

Participating Media & Vol. Scattering

Applications

- Clouds, smoke, water, ...
- Subsurface scattering: paint, skin, ...
- Scientific/medical visualization: CT, MRI, ...

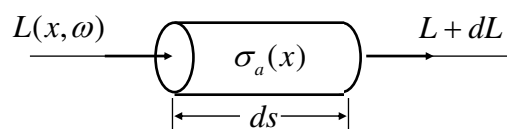
Topics

- Absorption and emission
- Scattering and phase functions
- Volume rendering equation
- Homogeneous media
- Ray tracing volumes

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Absorption



$$dL(x, \omega) = -\sigma_a(x)L(x, \omega) ds$$

Absorption cross-section: $\sigma_a(x)$

Probability of being absorbed per unit length

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Transmittance

$$dL(x, \omega) = -\sigma_a(x)L(x, \omega) ds$$

$$\frac{dL(x, \omega)}{L(x, \omega)} = -\sigma_a(x) ds$$

$$\ln L(x + s \omega, \omega) = -\int_0^s \sigma_a(x + s' \omega) ds' = -\tau(s)$$

Optical distance or depth

$$\tau(s) = \int_0^s \sigma_a(x + s' \omega) ds'$$

Homogenous media: constant σ_a

$$\sigma_a \rightarrow \tau(s) = \sigma_a s$$

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Transmittance and Opacity

$$dL(x, \omega) = -\sigma_a(x)L(x, \omega) ds$$

$$\frac{dL(x, \omega)}{L(x, \omega)} = -\sigma_a(x) ds$$

$$\ln L(x + s \omega, \omega) = -\int_0^s \sigma_a(x + s' \omega) ds' = -\tau(s)$$

$$L(x + s \omega, \omega) = e^{-\tau(s)} L(x, \omega) = T(s)L(x, \omega)$$

Transmittance

$$T(s) = e^{-\tau(s)}$$

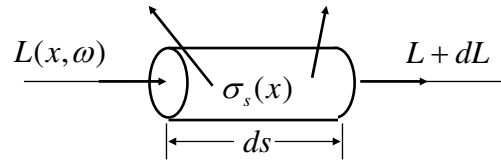
Opacity

$$\alpha(s) = 1 - T(s)$$

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Out-Scatter

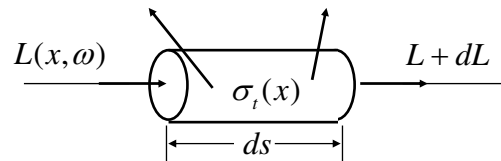


$$dL(x, \omega) = -\sigma_s(x)L(x, \omega) ds$$

Scattering cross-section: σ_s

Probability of being scattered per unit length

Extinction



$$dL(x, \omega) = -\sigma_t(x)L(x, \omega) ds$$

Total cross-section

$$\sigma_t = \sigma_a + \sigma_s$$

Albedo

$$W = \frac{\sigma_s}{\sigma_t} = \frac{\sigma_s}{\sigma_a + \sigma_s}$$

Attenuation due to both absorption and scattering

$$\tau(s) = \int_0^s \sigma_t(x + s' \omega) ds'$$

Black Clouds

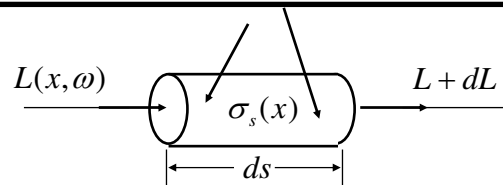


From Greenler, Rainbows, halos and glories

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In-Scatter



$$S(x, \omega) = \sigma_s(x) \int_{S^2} p(\omega' \rightarrow \omega) L(x, \omega') d\omega'$$

Phase function $p(\omega' \rightarrow \omega)$

Reciprocity

$$p(\omega \rightarrow \omega') = p(\omega' \rightarrow \omega)$$

Energy conserving

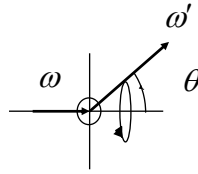
$$\int_{S^2} p(\omega' \rightarrow \omega) d\omega' = 1$$

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Phase Functions

Phase angle $\cos \theta = \omega \bullet \omega'$



Phase functions
(from the phase of the moon)

1. **Isotropic**
-simple
 2. **Rayleigh**
-molecules
 3. **Mie scattering**
- small spheres
- ... Huge literature ...

$$p(\cos \theta) = \frac{1}{4\pi}$$

$$p(\cos \theta) = \frac{3}{4} \frac{1 + \cos^2 \theta}{\lambda^4}$$

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Blue Sky = Red Sunset



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Coronas and Halos



Moon Corona



Sun Halos

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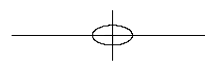
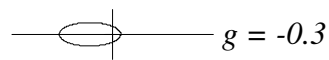
Henyey-Greenstein Phase Function

Empirical phase function

$$p(\cos \theta) = \frac{1}{4\pi} \frac{1-g^2}{(1+g^2-2g \cos \theta)^{3/2}}$$

$$2\pi \int_0^\pi p(\cos \theta) \cos \theta d\theta = g$$

g : average phase angle



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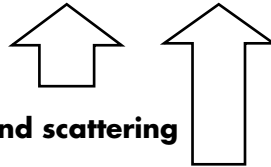
The Volume Rendering Equation

Integro-differential equation

$$\frac{\partial L(x, \omega)}{\partial s} = -\sigma_t(x)L(x, \omega) + S(x, \omega)$$

Integro-integral equation

$$L(x, \omega) = \int_0^{\infty} e^{-\int_0^{s'} \sigma_t(x+s''\omega) ds''} S(x+s'\omega) ds'$$



Attenuation: Absorption and scattering

Source: Scatter (+ emission)

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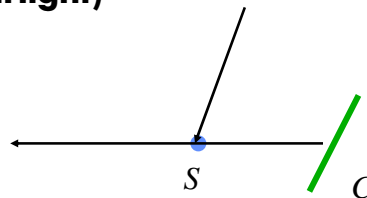
Simple Atmosphere Model

Assumptions

- Homogenous media
- Constant source term (airlight)

$$\frac{\partial L(s)}{\partial s} = -\sigma_t L(s) + S$$

$$L(s) = (1 - e^{-\sigma_t s}) S + e^{-\sigma_t s} C$$



Fog

Haze

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The Sky



Plate 5-16. Fisheye view of clear sky at the South Pole. (Photographed by the author)



Plate 5-17. View of slightly hazy sky in Wisconsin. (Photographed by the author)

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Atmospheric Perspective



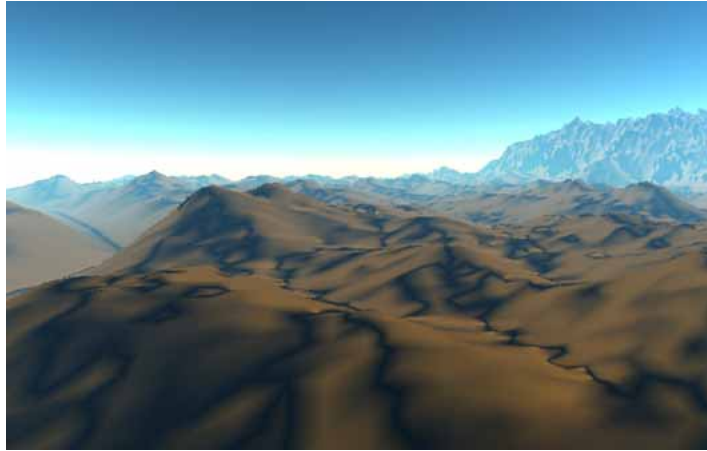
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Atmospheric Perspective

Aerial Perspective: loss of contrast and change in color



From Musgrave

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Semi-Infinite Homogenous Media

Reduced Intensity

$$L(z, \omega_i) = e^{-\tau(z, \omega_i)} L(0, \omega_i)$$

Effective source term

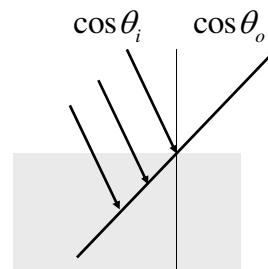
$$S(z, \omega_o) = \sigma_s p(\omega_i \rightarrow \omega_o) e^{-\tau(z, \omega_i)} L(0, \omega_i)$$

Volume rendering equation

$$\cos \theta_o \frac{\partial L(z, \omega_o)}{\partial z} = -\sigma_t L(z, \omega_o) + S(z, \omega_o)$$

Integrating over depths

$$\cos \theta_o L(\omega_o) = \int_0^{\infty} e^{-\sigma_t z / \cos \theta_o} \sigma_s p(\omega_i, \omega_o) e^{-\sigma_t z / \cos \theta_i} L(\omega_i) dz$$



$$z = s \cos \theta$$

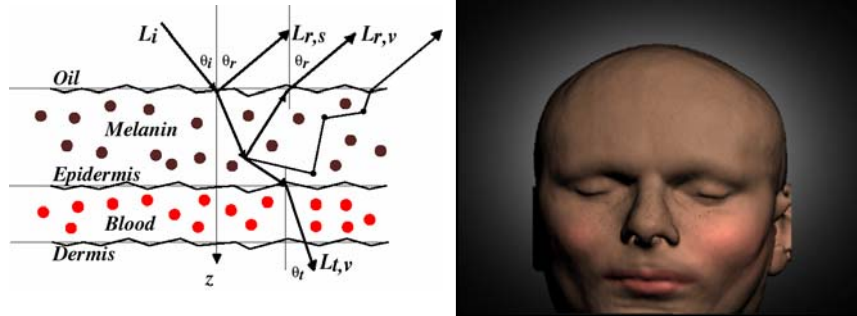
$$dz = ds \cos \theta$$

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Subsurface Scattering

Skin



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Volume Representations

3D arrays (uniform rectangular)

- CT data

3D meshes

- CFD, mechanical simulation

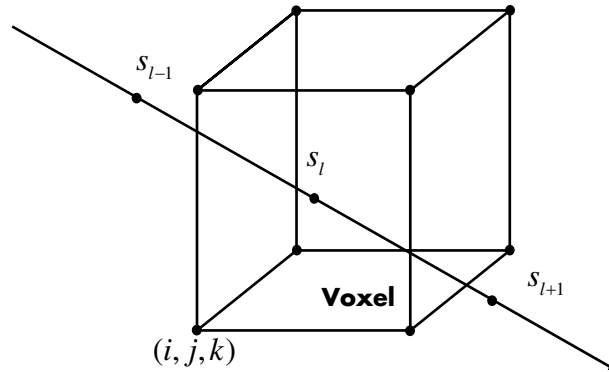
Simple shapes with solid texture

- Ellipsoidal clouds with sum-of-sines densities
- Hypertexture

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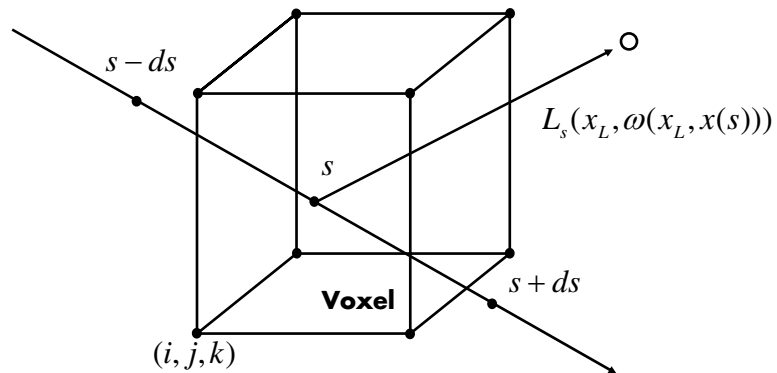
Scalar Volumes



Interpolation $v(s_l) = \text{trilinear}(v, i, j, k, x(s_l))$

Map scalars to optical properties $\sigma_s(v), \sigma_a(v)$

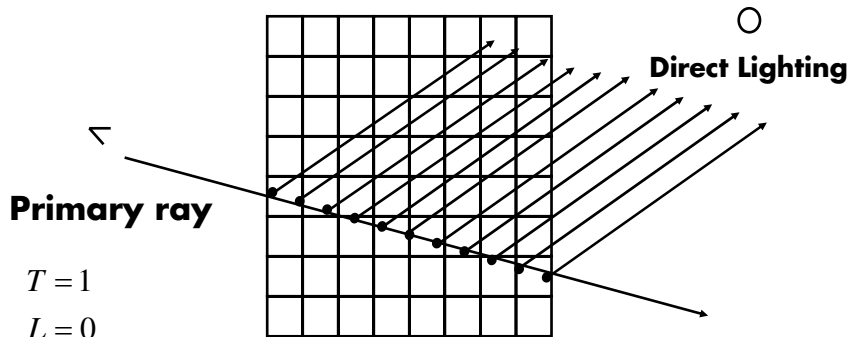
Scalar Volumes



Scatter

$$S(x(s), \omega) = \sigma_s(s) p(\omega, \omega(x(s), x_L)) L_s(x_L, \omega(x_L, x(s)))$$

Ray Marching



$$T = 1$$

$$L = 0$$

for($s = 0$; $s < 1$; $s += ds$)

$$S = \sigma_s(s) p(\omega, \omega(x(s), x_L)) L_s(x_L, \omega(x_L, x(s)))$$

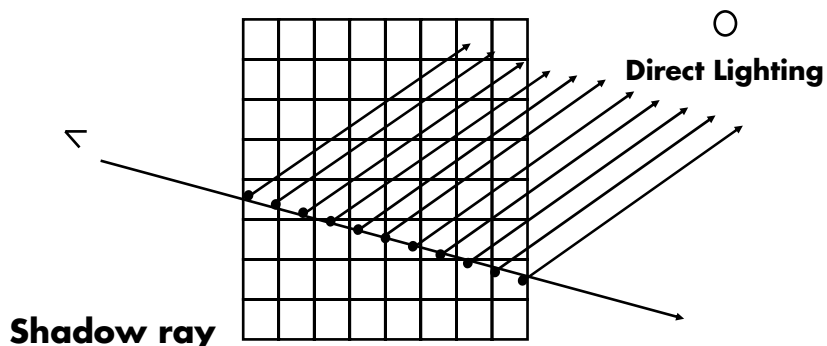
$$L = L + TS\Delta s$$

$$T = T[1 - \sigma_t(x(s))]\Delta s$$

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Ray Marching



$$T = 1$$

for($t = 0$; $t < 1$; $t += dt$)

$$T = T[1 - \sigma_t(x(t))]\Delta t$$

$$S(x(s)) = \sigma_s(s) p(\omega, \omega(x(s), x_L)) TL_s(x_L, \omega(x_L, x(s)))$$

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Beams of Light



From Greenler, Rainbows, halos and glories

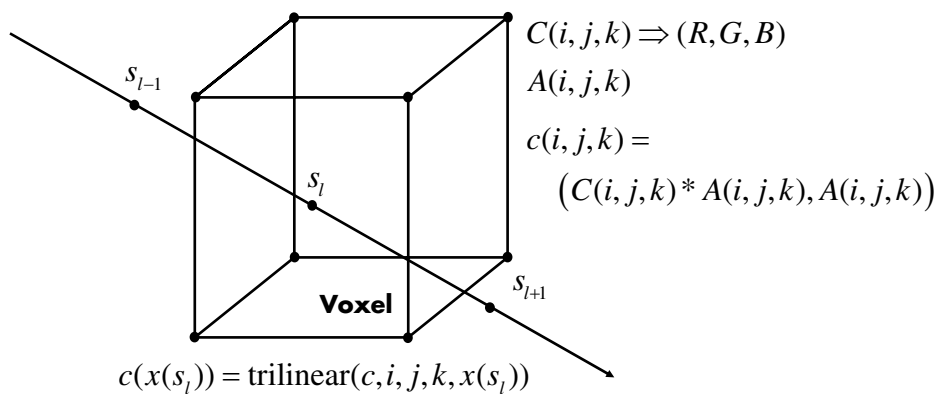
From Minneart, Color and light in the open air

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Color and Opacity Volumes

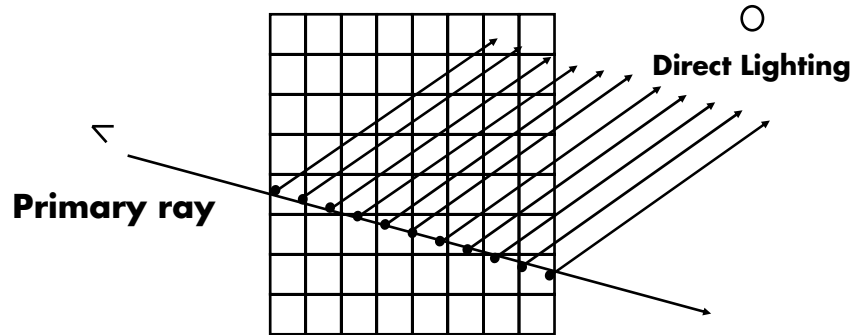
M. Levoy, Ray tracing volume densities



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Ray Marching



$$C = (0, 0, 0, 0)$$

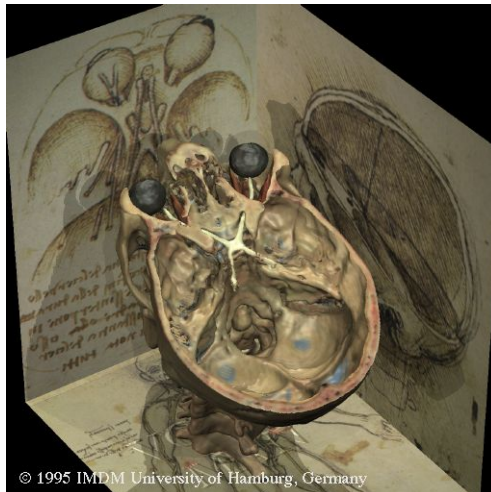
for $(s = 0; s < 1; s += ds)$

$$C = C + (1 - \alpha(C))c(s)$$

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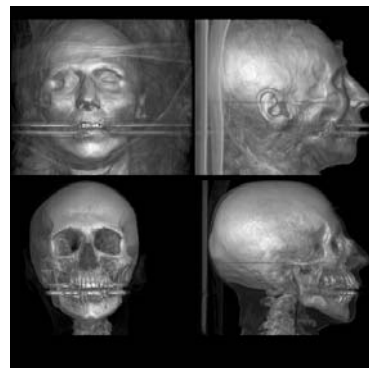
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Volume Rendering Examples



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From Karl Heinz Hoehne



From Marc Levoy

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