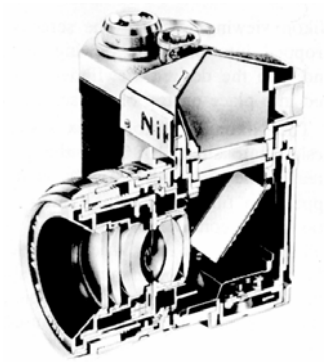
Camera Simulation



Effect	Cause
Field of view	Film size, stops and pupils
Depth of field	Aperture, focal length
Exposure	Film speed, aperture, shutter
Motion blur	Shutter

References

Photography, B. London and J. Upton
Optics in Photography, R. Kingslake
The Camera, The Negative, The Print, A. Adams

Topics

Ray tracing lenses

Focus

Depth of focus / depth of field

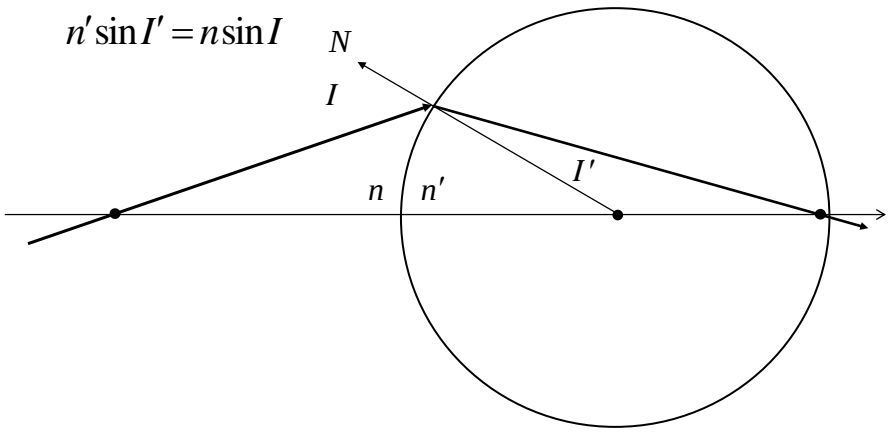
Exposure

Lenses

Refraction

Snell's Law

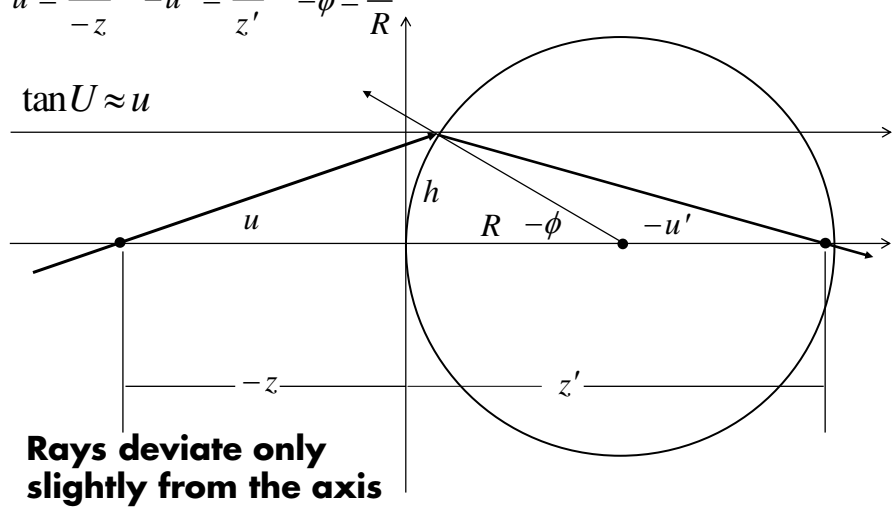
$$n' \sin I' = n \sin I$$



Paraxial Approximation

$$u = \frac{h}{-z} \quad -u' = \frac{h}{z'} \quad -\phi = \frac{h}{R}$$

$$\tan U \approx u$$



Rays deviate only slightly from the axis

Gauss' Formula

Paraxial approximation to Snell's Law

$$n'(u' - \phi) = n(u - \phi)$$

Ray coordinates

$$u' = -\frac{h}{z'} \qquad \phi = -\frac{h}{R} \qquad u = -\frac{h}{z}$$

Thin lens equation

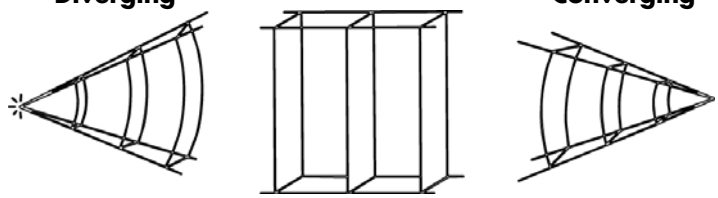
$$n'\left(\frac{h}{z'} - \frac{h}{R}\right) = n\left(\frac{h}{z} - \frac{h}{R}\right)$$

$$\frac{n'}{z'} = \frac{n}{z} + \frac{(n' - n)}{R}$$

⇦ **Holds for any height, any ray!**

Vergence

Diverging **Converging**



$V < 0$ $V = 0$ $V > 0$

Vergence $V \equiv \frac{n}{r} \approx \frac{n}{z} \quad \left[\frac{1}{m} = diopters\right]$

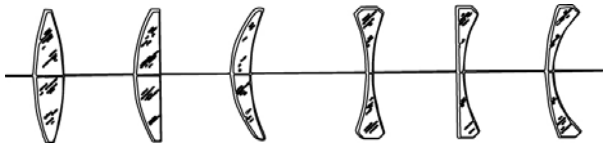
Thin lens equation $V' = V + P$

Surface Power equation $P \equiv (n' - n)\frac{1}{R}$

Lens-makers Formula

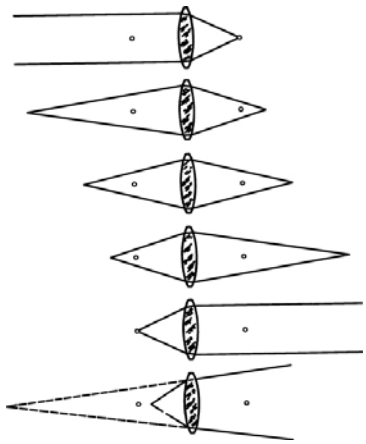
Refractive Power

$$P = (n' - n)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{f}$$



Converging **Diverging**

Conjugate Points

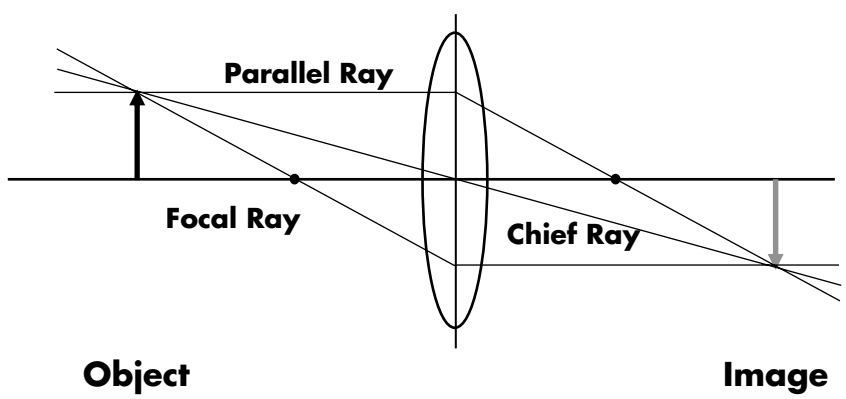


$$\frac{1}{z'} = \frac{1}{z} + \frac{1}{f}$$

To focus: move lens relative to backplane

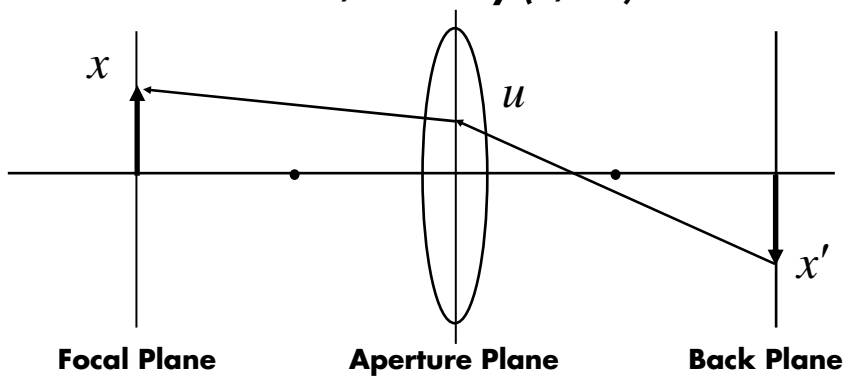
Horizontal rays converge on focal point in the focal plane

Gauss' Ray Tracing Construction

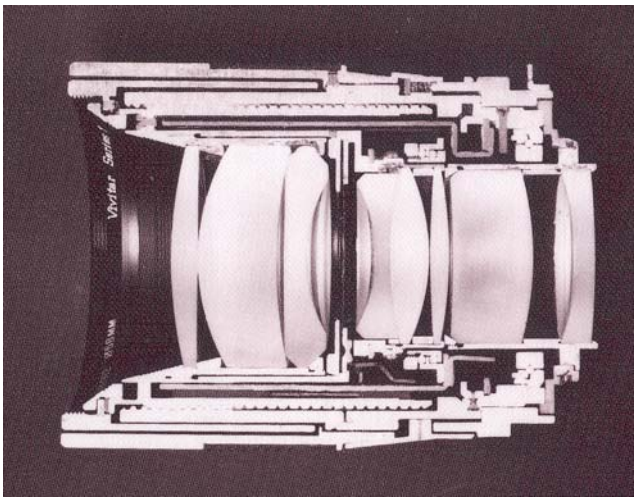


Ray Tracing: Finite Aperture

1. **Pick a point on image plane *x′***
2. **Pick a point on the lens *u***
3. **Transform *x′* to *x;* form ray *(u,x-u)***



Real Lens

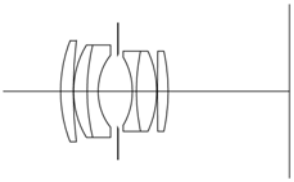


Cutaway section of a Vivitar Series 1 90mm f/2.5 lens
Cover photo, Kingslake, *Optics in Photography*

Double Gauss

Data from W. Smith,
Modern Lens Design, p 312

Radius (mm)	Thick (mm)	n_d	V-no	aperture
58.950	7.520	1.670	47.1	50.4
169.660	0.240			50.4
38.550	8.050	1.670	47.1	46.0
81.540	6.550	1.699	30.1	46.0
25.500	11.410			36.0
	9.000			34.2
-28.990	2.360	1.603	38.0	34.0
81.540	12.130	1.658	57.3	40.0
-40.770	0.380			40.0
874.130	6.440	1.717	48.0	40.0
-79.460	72.228			40.0



Ray Tracing Through Lenses

200 mm telephoto

35 mm wide-angle

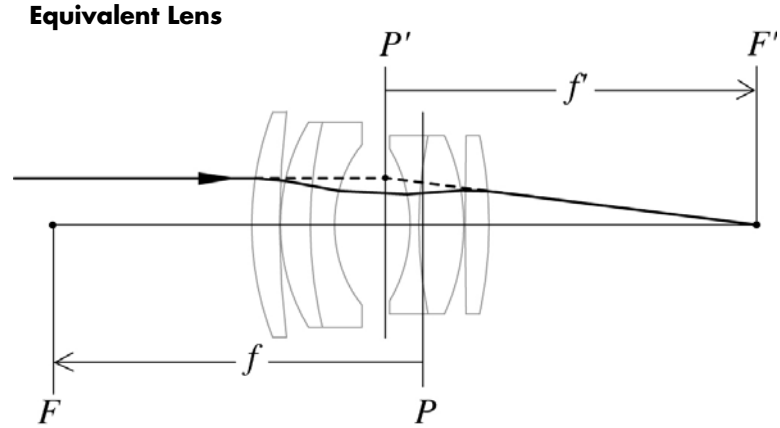
50 mm double-gauss

16 mm fisheye

From Kolb, Mitchell and Hanrahan (1995)

Thick Lenses

Equivalent Lens



Refraction occurs at the *principal planes*

Perspective Transformation

Thin lens equation

$$\frac{1}{z'} = \frac{1}{z} + \frac{1}{f} \Rightarrow z' = \frac{fz}{z+f}$$

$$\Rightarrow x' = \frac{fx}{z+f}$$

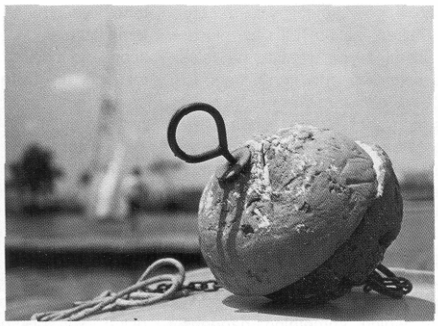
$$\Rightarrow y' = \frac{fy}{z+f}$$

Represent transformation as a 4x4 matrix

Depth of Field

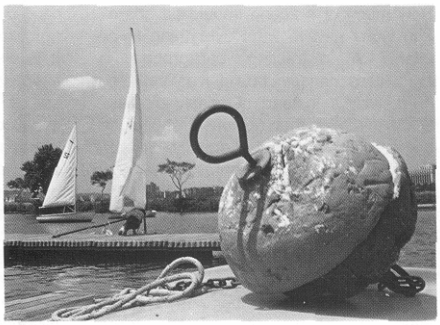
Depth of Field

less depth of field



wider aperture

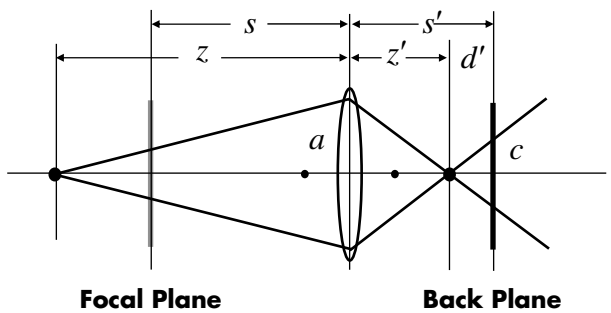
more depth of field



smaller aperture

From London and Upton

Circle of Confusion



Circle of confusion proportional to the size of the aperture

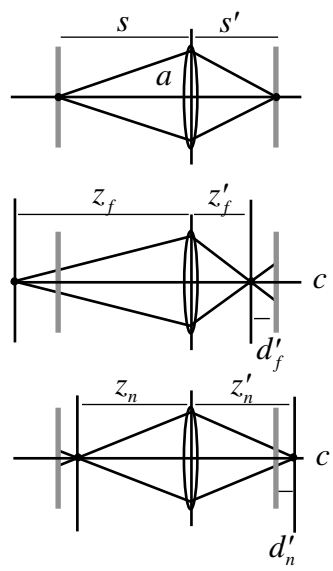
$$\frac{c}{a} = \frac{d'}{z'} = \frac{s' - z'}{z'}$$

Depth of Focus [Image Space]

Depth of focus ≡

Equal circles of confusion

Extreme planes: near and far



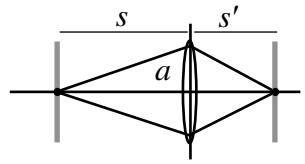
$$\frac{c}{a} = \frac{d'_f}{z'_f} = \frac{s' - z'_f}{z'_f}$$

$$\frac{c}{a} = \frac{d'_n}{z'_n} = \frac{z'_n - s'}{z'_n}$$

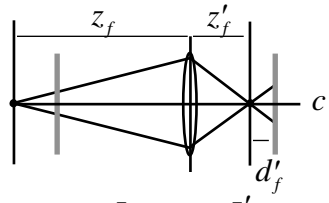
Depth of Focus [Image Space]

Depth of focus ≡

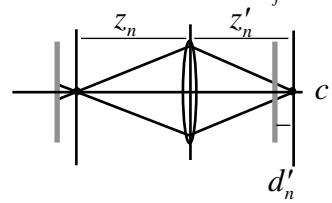
Equal circles of confusion



$$\frac{c}{a} = \frac{d'_f}{z'_f} = \frac{s' - z'_f}{z'_f} \quad \Rightarrow \quad \frac{1}{z'_f} = \frac{1}{s'}\left(1 + \frac{c}{a}\right)$$



$$\frac{c}{a} = \frac{d'_n}{z'_n} = \frac{z'_n - s'}{z'_n} \quad \Rightarrow \quad \frac{1}{z'_n} = \frac{1}{s'}\left(1 - \frac{c}{a}\right)$$



Depth of Focus [Image Space]

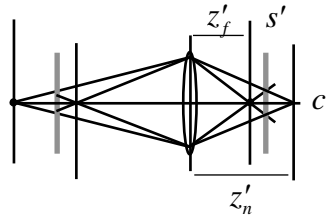
Depth of focus ≡

Equal circles of confusion

$$\frac{1}{z'_f} = \frac{1}{s'}\left(1 + \frac{c}{a}\right) \qquad \frac{1}{z'_n} = \frac{1}{s'}\left(1 - \frac{c}{a}\right)$$

$$\frac{1}{z'_f} + \frac{1}{z'_n} = 2\frac{1}{s'}$$

$$\frac{1}{z'_f} - \frac{1}{z'_n} = \frac{2c}{a}\frac{1}{s'}$$



Depth of Field [Object Space]

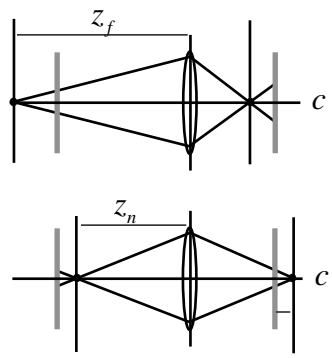
Depth of field ≡

Equal circles of confusion

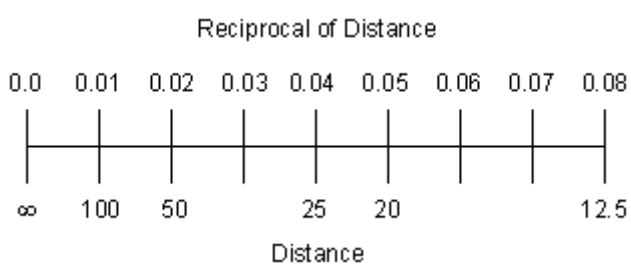
$$\frac{1}{s'} = \frac{1}{s} + \frac{1}{f} \qquad \frac{1}{z'_n} = \frac{1}{z_n} + \frac{1}{f} \qquad \frac{1}{z'_f} = \frac{1}{z_f} + \frac{1}{f}$$

$$\frac{1}{z_n} + \frac{1}{z_f} = 2\frac{1}{s}$$

$$\frac{1}{z_n} - \frac{1}{z_f} = \frac{2c}{a}\left(\frac{1}{f} - \frac{1}{s}\right) \approx \frac{2c}{a}\frac{1}{f}$$



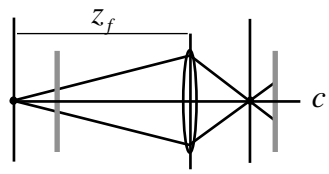
Depth of Field Scale

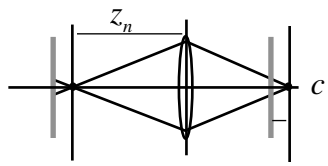


Hyperfocal Distance

$$\frac{1}{z_n}+\frac{1}{z_f}=2\frac{1}{s} \qquad N\equiv\frac{a}{f}$$

$$\frac{1}{z_n}-\frac{1}{z_f}=\frac{2c}{a}\frac{1}{f}=2\frac{cN}{f^2}\equiv 2\frac{1}{H}$$





When

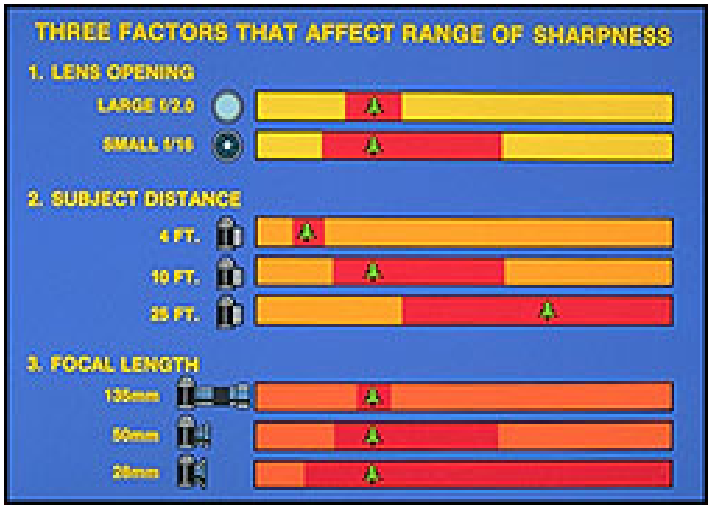
$$s\rightarrow H\Rightarrow z_n=\frac{H}{2},\ z_f=\infty$$

H is the hyperfocal distance

Factors Affecting DOF

From http://www.kodak.com/global/en/consumer/pictureTaking/cameraCare/cameCar6.shtml

$$\frac{1}{H}=\frac{cN}{f^2}$$



Resolving Power

- **Diffraction limit**

$$c = 1.22\frac{f}{a}\lambda \quad \left[= 1.22 \times 64 \times .500\,\mu\text{m} = 0.040\,\text{mm}\right]$$

- **35mm film (Leica standard)**

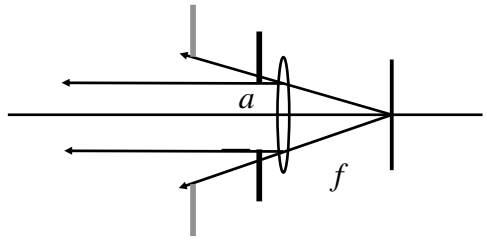
$$c = 0.025\,\text{mm}$$

- **CCD/CMOS pixel aperture**

$c = 0.0116\,\text{mm}$ (Nikon D1)

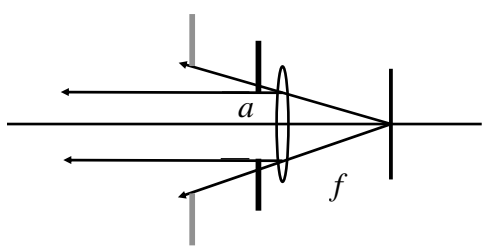
Exposure

Image Irradiance



$$E = \int_{\Omega} L \cos\theta \, d\omega = L \pi \sin^2\theta = L \frac{\pi}{4} \left(\frac{a}{f} \right)^2$$

Relative Aperture or F-Stop



$$a = \frac{f}{N}$$

F-Number and exposure: $E = L \frac{\pi}{4} \frac{1}{N^2}$

Fstops: 1.4 2 2.8 4.0 5.6 8 11 16 22 32 45 64

1 stop doubles exposure

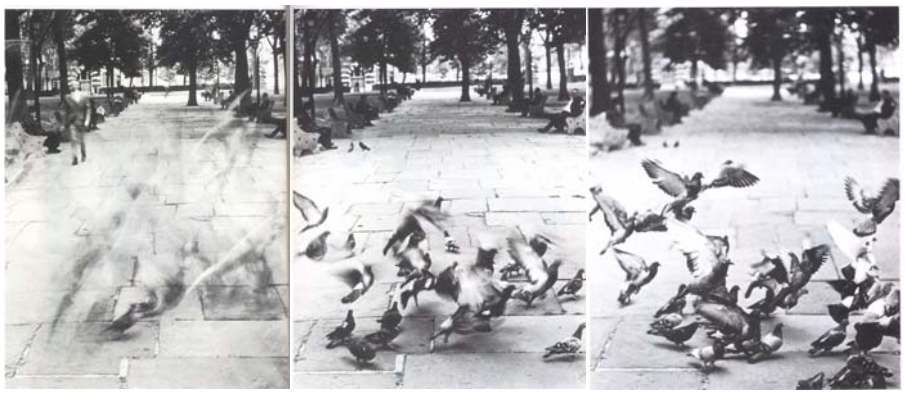
Camera Exposure

Exposure $H = E \times T$

Exposure overdetermined

- **Aperture: f-stop - 1 stop doubles *H***
  - **Decreases depth of field**
- **Shutter: Doubling the open time doubles *H***
  - **Increases motion blur**

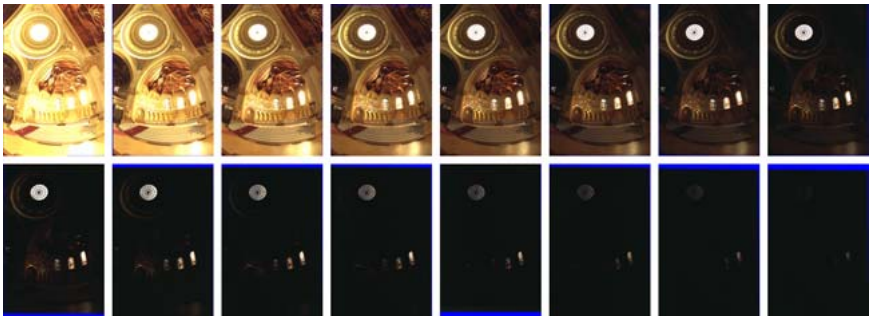
Aperture vs Shutter



f/16	f/4	f/2
1/8s	1/125s	1/500s

From London and Upton

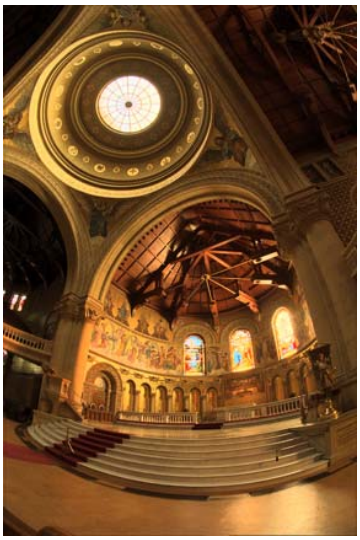
High Dynamic Range



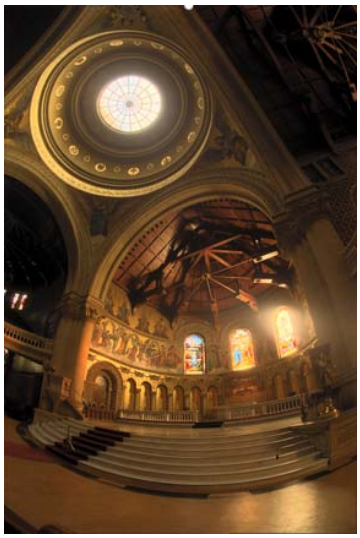
Sixteen photographs of the Stanford Memorial Church taken at 1-stop increments from 30s to 1/1000s.

From Debevec and Malik, High dynamic range photographs.

Simulated Photograph



Adaptive histogram

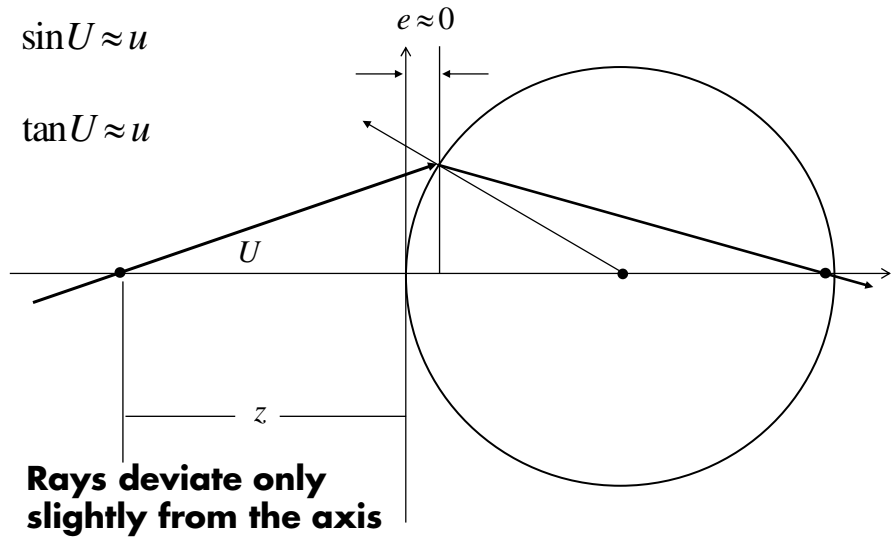


With glare, contrast, blur

Paraxial Approximation

$\sin U \approx u$

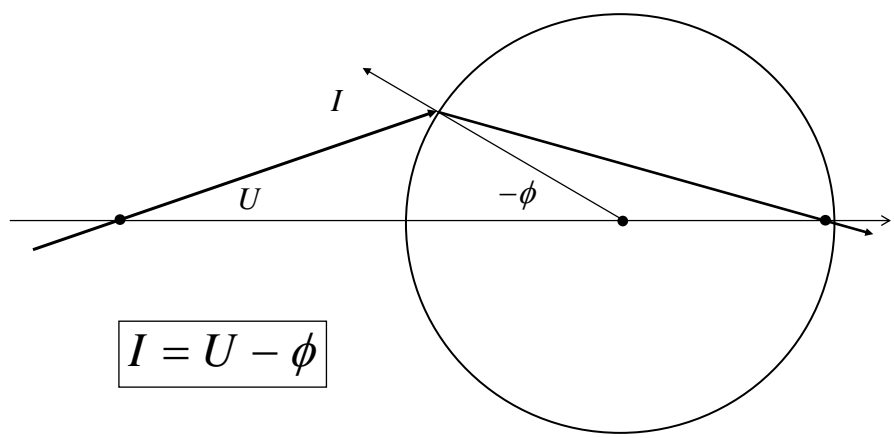
$\tan U \approx u$



Rays deviate only slightly from the axis

Incident Ray

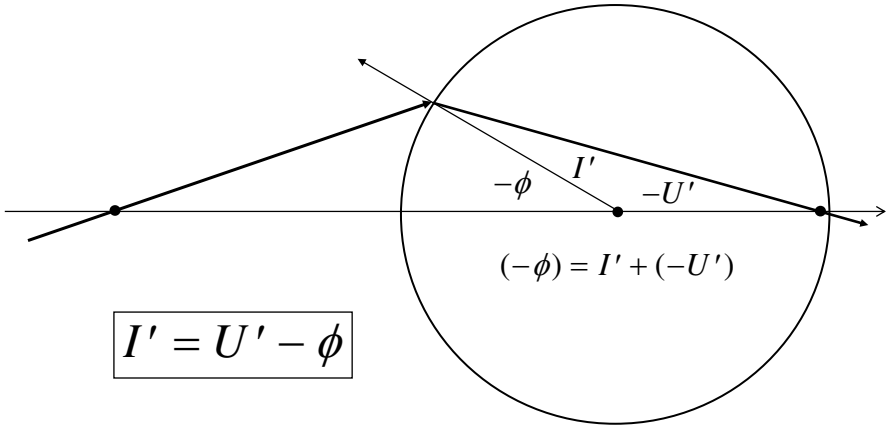
Angles: ccw is positive; cw is negative



$$I = U - \phi$$

The sum of the interior angles is equal to the exterior angle.

Refracted Ray



$$\boxed{I' = U' - \phi}$$

Derivation

Paraxial approximation

$$I = U - \phi \quad \Rightarrow \quad i = u - \phi$$
$$I' = U' - \phi \quad \Rightarrow \quad i' = u' - \phi$$

Derivation

Paraxial approximation

$$I = U - \phi \quad \Rightarrow \quad i = u - \phi$$
$$I' = U' - \phi \quad \Rightarrow \quad i' = u' - \phi$$

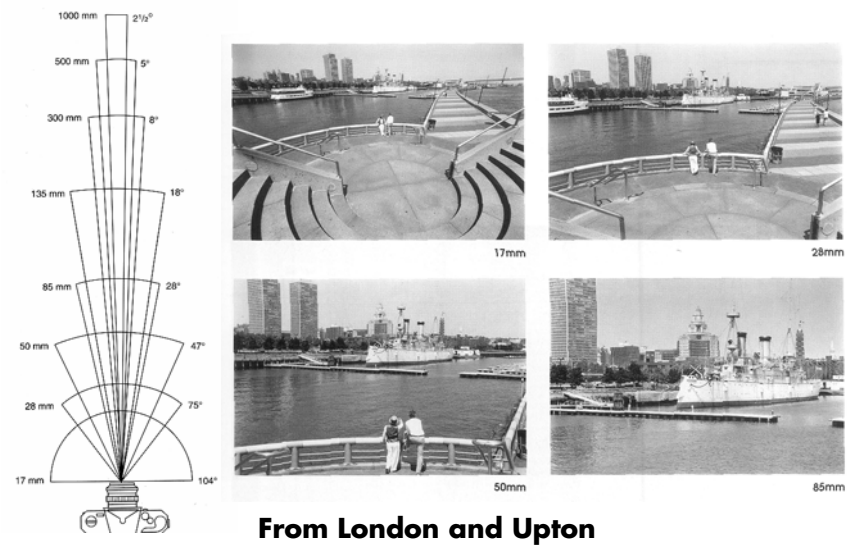
Snell's Law

$$n' \sin I' = n \sin I \quad \Rightarrow \quad n' i' = n i$$

$$\boxed{n'(u' - \phi) = n(u - \phi)}$$

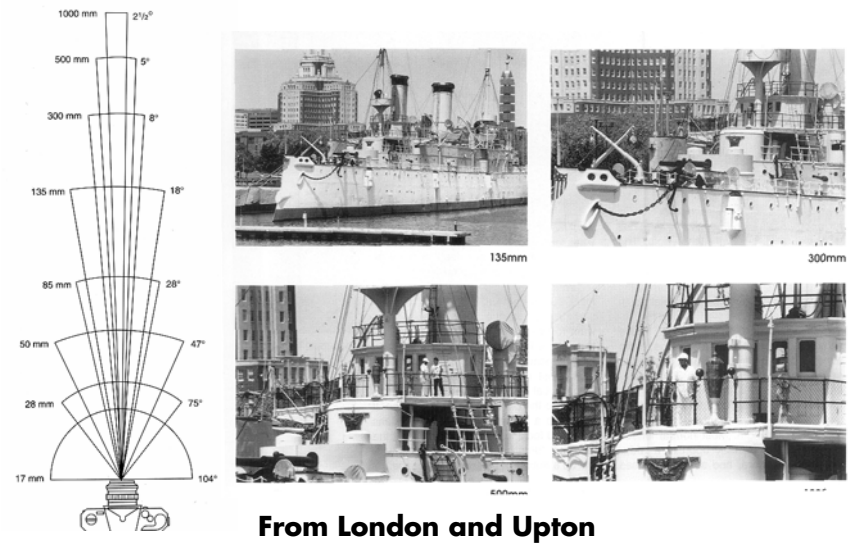
Field of View

Field of View



From London and Upton

Field of View



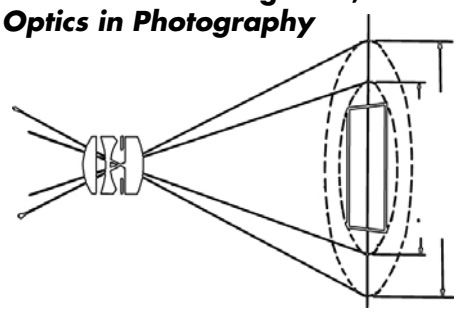
From London and Upton

Field of View

Field of view

$$\tan\frac{fov}{2} = \frac{filmsize}{f}$$

Redrawn from Kingslake, *Optics in Photography*



Types of lenses

- **Normal** **26°**

  Film diagonal ~ focal length
- **Wide-angle** **75-90°**
- **Narrow-angle** **10°**