

Reflection Models

Last lecture

- Reflection models
- The reflection equation and the BRDF
- Ideal reflection, refraction and diffuse

Today

- Phong model
- Microfacet models
- Torrance-Sparrow model
- Self-shadowing

Phong Model

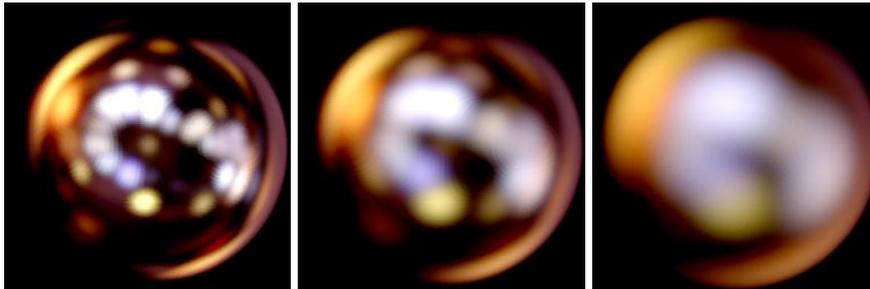
Glossy Surfaces



Mirror



Diffuse



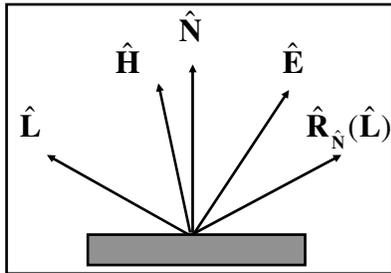
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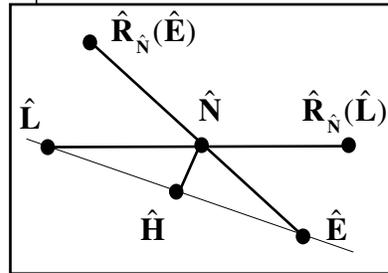
Reflection Geometry

$$\hat{H} = \frac{\hat{L} + \hat{E}}{|\hat{L} + \hat{E}|}$$



$$\cos \theta_i = \hat{L} \cdot \hat{N}$$

$$\cos \theta_r = \hat{E} \cdot \hat{N}$$



$$\cos \theta_s = \hat{E} \cdot \mathbf{R}_{\hat{N}}(\hat{L}) = \mathbf{R}_{\hat{N}}(\hat{E}) \cdot \hat{L}$$

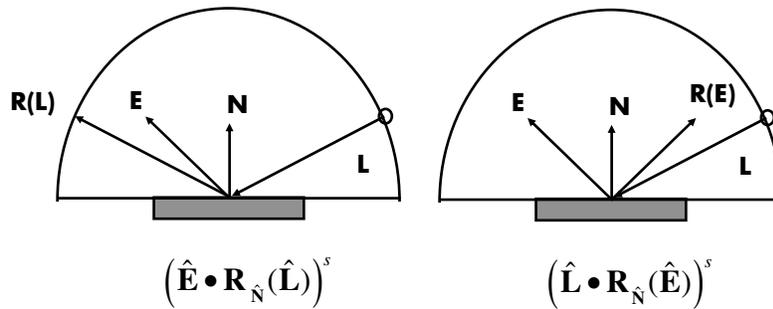
$$\cos \theta_g = \hat{E} \cdot \hat{L}$$

$$\cos \theta_{s'} = \hat{H} \cdot \hat{N}$$

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Phong Model



Reciprocity: $(\hat{\mathbf{E}} \cdot \mathbf{R}(\hat{\mathbf{L}}))^s = (\hat{\mathbf{L}} \cdot \mathbf{R}(\hat{\mathbf{E}}))^s$

Distributed light source!

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Energy Normalization

Energy normalize Phong Model

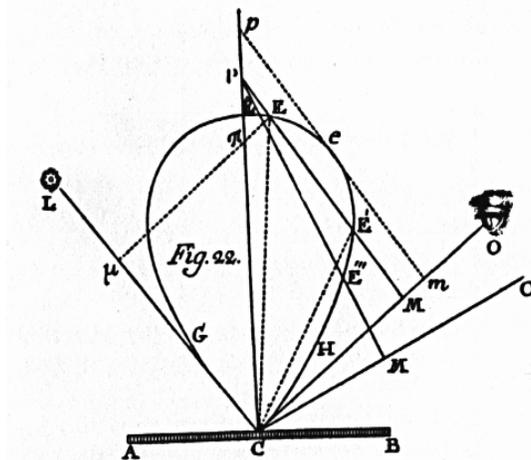
$$\begin{aligned}
 \rho(H^2 \rightarrow \omega_r) &= \int_{H^2(\hat{\mathbf{N}})} (\hat{\mathbf{L}} \cdot \mathbf{R}_{\hat{\mathbf{N}}}(\hat{\mathbf{E}}))^s \cos \theta_i d\omega_i \\
 &\leq \int_{H^2(\hat{\mathbf{N}})} (\hat{\mathbf{L}} \cdot \mathbf{R}_{\hat{\mathbf{N}}}(\hat{\mathbf{E}}))^s d\omega_i \\
 &\leq \int_{H^2(\mathbf{R}_{\hat{\mathbf{N}}}(\hat{\mathbf{E}}))} (\hat{\mathbf{L}} \cdot \mathbf{R}_{\hat{\mathbf{N}}}(\hat{\mathbf{E}}))^s d\omega_R \\
 &= \int_{H^2} \cos^s \theta d\omega = \frac{2\pi}{s+1}
 \end{aligned}$$

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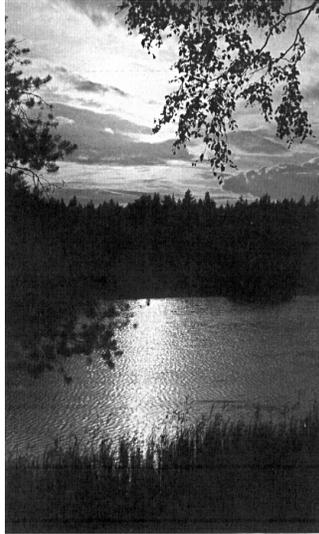
Microfacet Model

Bouguer's "little faces"

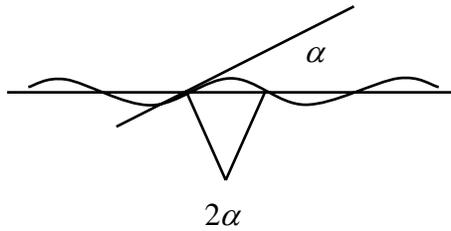


P. Bouguer, *Treatise on Optics*, 1760

Reflection of the Sun from the Sea



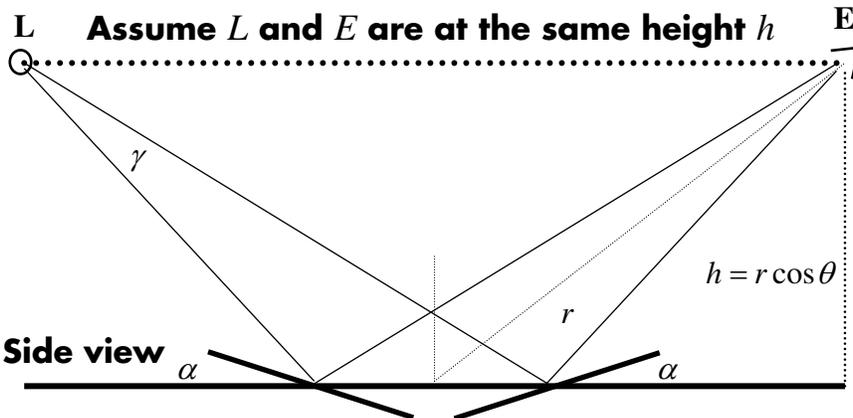
Minnaert, *Light and Color in the Outdoors*, p. 28



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Reflection Angles

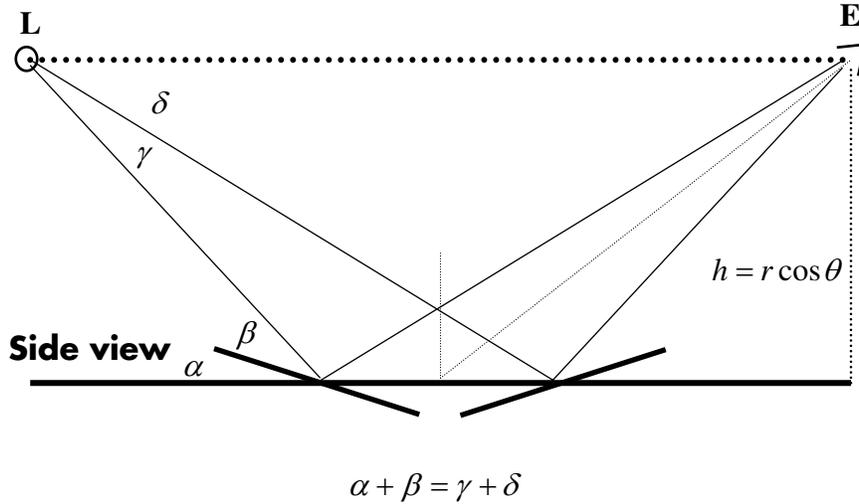


Calculate the angular height γ

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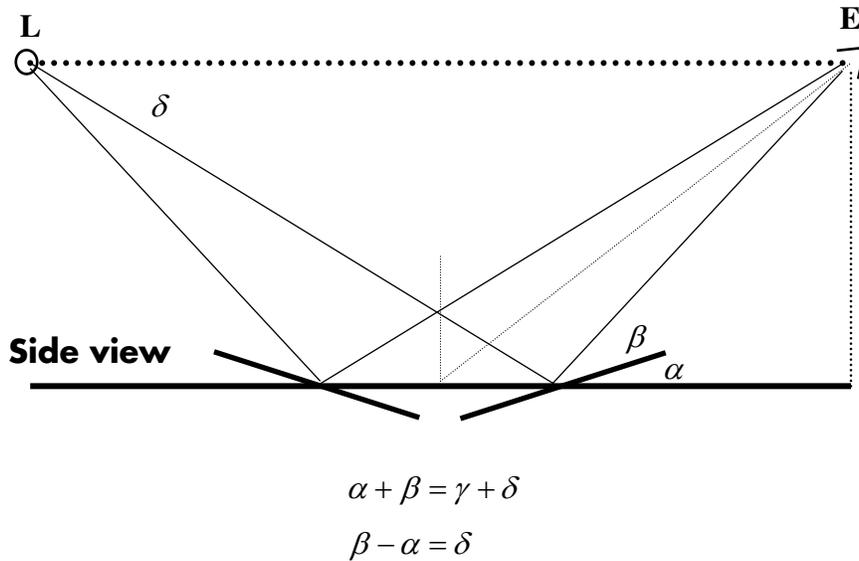
Reflection Angles



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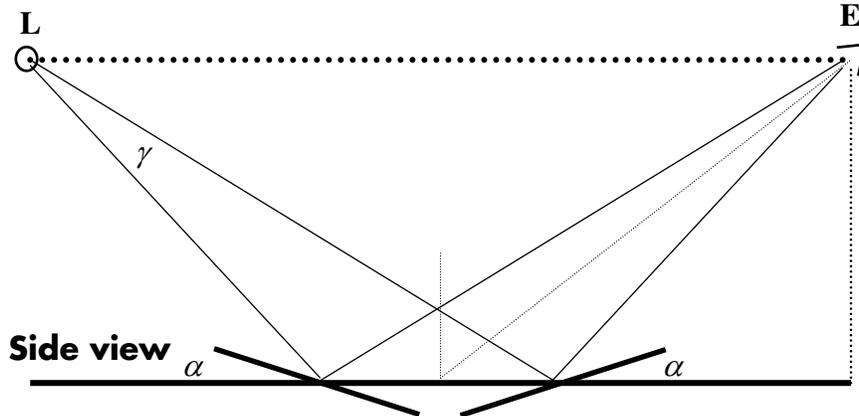
Reflection Angles



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Reflection Angles

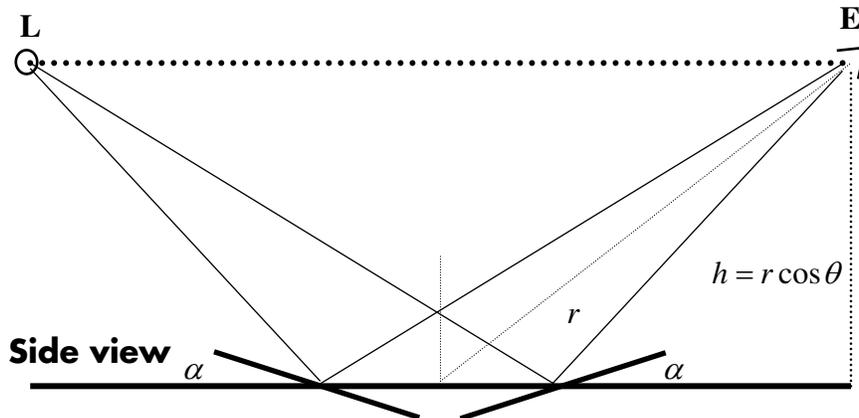


$$\begin{aligned} \alpha + \beta &= \gamma + \delta \\ \beta - \alpha &= \delta \end{aligned} \Rightarrow \gamma = 2\alpha$$

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Reflection Angles

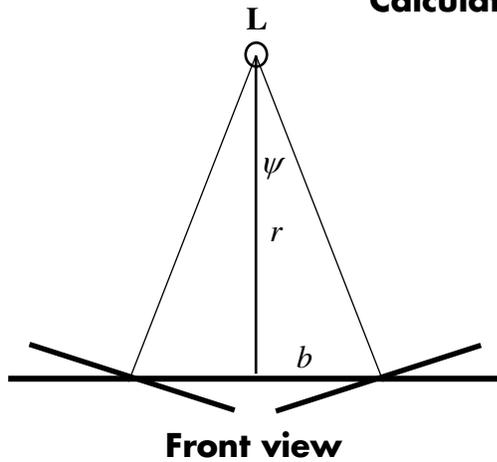


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Reflection Angles

Calculate the angular width



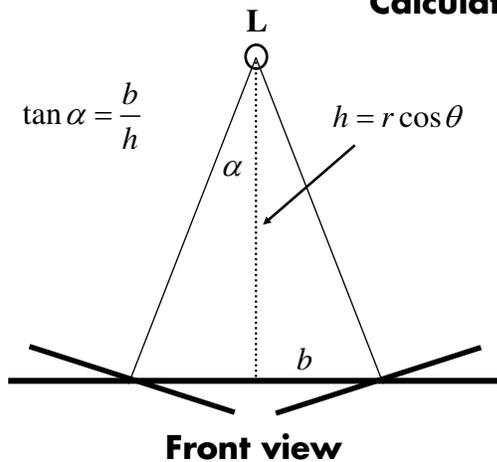
$$\tan \psi = \frac{b}{r}$$

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Reflection Angles

Calculate the angular width



$$\tan \alpha = \frac{b}{h}$$

$$h = r \cos \theta$$

$$\tan \psi = \frac{b}{r}$$

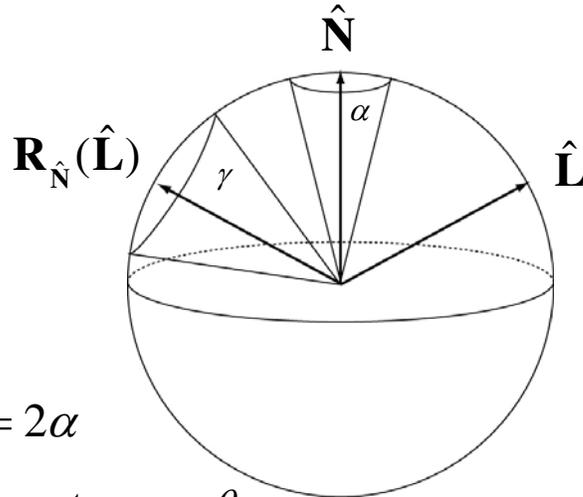
$$= \frac{h}{r} \tan \alpha$$

$$= \tan \alpha \cos \theta$$

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Analysis on the Sphere



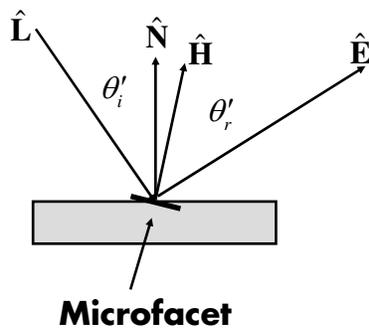
$$\gamma = 2\alpha$$

$$\tan \psi = \tan \alpha \cos \theta$$

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Microfacet Distributions



Total projected area

$$\int_{H^2} dA(\omega_h) \cos \theta_h d\omega_h = dA$$

Probability distribution

$$\int_{H^2} D(\omega_h) \cos \theta_h d\omega_h = 1$$

Area distribution $dA(\omega_h)$

Microfacet distribution $D(\omega_h) \equiv dA(\omega_h) / dA$

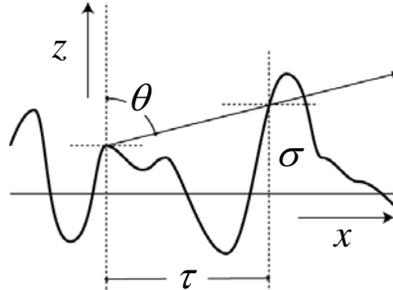
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Gaussian Rough Surface

Gaussian distribution of heights

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{z^2}{2\sigma^2}}$$



Gaussian distribution of slopes

$$D(\alpha) = \frac{1}{\sqrt{\pi}m^2 \cos^2 \alpha} e^{-\frac{\tan^2 \alpha}{m^2}} \quad m = \frac{2\sigma}{\tau}$$

Beckmann

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Microfacet Distribution Functions

Isotropic distributions

$$D(\omega_h) \Rightarrow D(\alpha)$$

Characterize by half-angle β

$$D(\beta) = \frac{1}{2}$$

Examples:

■ **Blinn**

$$D_1(\alpha) = \cos^{c_1} \alpha$$

$$c_1 = \frac{\ln 2}{\ln \cos \beta}$$

■ **Torrance-Sparrow**

$$D_2(\alpha) = e^{-(c_2\alpha)^2}$$

$$c_2 = \frac{\sqrt{2}}{\beta}$$

■ **Trowbridge-Reitz**

$$D_3(\alpha) = \frac{c_3^2}{(1-c_3^2)\cos^2 \alpha - 1}$$

$$c_3 = \left(\frac{\cos^2 \beta - 1}{\cos^2 \beta - \sqrt{2}} \right)^{1/2}$$

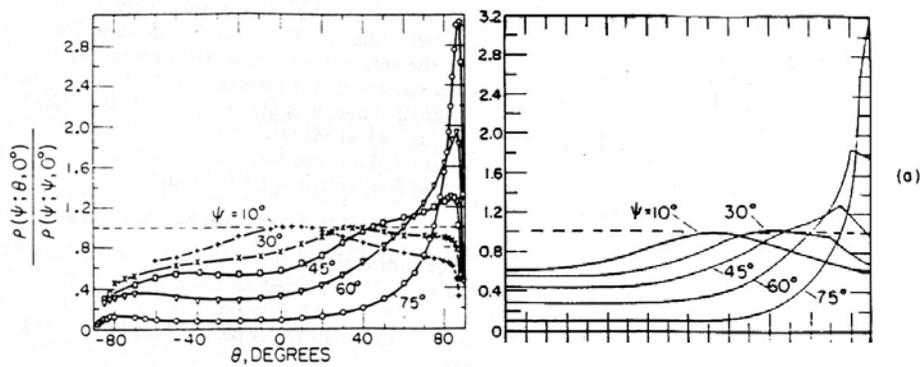
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Torrance-Sparrow Model

Torrance-Sparrow Comparison

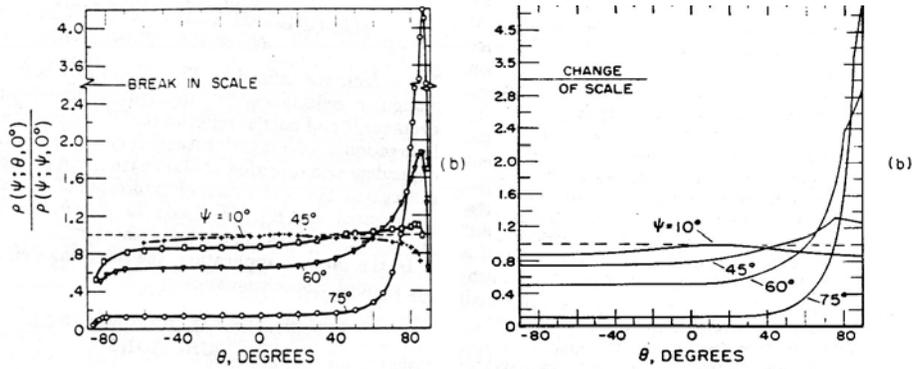
Found an off-specular peak



Aluminum

Torrance-Sparrow Comparison

Found an off-specular peak



Magnesium Oxide

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Explanation: Fresnel Term

$$f_r(\omega_i \rightarrow \omega_r) = \frac{F(\theta'_i)D(\alpha)}{4 \cos \theta_i \cos \theta_r}$$

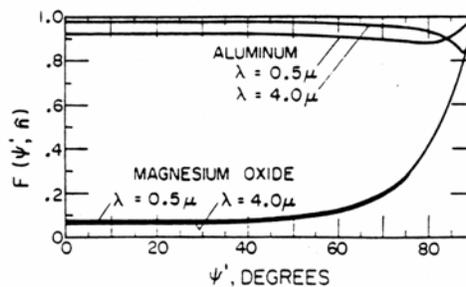
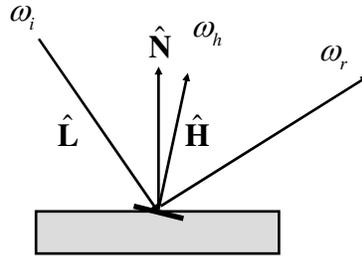


FIG. 6. Fresnel reflectance.

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Torrance-Sparrow Model



$$\begin{aligned}\cos \theta_i &= \hat{\mathbf{L}} \cdot \hat{\mathbf{N}} \\ \cos \theta'_i &= \hat{\mathbf{L}} \cdot \hat{\mathbf{H}} \\ d\omega'_i &= d\omega_i\end{aligned}$$

Prime indicates wrt H

$$d\Phi_h = L_i(\omega_i) \cos \theta'_i d\omega'_i dA(\omega_h)$$

$$dA(\omega_h) = D(\omega_h) d\omega_h dA$$

$$d\Phi_h = L_i(\omega_i) \cos \theta'_i d\omega'_i D(\omega_h) d\omega_h dA$$

$$d\Phi_r = dL_r(\omega_i \rightarrow \omega_r) \cos \theta_r d\omega_r dA$$

$$d\Phi_r = d\Phi_h$$

$$\therefore dL_r(\omega_i \rightarrow \omega_r) \cos \theta_r d\omega_r dA$$

$$= L_i(\omega_i) \cos \theta'_i d\omega'_i D(\omega_h) d\omega_h dA$$

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Torrance-Sparrow Model

$$dL_r(\omega_i \rightarrow \omega_r) \cos \theta_r d\omega_r dA$$

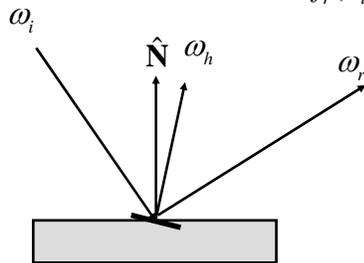
$$= L_i(\omega_i) \cos \theta'_i d\omega'_i D(\omega_h) d\omega_h dA$$

$$f_r(\omega_i \rightarrow \omega_r) \equiv \frac{dL_r(\omega_i \rightarrow \omega_r)}{dE_i(\omega_i)}$$

$$= \frac{L_i(\omega_i) \cos \theta'_i d\omega'_i D(\omega_h) d\omega_h dA}{(\cos \theta_r d\omega_r dA) (L_i(\omega_i) \cos \theta_i d\omega_i)}$$

$$= \frac{D(\omega_h)}{\cos \theta_i \cos \theta_r} \cos \theta'_i \frac{d\omega_h}{d\omega_r}$$

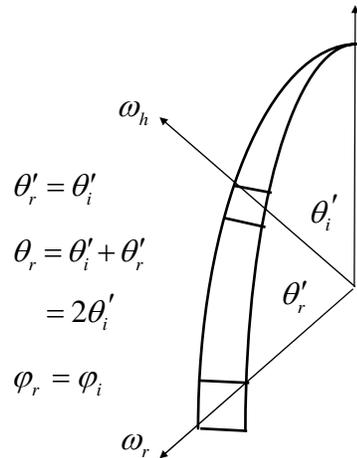
$$= \frac{D(\omega_h)}{4 \cos \theta_i \cos \theta_r}$$



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Solid Angle Distributions



$$\begin{aligned}
 d\omega_r &= \sin \theta_r d\theta_r d\varphi_r \\
 &= (\sin 2\theta_i') 2d\theta_i' d\varphi_i \\
 &= \left(2 \sin \theta_i' \cos \theta_i'\right) 2d\theta_i' d\varphi_i \\
 &= 4 \cos \theta_i' \sin \theta_i' d\theta_i' d\varphi_i \\
 &= 4 \cos \theta_i' d\omega_h
 \end{aligned}$$

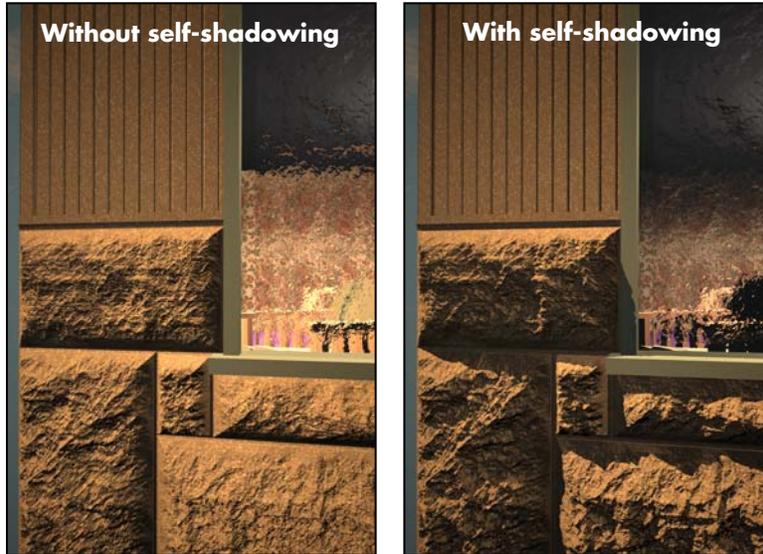
$$\frac{d\omega_h}{d\omega_r} = \frac{1}{4 \cos \theta_i'}$$

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Self-Shadowing

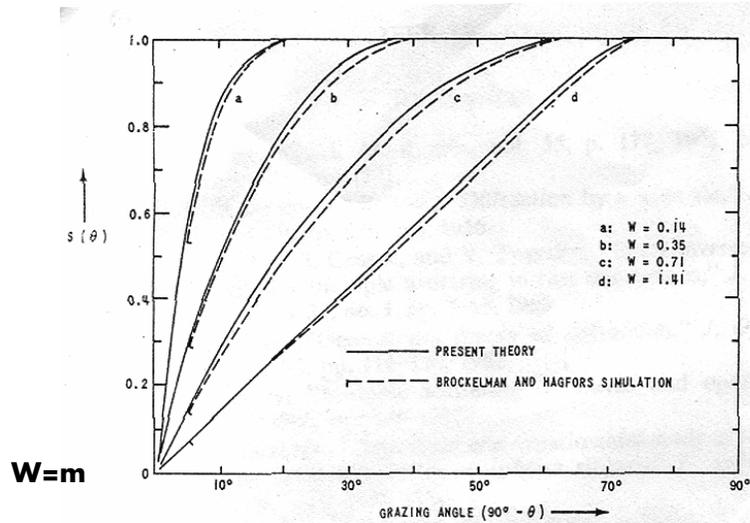
Problem: Conservation of Energy



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Self-Shadowing Function

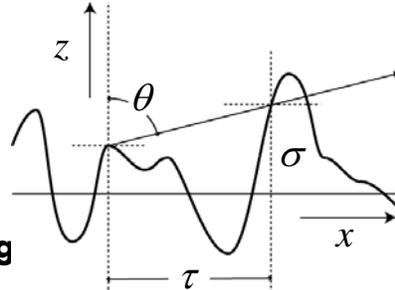


From Smith, 1967

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Self-Shadowing Function



Probability of shadowing

$$S(\theta) = \frac{\left[1 - \frac{1}{2} \operatorname{erfc}\left(\frac{\mu}{\sqrt{2}m}\right) \right]}{1 + \Lambda(\mu)}$$

$$2\Lambda(\mu) = \left(\sqrt{\frac{2}{\pi}} \right) \frac{m}{\mu} e^{-\mu^2/2m^2} - \operatorname{erfc}\left(\frac{\mu}{\sqrt{2}m}\right)$$

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Self-Consistency Condition

$$\int S(\theta) D(\alpha) \cos \theta' d\omega_\alpha = \cos \theta$$

The sum of the areas of the illuminated surface projected onto the plane normal to the direction of incidence is independent of the roughness of the surface, and equal to the projected area of the underlying mean plane.

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Torrance-Sparrow Theory

$$f_r(\omega_i \rightarrow \omega_r) = \frac{F(\theta_i)S(\theta_i)S(\theta_r)D(\alpha)}{4 \cos \theta_i \cos \theta_r}$$

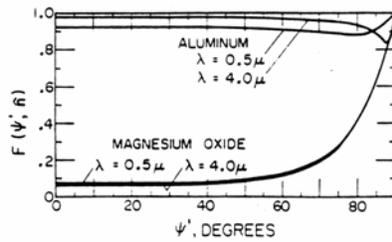


FIG. 6. Fresnel reflectance.

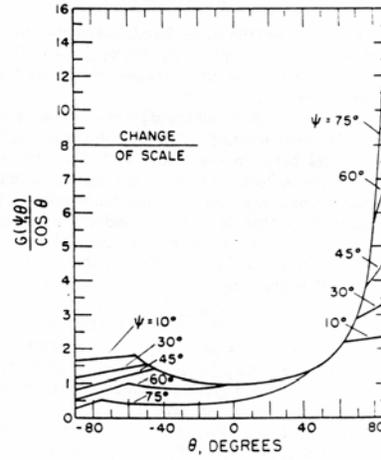


FIG. 7. The factor $G(\psi, \theta) / \cos \theta$ in the plane of incidence for various incidence angles ψ .