

# Participating Media & Vol. Scattering

## Applications

- Clouds, smoke, water, ...
- Subsurface scattering: paint, skin, ...
- Scientific/medical visualization: CT, MRI, ...

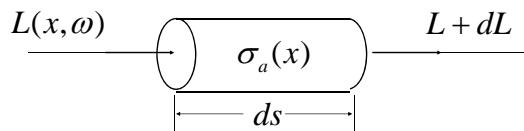
## Topics

- Absorption and emission
- Scattering and phase functions
- Volume rendering equation
- Homogeneous media
- Ray tracing volumes

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# Absorption



$$dL(x, \omega) = -\sigma_a(x)L(x, \omega)ds$$

**Absorption cross-section:**  $\sigma_a(x)$

**Probability of being absorbed per unit length**

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## Transmittance

$$dL(x, \omega) = -\sigma_a(x)L(x, \omega)ds$$

$$\frac{dL(x, \omega)}{L(x, \omega)} = -\sigma_a(x)ds$$

$$\ln L(x + s\omega, \omega) = -\int_0^s \sigma_a(x + s'\omega)ds' = -\tau(s)$$

### Optical distance or depth

$$\tau(s) = \int_0^s \sigma_a(x + s'\omega)ds'$$

**Homogenous media: constant  $\sigma_a$**

$$\sigma_a \rightarrow \tau(s) = \sigma_a s$$

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## Transmittance and Opacity

$$dL(x, \omega) = -\sigma_a(x)L(x, \omega)ds$$

$$\frac{dL(x, \omega)}{L(x, \omega)} = -\sigma_a(x)ds$$

$$\ln L(x + s\omega, \omega) = -\int_0^s \sigma_a(x + s'\omega)ds' = -\tau(s)$$

$$L(x + s\omega, \omega) = e^{-\tau(s)}L(x, \omega) = T(s)L(x, \omega)$$

### Transmittance

$$T(s) = e^{-\tau(s)}$$

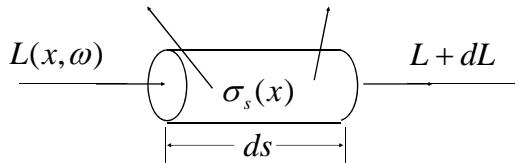
### Opacity

$$\alpha(s) = 1 - T(s)$$

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## Out-Scatter



$$dL(x, \omega) = -\sigma_s(x)L(x, \omega)ds$$

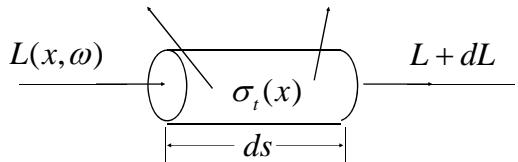
**Scattering cross-section:**  $\sigma_s$

**Probability of being scattered per unit length**

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## Extinction



$$dL(x, \omega) = -\sigma_t(x)L(x, \omega)ds$$

**Total cross-section**

$$\sigma_t = \sigma_a + \sigma_s$$

**Albedo**

$$W = \frac{\sigma_s}{\sigma_t} = \frac{\sigma_s}{\sigma_a + \sigma_s}$$

**Attenuation due to both absorption and scattering**

$$\tau(s) = \int_0^s \sigma_t(x + s' \omega) ds'$$

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## Black Clouds

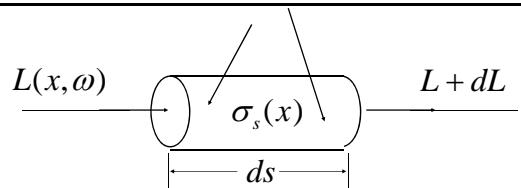


**From Greener, Rainbows, halos and glories**

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## In-Scatter



$$S(x, \omega) = \sigma_s(x) \int_{S^2} p(\omega' \rightarrow \omega) L(x, \omega') d\omega'$$

**Phase function**  $p(\omega' \rightarrow \omega)$

**Reciprocity**

$$p(\omega \rightarrow \omega') = p(\omega' \rightarrow \omega)$$

**Energy conserving**

$$\int_{S^2} p(\omega' \rightarrow \omega) d\omega' = 1$$

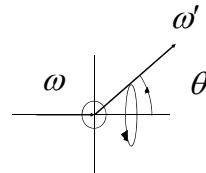
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## Phase Functions

**Phase angle**  $\cos \theta = \omega \bullet \omega'$

**Phase functions**  
(from the phase of the moon)



1. **Isotropic**  
-simple
2. **Rayleigh**  
-molecules
3. **Mie scattering**  
- small spheres  
... Huge literature ...

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## Blue Sky = Red Sunset



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## Coronas and Halos



Moon Corona



Sun Halos

From Greenler, Rainbows, halos and glories

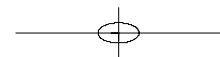
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## Henyey-Greenstein Phase Function

### Empirical phase function

$$p(\cos \theta) = \frac{1}{4\pi} \frac{1-g^2}{(1+g^2 - 2g \cos \theta)^{3/2}}$$



$$2\pi \int_0^\pi p(\cos \theta) \cos \theta d\theta = g$$

**$g$ : average phase angle**

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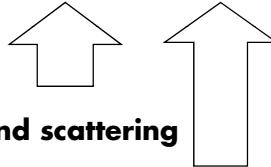
# The Volume Rendering Equation

## Integro-differential equation

$$\frac{\partial L(x, \omega)}{\partial s} = -\sigma_t(x)L(x, \omega) + S(x, \omega)$$

## Integro-integral equation

$$L(x, \omega) = \int_0^{\infty} e^{-\int_0^{s'} \sigma_t(x+s''\omega) ds''} S(x + s'\omega) ds'$$



**Attenuation: Absorption and scattering**

**Source: Scatter (+ emission)**

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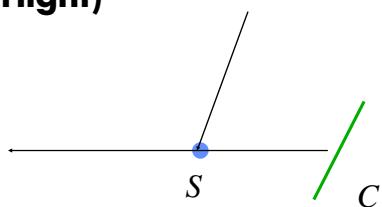
# Simple Atmosphere Model

## Assumptions

- Homogenous media
- Constant source term (airlight)

$$\frac{\partial L(s)}{\partial s} = -\sigma_t L(s) + S$$

$$L(s) = (1 - e^{-\sigma_t s})S + e^{-\sigma_t s}C$$



**Fog**

**Haze**

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## The Sky

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Plate 5-16. Fisheye view of clear sky at the South Pole. (Photographed by the author)



Plate 5-17. View of slightly hazy sky in Wisconsin. (Photographed by the author)

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## Atmospheric Perspective

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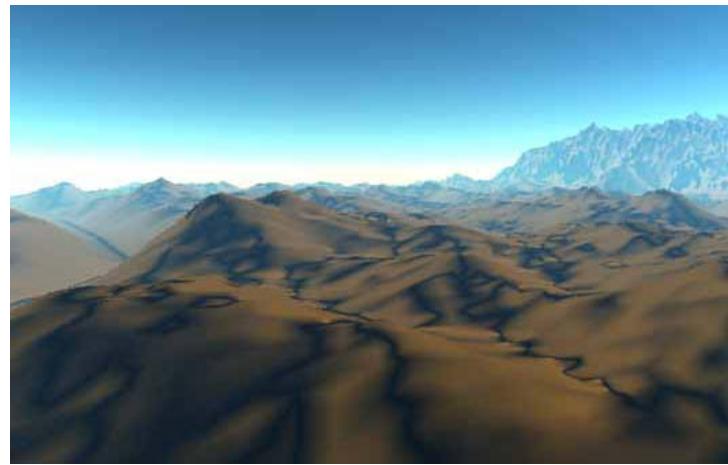
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## Atmospheric Perspective

Aerial Perspective: loss of contrast and change in color



From Musgrave

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## Semi-Infinite Homogenous Media

### Reduced Intensity

$$L(z, \omega_i) = e^{-\tau(z, \omega_i)} L(0, \omega_i)$$

### Effective source term

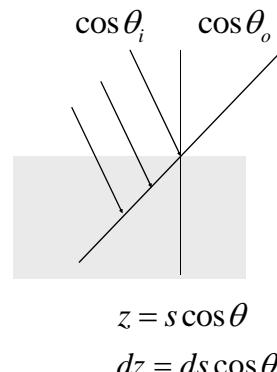
$$S(z, \omega_o) = \sigma_s p(\omega_i \rightarrow \omega_o) e^{-\tau(z, \omega_i)} L(0, \omega_i)$$

### Volume rendering equation

$$\cos \theta_o \frac{\partial L(z, \omega_o)}{\partial z} = -\sigma_t L(z, \omega_o) + S(z, \omega_o)$$

### Integrating over depths

$$\cos \theta_o L(\omega_o) = \int_0^{\infty} e^{-\sigma_i z / \cos \theta_o} \sigma_s p(\omega_i, \omega_o) e^{-\sigma_i z / \cos \theta_i} L(\omega_i) dz$$



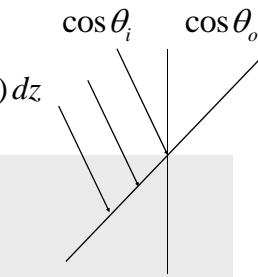
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## Semi-Infinite Homogenous Media

### Integrating over depths

$$\begin{aligned}\cos \theta_o L(\omega_o) &= \int_0^{\infty} e^{-\sigma_t z / \cos \theta_o} \sigma_s p(\omega_i, \omega_o) e^{-\sigma_t z / \cos \theta_i} L(\omega_i) dz \\ &= \sigma_s p(\omega_i, \omega_o) L(\omega_i) \int_0^{\infty} e^{-\sigma_t \left[ \frac{1}{\cos \theta_i} + \frac{1}{\cos \theta_o} \right] z} dz \\ &= \sigma_s p(\omega_i, \omega_o) L(\omega_i) \frac{1}{\sigma_t \left[ \frac{1}{\cos \theta_i} + \frac{1}{\cos \theta_o} \right]} \\ &= W p(\omega_i, \omega_o) L(\omega_i) \frac{\cos \theta_i \cos \theta_o}{\cos \theta_i + \cos \theta_o}\end{aligned}$$



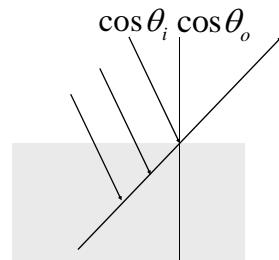
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## Semi-Infinite Homogenous Media

### BRDF

$$\begin{aligned}f_r(\omega_i, \omega_o) &= \frac{dL}{dE} = \frac{L(\omega_i, \omega_o)}{L(\omega_i) \cos \theta_i} \\ &= W p(\omega_i, \omega_o) \frac{1}{\cos \theta_i + \cos \theta_o}\end{aligned}$$



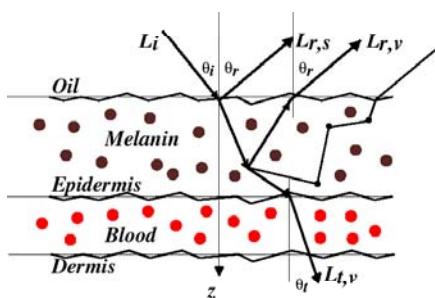
### Seeliger's Law or The Law of Diffuse Reflection

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# Subsurface Scattering

## Skin



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# Volume Representations

## 3D arrays (uniform rectangular)

- CT data

## 3D meshes

- CFD, mechanical simulation

## Simple shapes with solid texture

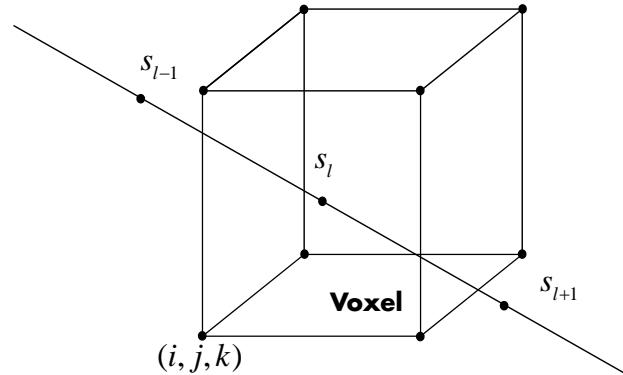
- Ellipsoidal clouds with sum-of-sines densities

- Hypertexture

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## Scalar Volumes



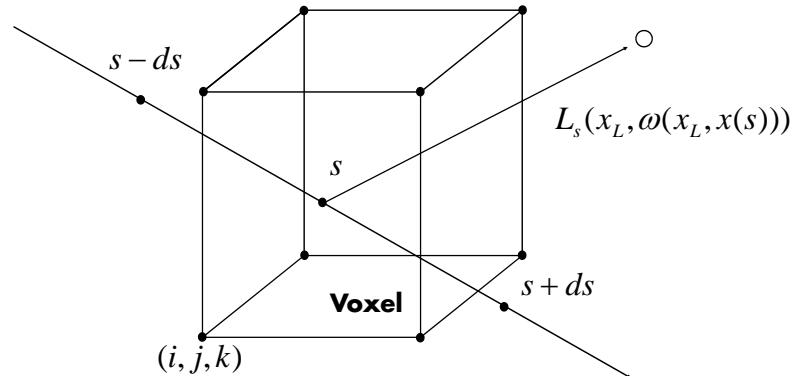
**Interpolation**  $v(s_l) = \text{trilinear}(v, i, j, k, x(s_l))$

**Map scalars to optical properties**  $\sigma_s(v), \sigma_a(v)$

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## Scalar Volumes



**Scatter**

$$S(x(s), \omega) = \sigma_s(s) p(\omega, \omega(x(s), x_L)) L_s(x_L, \omega(x_L, x(s)))$$

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## Ray Marching

**Primary ray**

$$T = 1$$

$$L = 0$$

for( $s = 0; s < 1; s+ = ds$ )

$$S = \sigma_s(s) p(\omega, \omega(x(s), x_L)) L_s(x_L, \omega(x_L, x(s)))$$

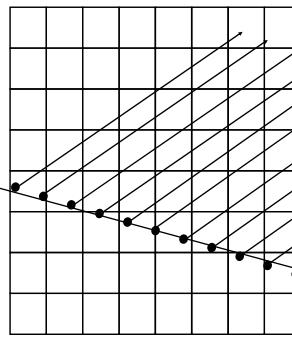
$$L = L + TS\Delta s$$

$$T = T[1 - \sigma_t(x(s))] \Delta s$$

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**Direct Lighting**



## Ray Marching

**Shadow ray**

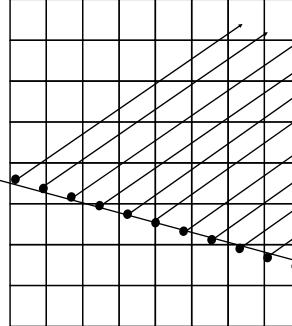
$$T = 1$$

for( $t = 0; t < 1; t+ = dt$ )

$$T = T[1 - \sigma_t(x(t))] \Delta t$$

$$S(x(s)) = \sigma_s(s) p(\omega, \omega(x(s), x_L)) T L_s(x_L, \omega(x_L, x(s)))$$

**Direct Lighting**



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## Beams of Light



From Greenler, Rainbows,  
halos and glories

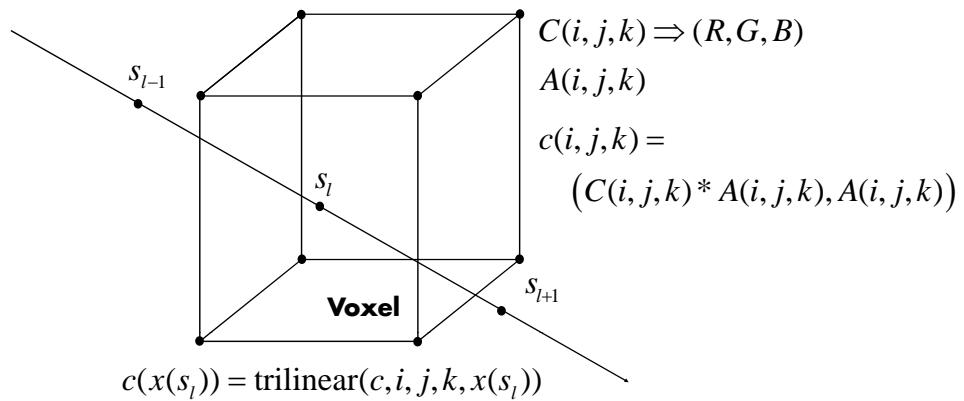
From Minneart, Color and light  
in the open air

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## Color and Opacity Volumes

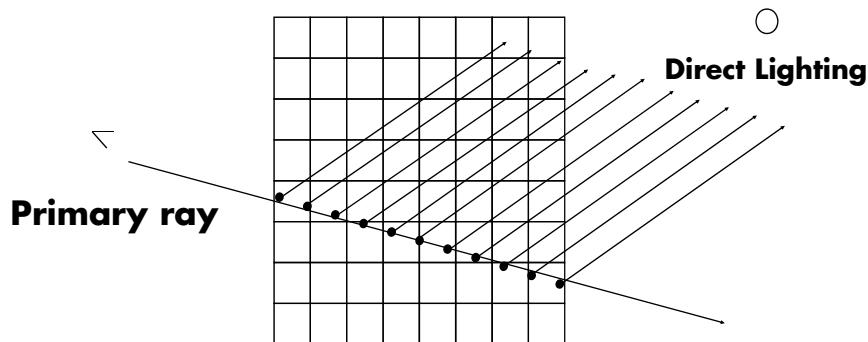
### M. Levoy, Ray tracing volume densities



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## Ray Marching



$$C = (0, 0, 0, 0)$$

for( $s = 0; s < 1; s+ = ds$ )

$$C = C + (1 - \alpha(C))c(s)$$

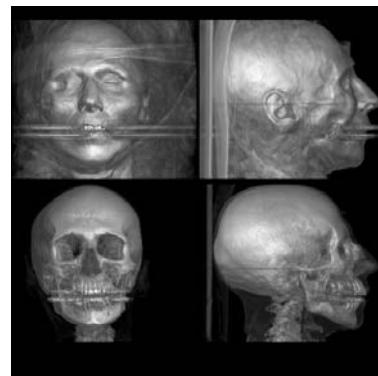
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## Volume Rendering Examples



From Karl Heinz Hoehne



From Marc Levoy

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