Volumes and Participating Media

Applications
1. Clouds, smoke, water, ...
2. Subsurface scattering: paint, skin, ...
3. Scientific and medical visualization: CT, MRI, ...

Topics
- Volume representations
- Absorption
- Scattering and phase functions
- Volume rendering equation
- Ray tracing volumes

Volume Representations

3D arrays (uniform rectangular)
- CT data

3D meshes
- CFD, mechanical simulation

Simple shapes with solid texture
- Ellipsoidal clouds with sum-of-sines densities
- Hypertexture
Absorption

\[ dL = -\sigma_a L \, ds \]

**Beer’s Law**

\[ L(s) = L(0) e^{-\sigma_a s} \]

Absorption probability

\[ L(s) = L(0) e^{-\int_0^s \sigma_a(s') \, ds'} \]

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Scatter

\[ dL = \sigma_s \left[ \int_{S^2} p(\omega' \rightarrow \omega) L(\omega') \, d\omega' - \int_{S^2} p(\omega \rightarrow \omega') L(\omega) \, d\omega \right] \]

\[ = \sigma_s \left[ \int_{S^2} p(\omega' \rightarrow \omega) L(\omega') \, d\omega' - L(\omega) \right] \]
Cross-sections

Total cross-section \( \sigma_t = \sigma_a + \sigma_s \)

Albedo \( W = \frac{\sigma_s}{\sigma_t} = \frac{\sigma_s}{\sigma_a + \sigma_s} \)

Micro vs. macro \( \Sigma = \rho \sigma \)
\[
\left[ \frac{1}{m} \right] = \left[ \frac{1}{m^3} \right] \left[ m^2 \right]
\]

Phase Functions

Phase angle \( \cos \theta = \omega \cdot \omega' \)

Phase functions \( 2\pi \int_0^\pi p(\cos \theta) d\theta = 1 \)
(From the phase of the moon)

1. Isotropic
   - simple
   \( p(\cos \theta) = \frac{1}{4\pi} \)

2. Rayleigh
   - molecules
   \( p(\cos \theta) = \frac{3}{4} + \frac{\cos^2 \theta}{\lambda^4} \)

3. Mie scattering
   - small spheres

... Huge literature ...
**Henyey-Greenstein Phase Function**

**Empirical phase function**

\[
p(\cos \theta) = \frac{1}{4\pi} \frac{1 - g^2}{\left(1 + g^2 - 2g \cos \theta \right)^{3/2}}
\]

\[
2\pi \int_0^\pi \! p(\cos \theta) \cos \theta \, d\theta = g
\]

\(g: \) average phase angle

**Volume Balance Equation**

\[
\text{[change in radiance along a direction]} = \text{[emission]} - \text{[absorption]} + \text{[scattered in]} - \text{[scattered out]}
\]
The Volume Rendering Equation

Integro-differential equation

\[
\frac{\partial L(x,\omega)}{\partial s} = -\sigma_i(x)L(x,\omega) + \sigma_s(x) \int_{s^2} p(\omega' \rightarrow \omega) L(x,\omega') \, d\omega'
\]

Integro-integral equation

\[
L(x,\omega) = \int_0^\infty e^{-\int_0^{s'} \sigma_i(x+s'\omega) \, ds'} \left[ \sigma_s(x+s'\omega) \int_{s^2} p(\omega' \rightarrow \omega) L(x+s'\omega,\omega') \, d\omega' \right] \, ds'
\]

Attenuation: Absorption and scattering

Source: Scatter (+ emission)

RGBA Formulation

Assume color and alpha are defined in the volume

- Unassociated
  \[(r(x), g(x), b(x), a(x)) = (C(x), a(x))\]

- Associated (premultiplied)
  \[(c(x), a(x)) = (a(x)C(x), a(x))\]

Use compositing operator

\[
T(0) = 1 - a(0) \quad A(0) = a(0)
\]
\[
L(0) = c(0) \quad L(0) = c(0)
\]
\[
L(x+1) = L(x) + T(x)c(x) \quad L(x+1) = L(x) + (1 - A(x))c(x)
\]
\[
T(x+1) = T(x)(1 - a(x+1)) \quad A(x+1) = A(x) + (1 - A(x))a(x+1)
\]
Ray Marching

\[ c(s_{i+1}) = c(s_i) + (1 - a(s_i))c_{i+1} \]
\[ a(s_{i+1}) = a(s_i) + (1 - a(s_i))a_{i+1} \]
\[ c(s_i) = \text{trilinear}(c, i, j, k, x(s_i)) \]

M. Levoy, Ray tracing volume densities

Should use premultiplied colors

Ray Marching with Shadows

Primary ray

Shadow rays
Simple Fog Model

\[ \frac{\partial L(s)}{\partial s} = -\sigma_a L(s) + c_f \]

\[ L(s) = (1 - e^{-\sigma_a s}) c_f + e^{-\sigma_a s} c_s \]

Examples

Participating media
- Sunset with beams of light
- Bohren example in the shower
- Henrik clouds
- Texels
- Hypertextures

Visualization
- Visible human
- Finite element
Clouds and Atmospheric Phenomena

Hogum Mountain
Sunrise and sunset

Modeling:
Simon Premoze
William Thompson

Rendering:
Henrik Wann Jensen

7am

6:30pm

9am