

# Reflection Models I

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## Today

- Types of reflection models
- The BRDF and reflectance
- The reflection equation
- Ideal reflection and refraction
- Fresnel effect
- Ideal diffuse

## Next lecture

- Glossy and specular reflection models
- Rough surfaces and microfacets

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# Reflection Models

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**Definition:** Reflection is the process by which light incident on a surface interacts with the surface such that it leaves on the incident side without change in frequency.

## Properties

- Spectra and Color
- Polarization
- Directional distribution

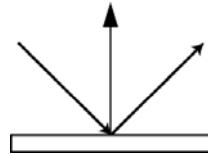
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## Types of Reflection Functions

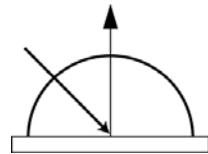
### Ideal Specular

- Reflection Law
- Mirror



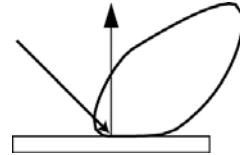
### Ideal Diffuse

- Lambert's Law
- Matte



### Specular

- Glossy
- Directional diffuse



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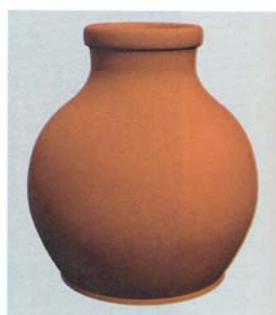
## Materials



Plastic



Metal



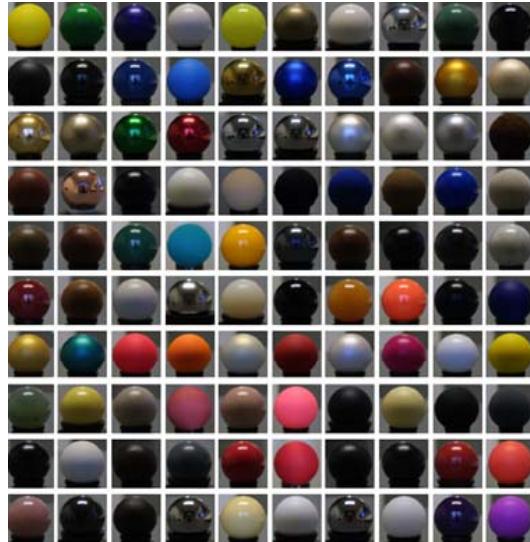
Matte

From Apodaca and Gritz, *Advanced RenderMan*

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## Spheres [Matusik et al.]

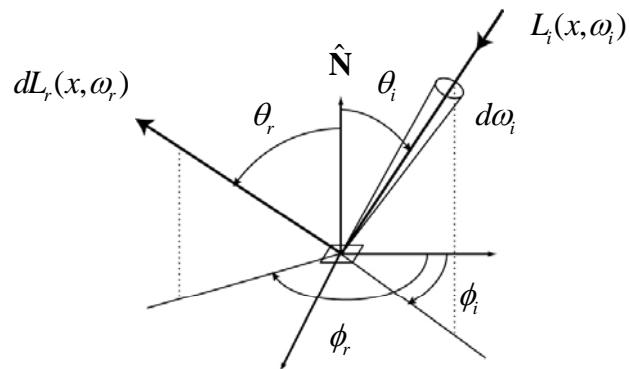


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## The BRDF

### Bidirectional Reflectance-Distribution Function

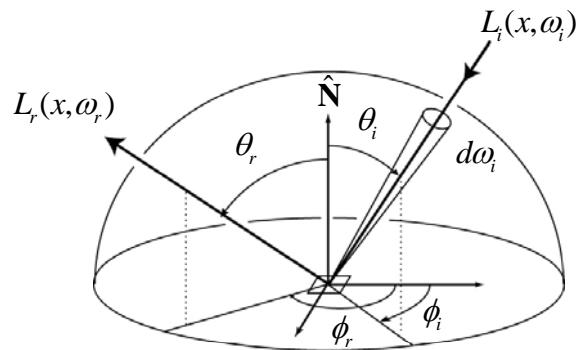


$$f_r(\omega_i \rightarrow \omega_r) \equiv \frac{dL_r(\omega_i \rightarrow \omega_r)}{dE_i} \left[ \frac{1}{sr} \right]$$

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## The Reflection Equation

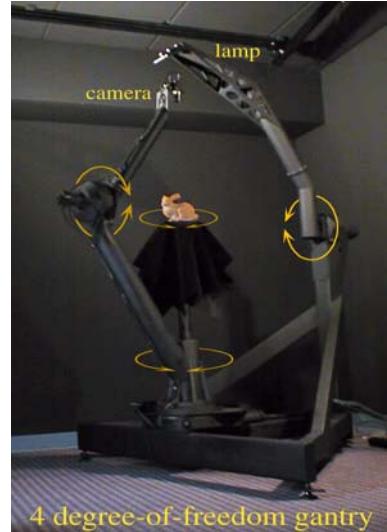
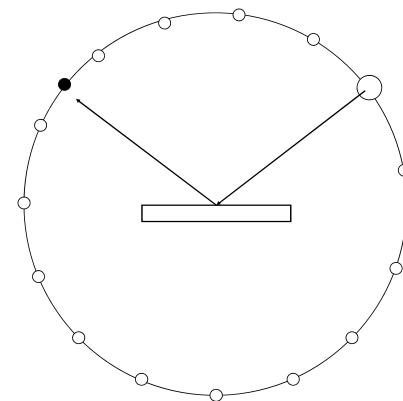


$$L_r(x, \omega_r) = \int_{H^2} f_r(x, \omega_i \rightarrow \omega_r) L_i(x, \omega_i) \cos \theta_i d\omega_i$$

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## Gonioreflectometer



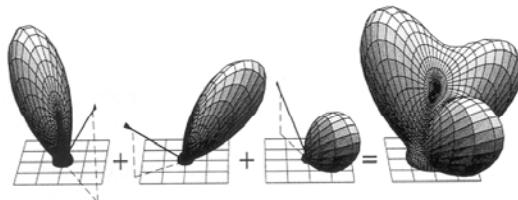
4 degree-of-freedom gantry

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## Properties of BRDF's

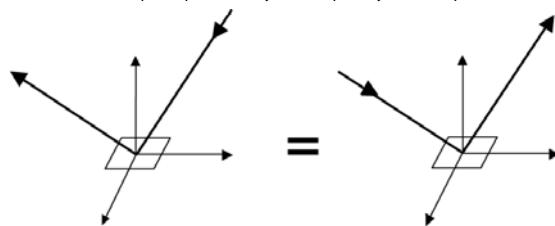
### 1. Linear



From Sillion, Arvo, Westin, Greenberg

### 2. Reciprocity principle

$$f_r(\omega_r \rightarrow \omega_i) = f_r(\omega_i \rightarrow \omega_r)$$



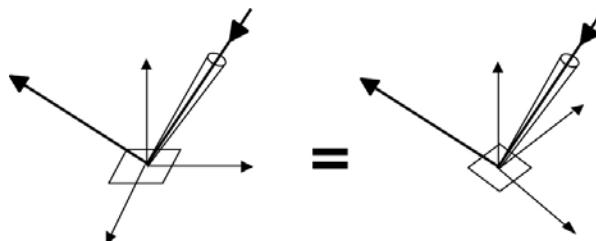
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## Properties of BRDF's

### 3. Isotropic vs. anisotropic

$$f_r(\theta_i, \varphi_i; \theta_r, \varphi_r) = f_r(\theta_i, \theta_r, \varphi_r - \varphi_i)$$



### Reciprocity and isotropy

$$f_r(\theta_i, \theta_r, \varphi_r - \varphi_i) = f_r(\theta_r, \theta_i, \varphi_i - \varphi_r) = f_r(\theta_i, \theta_r, |\varphi_r - \varphi_i|)$$

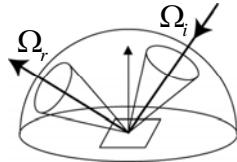
### 4. Energy conservation

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## Energy Conservation

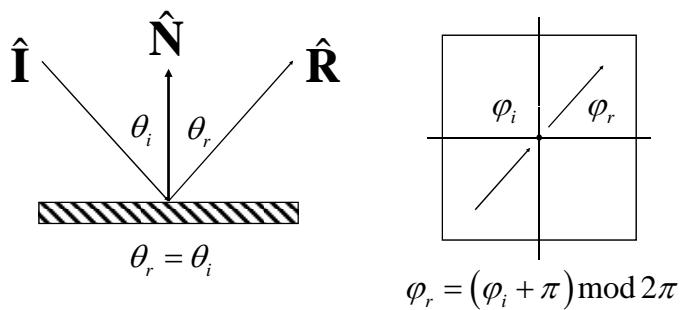
$$\begin{aligned} \frac{d\Phi_r}{d\Phi_i} &= \frac{\int_{\Omega_r} L_r(\omega_r) \cos \theta_r d\omega_r}{\int_{\Omega_i} L_i(\omega_i) \cos \theta_i d\omega_i} \\ &= \frac{\int_{\Omega_r} \int_{\Omega_i} f_r(\omega_i \rightarrow \omega_r) L_i(\omega_i) \cos \theta_i d\omega_i \cos \theta_r d\omega_r}{\int_{\Omega_i} L_i(\omega_i) \cos \theta_i d\omega_i} \\ &\leq 1 \end{aligned}$$



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## Law of Reflection



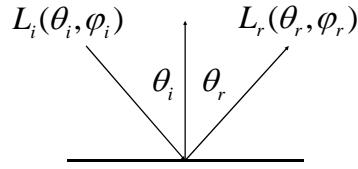
$$\hat{\mathbf{R}} + (-\hat{\mathbf{I}}) = 2 \cos \theta \hat{\mathbf{N}} = -2(\hat{\mathbf{I}} \bullet \hat{\mathbf{N}})\hat{\mathbf{N}}$$

$$\hat{\mathbf{R}} = \hat{\mathbf{I}} - 2(\hat{\mathbf{I}} \bullet \hat{\mathbf{N}})\hat{\mathbf{N}}$$

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## Ideal Reflection (Mirror)



$$L_{r,m}(\theta_r, \varphi_r) = L_i(\theta_r, \varphi_r \pm \pi)$$

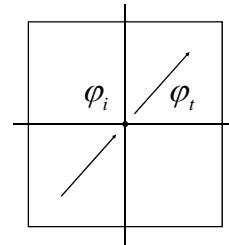
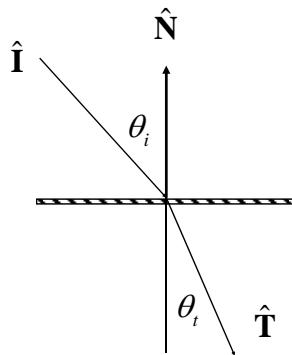
$$f_{r,m}(\theta_i, \varphi_i; \theta_r, \varphi_r) = \frac{\delta(\cos \theta_i - \cos \theta_r)}{\cos \theta_i} \delta(\varphi_i - \varphi_r \pm \pi)$$

$$\begin{aligned} L_{r,m}(\theta_r, \varphi_r) &= \int f_{r,m}(\theta_i, \varphi_i; \theta_r, \varphi_r) L_i(\theta_i, \varphi_i) \cos \theta_i d\cos \theta_i d\varphi_i \\ &= \int \frac{\delta(\cos \theta_i - \cos \theta_r)}{\cos \theta_i} \delta(\varphi_i - \varphi_r \pm \pi) L_i(\theta_i, \varphi_i) \cos \theta_i d\cos \theta_i d\varphi_i \\ &= L_i(\theta_r, \varphi_r \pm \pi) \end{aligned}$$

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## Snell's Law



$$\varphi_t = \varphi_i \pm \pi$$

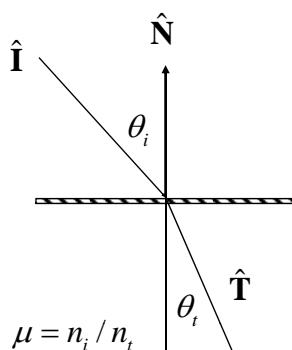
$$n_i \sin \theta_i = n_t \sin \theta_t$$

$$n_i \hat{\mathbf{N}} \times \hat{\mathbf{I}} = n_t \hat{\mathbf{N}} \times \hat{\mathbf{T}}$$

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## Law of Refraction



$$\hat{\mathbf{T}} = \mu \hat{\mathbf{I}} + \gamma \hat{\mathbf{N}}$$

$$\hat{\mathbf{T}}^2 = 1 = \mu^2 + \gamma^2 + 2\mu\gamma \hat{\mathbf{I}} \cdot \hat{\mathbf{N}}$$

Total internal reflection:

$$1 - \mu^2(1 - (\hat{\mathbf{I}} \cdot \hat{\mathbf{N}})^2) < 0$$

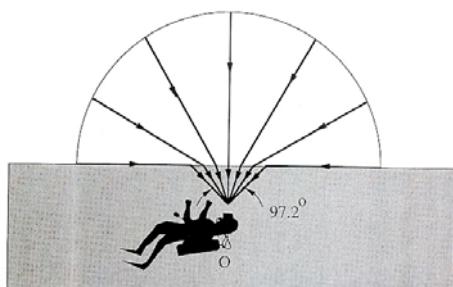
$$\begin{aligned}\gamma &= -\mu \hat{\mathbf{I}} \cdot \hat{\mathbf{N}} \pm \left\{ 1 - \mu^2 \left( 1 - (\hat{\mathbf{I}} \cdot \hat{\mathbf{N}})^2 \right) \right\}^{1/2} \\ &= \mu \cos \theta_i \pm \left\{ 1 - \mu^2 \sin^2 \theta_i \right\}^{1/2} \\ &= \mu \cos \theta_i \pm \cos \theta_t \quad \leftarrow \gamma = \mu - 1 \\ &= \mu \cos \theta_i - \cos \theta_t\end{aligned}$$

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## Optical Manhole

Total internal reflection



$$n_w = \frac{4}{3}$$



From Livingston and Lynch

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# Experiment

## Reflections from a shiny floor



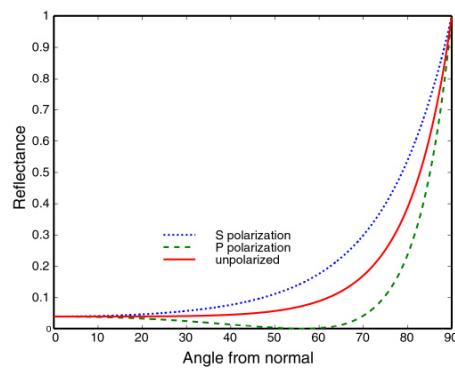
From Lafortune, Foo, Torrance, Greenberg, SIGGRAPH 97

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# Fresnel Reflectance

## Dielectric ( $N=1.5$ )



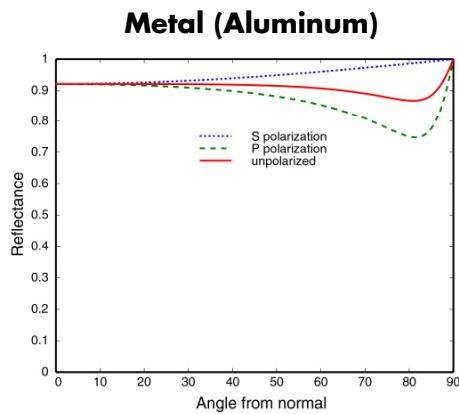
Glass  $n=1.5 F(0)=0.04$   
Diamond  $n=2.4 F(0)=0.15$

Schlick Approximation  $F(\theta) = F(0) + (1 - F(0))(1 - \cos \theta)^5$

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## Fresnel Reflectance



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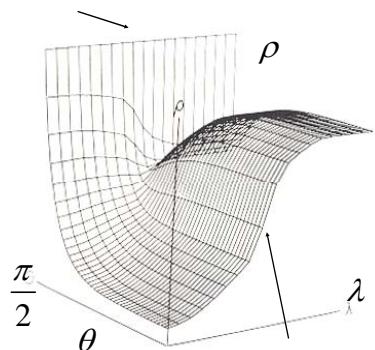
## Cook-Torrance Model for Metals

Reflectance of Copper as a function of wavelength and angle of incidence



Measured Reflectance

Light spectra



Copper spectra

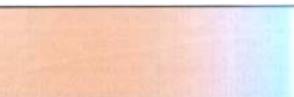
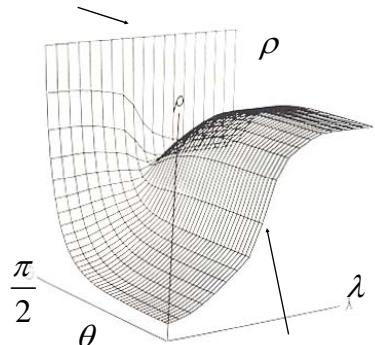
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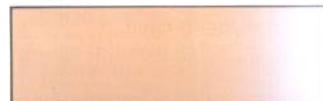
## Cook-Torrance Model for Metals

Reflectance of Copper as a function of wavelength and angle of incidence

Light spectra



Measured Reflectance



Approximated Reflectance

Cook-Torrance approximation

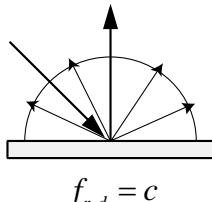
$$R = R(0) + R(\pi/2) \left[ \frac{F(\theta) - F(0)}{F(\pi/2) - F(0)} \right]$$

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## Ideal Diffuse Reflection

Assume light is equally likely to be reflected in any output direction



$$\begin{aligned} L_{r,d}(\omega_r) &= \int f_{r,d} L_i(\omega_i) \cos \theta_i d\omega_i \\ &= f_{r,d} \int L_i(\omega_i) \cos \theta_i d\omega_i \\ &= f_{r,d} E \end{aligned}$$

$$M = \int L_r(\omega_r) \cos \theta_r d\omega_r = L_r \int \cos \theta_r d\omega_r = \pi L_r$$

$$\rho_d = \frac{M}{E} = \frac{\pi L_r}{E} = \frac{\pi f_{r,d} E}{E} = \pi f_{r,d} \Rightarrow f_{r,d} = \frac{\rho_d}{\pi}$$

Lambert's Cosine Law  $M = \rho_d E = \rho_d E_s \cos \theta_s$

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## **“Diffuse” Reflection**

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### **Theoretical**

- Bouguer - Special micro-facet distribution
- Seeliger - Subsurface reflection
- Multiple surface or subsurface reflections

### **Experimental**

- Pressed magnesium oxide powder
- Almost never valid at high angles of incidence

***Paint manufacturers attempt to create ideal diffuse***