygon

Subdivision

↓

de Casteljau Subdivision

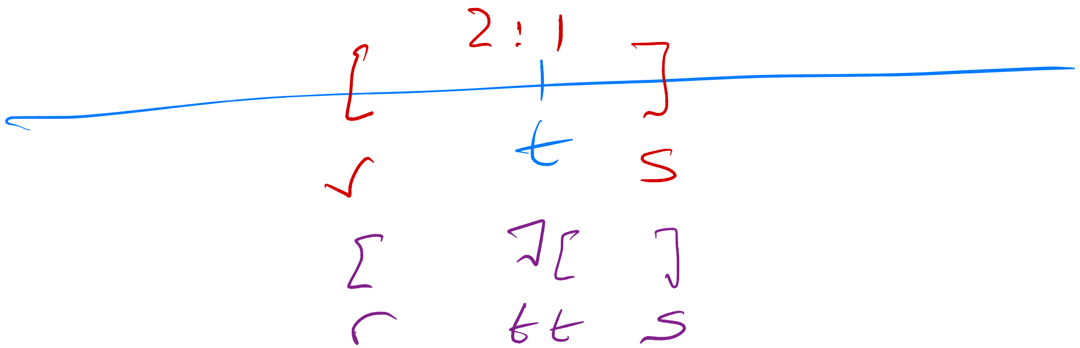


F(t) ?

Bézier Control Poly of [r,s] arc

Bézier CP [r,t)

Bézier CP [t,s)



degree $\underline{\underline{d}}$

$F(t)$ of degree d

Polar form
Blossom of F

$$F(t) \longleftrightarrow f(t_1, t_2, \dots t_d)$$

① diagonal $F(t) = f(t, t, \dots t)$

② symmetry $f(t_1, t_2, \dots t_n) = f(t_{\sigma(1)}, t_{\sigma(2)}, \dots, t_{\sigma(n)})$

③ multiaffine $\forall i$

$$f(\dots, t_i, \dots) = \alpha t_i + \beta$$

$F \longleftrightarrow f$

single var, high d

d variable, linear in each

$$F(t) = 3t^2 + 2t + 1 \qquad d=2 \qquad F \longleftrightarrow f$$

Linear

$$f(t_1, t_2) = \qquad \nexists \qquad \overrightarrow{\text{Polarization}}$$

$d=2$

$$1 \longrightarrow 1$$

$$t \longrightarrow \frac{1}{2}(t_1 + t_2)$$

$$t^2 \longrightarrow t_1 t_2$$

$$3t^2 + 2t + 1 \longrightarrow 3t_1t_2 + (t_1 + t_2) + 1$$

$$F(t) = t^3 + 2t^2 - 3t + 1$$

$d = 3$

t_1, t_2, t_3

$$1 \longrightarrow 1$$

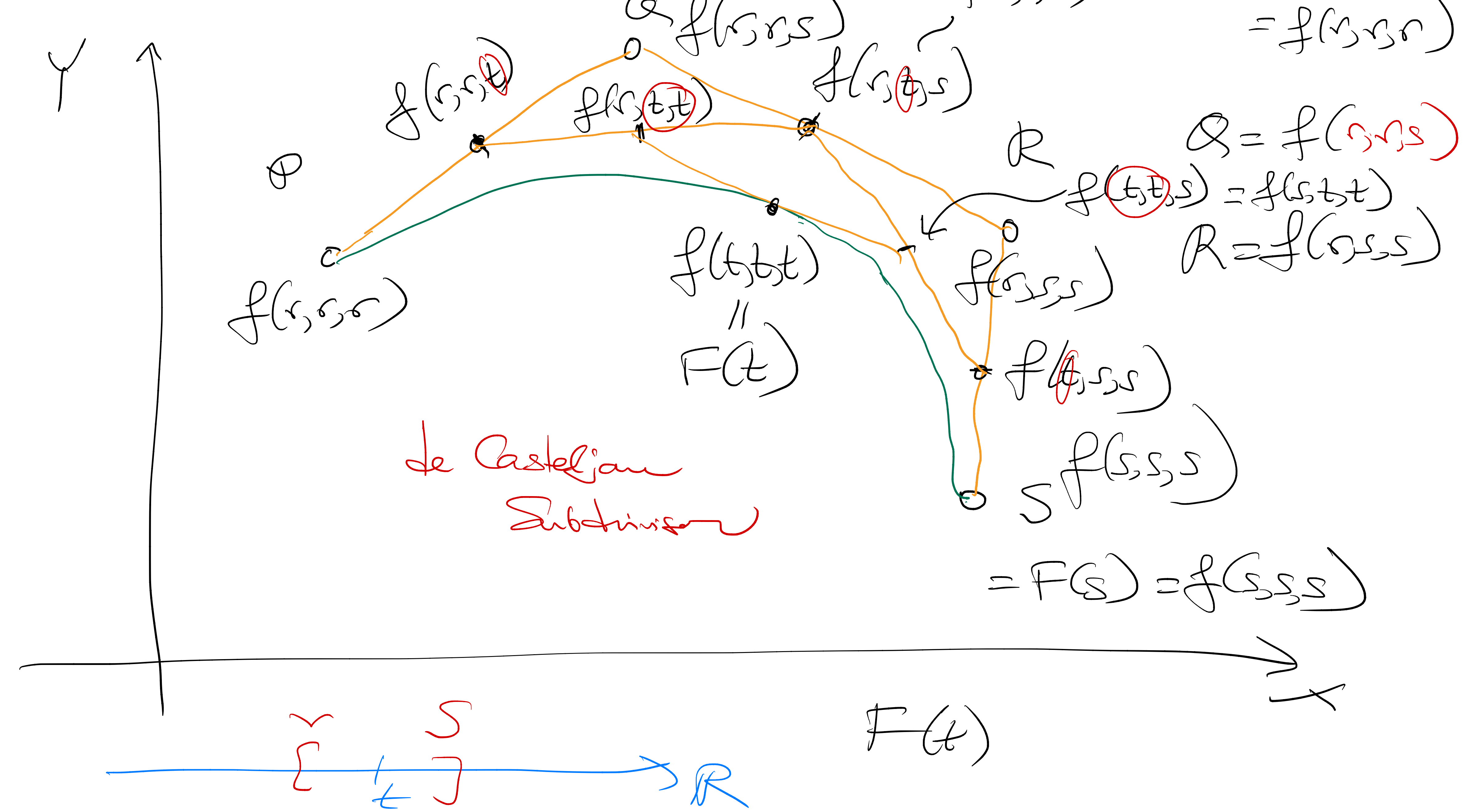
$$t \longrightarrow \frac{t_1 + t_2 + t_3}{3}$$

$$t^2 \longrightarrow \frac{t_1 t_2 + t_2 t_3 + t_3 t_1}{3}$$

$$t^3 \longrightarrow t_1 t_2 t_3$$

$$\longrightarrow t_1 t_2 t_3 + \frac{2}{3}(t_1 t_2 + t_2 t_3 + t_3 t_1) - (t_1 + t_2 + t_3) + 1$$

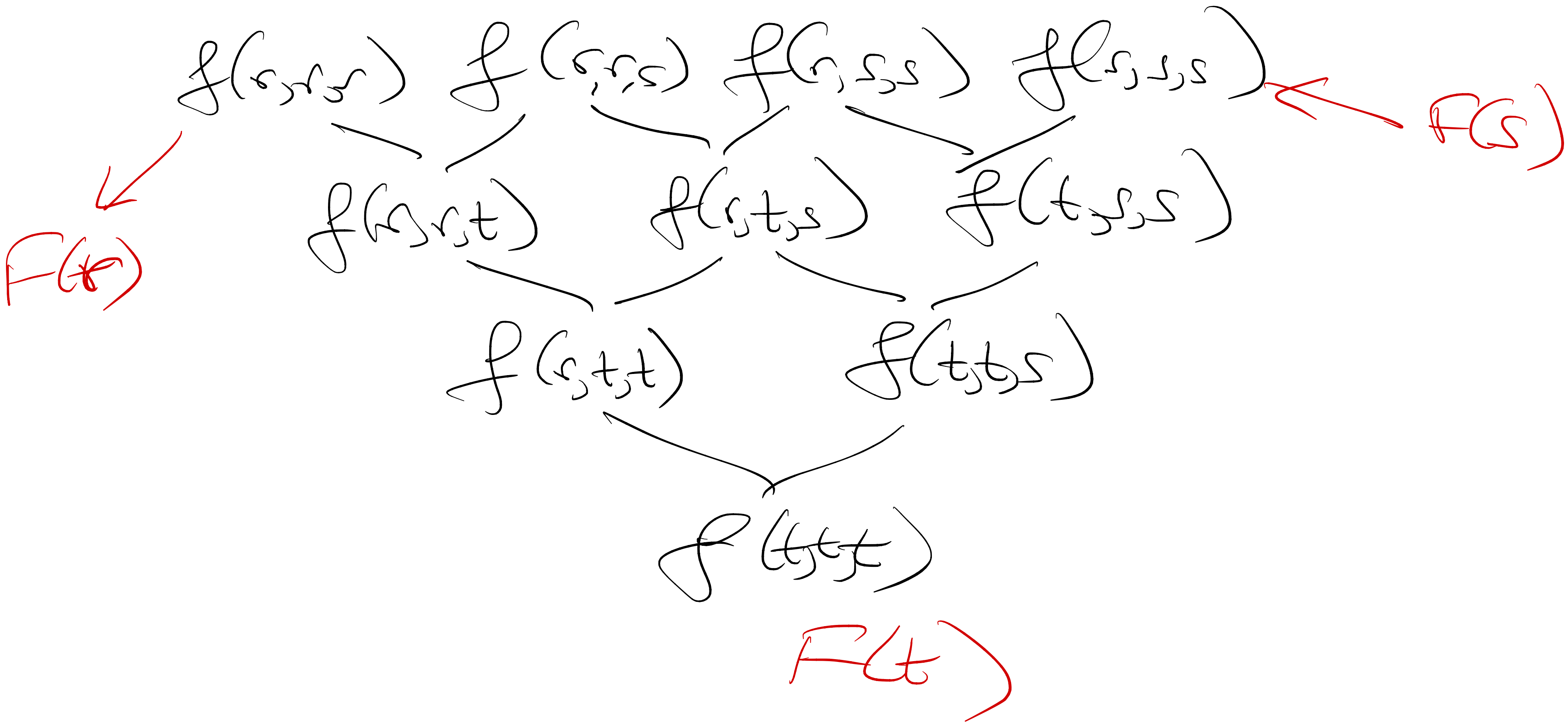
d=3 Cubic arcs



Bézier Control Pts

Bézier Control Poly

de Casteljau subdivision



$$F(t) = \left(\frac{s-t}{s-r}\right)^3 f(r,r,r) +$$

$$3\left(\frac{s-t}{s-r}\right)^2\left(\frac{t-r}{s-r}\right) f(r,r,s) +$$

$$3\left(\frac{s-t}{s-r}\right)\left(\frac{t-r}{s-r}\right)^2 f(r,s,s) +$$

$$\left(\frac{t-r}{s-r}\right)^3 f(s,s,s)$$

$r=0$

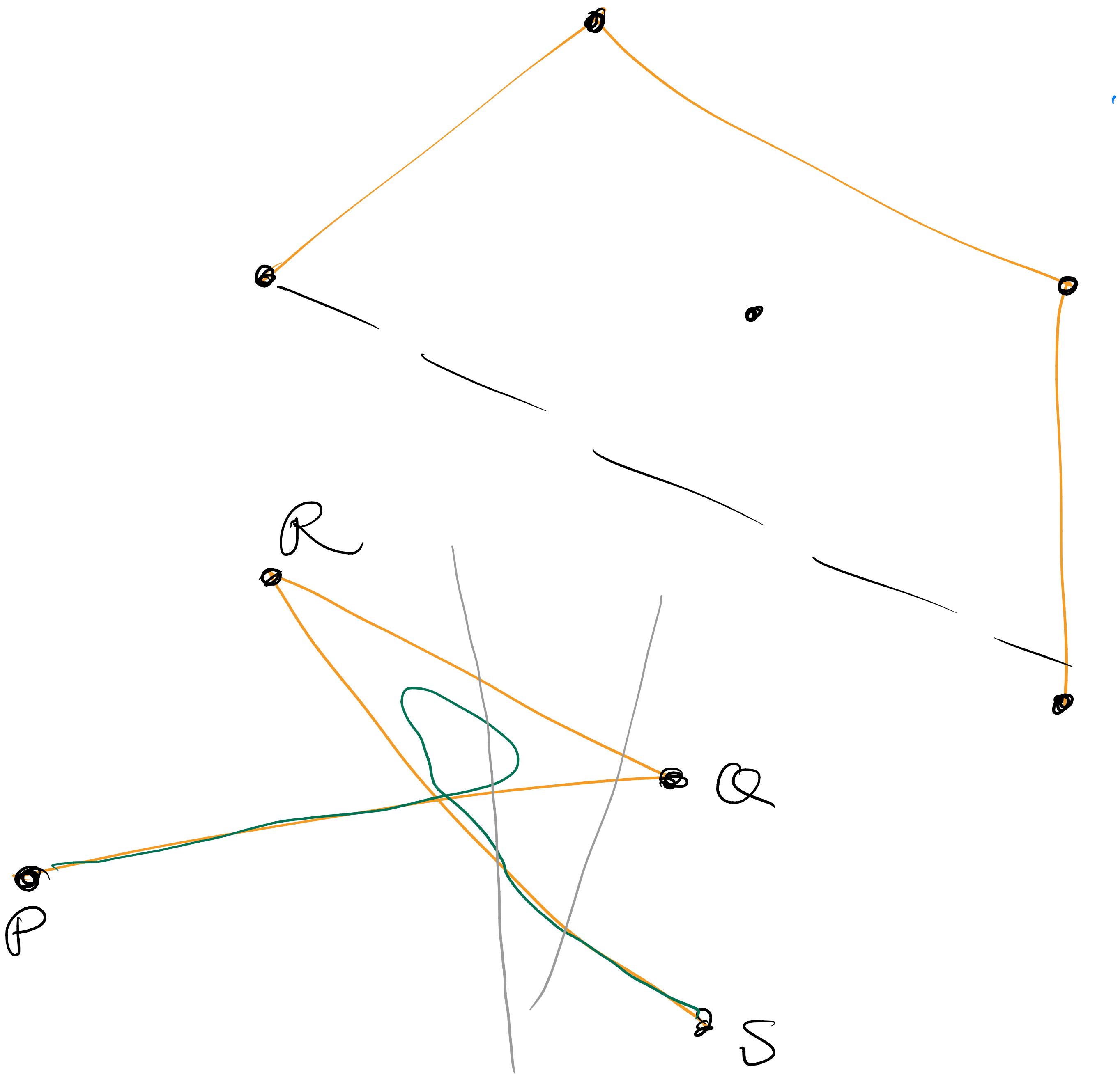
$s=1$

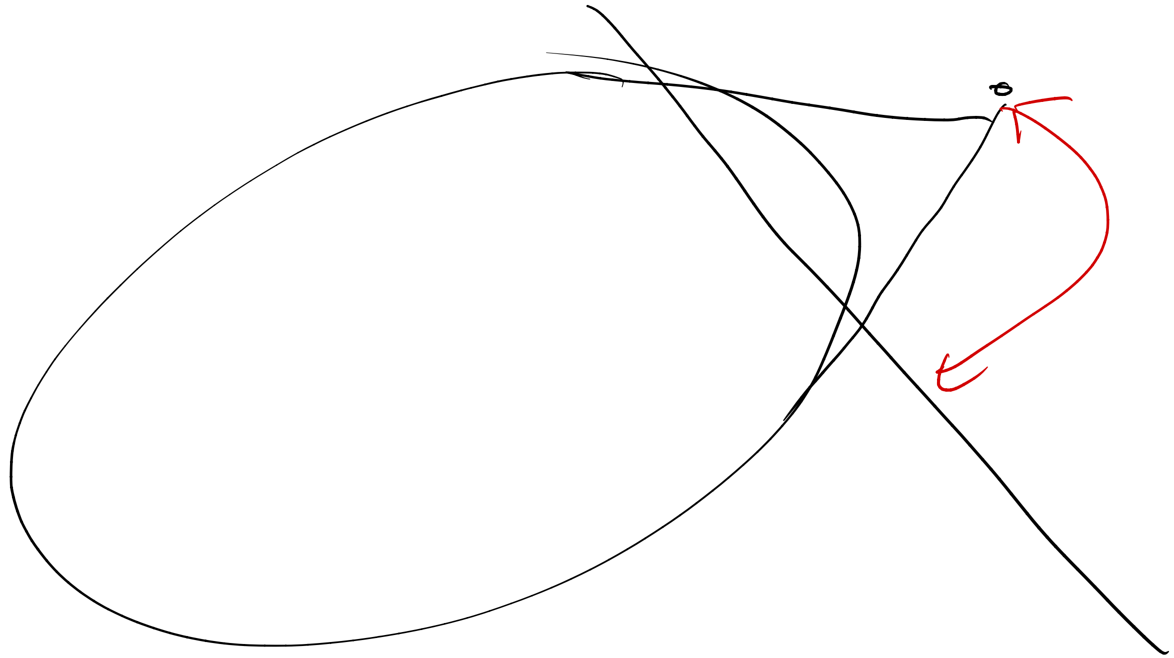
$$(1-t)^3 P + 3(1-t)^2 t\, Q + 3(1-t)t^2 R + t^3 S$$

$t \in [r,s]$

Bernstein poly

Variation diminishing property





That’s All