Why You Weight Your Camera Rays the Way You Probably Did

1 An Explanation

There appears to be some confusion about why the weighting factor for camera rays in assignment 3 should be $\frac{A cos^4 \theta}{Z^2}$. The reason *is not* the approximation made to produce equation 7 in the Kolb paper. In fact, the correct weighting of rays has nothing to do with this equation. The weighting is derived directly from the Monte Carlo estimate of the irradiance integral in equation 6. If this statement is surprising to you, then you got a bit lucky on assignment 3 and should definitely read along.

We'll start from the very beginning. We wish to compute the irradiance E at a point x' on the film plane. This results in the integral over radiance incident from all directions ω in the hemisphere above the film plane.

$$E(x') = \int_{\mathcal{H}^2} dE = \int_{\mathcal{H}^2} L \cos\theta' d\omega \tag{1}$$

Since the incoming radiance L is nonzero only for directions that intersect the back of the lens, we introduce a *change of variables* to integrate over the area of the back of the lens (in assignment 3's case, a disk with the same radius as the back lens element) instead of over the hemisphere of directions. Notice that the conversion from differential solid angle to differential area requires the solid angle to be projected onto the plane of the disk (or conversely, the differential patch of area on the disk to be projected into the direction of the solid angle), thus $d\omega = \frac{dA'' \cos\theta''}{\|x''-x'\|^2}$. Setting $\theta' = \theta''$ since the film plane is parallel to the back of the lens, this results in equation 5 from the Kolb paper (Notice now the domain of integration is the area D of the disk used to represent the back of the lens).

$$E(x') = \int_{x'' \in D} L(x'', x') \frac{\cos\theta'^2}{\|x'' - x'\|^2} dA'' \tag{2}$$

Since the perpendicular distance between the back of the lens and the film is Z, we know $\|x'' - x'\|^2 = \frac{Z^2}{\cos^2\theta'}$, and thus we get equation 6 from Kolb.

$$E(x') = \int_{x'' \in D} L(x'', x') \frac{\cos^4\theta'}{Z^2} dA'' \tag{3}$$

We now wish to compute a Monte Carlo estimate of this integral using N samples. Let E by the estimator for $E(x')$. E is given by:

$$E = \frac{1}{N} \sum_{i=0}^{N-1} \frac{f(X_i)}{p(X_i)} \tag{4}$$

Where X_i is a sample (in this case a randomly selected point x'' drawn from the domain D using the probability distribution $p(X_i)$). $f(X_i)$ is the value of the function being integrated for the sample X_i. More concretely, $f(x'') = L(x', x'')\frac{\cos^4\theta'}{Z^2}$.

I'll now assume that in assignment 3, you *uniformly sampled* the disk at the back of the lens with respect to area. Therefore $p(x'')$ is a constant, and normalization of the pdf requires $p(x'') = \frac{1}{A}$ (A is the total area of the disk). Therefore...

$$E = \frac{1}{N} \sum_{i=0}^{N-1} L(x', x'') \frac{A\cos^4\theta'}{Z^2} \tag{5}$$

PBRT takes care of performing this summation and weighting each term by $\frac{1}{N}$. The Scene::Li() method computes the value $L(x', x'')$ by shooting the ray into the scene. For the estimator to be unbiased, this value must be weighted by the term $\frac{A\cos^4\theta'}{Z^2}$.

That's where your weighting comes from. It derives from the decision to perform a uniform sampling of the disk with respect to area. If you chose a different sampling of the disk, then your ray weight would need to change accordingly.