Ray Tracing I: Basics

Today

- Basic algorithms
- Overview of pbrt
- Ray-surface intersection for single surface

Next lecture

- Techniques to accelerate ray tracing of large numbers of geometric primitives

Classic Ray Tracing

Greeks: Do light rays proceed from the eye to the light, or from the light to the eye?

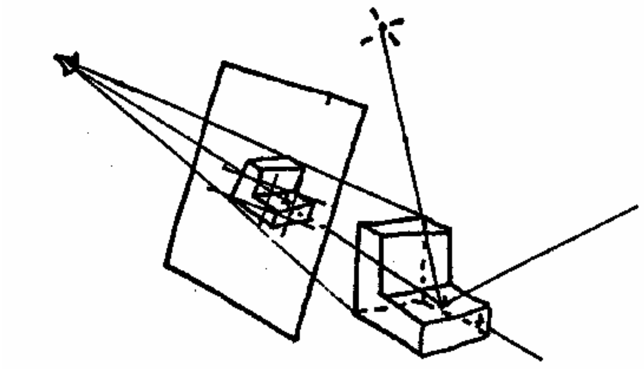
Three ideas about light

1. Light rays travel in straight lines (mostly)
2. Light rays do not interfere with each other if they cross (light is invisible!)
3. Light rays travel from the light sources to the eye (but the physics is invariant under path reversal - reciprocity).

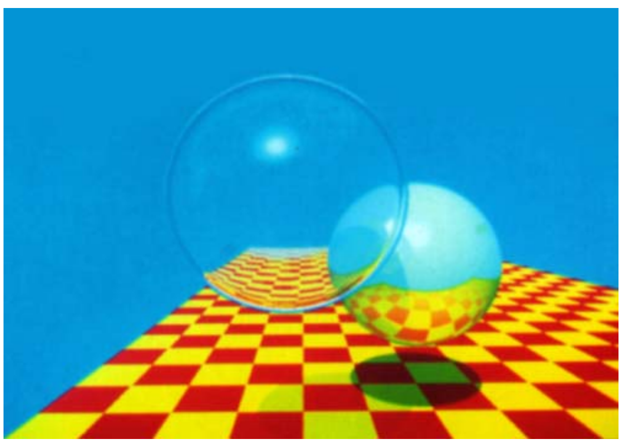
Ray Tracing in Computer Graphics

Appel 1968 - Ray casting

1. Generate an image by sending one ray per pixel
2. Check for shadows by sending a ray to the light



Ray Tracing in Computer Graphics



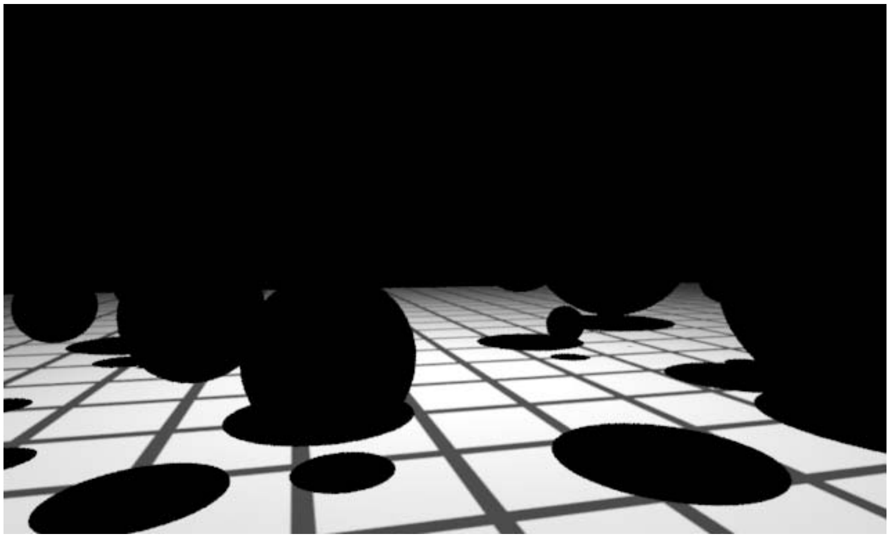
Whitted 1979

Recursive ray tracing (reflection and refraction)

Ray Tracing Video

Spheres-over-plane.pbrt (g/m 10)

Spheres-over-plane.pbrt (mirror 0)



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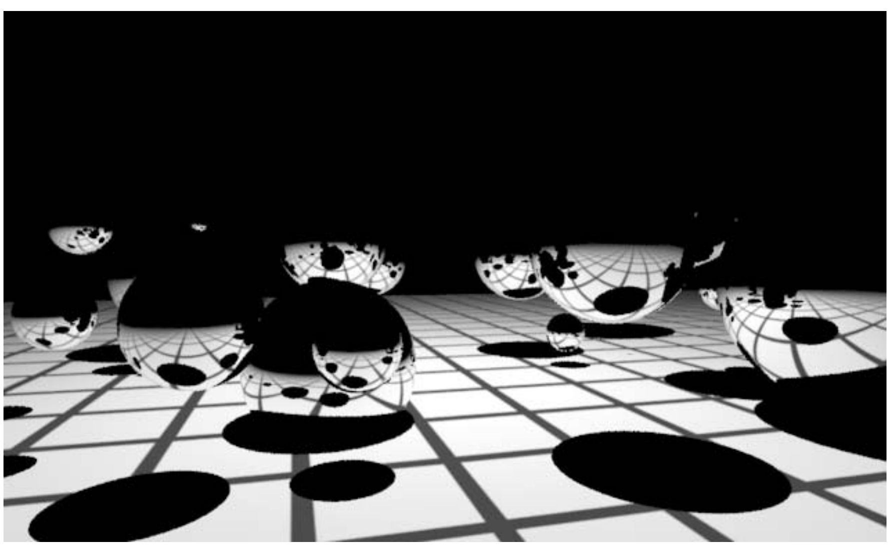
Spheres-over-plane.pbrt (mirror 1)



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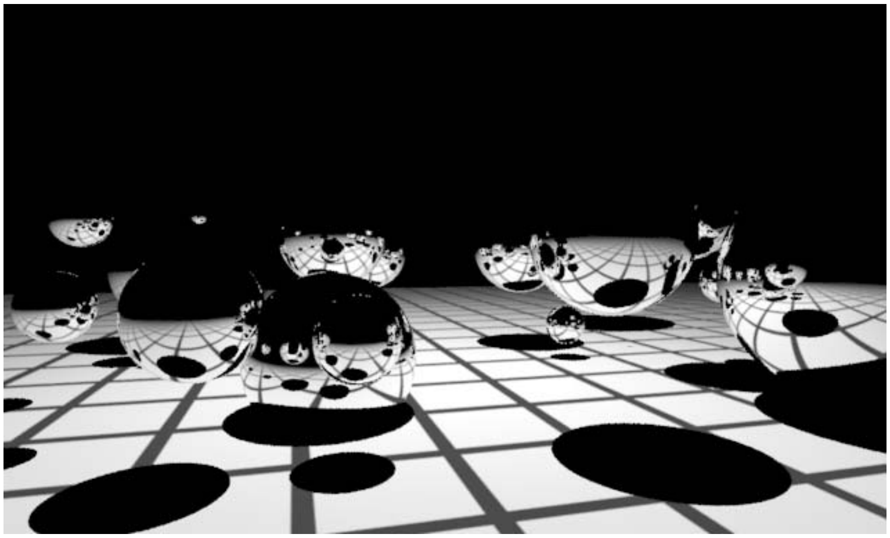
Spheres-over-plane.pbrt (mirror 2)



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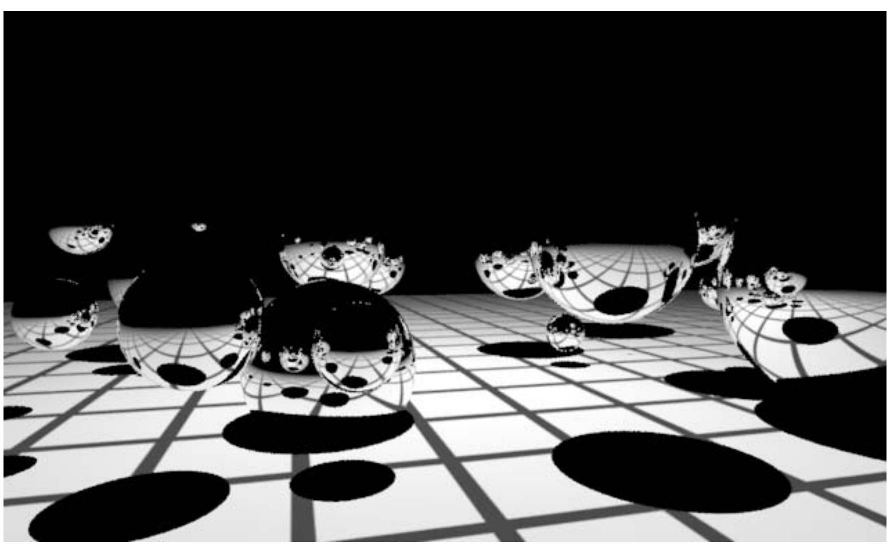
Spheres-over-plane.pbrt (mirror 5)



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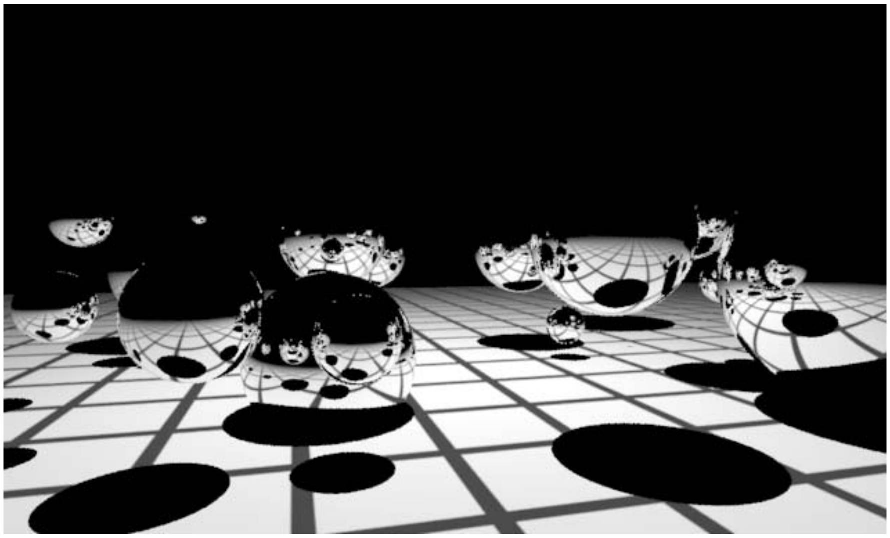
Spheres-over-plane.pbrt (mirror 10)



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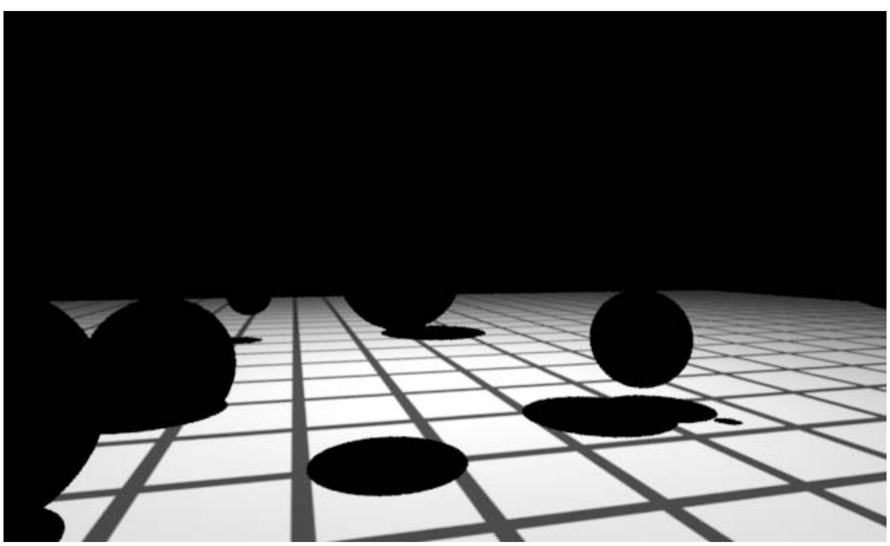
Spheres-over-plane.pbrt (glass 0)



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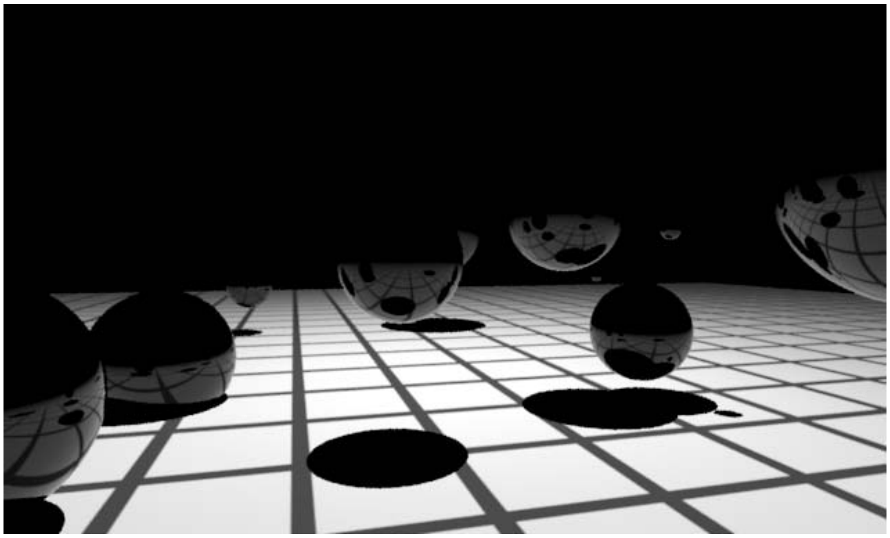
Spheres-over-plane.pbrt (glass 1)



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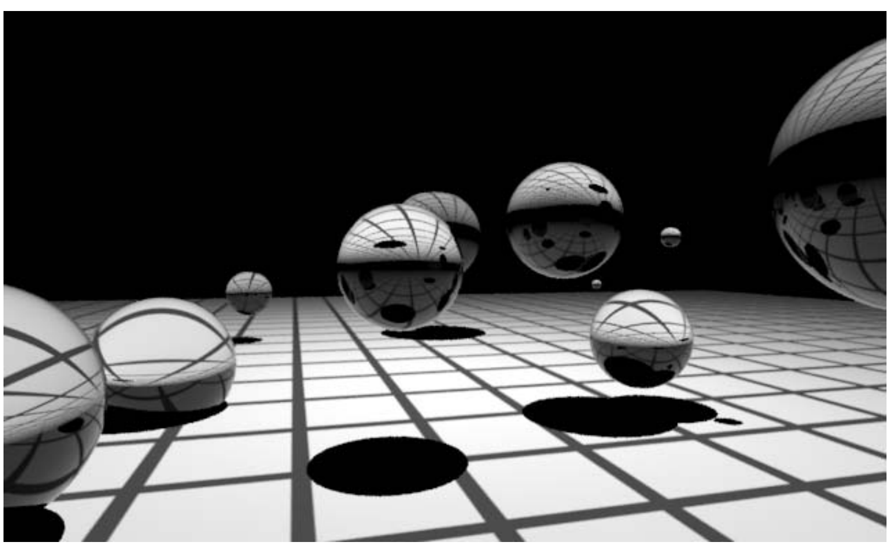
Spheres-over-plane.pbrt (glass 2)



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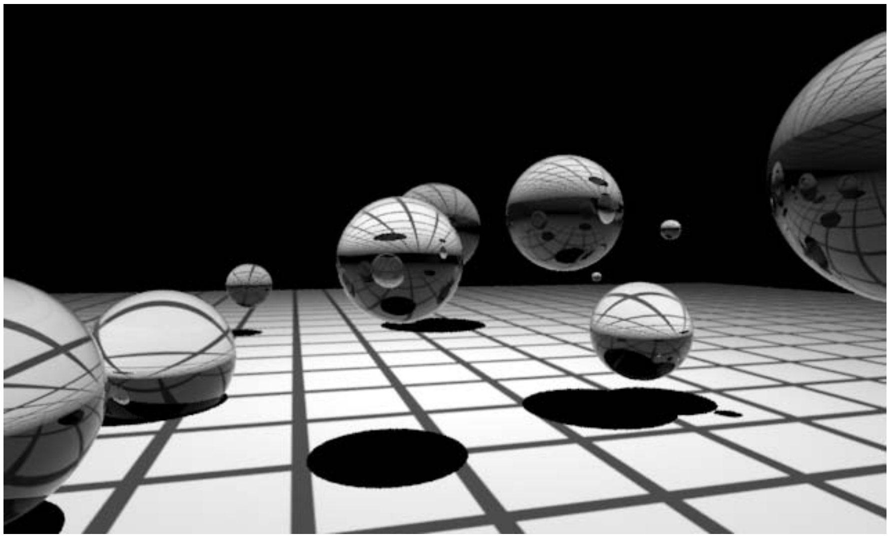
Spheres-over-plane.pbrt (glass 5)



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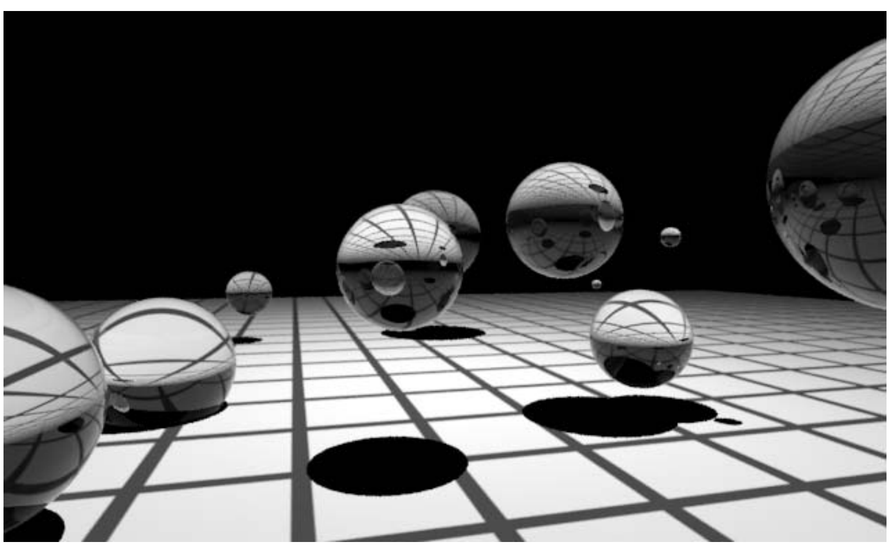
Spheres-over-plane.pbrt (glass 10)



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Spheres-over-plane.pbrt (glass 10)

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Spheres-over-plane.pbrt (g/m 10)

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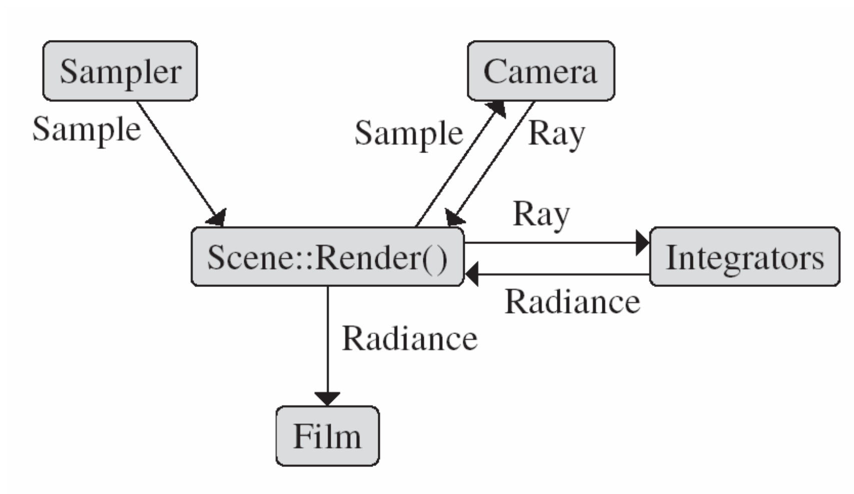
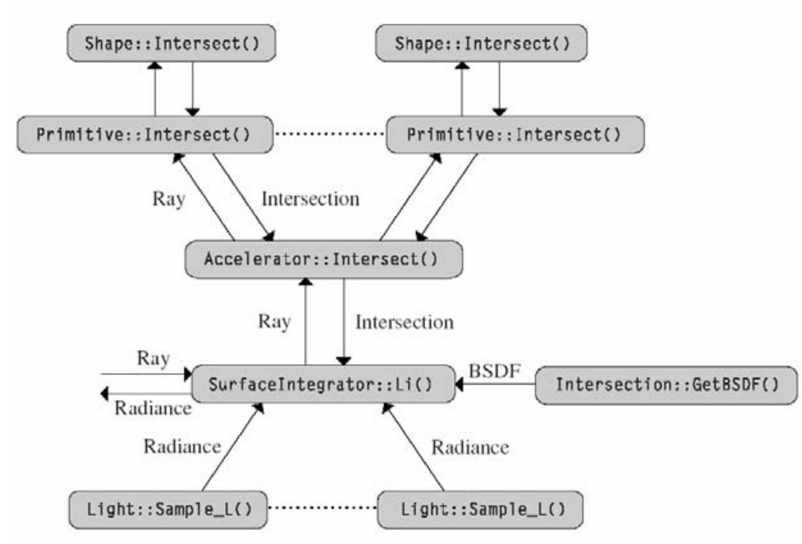
Table 1.1: Plug-ins. pbrt supports 13 types of plug-in objects that can be loaded at run time based on the contents of the scene description file. The system can be extended with new plug-ins, without needing to be recompiled itself.

Base class	Directory	Section
Shape	shapes/	3.1
Primitive	accelerators/	4.1
Camera	cameras/	6.1
Film	film/	8.1
Filter	filters/	7.6
Sampler	samplers/	7.2
ToneMap	tonemaps/	8.4
Material	materials/	10.2
Texture	textures/	11.3
VolumeRegion	volumes/	12.3
Light	lights/	13.1
SurfaceIntegrator	integrators/	16
VolumeIntegrator	integrators/	17

PBRT Architecture



PBRT Architecture



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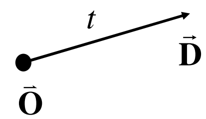
Ray-Surface Intersection

Ray-Plane Intersection

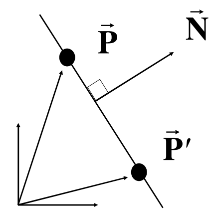
Ray: $\vec{\mathbf{P}} = \vec{\mathbf{O}} + t\,\vec{\mathbf{D}}$

$0 \le t < \infty$



Plane: $(\vec{\mathbf{P}} - \vec{\mathbf{P}}') \bullet \vec{\mathbf{N}} = 0$

$ax + by + cz + d = 0$



Solve for intersection

Substitute ray equation into plane equation

$$(\vec{\mathbf{P}} - \vec{\mathbf{P}}') \bullet \vec{\mathbf{N}} = (\vec{\mathbf{O}} + t\vec{\mathbf{D}} - \vec{\mathbf{P}}') \bullet \vec{\mathbf{N}} = 0$$

$$t = -\frac{(\vec{\mathbf{O}} - \vec{\mathbf{P}}') \bullet \vec{\mathbf{N}}}{\vec{\mathbf{D}} \bullet \vec{\mathbf{N}}}$$

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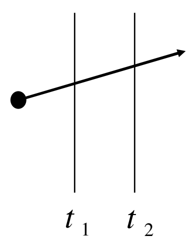
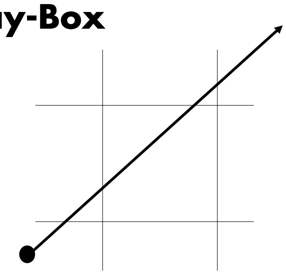
Ray-Polyhedra

Ray-Slab

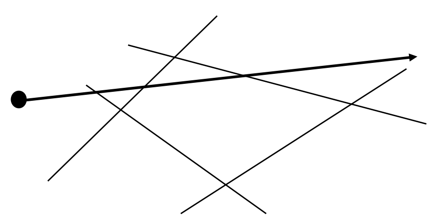


Note: Procedural geometry

Ray-Box



Ray-Convex Polyhedra



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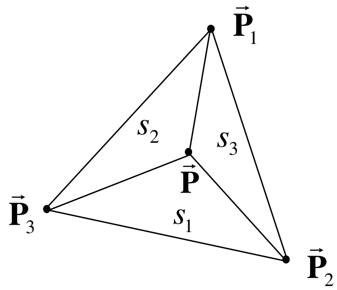
Ray-Triangle Intersection 1



$$s_1 = \text{area}(\Delta \mathbf{P}\mathbf{P}_2\mathbf{P}_3)$$

$$s_2 = \text{area}(\Delta \mathbf{P}\mathbf{P}_3\mathbf{P}_1)$$

$$s_3 = \text{area}(\Delta \mathbf{P}\mathbf{P}_1\mathbf{P}_2)$$

Barycentric coordinates

$$\vec{\mathbf{P}} = s_1\vec{\mathbf{P}}_1 + s_2\vec{\mathbf{P}}_2 + s_3\vec{\mathbf{P}}_3$$

Inside triangle criteria

$$0 \le s_1 \le 1$$

$$0 \le s_2 \le 1$$

$$0 \le s_3 \le 1$$

$$s_1 + s_2 + s_3 = 1$$

Ray-Triangle Intersection 2

$$\vec{\mathbf{P}} = s_1\vec{\mathbf{P}}_1 + s_2\vec{\mathbf{P}}_2 + s_3\vec{\mathbf{P}}_3$$

$$\begin{bmatrix} \mathbf{P}_1 & \mathbf{P}_2 & \mathbf{P}_3 \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} \mathbf{P} \end{bmatrix}$$

Ray-Triangle Intersection 3

$$s_1 = \frac{|\mathbf{P} \quad \mathbf{P}_2 \quad \mathbf{P}_3|}{|\mathbf{P}_1 \quad \mathbf{P}_2 \quad \mathbf{P}_3|} = \mathbf{P} \bullet \frac{\mathbf{P}_2 \times \mathbf{P}_3}{\Delta} \propto \mathbf{P} \bullet \mathbf{P}_2 \times \mathbf{P}_3$$

$$s_2 = \frac{|\mathbf{P}_1 \quad \mathbf{P} \quad \mathbf{P}_3|}{|\mathbf{P}_1 \quad \mathbf{P}_2 \quad \mathbf{P}_3|} = \mathbf{P} \bullet \frac{\mathbf{P}_3 \times \mathbf{P}_1}{\Delta} \propto \mathbf{P} \bullet \mathbf{P}_3 \times \mathbf{P}_1$$

$$s_3 = \frac{|\mathbf{P}_1 \quad \mathbf{P}_2 \quad \mathbf{P}|}{|\mathbf{P}_1 \quad \mathbf{P}_2 \quad \mathbf{P}_3|} = \mathbf{P} \bullet \frac{\mathbf{P}_1 \times \mathbf{P}_2}{\Delta} \propto \mathbf{P} \bullet \mathbf{P}_1 \times \mathbf{P}_2$$

$$\begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} \mathbf{P}_2 \times \mathbf{P}_3 \\ \mathbf{P}_3 \times \mathbf{P}_1 \\ \mathbf{P}_1 \times \mathbf{P}_2 \end{bmatrix} [\mathbf{P}]$$

⇧ **Precompute**

Triple Product

$$\vec{\mathbf{P}}_1 \bullet \vec{\mathbf{P}}_2 \times \vec{\mathbf{P}}_3 = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$$

$$\text{volume}(\mathbf{P}_1 \mathbf{P}_2 \mathbf{P}_3) \propto \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$$

Moller-Trumbore Algorithm

$$\vec{\mathbf{O}} + t\vec{\mathbf{D}} = (1 - b_1 - b_2)\vec{\mathbf{P}}_0 + b_1\vec{\mathbf{P}}_1 + b_2\vec{\mathbf{P}}_2$$

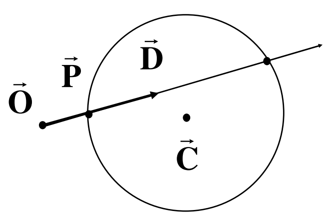
$$\begin{bmatrix} t \\ b_1 \\ b_2 \end{bmatrix} = \frac{1}{\vec{\mathbf{S}}_1 \bullet \vec{\mathbf{E}}_1} \begin{bmatrix} \vec{\mathbf{S}}_2 \bullet \vec{\mathbf{E}}_2 \\ \vec{\mathbf{S}}_1 \bullet \vec{\mathbf{S}} \\ \vec{\mathbf{S}}_2 \bullet \vec{\mathbf{D}} \end{bmatrix}$$

Where:

$$\vec{\mathbf{E}}_1 = \vec{\mathbf{P}}_1 - \vec{\mathbf{P}}_0$$
$$\vec{\mathbf{E}}_2 = \vec{\mathbf{P}}_2 - \vec{\mathbf{P}}_0$$
$$\vec{\mathbf{S}} = \vec{\mathbf{O}} - \vec{\mathbf{P}}_0$$
$$\vec{\mathbf{S}}_1 = \vec{\mathbf{D}} \times \vec{\mathbf{E}}_2$$
$$\vec{\mathbf{S}}_2 = \vec{\mathbf{S}} \times \vec{\mathbf{E}}_1$$

Cost = (1 div, 27 mul, 17 add)

Ray-Sphere Intersection



Ray: $\vec{\mathbf{P}} = \vec{\mathbf{O}} + t\vec{\mathbf{D}}$

Sphere: $\left(\vec{\mathbf{P}} - \vec{\mathbf{C}}\right)^2 - R^2 = 0$

$$\left(\vec{\mathbf{O}} + t\,\vec{\mathbf{D}} - \vec{\mathbf{C}}\right)^2 - R^2 = 0$$

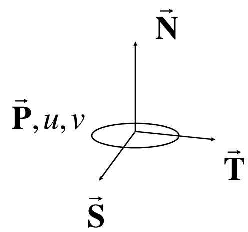
$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$at^2 + bt + c = 0$$
$$a = \vec{\mathbf{D}}^2$$
$$b = 2(\vec{\mathbf{O}} - \vec{\mathbf{C}}) \bullet \vec{\mathbf{D}}$$
$$c = (\vec{\mathbf{O}} - \vec{\mathbf{C}})^2 - R^2$$

Geometric Methods: Normals

e.g. Sphere

$$\vec{\mathbf{N}} = \vec{\mathbf{P}} - \vec{\mathbf{C}}$$

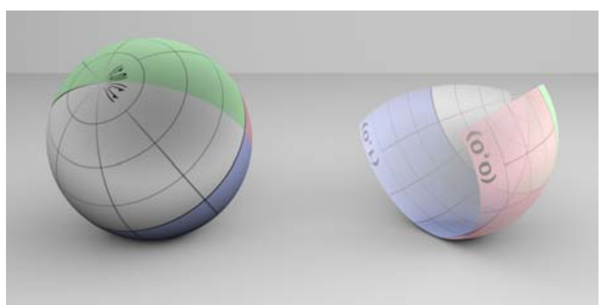


$$x = \sin\theta\cos\phi$$
$$y = \sin\theta\sin\phi$$
$$z = \cos\theta$$
$$\vec{\mathbf{P}} = (x, y, z)$$

$$\frac{\partial \vec{\mathbf{P}}}{\partial \theta} = (\cos\theta\cos\phi, \cos\theta\sin\phi, -\sin\theta)$$
$$\frac{\partial \vec{\mathbf{P}}}{\partial \phi} = (-\sin\theta\sin\phi, \sin\theta\cos\phi, 0)$$
$$\vec{\mathbf{N}} = \frac{\partial \vec{\mathbf{P}}}{\partial \theta} \times \frac{\partial \vec{\mathbf{P}}}{\partial \phi}$$

Geometric Methods: Parameters

e.g. Sphere



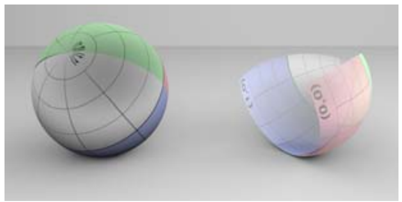
$$x = \sin\theta\cos\phi$$
$$y = \sin\theta\sin\phi$$
$$z = \cos\theta$$

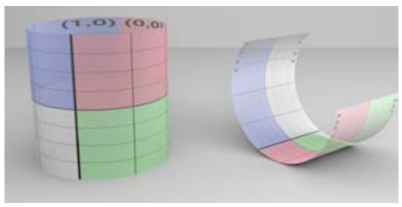
$$\phi = \tan^{-1}(x, y)$$
$$\theta = \cos^{-1} z$$

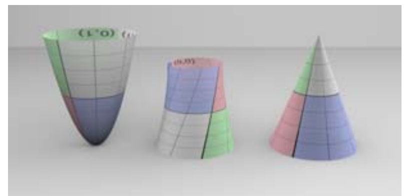
$$\theta = \theta_{\min} + v(\theta_{\max} - \theta_{\min})$$
$$\phi = u\phi_{\max}$$

$$v = (\theta - \theta_{\min}) / (\theta_{\max} - \theta_{\min})$$
$$u = \phi / \phi_{\max}$$

Quadrics







Ray-Implicit Surface Intersection

$$f(x,y,z)=0$$

$$x = x_0 + x_1 t$$
$$y = y_0 + y_1 t$$
$$z = z_0 + z_1 t$$

$$f^*(t)=0$$

1. Substitute ray equation
2. Find *positive, real* roots

Univariate root finding

- Newton's method
- *Regula-falsi*
- Interval methods
- Heuristics

Ray-Algebraic Surface Intersection

$$p_n(x,y,z)=0$$

$$x = x_0 + x_1\, t$$
$$y = y_0 + y_1\, t$$
$$z = z_0 + z_1\, t$$

$$p_n^*(t)=0$$

Degree *n*

***Linear*: Plane**

***Quadric*: Spheres, …**

***Quartic*: Tori**

Polynomial root finding

- **Quadratic, cubic, quartic**
- **Bezier/Bernoulli basis**
- **Descartes' rule of signs**
- **Sturm sequences**

History

Polygons	**Appel '68**
Quadrics, CSG	**Goldstein & Nagel '71**
Tori	**Roth '82**
Bicubic patches	**Whitted '80, Kajiya '82**
Superquadrics	**Edwards & Barr '83**
Algebraic surfaces	**Hanrahan '82**
Swept surfaces	**Kajiya '83, van Wijk '84**
Fractals	**Kajiya '83**
Height fields	**Coquillart & Gangnet '84, Musgrave '88**
Deformations	**Barr '86**
Subdivision surfs.	**Kobbelt, Daubert, Siedel, '98**

P. Hanrahan, A survey of ray-surface intersection algorithms