Camera Simulation

<table>
<thead>
<tr>
<th>Effect</th>
<th>Cause</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field of view</td>
<td>Film size, stops and pupils</td>
</tr>
<tr>
<td>Depth of field</td>
<td>Aperture, focal length</td>
</tr>
<tr>
<td>Exposure</td>
<td>Film speed, aperture, shutter</td>
</tr>
<tr>
<td>Motion blur</td>
<td>Shutter</td>
</tr>
</tbody>
</table>

References
Photography, B. London and J. Upton
Optics in Photography, R. Kingslake
The Camera, The Negative, The Print, A. Adams

Topics

<table>
<thead>
<tr>
<th>Lenses</th>
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<tbody>
<tr>
<td>Focus</td>
</tr>
<tr>
<td>Depth of focus / depth of field</td>
</tr>
<tr>
<td>Exposure</td>
</tr>
</tbody>
</table>
Lenses

Refraction

$\vec{N}$

$\vec{I}$

$\vec{I}'$

$n$

$n'$
**Incident Ray**

Angles: ccw is positive; cw is negative

\[ I = U - \phi \]

The sum of the interior angles is equal to the exterior angle.

**Refraction**

Snell’s Law

\[ n' \sin I' = n \sin I \]
**Paraxial Approximation**

\[ u = \frac{h}{-z} \quad -u' = \frac{h}{z'} \quad -\phi = \frac{h}{R} \]

\[ \tan U \approx u \]

Rays deviate only slightly from the axis

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**Gauss’ Formula**

Paraxial approximation to Snell’s Law

\[ n'(u' - \phi) = n(u - \phi) \]

Ray coordinates

\[ u' = -\frac{h}{z'} \quad \phi = -\frac{h}{R} \quad u = -\frac{h}{z} \]

Thin lens equation

\[ n'(\frac{h}{z'} - \frac{h}{R}) = n(\frac{h}{z} - \frac{h}{R}) \]

\[ \frac{n'}{z'} = \frac{n}{z} + \frac{(n' - n)}{R} \quad \text{Holds for any height, any ray!} \]
Conjugate Points

To focus: move lens relative to backplane
Horizontal rays converge on focal point in the focal plane

Perspective Transformation

Thin lens equation

\[
\frac{1}{z'} = \frac{1}{z} + \frac{1}{f} \Rightarrow z' = \frac{fz}{z + f}
\]

\[
\Rightarrow x' = \frac{fx}{z + f}
\]

\[
\Rightarrow y' = \frac{fy}{z + f}
\]

Represent transformation as a 4x4 matrix
Ray Tracing: Finite Aperture

1. Pick a point on image plane \( x' \)
2. Pick a point on the lens \( u \)
3. Transform \( x' \) to \( x \); form ray \( (u, x-u) \)
Real Lens

Cutaway section of a Vivitar Series 1 90mm f/2.5 lens
Cover photo, Kingslake, Optics in Photography

Double Gauss

Data from W. Smith,
Modern Lens Design, p 312

<table>
<thead>
<tr>
<th>Radius (mm)</th>
<th>Thick (mm)</th>
<th>n_d</th>
<th>V-no</th>
<th>aperture</th>
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<td>1.670</td>
<td>47.1</td>
<td>46.0</td>
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<tr>
<td>81.540</td>
<td>6.550</td>
<td>1.699</td>
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<td>46.0</td>
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<td>25.500</td>
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<td>9.000</td>
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<td>-79.460</td>
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<td>40.0</td>
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</table>
Ray Tracing Through Lenses

From Kolb, Mitchell and Hanrahan (1995)

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Thick Lenses

Equivalent Lens

Refraction occurs at the principal planes
Depth of Field

From London and Upton
**Circle of Confusion**

![Diagram of Focal Plane and Back Plane](image)

Circle of confusion proportional to the size of the aperture

\[ \frac{c}{a} = \frac{d'}{z'} = \frac{s' - z'}{z'} \]

---

**Depth of Focus [Image Space]**

Depth of focus \( \equiv \)

Equal circles of confusion

Extreme planes: near and far

\[ \frac{c}{a} = \frac{d'_f}{z'_f} = \frac{s' - z'_f}{z'_f} \]

\[ \frac{c}{a} = \frac{d'_n}{z'_n} = \frac{z'_n - s'}{z'_n} \]
**Depth of Focus [Image Space]**

Depth of focus $\equiv$ Equal circles of confusion

\[
\frac{c}{a} = \frac{d'_f}{z'_f} = \frac{s'-z'_f}{z'_f} \Rightarrow \frac{1}{z'_f} = \frac{1}{s'} \left( 1 + \frac{c}{a} \right)
\]

\[
\frac{c}{a} = \frac{d'_n}{z'_n} = \frac{z'_n-s'}{z'_n} \Rightarrow \frac{1}{z'_n} = \frac{1}{s'} \left( 1 - \frac{c}{a} \right)
\]
Resolving Power

- **Diffraction limit**
  \[ c = 1.22 \frac{f}{\lambda} \quad [= 1.22 \times 64 \times .500 \mu m = 0.040 \text{ mm}] \]

- **35mm film (Leica standard)**
  \[ c = 0.025 \text{ mm} \]

- **CCD/CMOS pixel aperture**
  \[ c = 0.0116 \text{ mm (Nikon D1)} \]

Depth of Field [Object Space]

Depth of field ≡

**Equal circles of confusion**

\[
\frac{1}{s^f} = \frac{1}{s} + \frac{1}{f} \quad \frac{1}{z_n^f} = \frac{1}{z_n} + \frac{1}{f} \quad \frac{1}{z_f^f} = \frac{1}{z_f} + \frac{1}{f}
\]

\[
\frac{1}{z_n} + \frac{1}{z_f} = \frac{2}{s}
\]

\[
\frac{1}{z_n} \frac{1}{z_f} = \frac{2c}{a} \left( \frac{1}{f} - \frac{1}{s} \right) \approx \frac{2c}{a} \frac{1}{f}
\]
### Depth of Field Scale

Reciprocal of Distance

<table>
<thead>
<tr>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.5</td>
</tr>
<tr>
<td>20</td>
</tr>
<tr>
<td>25</td>
</tr>
<tr>
<td>50</td>
</tr>
<tr>
<td>100</td>
</tr>
<tr>
<td>∞</td>
</tr>
</tbody>
</table>

### Hyperfocal Distance

\[
\frac{1}{z_n} + \frac{1}{z_f} = 2 \frac{1}{s} \quad \quad N = \frac{a}{f}
\]

\[
\frac{1}{z_n} - \frac{1}{z_f} = 2c \frac{1}{a} \frac{1}{f} = 2 \frac{cN f}{f^2} = 2 \frac{1}{H}
\]

When \( s \to H \Rightarrow z_n = \frac{H}{2}, z_f = \infty \)

**H is the hyperfocal distance**
Factors Affecting DOF


\[
\frac{1}{H} = \frac{cN}{f^2}
\]

THREE FACTORS THAT AFFECT RANGE OF SHARPNESS

1. LENS OPENING
   - LARGE f/2.0
   - SMALL f/16

2. SUBJECT DISTANCE
   - 4 FT.
   - 10 FT.
   - 25 FT.

3. FOCAL LENGTH
   - 120mm
   - 50mm
   - 28mm

Exposure
**Image Irradiance**

\[ E = \int_{\Omega} L \cos \theta \, d\omega = L \pi \sin^2 \theta = L \frac{\pi}{4} \left( \frac{a}{f} \right)^2 \]

**Relative Aperture or F-Stop**

\[ a = \frac{f}{N} \]

**F-Number and exposure:**

\[ E = L \frac{\pi}{4} \frac{1}{N^2} \]

**F-stops:** 1.4  2  2.8  4.0  5.6  8  11  16  22  32  45  64

1 stop doubles exposure
Camera Exposure

Exposure \[ H = E \times T \]

Exposure overdetermined

Aperture: f-stop - 1 stop doubles \( H \)
- Decreases depth of field

Shutter: Doubling the open time doubles \( H \)
- Increases motion blur

Aperture vs Shutter

From London and Upton

<table>
<thead>
<tr>
<th>Aperture</th>
<th>Shutter</th>
</tr>
</thead>
<tbody>
<tr>
<td>f/16</td>
<td>1/8s</td>
</tr>
<tr>
<td>f/4</td>
<td>1/125s</td>
</tr>
<tr>
<td>f/2</td>
<td>1/500s</td>
</tr>
</tbody>
</table>
High Dynamic Range

Sixteen photographs of the Stanford Memorial Church taken at 1-stop increments from 30s to 1/1000s.
From Debevec and Malik, High dynamic range photographs.

Simulated Photograph

Adaptive histogram  With glare, contrast, blur

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**Paraxial Approximation**

\[ \sin U \approx u \]
\[ \tan U \approx u \]

Rays deviate only slightly from the axis

---

**Incident Ray**

Angles: ccw is positive; cw is negative

\[ I = U - \phi \]

The sum of the interior angles is equal to the exterior angle.
Refracted Ray

\[ I' = U' - \phi \]

Derivation

Paraxial approximation

\[ I = U - \phi \Rightarrow i = u - \phi \]
\[ I' = U' - \phi \Rightarrow i' = u' - \phi \]
Derivation

Paraxial approximation

\[ I = U - \phi \quad \Rightarrow \quad i = u - \phi \]
\[ I' = U' - \phi \quad \Rightarrow \quad i' = u' - \phi \]

Snell’s Law

\[ n' \sin I' = n \sin I \quad \Rightarrow \quad n'i' = ni \]

\[ n'(u' - \phi) = n(u - \phi) \]

Field of View
Field of View

CS348B Lecture 7  Pat Hanrahan, 2007

From London and Upton
Field of View

Field of view

Redrawn from Kingslake, Optics in Photography

\[ \tan \frac{fov}{2} = \frac{filmsize}{f} \]

Types of lenses

- **Normal** \(26^\circ\)
  
  Film diagonal ~ focal length

- **Wide-angle** \(75-90^\circ\)

- **Narrow-angle** \(10^\circ\)