Camera Simulation

<table>
<thead>
<tr>
<th>Effect</th>
<th>Cause</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field of view</td>
<td>Film size, stops and pupils</td>
</tr>
<tr>
<td>Depth of field</td>
<td>Aperture, focal length</td>
</tr>
<tr>
<td>Exposure</td>
<td>Film speed, aperture, shutter</td>
</tr>
<tr>
<td>Motion blur</td>
<td>Shutter</td>
</tr>
</tbody>
</table>

References
Photography, B. London and J. Upton
Optics in Photography, R. Kingslake
The Camera, The Negative, The Print, A. Adams

Topics
Ray tracing lenses
Focus
Depth of focus / depth of field
Exposure
Lenses

Refraction

Snell’s Law

\[ n' \sin I' = n \sin I \]
Paraxial Approximation

\[ u = \frac{h}{-z} \quad -u' = \frac{h}{z'} \quad -\phi = \frac{h}{R} \]

\[ \tan U \approx u \]

Rays deviate only slightly from the axis

Gauss’ Formula

Paraxial approximation to Snell’s Law

\[ n'(u' - \phi) = n(u - \phi) \]

Ray coordinates

\[ u' = -\frac{h}{z'} \quad \phi = -\frac{h}{R} \quad u = -\frac{h}{z} \]

Thin lens equation

\[ n'(\frac{h}{z'} - \frac{h}{R}) = n(\frac{h}{z} - \frac{h}{R}) \]

\[ \frac{n'}{z'} = \frac{n}{z} + \frac{(n' - n)}{R} \quad \text{Holds for any height, any ray!} \]
Vergence

Diverging

\[ V < 0 \]

Converging

\[ V > 0 \]

Vergence

\[ V \equiv \frac{n}{r} \approx \frac{n}{z} \left[ \frac{1}{m} = \text{diopters} \right] \]

Thin lens equation

\[ V' = V + P \]

Surface Power equation

\[ P \equiv (n' - n) \frac{1}{R} \]

Lens-makers Formula

Refractive Power

\[ P = (n' - n) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) = \frac{1}{f} \]

Converging

Diverging
Conjugate Points

To focus: move lens relative to backplane
Horizontal rays converge on focal point in the focal plane

Gauss’ Ray Tracing Construction

Parallel Ray

Focal Ray

Chief Ray

Object

Image
Ray Tracing: Finite Aperture

1. Pick a point on image plane $x'$
2. Pick a point on the lens $u$
3. Transform $x'$ to $x$; form ray $(u, x - u)$

Real Lens

Cutaway section of a Vivitar Series 1 90mm f/2.5 lens
Cover photo, Kingslake, Optics in Photography
Double Gauss

Data from W. Smith, *Modern Lens Design*, p 312

<table>
<thead>
<tr>
<th>Radius (mm)</th>
<th>Thick (mm)</th>
<th>1/d</th>
<th>V-no</th>
<th>Aperture</th>
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<tbody>
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<td>7.520</td>
<td>1.670</td>
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</tbody>
</table>

Ray Tracing Through Lenses

200 mm telephoto

35 mm wide-angle

50 mm double-gauss

16 mm fisheye

From Kolb, Mitchell and Hanrahan (1995)
Thick Lenses

Equivalent Lens

Refraction occurs at the principal planes

Perspective Transformation

Thin lens equation

\[ \frac{1}{z'} = \frac{1}{z} + \frac{1}{f} \Rightarrow z' = \frac{fz}{z + f} \]

\[ \Rightarrow x' = \frac{fx}{z + f} \]

\[ \Rightarrow y' = \frac{fy}{z + f} \]

Represent transformation as a 4x4 matrix
Depth of Field

From London and Upton
**Circle of Confusion**

![Diagram of Circle of Confusion](image)

Circle of confusion proportional to the size of the aperture
\[
\frac{c}{a} = \frac{d'}{z'} = \frac{s' - z'}{z'}
\]

**Depth of Focus [Image Space]**

Depth of focus \[=\] Equal circles of confusion

Extreme planes: near and far
\[
\frac{c}{a} = \frac{d'_f}{z'_f} = \frac{s' - z'_f}{z'_f}
\]
\[
\frac{c}{a} = \frac{d'_n}{z'_n} = \frac{z'_n - s'}{z'_n}
\]
Depth of Focus [Image Space]

Depth of focus ≡

Equal circles of confusion

\[
\frac{c}{a} = \frac{d_f}{z_f} = \frac{s' - z_f'}{z_f'} \Rightarrow \frac{1}{z_f'} = \frac{1}{s'} \left( 1 + \frac{c}{a} \right)
\]

\[
\frac{c}{a} = \frac{d_n}{z_n} = \frac{z_n' - s'}{z_n'} \Rightarrow \frac{1}{z_n'} = \frac{1}{s'} \left( 1 - \frac{c}{a} \right)
\]
Depth of Field [Object Space]

Depth of field ≡

Equal circles of confusion

\[
\frac{1}{s'} = \frac{1}{s} + \frac{1}{f} \quad \frac{1}{z'_n} = \frac{1}{z_n} + \frac{1}{f} \quad \frac{1}{z'_f} = \frac{1}{z_f} + \frac{1}{f}
\]

\[
\frac{1}{z_n} + \frac{1}{z_f} = 2 \frac{1}{s}
\]

\[
\frac{1}{z_n} - \frac{1}{z_f} = 2c \left( \frac{1}{a} - \frac{1}{sf} \right) \approx \frac{2c}{a} \frac{1}{sf}
\]
**Hyperfocal Distance**

\[
\frac{1}{z_n} + \frac{1}{z_f} = \frac{1}{s} \quad N \equiv \frac{a}{f}
\]

\[
\frac{1}{z_n} - \frac{1}{z_f} = \frac{2c}{a} \frac{1}{f} = 2 \frac{cN}{f^2} \equiv 2 \frac{1}{H}
\]

**When**

\[s \to H \Rightarrow z_n = \frac{H}{2}, \quad z_f = \infty\]

\(H\) is the hyperfocal distance

---

**Factors Affecting DOF**


\[
\frac{1}{H} = \frac{cN}{f^2}
\]
Resolving Power

- Diffraction limit
  \[ c = 1.22 \frac{f}{\lambda} \quad [= 1.22 \times 64 \times .500\mu m = 0.040\ mm] \]

- 35mm film (Leica standard)
  \[ c = 0.025\ mm \]

- CCD/CMOS pixel aperture
  \[ c = 0.0116\ mm \text{ (Nikon D1)} \]
**Image Irradiance**

\[
E = \int_{\Omega} L \cos \theta \, d\omega = L \pi \sin^2 \theta = L \frac{\pi}{4} \left( \frac{a}{f} \right)^2
\]

**Relative Aperture or F-Stop**

F-Number and exposure: 

\[
E = L \frac{\pi}{4} \frac{1}{N^2}
\]

F-stops: 1.4 2 2.8 4.0 5.6 8 11 16 22 32 45 64

1 stop doubles exposure
Camera Exposure

**Exposure** \( H = E \times T \)

Exposure overdetermined
- **Aperture:** f-stop - 1 stop doubles \( H \)
  - Decreases depth of field
- **Shutter:** Doubling the open time doubles \( H \)
  - Increases motion blur

Aperture vs Shutter

![Images showing different aperture and shutter settings](Image)

- **f/16** 1/8s
- **f/4** 1/125s
- **f/2** 1/500s

*From London and Upton*
High Dynamic Range

Sixteen photographs of the Stanford Memorial Church taken at 1-stop increments from 30s to 1/1000s.
From Debevec and Malik, High dynamic range photographs.

Simulated Photograph

Adaptive histogram
With glare, contrast, blur

Page 17
**Paraxial Approximation**

\[ \sin U \approx u \]
\[ \tan U \approx u \]

Rays deviate only slightly from the axis

**Incident Ray**

Angles: ccw is positive; cw is negative

\[ I = U - \phi \]

The sum of the interior angles is equal to the exterior angle.
**Refracted Ray**

\[ I' = U' - \phi \]

\[ (-\phi) = I' + (-U') \]

**Derivation**

**Paraxial approximation**

\[ I = U - \phi \Rightarrow i = u - \phi \]
\[ I' = U' - \phi \Rightarrow i' = u' - \phi \]
Derivation

Paraxial approximation

\[ I = U - \phi \quad \Rightarrow \quad i = u - \phi \]
\[ I' = U' - \phi \quad \Rightarrow \quad i' = u' - \phi \]

Snell’s Law

\[ n' \sin I' = n \sin I \quad \Rightarrow \quad n'i' = ni \]

\[ n'(u' - \phi) = n(u - \phi) \]

Field of View
Field of View

From London and Upton

CS348B Lecture 7
Pat Hanrahan, 2006
Field of View

Field of view

\[
\tan \left( \frac{\text{fov}}{2} \right) = \frac{\text{filmsize}}{f}
\]

Types of lenses

- **Normal** 26°
  - Film diagonal \~ focal length
- **Wide-angle** 75-90°
- **Narrow-angle** 10°

Redrawn from Kingslake, *Optics in Photography*