The Light Field

Electromagnetic waves; photons

Frequency spectra and color

Polarization

Spatial distribution

Radiometry

1. How is light measured?

2. How is the spatial distribution of light energy described?

3. How is reflection from a surface characterized?

4. What are the conditions for equilibrium flow of light in an environment?
Radiant Energy and Power

Power: Watts vs. Lumens
- Energy efficiency
- Spectral efficacy

Energy: Joules vs. Talbot
- Since the velocity of light is so fast, may typically ignore distinction between power and energy
- Exposure - Reaction rates
  - Film response
  - Skin - sunburn

Radiometry vs. Photometry

- Radiance & Radiometry [Units = Watts]
  
  Physical measurement of electromagnetic energy.

- Luminance & Photometry/Colorimetry [Lumen]
  
  Perceptual measurement of relative subjective sensation due to light of different wavelengths.

- Brightness [Units = Brils] \( B = Y^{\frac{1}{3}} \)
  
  Perceptual measurement of the relative perceived sensation of light of different intensities.
Transport Theory

Transport theory is concerned with calculating how stuff $Q$ flows in the environment.

- Mass $m$
- Charge $q$
- Radiant energy $\Phi$

Transport quantities are built around a core of basic geometric ideas. These are tricky!

The easiest way to learn transport theory is to think in terms of particles (photons).

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Particle Density

Phase Space

Particle characterized by position and velocity

Particle densities

Ratio of number of particles to volumes (phase space)

$$ n(x, v, t) = \lim_{\Delta x \Delta v \to 0} \frac{N(t)}{\Delta^3 x \Delta^3 v} $$

$$ n(x, v, t) = \frac{1}{\Delta^3 x \Delta^3 v} \frac{N(t)}{\Delta^3 x \Delta^3 v} $$

$\mathbb{R}^3 \times \mathbb{R}^3$
Flows

Count particles crossing a surface

\[ d^3x = v \cos \theta \, dA = (\vec{v} \, dt) \cdot dA \]

\[ Q(x, v) = n(x, v) d^3x = n(x, v)(\vec{v} \, dt) \cdot dA \]

- Flux [Stuff/Time] (or Rate or Flow)

\[ \Phi = \frac{dQ}{dt} = n(x, v) \vec{v} \cdot dA \]

- Flux density [Stuff/(Time • Area)]

\[ \Phi = \frac{d^2Q}{dt \, dA} = n(x, v) \vec{v} \cdot dA = n(x, v) \nu \cos \theta \]

Angles and Solid Angles

- Angle

\[ \theta = \frac{\text{length}}{\text{radius}} \]

⇒ circle has \(2\pi\) radians

- Solid angle

\[ \Omega = \frac{\text{area}}{\text{radius}^2} \]

⇒ sphere has \(4\pi\) steradians

*If the area is not on the sphere, then the solid angle subtended by the area is equal to the area projected onto the unit sphere.*
Differential Solid Angles

\[ dA = (r \, d\theta)(r \sin \theta \, d\phi) = r^2 \sin \theta \, d\theta \, d\phi \]

\[ d\omega = \frac{dA}{r^2} = \sin \theta \, d\theta \, d\phi \]

\[ S = \int_0^{\pi/2} \int_0^{2\pi} \sin \theta \, d\theta \, d\phi = 4\pi \]

Radiance and Luminance

Definition: The radiance (luminance) is the power per unit projected area perpendicular to the ray per unit solid angle in the direction of the ray.

\[ d^2\Phi = L(x, \omega) \, d\vec{\omega} \cdot d\vec{A} \]

\[ L(x, \omega) \equiv \frac{d^2\Phi}{d\vec{\omega} \cdot d\vec{A}} \]

\[ \begin{bmatrix} \frac{W}{m^2 \text{sr}} \\ \frac{lm}{m^2 \text{sr}} \end{bmatrix} = \text{nit} \]
Environment Maps = Radiance at a Point

Miller and Hoffman, 1984

Environment Maps

*Interface*, Chou and Williams (ca. 1985)
Environment Maps

Cubical Environment Map

180 degree fisheye
Photo by R. Packo

Cylindrical Panoramas

Spherical Light Field

$L(x, y, \theta, \varphi)$

Capture all the light leaving an object - like a hologram

4 degree-of-freedom gantry
Two-Plane Light Field

2D Array of Cameras  
2D Array of Images

Properties of Radiance

1. Fundamental field quantity that characterizes the distribution of light in an environment.
   
   -> All other quantities are derived from it.

2. Radiance invariant along a ray.
   
   -> Radiance is what is propagated in a ray tracer

3. Response of a sensor proportional to radiance.
   
   -> Image is a 2D set or rays
Radiance: 1st Law

The radiance in the direction of a light ray remains constant as the ray propagates.

\[ d\Phi_1 = L_1 d\omega_1 dA_1 = L_2 d\omega_2 dA_2 = d\Phi_2 \]

\[ d\omega_1 = \frac{dA_1}{r^2} \]
\[ d\omega_2 = \frac{dA_2}{r^2} \]

\[ d\omega_1 dA_1 = \frac{dA_1 dA_2}{r^2} = d\omega_2 dA_2 \]

\[ \therefore L_1 = L_2 \]

*L is the numeric quantity that should be associated with rays in ray tracers.*

Radiance: 2nd Law

The response of a sensor is proportional to the radiance of the surface visible to the sensor.

L is what should be computed and displayed.

\[ R = \int \int_A L \cos \theta \, d\omega \, dA = L \int \int_A \cos \theta \, d\omega \, dA = LT \]

Throughput

\[ T = \int \int_A \cos \theta \, d\omega \, dA \]
Throughput

Throughput: \( T = \int \int d^2 T = \int \int d\hat{\omega} \cdot d\bar{A} \)

Properties:

1. Throughput measures or counts the number of lines or rays in beam of light.

2. Throughput is conserved in an optical system; that is, throughput is unchanged under the laws of geometric optics (straight lines, reflection, refraction, mirages).

Radiance = energy [conserved] / throughput [conserved]

\[
L(x, \omega) = \frac{d^2 \Phi}{d\hat{\omega} \cdot d\bar{A}} \quad [\text{Conserved}]
\]

Quiz 1

Does radiance increase under a magnifying glass?

\[\text{NO!!}\]

\[\text{CS348B Lecture 4} \quad \text{Pat Hanrahan, Spring 2001}\]
Quiz 2

Does the brightness that a wall appears to the eye depend on the distance of the viewer to the wall?

NO!!