Light

Visible electromagnetic radiation

Power spectrum

![Power spectrum diagram](image)

Polarization

Photon (quantum effects)

Wave (interference, diffraction)

Topics

Radiometry and photometry

Measuring spatial properties of light

- Radiant intensity
- Radiance
- Radiant exitance (radiosity)
- Irradiance
  - Inverse square law and cosine law

Illumination calculations

- Irradiance from environment
Radiometry and Photometry

Radiant Energy and Power

Power: Watts (radiometry) vs. Lumens (photometry)

\( \Phi \)
- Energy efficiency
- Spectral efficacy

Energy: Joules vs. Talbot

- Exposure
  - Film response
  - Skin - sunburn

Luminance

\[ Y = \int V(\lambda) L(\lambda) d\lambda \]
Radiant Intensity

Definition: The radiant (luminous) intensity is the power per unit solid angle emanating from a point source.

\[ I(\omega) \equiv \frac{d\Phi}{d\omega} \]

\[
\left[ \frac{W}{sr} \right] \left[ \frac{Im}{sr} = cd = candela \right]
\]
### Angles and Solid Angles

- **Angle**  \[ \theta = \frac{l}{r} \]
  
  \( \Rightarrow \) circle has \( 2\pi \) radians

- **Solid angle**  \[ \Omega = \frac{A}{R^2} \]
  
  \( \Rightarrow \) sphere has \( 4\pi \) steradians

### Differential Solid Angles

\[ dA = (r \, d\theta)(r \sin \theta \, d\phi) = r^2 \sin \theta \, d\theta \, d\phi \]

\[ d\omega = \frac{dA}{r^2} = \sin \theta \, d\theta \, d\phi \]
Differential Solid Angles

\[ d\omega = \sin \theta \, d\theta \, d\phi \]

\[ \Omega = \int_{S^2} d\omega = \int_{0}^{\frac{\pi}{2}} \int_{0}^{2\pi} \sin \theta \, d\theta \, d\phi = \int_{0}^{\frac{\pi}{2}} \int_{-1}^{1} d\cos \theta \, d\phi = 4\pi \]

Isotropic Point Source

\[ \Phi = \int_{S^2} I \, d\omega = 4\pi I \]

\[ I = \frac{\Phi}{4\pi} \]
Warn’s Spotlight

\[ I(\omega) = \cos^s \theta = (\mathbf{\omega} \cdot \mathbf{\hat{A}})^s \]

\[ \Phi = \int_0^{2\pi} \int_0^1 I(\omega) \, d\cos \theta \, d\varphi \]

\[ I(\omega) = \frac{s+1}{2\pi} \cos^s \theta \]
Light Source Goniometric Diagrams

1. Porcelain-enamled ventilated standard dome with incandescent lamp
2. Pendant diffusing sphere with incandescent lamp
3. Concentric ring unit with incandescent silved-bowl lamp
4. R-40 flood with specular anodized reflector skirt, 45° cut-off

PIXAR Point Light Source

UberLight()
{
    Clip to near/far planes
    Clip to shape boundary
    foreach superelliptical blocker
        atten *= ...
    foreach cookie texture
        atten *= ...
    foreach slide texture
        color *= ...
    foreach noise texture
        atten, color *= ...
    foreach shadow map
        atten, color *= ...
    Calculate intensity fall-off
    Calculate beam distribution
}
Irradiance

Definition: The irradiance (illuminance) is the power per unit area incident on a surface.

\[ E(x) \equiv \frac{d\Phi_i}{dA} \]

\[
\begin{bmatrix}
W \\
\text{m}^2
\end{bmatrix}
\begin{bmatrix}
lm \\
\text{m}^2 = \text{lux}
\end{bmatrix}
\]

Sometimes referred to as the radiant (luminous) incidence.
Lambert’s Cosine Law

\[ \Phi = E A \]

\[ E = \frac{\Phi}{A} \]

Lambert’s Cosine Law

\[ \Phi \]

\[ A \]

\[ \frac{A}{\cos \theta} \]

\[ \theta \]

\[ E = \frac{\Phi}{A/\cos \theta} = \frac{\Phi}{A} \cos \theta \]
Irradiance: Isotropic Point Source

\[ I = \frac{\Phi}{4\pi} \]

\[ d\Phi = I \, d\omega \]
Irradiance: Isotropic Point Source

\[ I = \frac{\Phi}{4\pi} \]

\[ d\omega = \frac{\cos \theta}{r^2} dA \]

\[ I d\omega = \frac{\Phi}{4\pi} \frac{\cos \theta}{r^2} dA \]
Irradiance: Isotropic Point Source

\[ I = \frac{\Phi}{4\pi} \]

\[ I \, d\omega = \frac{\Phi \cos \theta}{4\pi} \frac{dA}{r^2} = E \, dA \]

\[ E = \frac{\Phi \cos \theta}{4\pi} \frac{1}{r^2} \]

Irradiance: Isotropic Point Source

\[ I = \frac{\Phi}{4\pi} \]

\[ h = r \cos \theta \]

\[ E = \frac{\Phi \cos \theta}{4\pi} \frac{1}{r^2} = \frac{\Phi}{4\pi} \frac{\cos^3 \theta}{h^2} \]
The Invention of Photometry

Bouguer’s classic experiment
- Compare a light source and a candle
- Move until they both appear equally bright
- Intensity is proportional to ratio of distances squared

Definition of a candela
- Originally a “standard” candle
- Currently 550 nm laser w/ 1/683 W/sr
- 1 of 6 fundamental SI units

Typical Values of Illuminance [lm/m²]

<table>
<thead>
<tr>
<th>Condition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sunlight plus skylight</td>
<td>100,000 lux</td>
</tr>
<tr>
<td>Sunlight plus skylight (overcast)</td>
<td>10,000</td>
</tr>
<tr>
<td>Interior near window (daylight)</td>
<td>1,000</td>
</tr>
<tr>
<td>Artificial light (minimum)</td>
<td>100</td>
</tr>
<tr>
<td>Moonlight (full)</td>
<td>0.02</td>
</tr>
<tr>
<td>Starlight</td>
<td>0.0003</td>
</tr>
</tbody>
</table>
Radiance

**Definition:** The surface *radiance* (*luminance*) is the intensity per unit area leaving a surface

\[ L(x, \omega) = \frac{dI(x, \omega)}{dA} = \frac{d^2\Phi(x, \omega)}{d\omega dA} \]

\[ L(x, \omega) \equiv \frac{W}{sr \; m^2} \left[ \frac{cd}{m^2} = \frac{lm}{sr \; m^2} = nit \right] \]
## Typical Values of Luminance [cd/m²]

<table>
<thead>
<tr>
<th>Surface of the sun</th>
<th>2,000,000,000 nit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sunlight clouds</td>
<td>30,000</td>
</tr>
<tr>
<td>Clear day</td>
<td>3,000</td>
</tr>
<tr>
<td>Overcast day</td>
<td>300</td>
</tr>
<tr>
<td>Moon</td>
<td>0.03</td>
</tr>
</tbody>
</table>

---

**Radiant Exitance**

(Radiosity)
Radiant Exitance

Definition: The radiant (luminous) exitance is the energy per unit area leaving a surface.

\[ M(x) \equiv \frac{d\Phi_o}{dA} \]

\[
\begin{bmatrix}
\frac{W}{m^2} \\
\frac{lm}{m^2} = lux
\end{bmatrix}
\]

In computer graphics, this quantity is often referred to as the radiosity (B)

Directional Power Leaving a Surface

\[
L_o(x, \omega)
\]

\[
d\omega
\]

\[
dA
\]

\[
\theta
\]

\[
d^2\Phi_o(x, \omega) = L_o(x, \omega) \cos \theta dA d\omega
\]

\[
d^2\Phi_o(x, \omega)
\]
**Area Light Source**

\[
d^2 \Phi_o(x, \omega) = L_o(x, \omega) \cos \theta dA d\omega
\]

\[
d^2 \Phi_o(x, \omega) = \frac{d^2 \Phi_o(x, \omega)}{dA} = L_o(x, \omega) \cos \theta d\omega
\]

Same \(dA\) for all directions

**Uniform Diffuse Emitter**

\[
M = \int_{H^2} L_o \cos \theta d\omega
\]

\[
= L_o \int_{H^2} \cos \theta d\omega
\]

\(H^2\) Hemisphere

\(L_o(x, \omega) = L_o\)
Projected Solid Angle

\[ \tilde{\Omega} \equiv \int \cos \theta \, d\omega \]

\[ \tilde{\Omega} = \int \cos \theta \, d\omega = \pi \]

Uniform Diffuse Emitter

\[ M = \int_{H^2} L_o \cos \theta \, d\omega \]
\[ = L_o \int_{H^2} \cos \theta \, d\omega \]
\[ = \pi L_o \]

\[ L_o = \frac{M}{\pi} \]
**Directional Power Arriving at a Surface**

\[
\frac{L_i(x, \omega)}{d\omega} \int dA d\omega 
\]

\[
d^2\Phi_i(x, \omega) = L_i(x, \omega) \cos \theta dA d\omega
\]

---

**The Sky Radiance Distribution**

*From Greenler, Rainbows, halos and glories*
Gazing Ball $\Rightarrow$ Environment Maps

Miller and Hoffman, 1984

- Photograph of mirror ball
- Reflection direction indexed by normal
- Image is the radiance in the reflected dir.

Environment Maps

Interface, Chou and Williams (ca. 1985)
Irradiance from the Environment

\[ d^2 \Phi_i(x, \omega) = L_i(x, \omega) \cos \theta \, dA \, d\omega \]
\[ dE(x, \omega) = L_i(x, \omega) \cos \theta \, d\omega \]

\[ E(x) = \int_{H^2} L_i(x, \omega) \cos \theta \, d\omega \]

Irradiance Environment Maps

\[ L(\theta, \varphi) \]  \hspace{1cm}  \[ E(\theta, \varphi) \]

Radiance Environment Map  \hspace{1cm}  Irradiance Environment Map
Irradiance Map or Light Map

Isolux contours

Radiometry and Photometry

Summary
Radiometric and Photometric Terms

<table>
<thead>
<tr>
<th>Physics</th>
<th>Radiometry</th>
<th>Photometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy</td>
<td>Radiant Energy</td>
<td>Luminous Energy</td>
</tr>
<tr>
<td>Flux (Power)</td>
<td>Radiant Power</td>
<td>Luminous Power</td>
</tr>
<tr>
<td>Flux Density</td>
<td>Irradiance</td>
<td>Illuminance</td>
</tr>
<tr>
<td></td>
<td>Radiosity</td>
<td>Luminosity</td>
</tr>
<tr>
<td>Angular Flux Density</td>
<td>Radiance</td>
<td>Luminance</td>
</tr>
<tr>
<td>Intensity</td>
<td>Radiant Intensity</td>
<td>Luminous Intensity</td>
</tr>
</tbody>
</table>

Photometric Units

<table>
<thead>
<tr>
<th>Photometry</th>
<th>Units</th>
<th>MKS</th>
<th>CGS</th>
<th>British</th>
</tr>
</thead>
<tbody>
<tr>
<td>Luminous Energy</td>
<td>Talbot</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Luminous Power</td>
<td>Lumen</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Illuminance</td>
<td>Lux</td>
<td></td>
<td>Phot</td>
<td>Footcandle</td>
</tr>
<tr>
<td>Luminosity</td>
<td>Nit</td>
<td>Apostilb, Blondel</td>
<td>Stilb</td>
<td>Lambert</td>
</tr>
<tr>
<td>Luminance</td>
<td>Nit</td>
<td>Apostilb, Blondel</td>
<td>Stilb</td>
<td>Lambert</td>
</tr>
<tr>
<td>Luminous Intensity</td>
<td>Candela (Candle, Candlepower, Carcel, Hefner)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

“Thus one nit is one lux per steradian is one candela per square meter is one lumen per square meter per steradian. Got it?”, James Kajiya