Ray Tracing

Ray Tracing 1

- Basic algorithm
- Overview of pbrt
- Ray-surface intersection (triangles, ...)

Ray Tracing 2

- Problem: brute force = |Image| x |Objects|
- Acceleration data structures

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Primitives

`pbrt` primitive base class

- Shape
- Material (reflection and emission)

Subclasses

- Primitive instance
  - Transformation and pointer to a primitive
- Aggregate (collection)
  - Treat collections just like single primitives
  - Incorporate acceleration structures into collections
  - May nest accelerators of different types
  - Types: grid.cpp and kdtree.cpp

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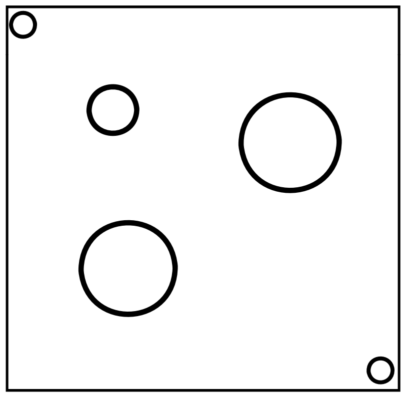
Uniform Grids



Preprocess scene

1. Find bounding box

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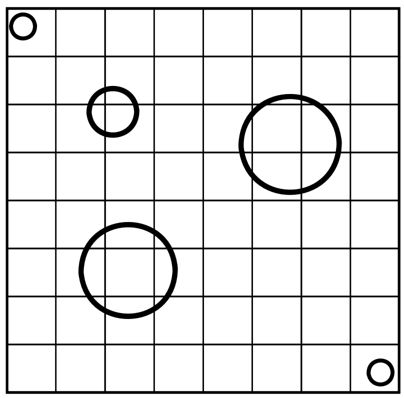
Uniform Grids



Preprocess scene

1. Find bounding box
2. Determine resolution

$$n_v = n_x n_y n_z \propto n_o$$

$$\max(n_x, n_y, n_z) = d\sqrt[3]{n_o}$$

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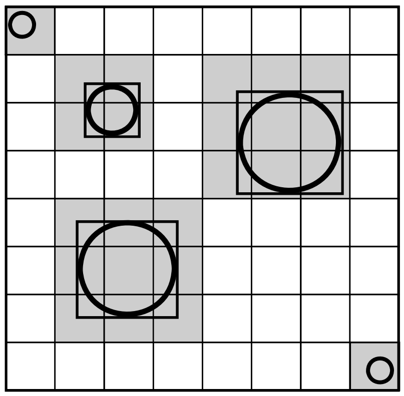
Uniform Grids



Preprocess scene

1. **Find bounding box**
2. **Determine resolution**

$$\max(n_x, n_y, n_z) = d\sqrt[3]{n_o}$$

3. **Place object in cell, if object overlaps cell**

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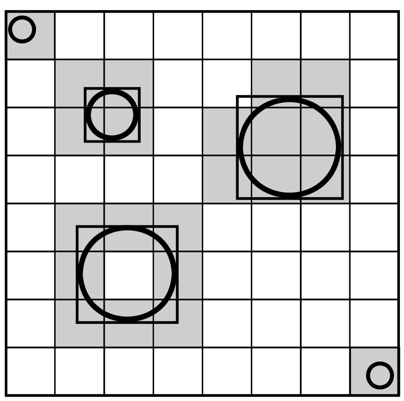
Uniform Grids



Preprocess scene

1. **Find bounding box**
2. **Determine resolution**

$$\max(n_x, n_y, n_z) = d\sqrt[3]{n_o}$$

3. **Place object in cell, if object overlaps cell**
4. **Check that object's surface intersects cell**

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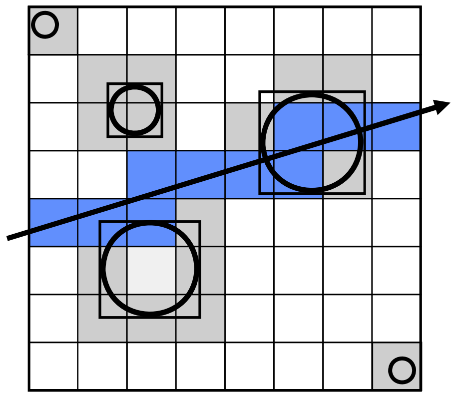
Uniform Grids



Preprocess scene

Traverse grid

- 3D line – 3D-DDA
- 6-connected line

Section 4.3

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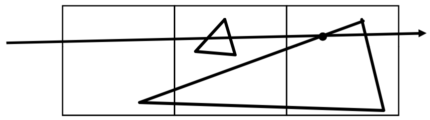
Caveat: Overlap

Problem: Don't output first intersection found!



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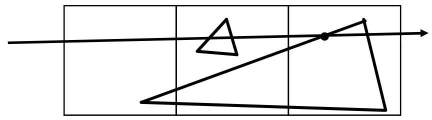
Caveat: Overlap

Problem: Don't output first intersection found!



Problem: Redundant intersection tests

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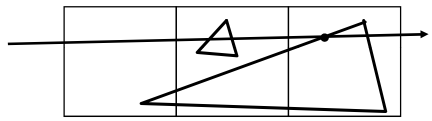
Caveat: Overlap

Problem: Don't output first intersection found!



Problem: Redundant intersection tests

Solution: Mailboxes

- Assign each ray an increasing number
- Primitive intersection cache (mailbox)
  - Store last ray number tested in mailbox
  - Only intersect if ray number is greater

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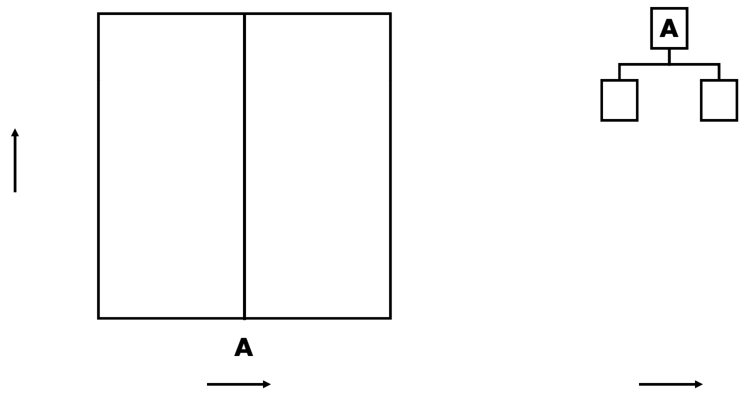
Spatial Hierarchies



Letters correspond to planes (A)

Point Location by recursive search

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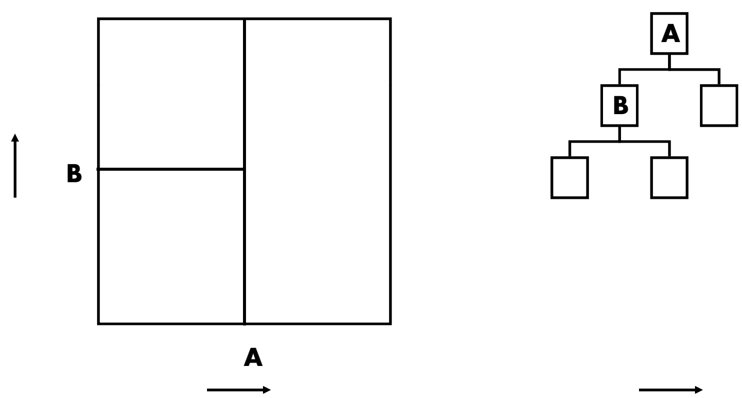
Spatial Hierarchies



Letters correspond to planes (A, B)

Point Location by recursive search

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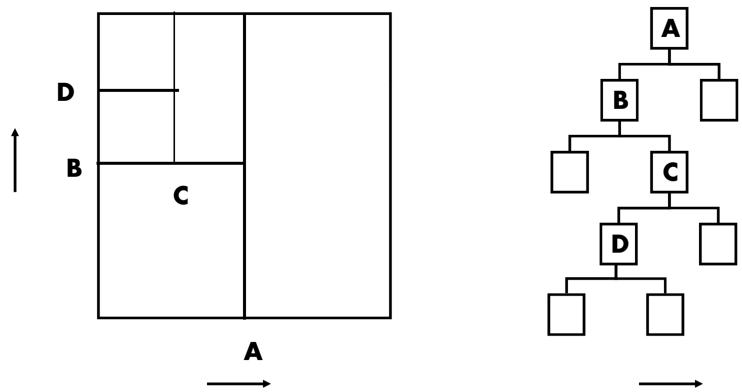
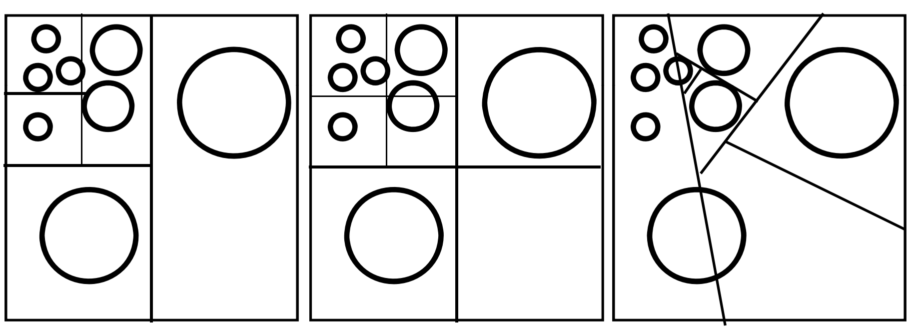
Spatial Hierarchies



Letters correspond to planes (A, B, C, D)

Point Location by recursive search

Variations



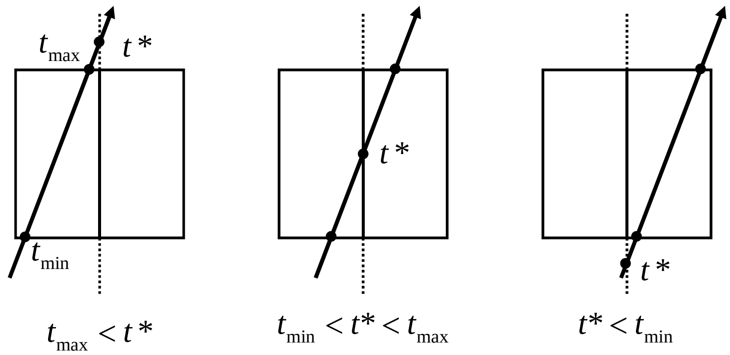
kd-tree **oct-tree** **bsp-tree**

Ray Traversal Algorithms

Recursive inorder traversal

[Kaplan, Arvo, Jansen]

$$t^* = (S - \mathbf{O}[a]) / \mathbf{D}[a]$$




```
Intersect(L,tmin,tmax)  Intersect(L,tmin,t*) Intersect(R,tmin,tmax)
                        Intersect(R,t*,tmax)
```

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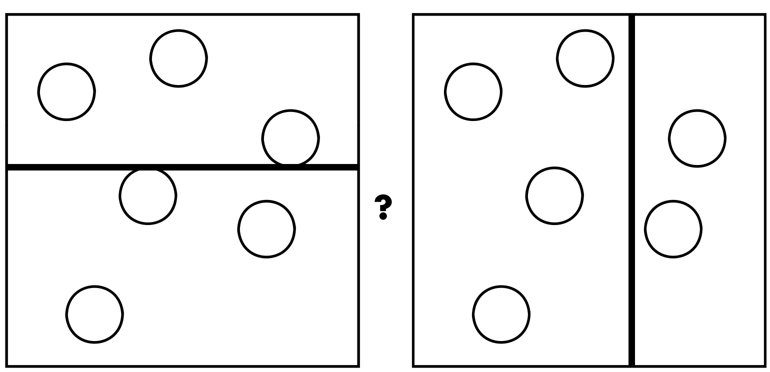
How to Build the Hierarchy?



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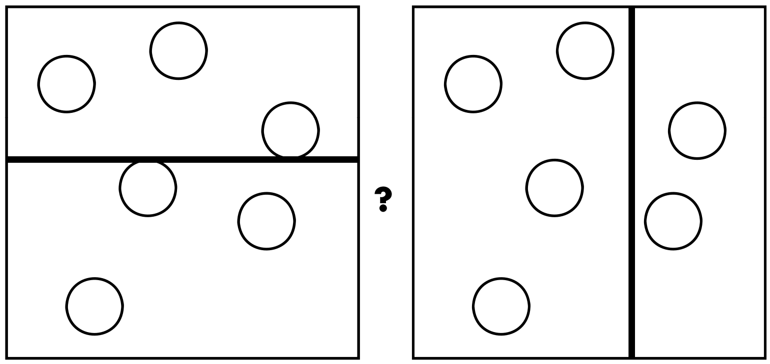
Build Hierarchy Top-Down



Methods to choose axis and splitting plane

- **Midpoint**
- **Median cut (balanced)**
- **Surface area heuristic**

Cost

What is the cost of tracing a ray through a node?

Cost(node) = C_trav + Prob(hit L) * Cost(L) + Prob(hit R) * Cost(R)

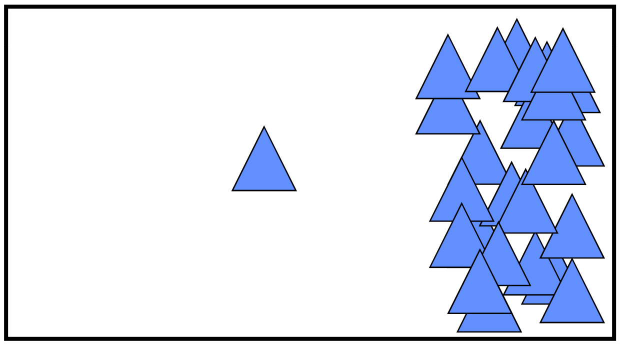
C_trav = cost of traversing a cell

Cost(L) = cost of traversing left child

Cost(R) = cost of traversing right child

Splitting with Cost in Mind

From Gordon Stoll



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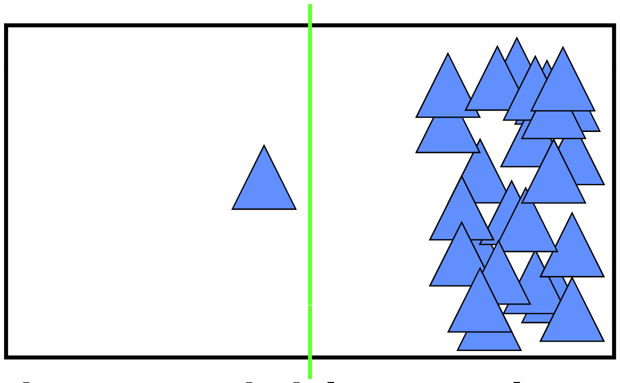
Split in the Middle = Bad!

From Gordon Stoll



Makes the L & R probabilities equal

Pays no attention to the L & R costs

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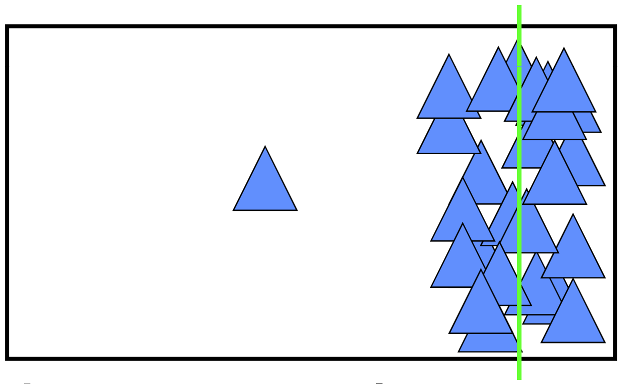
Split at the Median = Bad!

From Gordon Stoll



Makes the L & R costs equal

Pays no attention to the L & R probabilities

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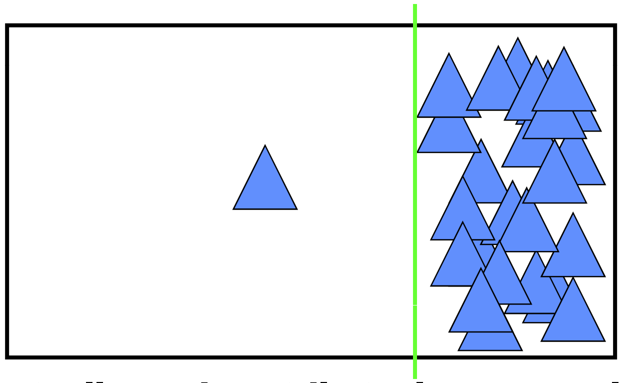
Cost-Optimized Split = Good!

From Gordon Stoll



Automatically and rapidly isolates complexity

Produces large chunks of empty space

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Cost

Need the probabilities

- **Turns out to be proportional to surface area**

Need the child cell costs

- **Triangle count is a good approximation**

Cost(cell) = C_trav + Prob(hit L) * Cost(L) + Prob(hit R) * Cost(R)

= C_trav + SA(L) * TriCount(L) + SA(R) * TriCount(R)

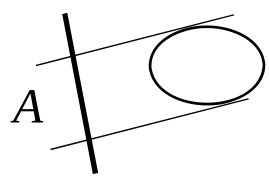
C_trav is the ratio of the cost to traverse to the cost to intersect

C_trav = 1:80 in pbrt

C_trav = 1:1.5 in a highly optimized version

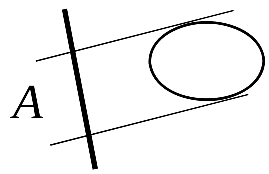
Projected Area and Ray Intersection

Number of rays in a given direction that hit an object is proportional to its projected area



Projected Area and Surface Area

Number of rays in a given direction that hit an object is proportional to its projected area



The total number of rays hitting an object is $4\pi\overline{A}$

Crofton's Theorem:

For a convex body $\overline{A} = \frac{S}{4}$

For a sphere $S = 4\pi r^2$ and $\overline{A} = A = \pi r^2$

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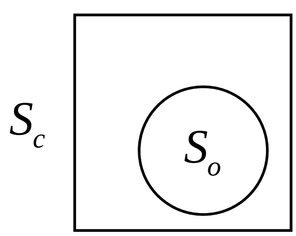
Surface Area and Ray Intersection

The probability of a ray hitting a convex shape enclosed by another convex shape is



$$\Pr[r \cap S_o \,|\, r \cap S_c] = \frac{S_o}{S_c}$$

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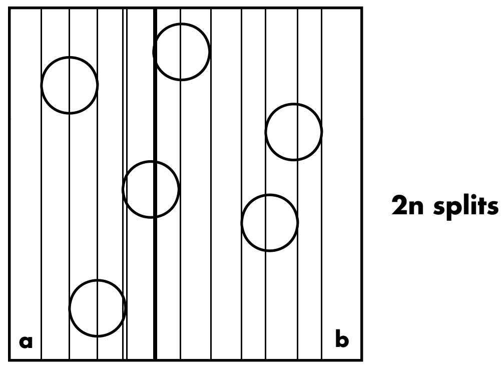
Sweep Build Algorithm



$$p_a = \frac{S_a}{S} \qquad p_b = \frac{S_b}{S}$$

$$N_a \qquad N_b$$

Basic Build Algorithm (Triangles)

1. Pick an axis, or optimize across all three
2. Build a set of "candidate" split locations
   Note: Cost extrema must be at bbox vertices
   - Vertices of triangle
   - Vertices of triangle clipped to node bbox
3. Sort or bin the triangles
4. Sweep to incrementaly track L/R counts, cost
5. Output position of minimum cost split

Running time: $T(N) = N \log N + 2T(N/2)$

$$T(N) = N \log^2 N$$

Termination Criteria

When should we stop splitting?

- Bad: depth limit, number of triangles
- Good: When split does not lower the cost

Threshold of cost improvement

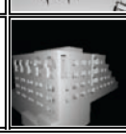
- Stretch over multiple levels
- For example, if cost doesn't go down after three splits in a row, terminate

Threshold of cell size

- Absolute probability SA(node)/SA(scene) small

Best Reported Timings

Millions of Rays per Second

scene # of triangles and shader (+/-) \ Framerate (FPS) @ 1024x1024 resolution			OpenRT @ 2.5 GHz P4 1 thread	MLRTA @ 2.4 GHz P4 1 thread	MLRTA @ 3.2 GHz P4 with HT 2 threads
Erw6 804		– shader	7.1	70.2	109.8
		+ shader	2.3	37.8	50.7
Confe-rence 274K		– shader	4.55	11.2	19.5
		+ shader	1.93	9.5	15.6
Soda Hall 2195K		– shader	4.12	21.1	35.5
		+ shader	1.8	15.3	24.1

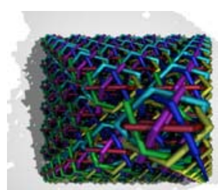
Reshetov, Soupikov, Hurley, SIGGRAPH 2005

Superoptimizations

Lots of optimizations

- Carefully written inner loop (no recursion)
- Use vector instructions SSE2
- 64 bits per kd-tree node
  - 32 bit position
  - 32 bit pointer to pair of child nodes
  - 2 bits for split plane direction (x, y, or z)
- Trace packet of rays
  - 4 or more rays at a time
- Intersect beam at top of tree
- Encourage empty nodes
- Special case axis-aligned triangles
- ...

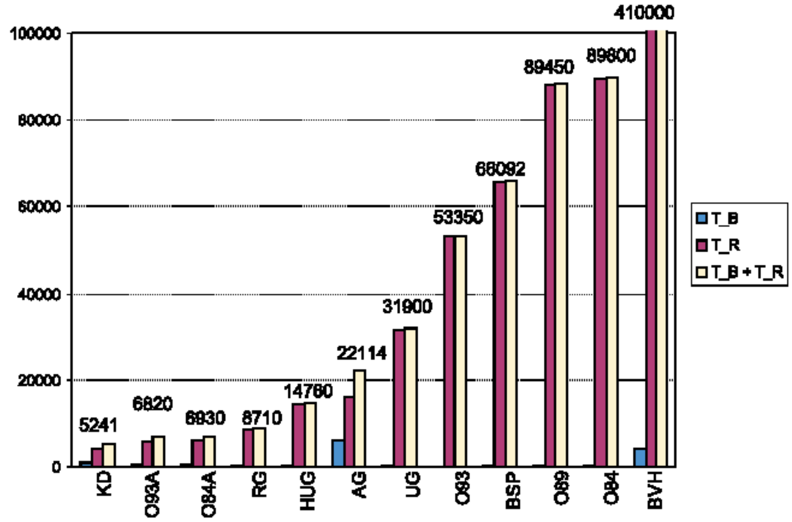
Comparison


Time		Spheres	Rings	Tree
Uniform Grid	d=1	244	129	1517
	d=20	38	83	781
Hierarchical Grid		34	116	34

V. Havran, Best Efficiency Scheme Project
http://www.cgg.cvut.cz/BES

Comparison

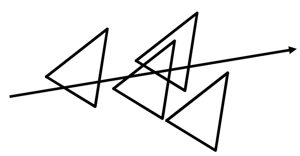


T_R – Time to Ray Trace; T_B – Time to Build

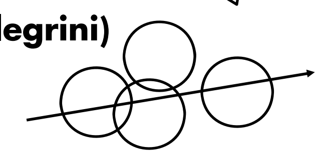
Theoretical Nugget 1

Computational geometry of ray shooting

1. **Triangles (Pellegrini)**
   - **Time:** $O(\log n)$
   - **Space:** $O(n^{5+\varepsilon})$



2. **Sphere (Guibas and Pellegrini)**
   - **Time:** $O(\log^2 n)$
   - **Space:** $O(n^{5+\varepsilon})$

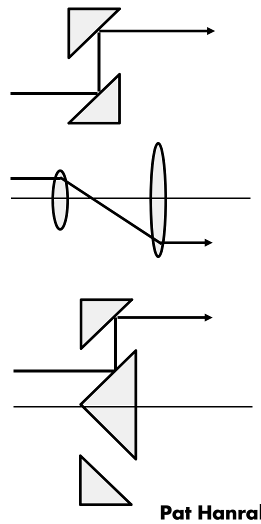


Theoretical Nugget 2

Optical computer = Turing machine

Reif, Tygar, Yoshida

Determining if a ray starting at y0 arrives at yn is undecidable



y = y+1

y = -2*y

if(y>0)