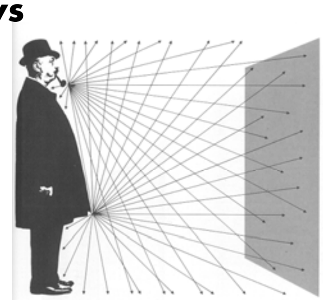
The Light Field

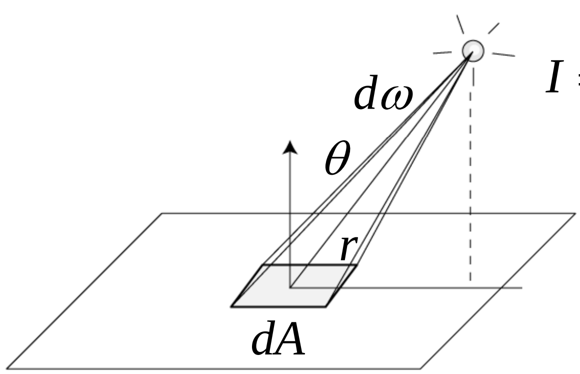
Concepts

- Light field = radiance function on rays
- Conservation of radiance
- Throughput and counting rays
- Measurement equation
- Radiosity
- Irradiance



From London and Upton

Radiant Power/Intensity/Irradiance



$$I = \frac{\Phi}{4\pi}$$

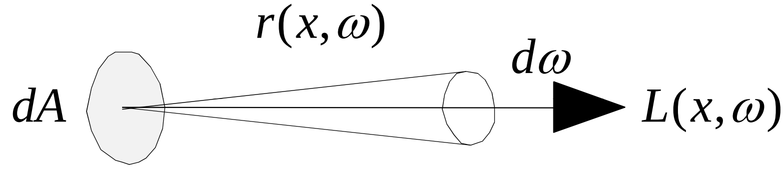
$$E dA = I d\omega$$

$$E = \frac{\Phi}{4\pi} \frac{\cos\theta}{r^2}$$

Light Field = Radiance(Ray)

Field Radiance

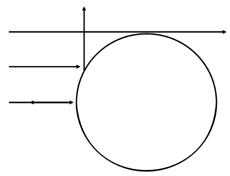
Definition: The field *radiance* (*luminance*) at a point in space in a given direction is the power per unit solid angle per unit area perpendicular to the direction



Radiance is the quantity associated with a ray

Gazing Ball ⇒ Environment Maps

Miller and Hoffman, 1984



- **Photograph of mirror ball**
- **Reflection direction indexed by normal**
- **Image is the radiance in the reflected dir.**

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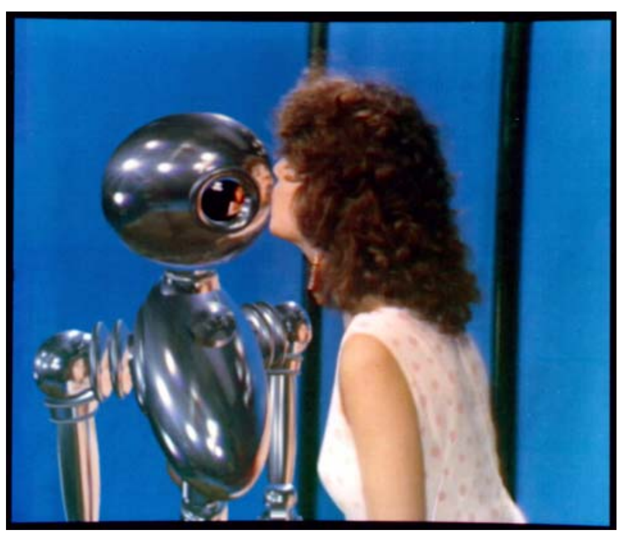
Environment Maps



***Interface*, Chou and Williams (ca. 1985)**

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The Sky Radiance Distribution

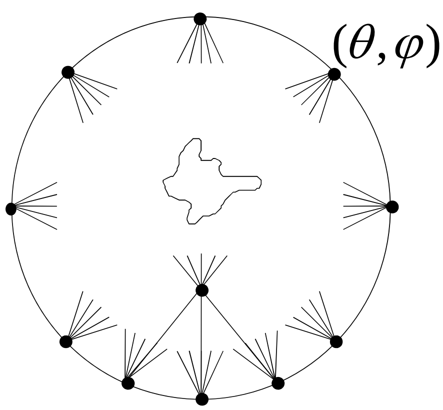
Plate 5-16. Fisheye view of clear sky at the South Pole. (Photographed by the author)

Plate 5-17. View of slightly hazy sky in Wisconsin. (Photographed by the author)

From Greenler, Rainbows, halos and glories

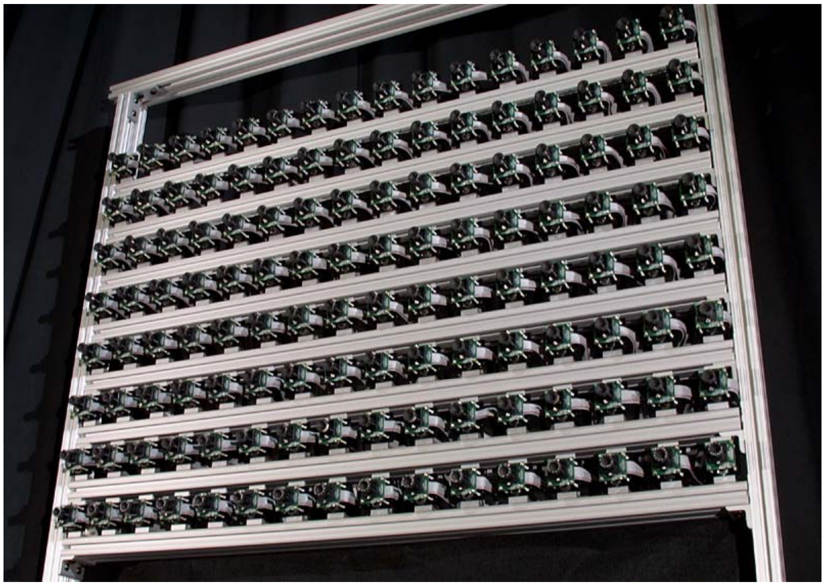
Spherical Gantry ⇒ 4D Light Field

$$L(x, y, \theta, \varphi)$$



Capture all the light leaving an object - like a hologram

Multi-Camera Array ⇒ Light Field



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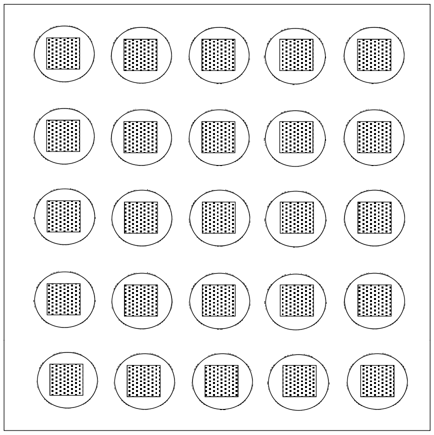
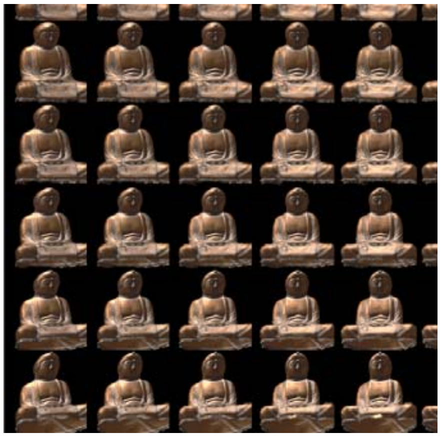
Two-Plane Light Field



2D Array of Cameras



2D Array of Images

$$L(u,v,s,t)$$

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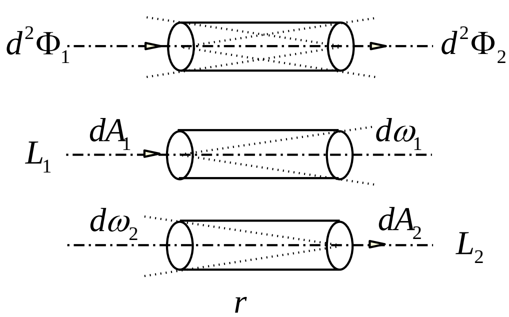
Properties of Radiance

Properties of Radiance

1. Fundamental field quantity that characterizes the distribution of light in an environment.
   - ∴ Radiance is a function on rays
   - ∴ All other field quantities are derived from it
2. Radiance invariant along a ray.
   - ∴ 5D ray space reduces to 4D
3. Response of a sensor proportional to radiance.

1st Law: Conversation of Radiance

The radiance in the direction of a light ray remains constant as the ray propagates



$$d^2\Phi_1 = d^2\Phi_2$$

$$d^2\Phi_1 = L_1 d\omega_1 dA_1$$

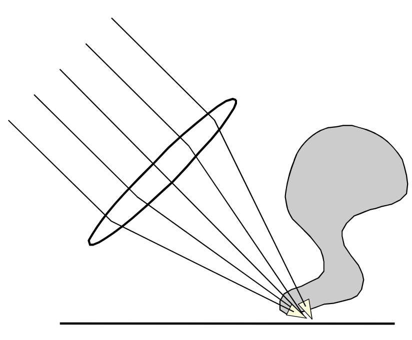
$$d^2\Phi_2 = L_2 d\omega_2 dA_2$$

$$d\omega_1 dA_1 = \frac{dA_1 dA_2}{r^2} = d\omega_2 dA_2$$

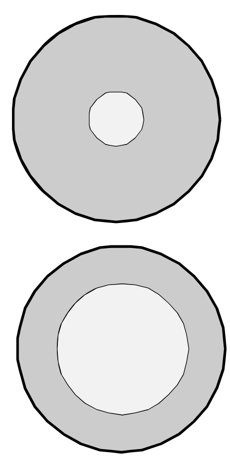
$$\therefore L_1 = L_2$$

Quiz

Does radiance increase under a magnifying glass?



No!!

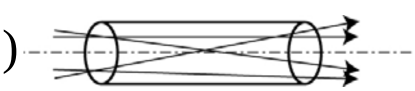


Measuring Rays = Throughput

Throughput Counts Rays

Define an infinitesimal beam as the set of rays intersecting two infinitesimal surface elements

$r(u_1, v_1, u_2, v_2)$

$dA_1(u_1, v_1)$  $dA_2(u_2, v_2)$

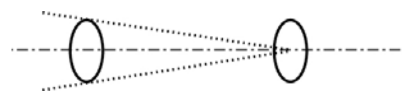
The differential throughput measures size of the beam:

$$d^2T = \frac{dA_1 dA_2}{|x_1 - x_2|^2}$$

Parameterizing Rays

Parameterize rays wrt to receiver $r(u_2, v_2, \theta_2, \phi_2)$

$d\omega_2(\theta_2, \phi_2)$



$dA_2(u_2, v_2)$

$$d^2T = \frac{dA_1}{|x_1 - x_2|^2} dA_2 = d\omega_2 dA_2$$

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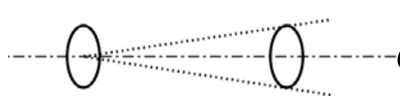
Parameterizing Rays

Parameterize rays wrt to source $r(u_1, v_1, \theta_1, \phi_1)$

$dA_1(u_1, v_1)$



$d\omega_1(\theta_1, \phi_1)$

$$d^2T = dA_1 \frac{dA_2}{|x_1 - x_2|^2} = dA_1 d\omega_1$$

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Parameterizing Rays

Tilting the surfaces reparameterizes the rays!

$$r(u_1, v_1, u_2, v_2)$$

$dA_1(u_1, v_1)$  $dA_2(u_2, v_2)$

$$d^2T = \frac{\cos\theta_1 \cos\theta_2}{\left|x_1 - x_2\right|^2} dA_1 dA_2$$

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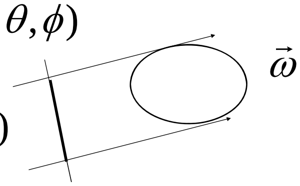
Parameterizing Rays: $S^2 \times R^2$

Parameterize rays by $r(x, y, \theta, \phi)$

Projected area $\tilde{A}(\vec{\omega})$



Measuring the number or rays that hit a shape

$$T = \int_{S^2} d\omega(\theta,\varphi) \int_{R^2} dA(x,y)$$

$$= \int_{S^2} \tilde{A}(\theta,\varphi)\, d\omega(\theta,\varphi)$$

$$= 4\pi\bar{\tilde{A}}$$

Sphere:

$$T = 4\pi\bar{\tilde{A}} = 4\pi^2 R^2$$

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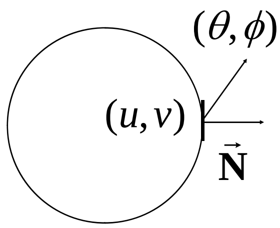
Parameterizing Rays: M² × S²

Parameterize rays by $r(u,v,\theta,\phi)$



$$T = \underbrace{\left[\int_{M^2} dA(u,v) \right]}_{S} \underbrace{\left[\int_{H^2(\vec{\mathbf{N}})} \cos\theta \, d\omega(\theta,\varphi) \right]}_{\pi}$$

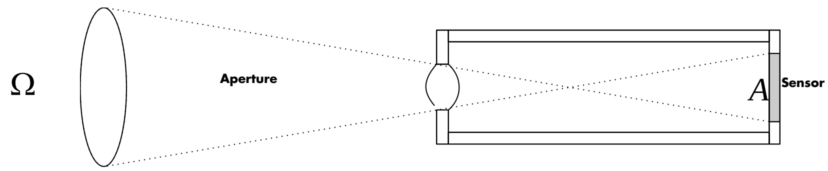
Sphere: $T = \pi S = 4\pi^2 R^2$

Crofton's Theorem: $4\pi \bar{\tilde{A}} = \pi S \Rightarrow \bar{\tilde{A}} = \dfrac{S}{4}$

The Measurement Equation

Radiance: 2nd Law

The response of a sensor is proportional to the radiance of the surface visible to the sensor.



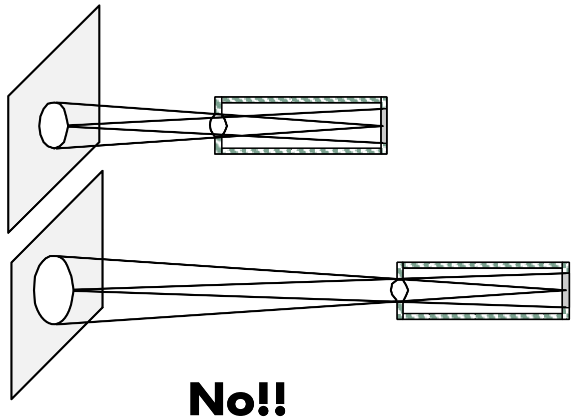
$$R = \int_A \int_\Omega L \, d\omega \, dA = \bar{L} T \qquad T = \int_A \int_\Omega d\omega \, dA$$

L is what should be computed and displayed.

T quantifies the gathering power of the device; the higher the throughput the greater the amount of light gathered

Quiz

Does the brightness that a wall appears to the sensor depend on the distance?



No!!

Radiant Exitance

(Radiosity)

Radiant Exitance

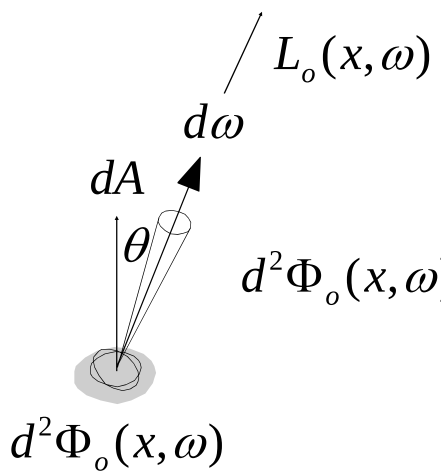
<u>Definition</u>: The *radiant* (*luminous*) *exitance* is the energy per unit area leaving a surface.

$$M(x) \equiv \frac{d\Phi_o}{dA}$$

$$\left[\frac{W}{m^2}\right] \left[\frac{lm}{m^2} = lux\right]$$

In computer graphics, this quantity is often referred to as the *radiosity (B)*

Directional Power Leaving a Surface

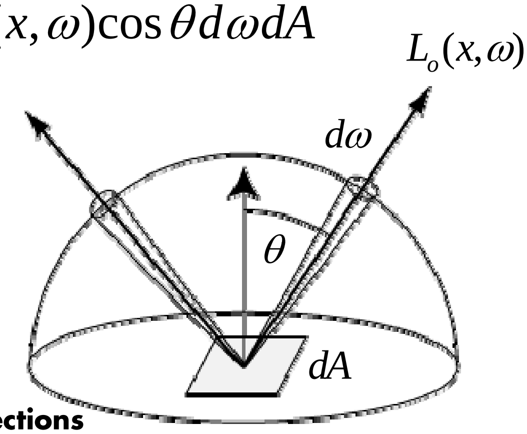


$$d^2\Phi_o(x,\omega) = L_o(x,\omega)\cos\theta\, dA\, d\omega$$

Area Light Source

$$d^2\Phi(x,\omega) = L_o(x,\omega)\cos\theta\, d\omega\, dA = dM(x,\omega)\, dA$$

$$dM(x,\omega) = L_o(x,\omega)\cos\theta\, d\omega\, dA$$



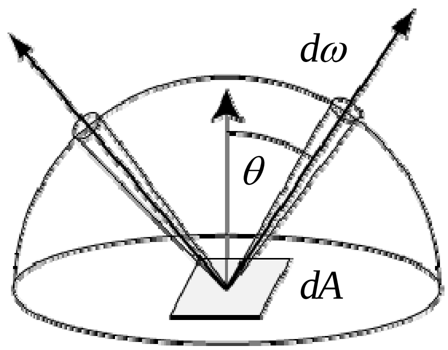
Same *dA* for all directions

Uniform Diffuse Emitter

$$M = \int_{H^2} L_o \cos\theta \, d\omega$$

$$= L_o \int_{H^2} \cos\theta \, d\omega$$

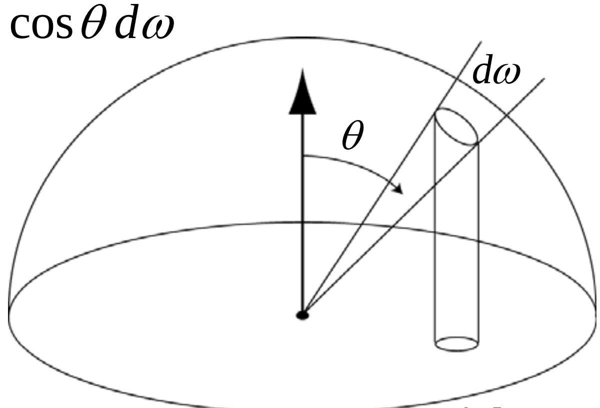
$$L_o(x,\omega) = L_o$$



H^2 **Hemisphere**

Projected Solid Angle

$$\tilde{\Omega} \equiv \int_{\Omega} \cos\theta \, d\omega$$



$$\cos\theta \, d\omega$$

$$\tilde{\Omega} = \int_{H^2} \cos\theta \, d\omega = \pi$$

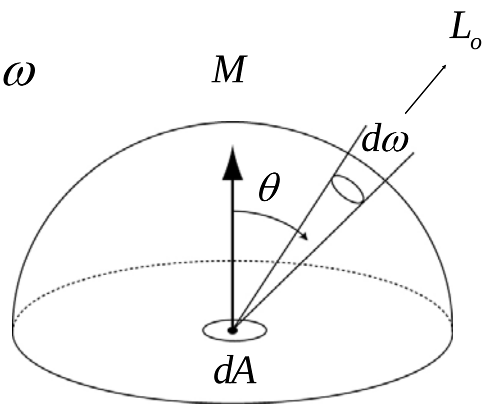
Uniform Diffuse Emitter

$$M = \int_{H^2} L_o \cos\theta \, d\omega$$

$$= L_o \int_{H^2} \cos\theta \, d\omega$$

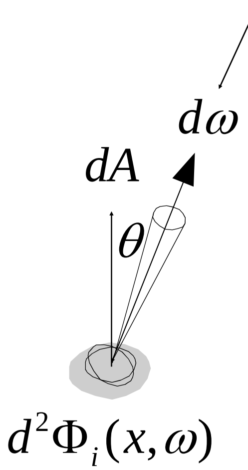
$$= \pi L_o$$

$$L_o = \frac{M}{\pi}$$



Irradiance

Directional Power Arriving at a Surface



$$d^2\Phi_i(x,\omega) = L_i(x,\omega)\cos\theta \, dA \, d\omega$$

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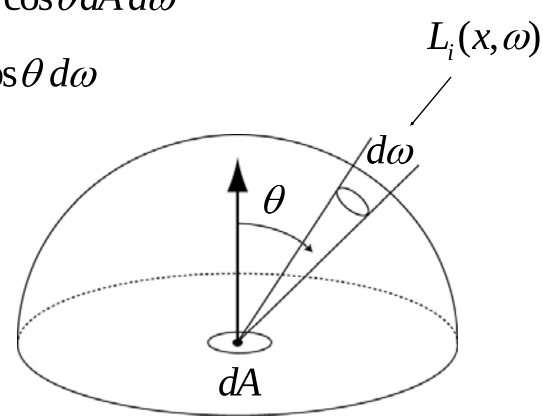
Irradiance from the Environment

$$d^2\Phi_i(x,\omega) = L_i(x,\omega)\cos\theta \, dA \, d\omega$$

$$dE(x,\omega) = L_i(x,\omega)\cos\theta \, d\omega$$

Light meter



$$E(x) = \int_{H^2} L_i(x,\omega)\cos\theta \, d\omega$$

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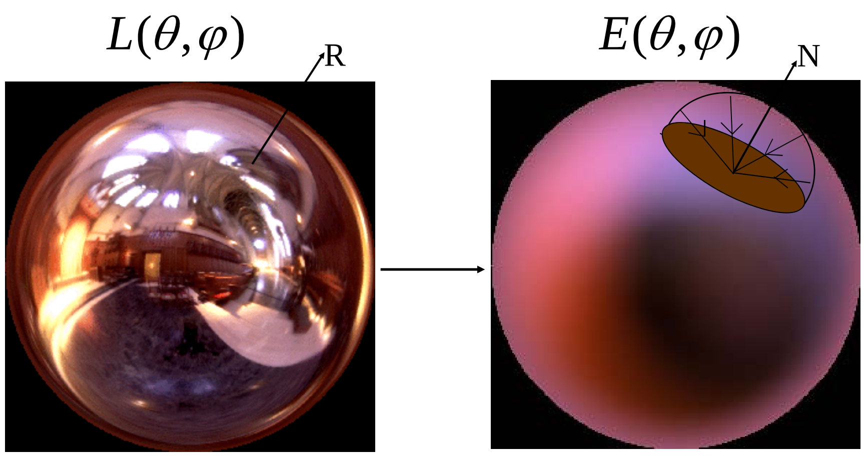
Irradiance Environment Maps



Radiance Environment Map | **Irradiance Environment Map**

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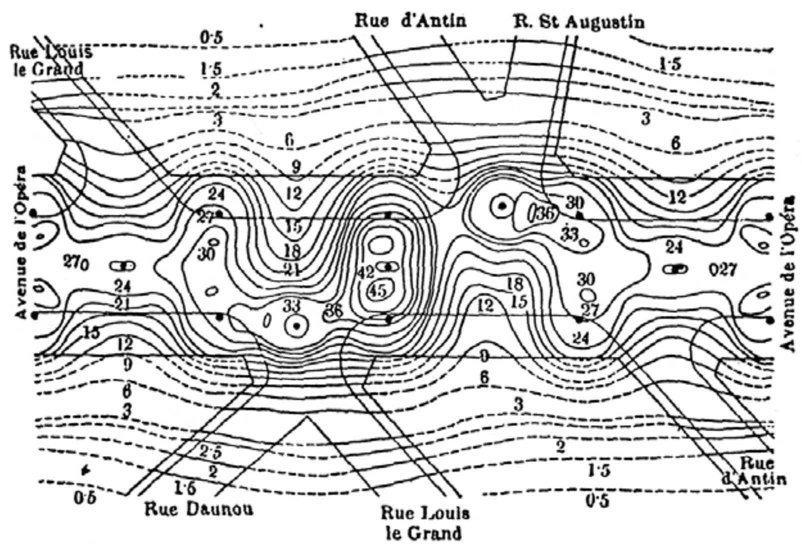
Irradiance Map or Light Map



Isolux contours

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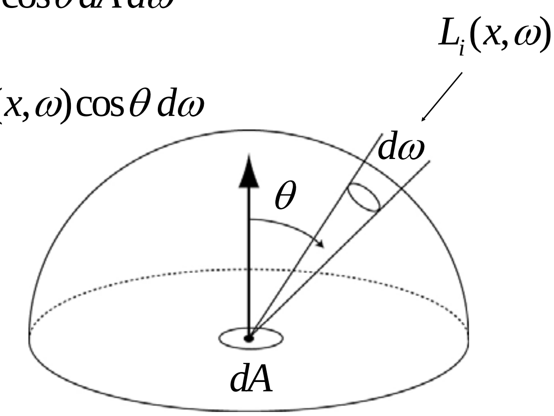
Irradiance from the Environment

$$d^2\Phi_i(x,\omega) = L_i(x,\omega)\cos\theta\, dA\, d\omega$$

$$dE(x,\omega) \equiv \frac{d^2\Phi}{dA} = L_i(x,\omega)\cos\theta\, d\omega$$



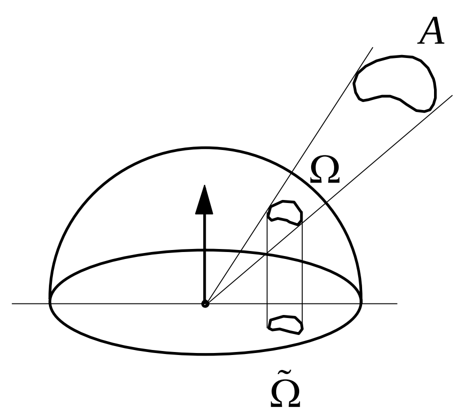
$$E(x) = \int_{H^2} L_i(x,\omega)\cos\theta\, d\omega$$

Uniform Area Source

$$E(x) = \int_{H^2} L\cos\theta d\omega$$

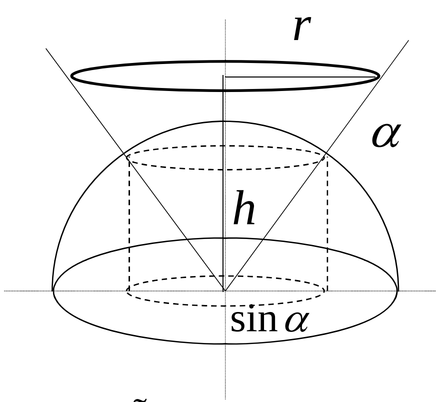
$$= L\int_{\Omega}\cos\theta d\omega$$

$$= L\tilde{\Omega}$$



Uniform Disk Source

Geometric Derivation



$$\tilde{\Omega} = \pi \sin^2 \alpha$$

Algebraic Derivation

$$\tilde{\Omega} = \int_1^{\cos\alpha} \int_0^{2\pi} \cos\theta \, d\phi \, d\cos\theta$$

$$= 2\pi \left.\frac{\cos^2\theta}{2}\right|_1^{\cos\alpha}$$

$$= \pi \sin^2 \alpha$$

$$= \pi \frac{r^2}{r^2 + h^2}$$

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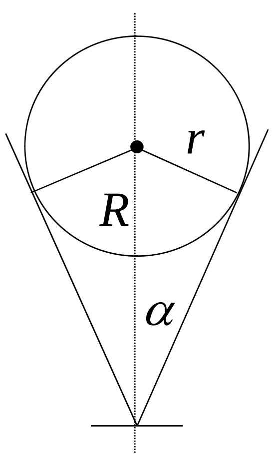
Spherical Source

Geometric Derivation



$$\tilde{\Omega} = \pi \sin^2 \alpha$$

Algebraic Derivation

$$\tilde{\Omega} = \int \cos\theta \, d\omega$$

$$= \pi \sin^2 \alpha$$

$$= \pi \frac{r^2}{R^2}$$

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The Sun

Solar constant (normal incidence at zenith)

Irradiance 1353 W/m²

Illuminance 127,500 lm/m² = 127.5 kilolux

Solar angle

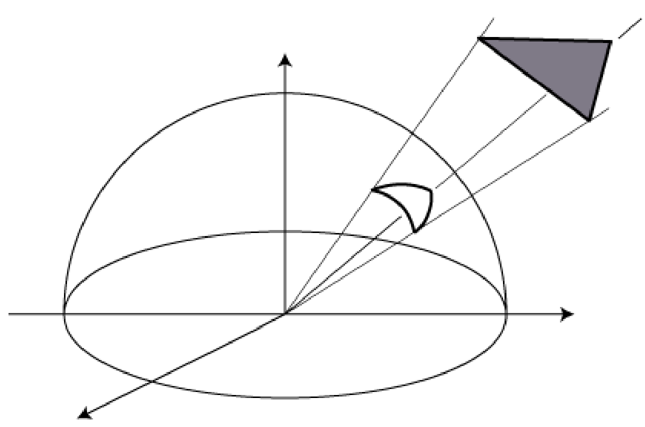
α **= .25 degrees = .004 radians (half angle)**

$\tilde{\Omega} = \pi \sin^2 \alpha \approx \pi\alpha^2$ **= 6 x 10^{-5} steradians**

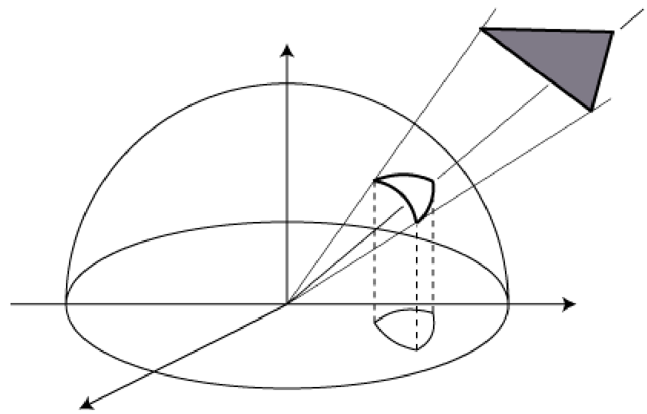
Solar radiance

$$L = \frac{E}{\tilde{\Omega}} = \frac{1.353 \times 10^3 \, W/m^2}{6 \times 10^{-5} \, sr} = 2.25 \times 10^7 \, \frac{\mathrm{W}}{m^2 \cdot sr}$$

Polygonal Source



Polygonal Source



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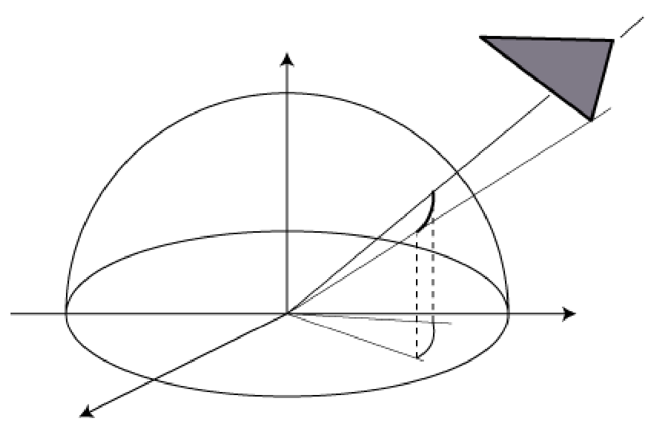
Polygonal Source



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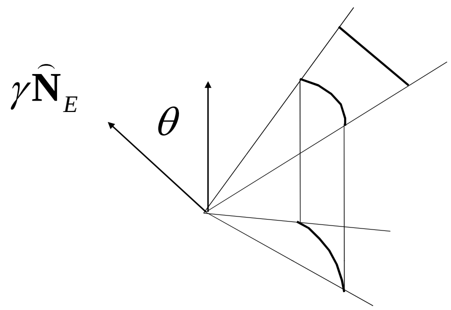
Consider 1 Edge

γ_i **Area of sector**



$$A = \gamma \cos\theta = \gamma \vec{\mathbf{N}}_E \bullet \vec{\mathbf{N}}$$

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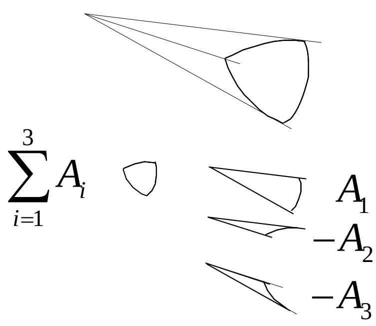
Lambert's Formula



$$\sum_{i=1}^{n} A_i = \sum_{i=1}^{n} \gamma_i \vec{\mathbf{N}}_i \bullet \vec{\mathbf{N}}$$

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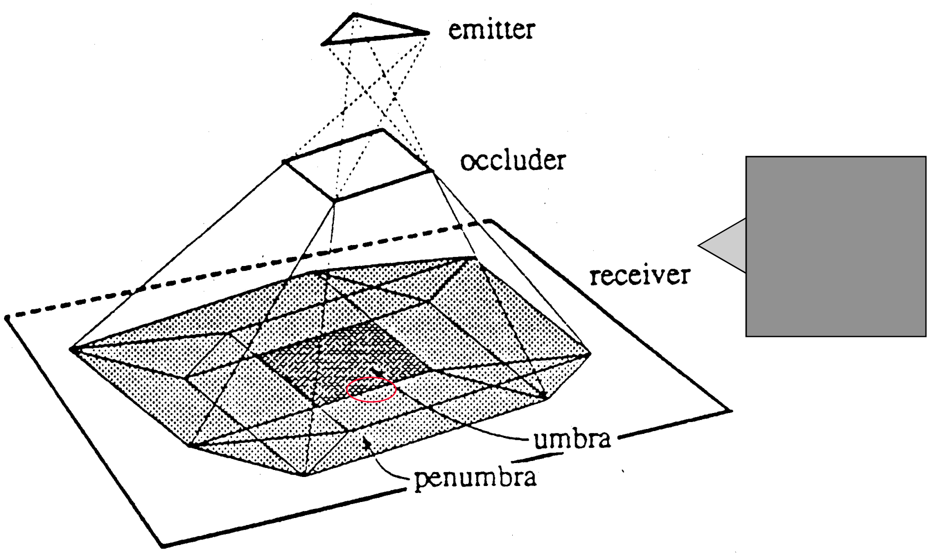
Penumbras and Umbras



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