The Light Field

Concepts
- Light field = radiance function on rays
- Conservation of radiance
- Throughput and counting rays
- Measurement equation
- Radiosity
- Irradiance

From London and Upton

Radiant Power/Intensity/Irradiance

\[ I = \frac{\Phi}{4\pi} \]

\[ EdA = Id\omega \]

\[ E = \frac{\Phi \cos \theta}{4\pi \ r^2} \]
Light Field = Radiance(Ray)

**Field Radiance**

**Definition:** The field *radiance* (*luminance*) at a point in space in a given direction is the power per unit solid angle per unit area perpendicular to the direction.

$$dA \quad r(x, \omega) \quad d\omega \quad L(x, \omega)$$

Radiance is the quantity associated with a ray.
Gazing Ball ⇒ Environment Maps

Miller and Hoffman, 1984

- Photograph of mirror ball
- Reflection direction indexed by normal
- Image is the radiance in the reflected dir.

Environment Maps

*Interface, Chou and Williams (ca. 1985)*
The Sky Radiance Distribution

From Greenler, Rainbows, halos and glories

Spherical Gantry $\Rightarrow$ 4D Light Field

Capture all the light leaving an object - like a hologram
Multi-Camera Array ⇒ Light Field

Two-Plane Light Field

\[ L(u, v, s, t) \]
Properties of Radiance

1. Fundamental field quantity that characterizes the distribution of light in an environment.
   - Radiance is a function on rays
   - All other field quantities are derived from it
2. Radiance invariant along a ray.
   - 5D ray space reduces to 4D
3. Response of a sensor proportional to radiance.
1st Law: Conversation of Radiance

The radiance in the direction of a light ray remains constant as the ray propagates

\[ d^2 \Phi_1 = d^2 \Phi_2 \]

\[ d^2 \Phi_1 = L_1 d\omega_1 dA_1 \]

\[ d^2 \Phi_2 = L_2 d\omega_2 dA_2 \]

\[ d\omega_1 dA_1 = \frac{dA_1 dA_2}{r^2} = d\omega_2 dA_2 \]

\[ \therefore L_1 = L_2 \]

Quiz

Does radiance increase under a magnifying glass?

No!!
Measuring Rays = Throughput

Throughput Counts Rays

Define an infinitesimal beam as the set of rays intersecting two infinitesimal surface elements

\[ r(u_1, v_1, u_2, v_2) \]

\[ dA_1(u_1, v_1) \quad \text{and} \quad dA_2(u_2, v_2) \]

The differential throughput measures size of the beam:

\[ d^2T = \frac{dA_1 dA_2}{|x_1 - x_2|^2} \]
Parameterizing Rays

Parameterize rays wrt to receiver \( r(u_2, v_2, \theta_2, \phi_2) \)

\[
d\omega_2(\theta_2, \phi_2) \quad \quad \quad \quad \quad dA_2(u_2, v_2)
\]

\[
d^2T = \frac{dA_1}{|x_1 - x_2|^2} dA_2 = d\omega_2 dA_2
\]

Parameterizing Rays

Parameterize rays wrt to source \( r(u_1, v_1, \theta_1, \phi_1) \)

\[
dA_1(u_1, v_1) \quad \quad \quad \quad \quad d\omega_1(\theta_1, \phi_1)
\]

\[
d^2T = dA_1 \frac{dA_2}{|x_1 - x_2|^2} = dA_1 d\omega_1
\]
Parameterizing Rays

Tilting the surfaces reparameterizes the rays!

\[ dA_1(u_1, v_1) \rightarrow dA_2(u_2, v_2) \]

\[ d^2T = \frac{\cos \theta_1 \cos \theta_2}{|x_1 - x_2|^2} dA_1 dA_2 \]

Parameterizing Rays: \( S^2 \times \mathbb{R}^2 \)

Parameterize rays by \( r(x, y, \theta, \phi) \)

Projected area \( \tilde{A}(\tilde{\omega}) \)

Measuring the number or rays that hit a shape

\[ T = \int_{S^2} d\omega(\theta, \phi) \int_{\mathbb{R}^2} dA(x, y) \]

\[ = \int_{S^2} \tilde{A}(\theta, \phi) d\omega(\theta, \phi) \]

\[ = 4\pi \tilde{A} \]

Sphere:

\[ T = 4\pi \tilde{A} = 4\pi^2 R^2 \]
Parameterizing Rays: $M^2 \times S^2$

Parameterize rays by $r(u, v, \theta, \phi)$

\[
T = \int_{M^2} dA(u, v) \int_{H^2(N)} \cos \theta d\omega(\theta, \phi)
\]

**Sphere:** $T = \pi S = 4\pi^2 R^2$

**Crofton’s Theorem:** $4\pi \bar{A} = \pi S \Rightarrow \bar{A} = \frac{S}{4}$

**The Measurement Equation**
Radiance: 2nd Law

The response of a sensor is proportional to the radiance of the surface visible to the sensor.

\[
R = \int\int_A L d\omega dA = \bar{L}T
\]

\[
T = \int\int_A d\omega dA
\]

\(L\) is what should be computed and displayed.

\(T\) quantifies the gathering power of the device; the higher the throughput the greater the amount of light gathered.

Quiz

Does the brightness that a wall appears to the sensor depend on the distance?

\[\text{No!!}\]
Radiant Exitance

(Radiosity)

Definition: The radiant (luminous) exitance is the energy per unit area leaving a surface.

\[
M(x) \equiv \frac{d\Phi_o}{dA}
\]

\[
\begin{bmatrix}
W \\
\frac{m^2}{m^2}
\end{bmatrix}
\begin{bmatrix}
\frac{lm}{m^2} = lux
\end{bmatrix}
\]

In computer graphics, this quantity is often referred to as the radiosity (B)
**Directional Power Leaving a Surface**

\[
\frac{L_o(x, \omega)}{d\omega} = \frac{d^2 \Phi_o(x, \omega)}{dA} = L_o(x, \omega) \cos \theta dA d\omega
\]

**Area Light Source**

\[
d^2 \Phi(x, \omega) = L_o(x, \omega) \cos \theta d\omega dA = dM(x, \omega) dA
\]

\[
dM(x, \omega) = L_o(x, \omega) \cos \theta d\omega dA
\]

Same \( dA \) for all directions
Uniform Diffuse Emitter

\[ M = \int_{H^2} L_o \cos \theta \, d\omega \]

\[ = L_o \int_{H^2} \cos \theta \, d\omega \]

\[ L_o(x, \omega) = L_o \]

\[ H^2 \text{ Hemisphere} \]

Projected Solid Angle

\[ \tilde{\Omega} \equiv \int_{\Omega} \cos \theta \, d\omega \]

\[ \tilde{\Omega} = \int_{H^2} \cos \theta \, d\omega = \pi \]
**Uniform Diffuse Emitter**

\[
M = \int_{H^2} L_o \cos \theta \, d\omega \\
= L_o \int_{H^2} \cos \theta \, d\omega \\
= \pi L_o \\
L_o = \frac{M}{\pi}
\]

**Irradiance**
Directional Power Arriving at a Surface

\[ d^2 \Phi_i(x, \omega) = L_i(x, \omega) \cos \theta dA d\omega \]

Irradiance from the Environment

\[ d^2 \Phi_i(x, \omega) = L_i(x, \omega) \cos \theta dA d\omega \]
\[ dE(x, \omega) = L_i(x, \omega) \cos \theta d\omega \]

\[ E(x) = \int_{4\pi} L_i(x, \omega) \cos \theta d\omega \]
Irradiance Environment Maps

\[ L(\theta, \varphi) \quad R \]
\[ E(\theta, \varphi) \quad N \]

Radiance Environment Map

Irradiance Environment Map

Isolux contours

Irradiance Map or Light Map
Irradiance from the Environment

\[ d^2 \Phi_i(x, \omega) = L_i(x, \omega) \cos \theta \, dA \, d\omega \]

\[ dE(x, \omega) \equiv \frac{d^2 \Phi}{dA} = L_i(x, \omega) \cos \theta \, d\omega \]

\[ E(x) = \int_{iH} L_i(x, \omega) \cos \theta \, d\omega \]

Uniform Area Source

\[ E(x) = \int_{iH} L \cos \theta \, d\omega \]

\[ = L \int_{\tilde{\Omega}} \cos \theta \, d\omega \]

\[ = L \tilde{\Omega} \]
Uniform Disk Source

Geometric Derivation

\[ \Omega = \pi \sin^2 \alpha \]

Algebraic Derivation

\[ \Omega = \int_0^{2\pi} \int_0^\cos \alpha \cos \theta \, d\phi \, d\cos \theta \]
\[ = 2\pi \left[ \cos^2 \theta \right]_1^\cos \alpha \]
\[ = \pi \sin^2 \alpha \]
\[ = \pi \frac{r^2}{r^2 + h^2} \]

Spherical Source

Geometric Derivation

\[ \Omega = \pi \sin^2 \alpha \]

Algebraic Derivation

\[ \Omega = \int \cos \theta \, d\omega \]
\[ = \pi \sin^2 \alpha \]
\[ = \pi \frac{r^2}{R^2} \]
The Sun

Solar constant (normal incidence at zenith)

Irradiance 1353 W/m²
Illuminance 127,500 lm/m² = 127.5 kilolux

Solar angle

\[ \alpha = 0.25 \text{ degrees} = 0.004 \text{ radians (half angle)} \]

\[ \tilde{\Omega} = \pi \sin^2 \alpha \approx \pi \alpha^2 = 6 \times 10^{-5} \text{ steradians} \]

Solar radiance

\[ L = \frac{E}{\tilde{\Omega}} = \frac{1.353 \times 10^3 \text{ W/m}^2}{6 \times 10^{-5} \text{ sr}} = 2.25 \times 10^7 \frac{\text{W}}{m^2 \cdot \text{sr}} \]

Polygonal Source
Consider 1 Edge

\( \gamma_i \) Area of sector

\[ A = \gamma \cos \theta = \gamma \vec{N}_E \cdot \vec{N} \]

Lambert’s Formula

\[ \sum_{i=1}^{3} A_i = A_1 - A_2 - A_3 \]

\[ \sum_{i=1}^{n} A_i = \sum_{i=1}^{n} \gamma_i \vec{N}_i \cdot \vec{N} \]
Penumbras and Umbras

emitter

occluder

receiver

umbra

penumbra